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# Testing for Stationarity in Panel Data when Errors are Serially Correlated. Finite-Sample Results.*

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### Abstract

In this paper, we study the small sample properties of the panel data stationarity test of Hadri (2000). We find that the previously suggested moments, that are to be used when standardizing the panel data stationarity test, cause size distortions when samples are small and serial correlation in the disturbance terms is allowed for. Instead, we supply standardizing moments that are to be used in a panel data stationarity test when samples are small and serial correlation in the disturbances may be an issue. We also document a serious small-sample bias in the panel data stationarity test when a linear trend is present in the data.

**JEL Classification**: C15; C23; C32; C33
**Keywords**: Panel Data; Stationarity; Serial Correlation; Monte Carlo Simulation

# 1 Introduction

Ever since the seminal papers by Levin and Lin (1992, 1993), Quah (1994) and Im et al. (1997), the development of time-series methods in the panel data setting has flourished. More recently, Hadri (2000) considers a stationarity test applied to panels. When performing the panel data stationarity test of Hadri (2000), a univariate stationarity test is performed on each of the time series in the panel. By standardizing the univariate test statistics by appropriate moments and then calculating the average of the univariate test statistics, a panel test statistic with a standard normal limit can be obtained. The appropriate asymptotic moments for the standardization has been supplied by Hadri (2000), while Hadri and Larsson (2003) provide finite-sample moments. The asymptotic behavior of the test statistic is independent of the presence or absence of serial correlation in the stationary

disturbance terms, as long as the long-run variance of the disturbance term i consistently estimated, while the finite-sample moments, supplied by Hadri and Larsson (2003), are obtained under the assumption that the stationary noise process are serially uncorrelated.

In this paper, we investigate if the finite-sample or asymptotic moments can be applied in small-sample situations where serial correlation in the disturbances might be an issue. We find that the moments supplied by Hadri (2000) and Hadri and Larsson (2003), cause size distortions when the panel data stationarity test is performed while allowing for serially correlated disturbances. We provide standardizing moments that can be used under a wide range of small-sample situations and autocorrelation structures without disturbing the size of the test. However, when investigating the power of the size-corrected panel data stationarity test, we find that the test has a severe small-sample bias, especially when detreding the cross-section time series with a linear trend. The loss of power in the panel data setting is in line with previous results for univariate stationarity tests, that suggest that the power of this test can be severely affected when serial correlation in the disturbance terms is an issue (see e.g. Lee, 1996; Caner and Kilian, 2001).

The rest of this paper is organized as follows. In Section 2, we introduce the panel data stationarity test of Hadri (2000) and discuss the standardizing moments previously suggested. We then continue, in Section 3, by studying the performance of the panel data stationarity test when applying the currently available standardizing moments. We investigate the size of the test under a variety of circumstances regarding the autocorrelation structure. In Section 4, we present appropriate standardizing moments that can be used under different autocorrelation structures. We also study the size and power properties of the suggested moments. Finally, Section 5 concludes the paper.

## 2 The panel data stationarity test

To test for stationarity in panel data, Hadri (2000) considers a panel data model containing $N$ different time series each consisting of $T$ time series observations. More specifically, the panel data model is described by (1) and (2) below.

$$y_{it} = \alpha_i + \delta_i t + \xi_{it} + \varepsilon_{it} \tag{1}$$

$$\xi_{it} = \xi_{it-1} + \eta_{it} \tag{2}$$

In (1), $y_{it}$ is an observation for cross section $i$ at time $t$. $\{\alpha_i, \delta_i t\}$ is an intercept and a time trend, respectively, which are specific to cross section $i$. $\xi_{it}$ describes a random walk component. Finally, $\varepsilon_{it}$ is a disturbance term, with a $N(0, \sigma_{\varepsilon,i}^2)$ distribution, which is assumed to be uncorrelated over cross sections. The evolvement of the random walk component is described in (2), where $\eta_{it} \sim N(0, \sigma_{\eta,i}^2)$.

The null hypothesis of panel data stationarity is represented by a zero variance of the disturbance that drives the random walk, that is $H_0$: $\sigma_{\eta,i}^2 = 0 \ \forall \ i$. Kwiatkowski et al. (1992) suggest that univariate stationarity, for cross section $i$, should be tested

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using the test statistic in (3), which have the limiting distribution in (4).

$$LM_i = \frac{T^{-2}\sum_{t=1}^{T} S_{it}^2}{\sigma_i^2} \tag{3}$$

$$LM_i \to \frac{\sigma_i^2 \int_0^1 V(r)^2 dr}{\sigma_i^2} = \int_0^1 V(r)^2 dr \tag{4}$$

In (3), $S_{it} = \sum_{j=1}^{t} \varepsilon_{ij}$ is the partial sum process. $\sigma_i^2$ is the long-run variance of $\varepsilon_{it}$, while $\int_0^1 V(r)dr$ in (4) is a standard Brownian bridge. In practice, $\varepsilon_{it}$ and $\sigma_i^2$ are not known and these theoretical quantities have to be replaced by its estimated counterparts, $\hat{\varepsilon}_{it}$ and $\hat{\sigma}_i^2$. $\hat{\varepsilon}_{it}$ is found by demeaning and detrending each individual time series using the deterministic components in (1), while the estimator $\hat{\sigma}_i^2$ depends of the nature of the disturbances, $\varepsilon_{it}$. If the disturbances are serially uncorrelated, $\hat{\sigma}_i^2$ in (5) can be used. If $\varepsilon_{it}$ is stationary but autocorrelated, Kwiatkowski et al. (1992) and Hadri (2000) suggest that the estimator $\hat{\sigma}_i^2(l)$ of the long-run variance in (6), with the weighting function $w(s,l)$ in (7), can be used.

$$\hat{\sigma}_i^2 = T^{-1}\sum_{t=1}^{T} \hat{\varepsilon}_{it}^2 \tag{5}$$

$$\hat{\sigma}_i^2(l) = T^{-1}\sum_{t=1}^{T} \hat{\varepsilon}_{it}^2 + 2T^{-1}\sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} \hat{\varepsilon}_{it}\hat{\varepsilon}_{it-s} \tag{6}$$

$$w(s,l) = 1 - \frac{s}{l+1} \tag{7}$$

Since all test statistics, $LM_i$ for $i \in \{1,...,N\}$, are independent by assumption, Hadri (2000) suggests that a panel data stationarity test is based on the average of the univariate test statistics. By applying the sequential limit theory discussed by Phillips and Moon (1999), and appropriately standardizing the average of the individual test statistics, a panel data stationarity test with a standard normal limit is achieved. The panel data test statistic, denoted $LM_{(T,N\to\infty)_{seq}}$, and its distribution is given in (8) below.

$$LM_{(T,N\to\infty)_{seq}} = \frac{N^{-1}\sum_{i=1}^{N}[LM_i - E(LM^\infty)]}{\sqrt{Var(LM^\infty)/N}} \Rightarrow N(0,1) \tag{8}$$

In (8), $E(LM^\infty)$ and $Var(LM^\infty)$ are the expected value and the variance of the functional of the Brownian bridge in (4). These moments depend on the detrending procedure used to obtain the residuals for the the individual stationarity test statistics, $LM_i$. If a only a an intercept is used when detrending the data, the appropriate asymptotic moments are given by $E(LM^\infty) = 1/6$ and $Var(LM^\infty) = 1/45$, while $E(LM^\infty) = 1/15$ and $Var(LM^\infty) = 11/6300$ should be used when both an intercept and a time trend is used to obtain the residuals.

The use of asymptotic moments can be problematic in empirical applications since we always have to rely on finite samples that in addition often are small. This lead Hadri and Larsson (2003) to derive standardizing moments to be used for fixed, and finite, $T$. Let these moments be denoted $E(LM^T)$ and $Var(LM^T)$. With such moments at hand, we do not have to consider joint limit theory anymore

since the moments for finite $T$ is available. Instead we only have to consider cases where $N \to \infty$, while $T$ is fixed. Hadri and Larsson (2003) hence supplied a new panel data stationarity test, denoted $LM_{(T \text{ fixed}, N \to \infty)}$, as in (9).

$$LM_{(T \text{ fixed}, N \to \infty)} = \frac{N^{-1} \sum_{i=1}^{N} [LM_i - E(LM^T)]}{\sqrt{Var(LM^T)/N}} \Rightarrow N(0, 1) \qquad (9)$$

Hadri and Larsson (2003) show that the appropriate finite sample moments are $E(LM^T) = (T + 1)/6T$ and $Var(LM^T) = (T^2 + 1)/20T^2 - (T + 1)^2/(6T)^2$ for the case where an intercept is present and $E(LM^T) = (T + 2)/15T$ and $Var(LM^T) = (T + 2)(13T^2 + 23)/2100T^3 - ((T + 2)/15T)^2$ in the case where both an intercept and a time trend is present. Simulation results, provided by Hadri and Larsson (2003), show that the test statistic using these performs excellent when $\varepsilon_{it}$ is white noise. The key question that we address in this paper is whether the asymptotic and finite-sample moments are appropriate also in in the presence of serially correlated disturbances. This issue is investigated in the next section.[1]

# 3 The performance of the panel data stationarity test

In this section we investigate which standardizing moments that are appropriate, and which that are not, when performing the previously discussed panel data stationarity test.

We can note that the moments that are supplied by Hadri (2000) are asymptotic moments. As such, they can be used regardless of whether or not serial correlation is allowed for.[2] However, in finite samples these moments are not appropriate. Instead, in finite-sample situations, the moments supplied by Hadri and Larsson (2003) can be used. However, these moments are obtained under the assumption that the estimator of the error variance, given by (5), is valid. But when disturbances are possibly serially correlated this estimator is not valid. Instead, we have to apply the estimator for the long-run variance in (6) and (7). This will imply that the small-sample behavior of long-run variance estimator will affect the panel data test statistic in such a way that the finite-sample moments become may be inappropriate. However, the effects of using either asymptotic or finite-sample moments, while allowing for serially correlated disturbances, have not been greatly investigated previously. Hence, a first step is to study whether moments currently available can be used for inference about the stationarity hypothesis in the panel data context.

To study whether the available moments can be used in the panel data stationarity test, while allowing for serially correlated disturbances, we set up a Monte Carlo study. We generate data according to (1) under different assumption regarding the deterministic components. When only an intercept is considered, we let $\alpha_i$

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[1]It can be noted that Tiffin (1999) obtain some preliminary results on the panel data test of Hadri (2000). However, the results are somewhat unclear and provide no specific solution to all of the problems encountered.

[2]Hadri (2004) provides simulation evidence that indicate that asymptotic moments can be applied without problem when samples are large and serial correlation is allowed for.

be uniformly distributed over the interval $[0, 10]$, i.e. $\alpha_i \in U[0, 10]$, while $\beta_i = 0$. In the case where both an intercept and a time trend is considered we let $\alpha_i \in U[0, 10]$ and $\beta_i \in U[0, 2]$. The distribution of the error term, $\varepsilon_i$, is set to $N(0, 1)$. Under the null hypothesis of stationarity, $\sigma_{\eta, it}^2 = 0 \ \forall \ i$. This implies that $\xi_{it} = \xi_{i0} \ \forall \ i$. Without loss of generality we can let $\xi_{i0} = 0$ across all $i$.

To investigate the size of the panel data stationarity test, we generate data with different time-series and cross-section dimension. More specifically we consider the cases where $T \in \{10, 20, 30, 40, 50, 75, 100\}$ and $N \in \{10, 25, 50\}$. For each of the samples, we then apply the test in (3). However, since we are interested in the cases where the error term is possibly serially correlated, we apply the variance estimator given by (6) and (7) where $l = int[k(\frac{T}{100})^{0.25}]$ with $k \in \{4, 8, 12, 16, 20, 24\}$.[3,4] We generate 10,000 test statistics for each sample size and choice of $k$ and then calculate the mean over the cross-sectional dimensions for these 10,000 test statistics. Using the finite-sample and asymptotic moments, together with the 5% critical value from the normal distribution, we then calculate the size of the panel data stationarity test. The results are presented in Table 1 and Table 2.

In Table 1, we see the size of the panel data stationarity test when applying the asymptotic moments of Hadri (2000). From the tables we see that the size of the panel data stationarity test is severely distorted when we consider fine-sample situations and allow for serially correlated disturbances. When we consider the model with only an intercept, we see from Table 1 that the time-series dimension has to be large, $T \geq 75$, and the parameter determining the lag window has to be small, $k = 4$, for the asymptotic moments to be applicable without causing any size distortions. Under these circumstances, the asymptotic approximation for the moments seems to work well, rendering a size close to the significance level. However, for smaller values of $T$ and/or larger values of $k$, the error arising from the poor approximation of the moments causes a size distortion in the panel data stationarity test. As seen from Table 1, the size distortion becomes larger as the number of cross sections, $N$, increases. When we consider the model with both an intercept and a time trend, we see from Table 1 that none of the parameter combinations in the simulation setup renders a situation where the asymptotic moments can be used for inference about the stationarity hypothesis in the panel data context without causing size distortions.

When we consider the size of the panel data stationarity test, standardized with the finite-sample moments supplied by Hadri and Larsson (2003), we see from Table 2 that the basic result is the same. When we consider the model with an intercept only, and $k \geq 12$, the test is size-distorted for all time-series dimensions where $T \leq 100$. For the model including both an intercept and a time trend, the panel data stationarity test is size-distorted for all of combination of $N$, $T$ and $k$ considered.

To come to terms with the size distortions that arise as a consequence of inappropriate standardizing moments, we suggest that moments should be supplied for a specific model. That is, we argue that the standardizing moments that are to be used in the panel data stationarity test should depend on the deterministic

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[3]In the choice of weighting function and spectral window, we follow Kwiatkowski et al. (1992) and Hadri (2000).

[4]Due to the insufficient number of degrees of freedom, we do not study the case where $k \in \{16, 20, 24\}$ while $T = 10$.

component, time-series dimension and choice of lag window. In the next section, we provide such moments and investigate the size and power characteristics of the panel data stationarity test using the suggested moments.

# 4 Monte Carlo simulation

## 4.1 Appropriate moments

As seen from in previous section, the moments that have been previously suggested for use in the panel data stationarity test of Hadri (2000) are inappropriate when serial correlation is allowed for under the null hypothesis. We saw that the null hypothesis was grossly over-rejected at the 5% significance level, leading to the faulty conclusion that a panel of time series is not stationary. In this section we try to alleviate the problems that arise as a consequence of the inappropriate moments by providing a new set of moments that can be used when performing the panel data stationarity test in finite samples when serial correlation is allowed for.

The moments that we supply in this paper are obtained through stochastic simulation. To obtain the moments, we generate data according to (1) under different assumption regarding the deterministic components. When only an intercept is considered, we let $\alpha_i$ be uniformly distributed over the interval $[0, 10]$, i.e. $\alpha_i \in U[0, 10]$, while $\beta_i = 0$. In the case where both an intercept and a time trend is considered we let $\alpha_i \in U[0, 10]$ and $\beta_i \in U[0, 2]$. The distribution of the error term, $\varepsilon_{it}$, is set to $N(0, 1)$.

We generate data with different time series dimension, more specifically we consider the cases where $T \in \{10, 20, 30, 40, 50, 75, 100\}$. For each of the samples we then apply the test in (3). Since we want to obtain moments that varies across different values of $k$, we apply the variance estimator given by (6) and (7) where $l = int[k(\frac{T}{100})^{0.25}]$ with $k \in \{4, 8, 12, 16, 20, 24\}$. We generate 10,000 test statistics for each sample size. and calculate the mean and the variance of these 10,000 test statistics. To distinguish the small sample moments for autocorrelated residuals from the small sample moments that are to be used when errors are serially uncorrelated, we denote the simulated moments $E(LM^{T,k})$ and $Var(LM^{T,k})$, where $T$ and $k$ indicate the time series dimension and lag window parameter. To reduce simulation error we repeat this procedure 100 times and hence get 100 estimated means and 100 estimated variances for each sample size and choice of $k$. The average over the 100 different moments, for the different choices of $T$ and $k$, is presented in Table 3.[5]

With the simulated moments at hand we can once again study the performance of the panel data stationarity test. To this end we generate 10,000 data sets under the null hypothesis, for different choices of $T$, $N$ and $k$, and calculate the individual stationarity tests as in (3), with the long-run variance estimated as in (6) and (7). We then obtain the panel data test statistic by standardizing the mean of the individual test statistics using the different moments presented in Table 3. In Table 4, we present the size properties of the test using these moments.

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[5]It can be noted that we tried to fit response surface regressions to the moments. However, this attempt proved fruitless. The smallest regression error in the response surface regressions accumulates across the cross-sectional units and renders severe size distortions that becomes worse as $N$ increases.

As seen in the table, the size properties are very good when using the simulated moments provided in Table 3. The conclusion is that the simulated moments provided in this paper are the appropriate moments to use when testing for stationarity in panel data when disturbances are potentially serially correlated. The next thing we want to investigate is how well the panel data stationarity test can distinguish between the null and alternative hypothesis when the null is in fact false. That is, we want to study the power of the test.

## 4.2   Power of the panel data stationarity test

To simulate the power of the panel data stationarity test, we generate data as described in the previous section with the modification that we add a random walk component to each of the time series in the panel. The random walk components of different cross-sections are independent and driven by the disturbance term $\eta_{it}$ which, in addition to being independently distributed across cross sections, is serially uncorrelated in all of our simulations. Moreover, we let that $\eta_{it} \sim N(0,1)$.[6] Finally, since the panel data stationarity test of Hadri (2000) is a heterogeneous panel data test, we consider five different situations when investigating the power. More specifically, we let the fraction of non-stationary cross-sections, $\Psi$, take the five values 0.20, 0.40, 0.60, 0.80 and 1.00, respectively. In Table 5 and Table 6, we present the power of the panel data stationarity test for the models with an intercept and an intercept and a time trend, respectively, and for different choices of sample size and lag window.

From Table 5 we see that the power of the test is good when an intercept is the only deterministic component considered. For all sample sizes where $T \geq 30$ and $k \in \{4, 8, 12, 16, 20\}$, we see that the panel data stationarity test has acceptable power properties. However, in some cases when $T$ is small and $k$ is large, an unfortunate characteristic of the test is displayed. Consider for example that case where $T = 20$ and $k = 24$. For this specific choice of parameter values, we see that the power of the panel data stationarity test falls below the size of the test. That is, the panel data stationarity test is biased. Moreover, the bias seems to aggregate over $N$ so that when $N$ increases, the bias becomes more severe. Once we consider the the cases where $T$ increases, while keeping $k$ fixed, we see that the bias disappears. However, the results indicate that we should be aware of the low power of the test in small-sample situations where severe autocorrelation is an issue.

From Table 6 we see that the bias of the test becomes even more accentuated when we allow for both an intercept and a time trend. When $k = 24$, we must have as much as 100 observations in order to get a power of the panel data stationarity test that is higher than the size of the test at the 5% significance level. Also, contrary to the case where we have an intercept only, the bias of the panel data stationarity test occurs across all choices of $k$.

One key question, that arises in the light of the size and power properties of the panel data stationarity test, is whether $k$ has to be chosen as large as 16, 20

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[6]The signal-to-noise ratio $\sigma^2_{\eta,i}/\sigma^2_{\varepsilon,i}$ affects the power of the panel data stationarity test. However, since we are not aiming to portray the power of the test for different signal-to-noise ratios, we only consider the case where the ratio is equal to one and instead focus on the effects of the choice of lag window.

or 24 in order to account for possible serial correlation in the disturbances. To answer this question, we study the size properties of the panel data stationarity test under various degrees of serial correlation. We generate data under the null hypothesis, according to the procedure describes above, with the modification that we now let the disturbance term, $\varepsilon_{it}$, be serially correlated. More specifically, the process for the disturbance term is given by $\varepsilon_{it} = \rho_i \varepsilon_{it-1} + \nu_{it}$, where $\nu_{it}$ is set to be distributed $N(0,1)$. We perform the simulation for the case where $N = 10$ and $T = 50$.[7] The autoregressive parameter, $\rho_i$, is uniformly distributed over the interval $(\mu_\rho - 0.01, \mu_\rho + 0.01)$.[8] To assess how the different choices of lag window are able to accommodate different degrees of serial correlation, we let $\mu_\rho \in \{0.00, 0.05, \ldots, 0.90, 0.95\}$. Using 10,000 replications for each choice of $\mu_\rho$, we calculate the size of the panel data stationarity test.

In Figure 5, we present the size of the panel data stationarity test when the only deterministic component allowed for is an intercept. As seen in the figure, the size of the panel data stationarity test has large distortions already for $\mu_\rho = 0.50$ when $k \in \{4, 8, 12\}$. For $\mu_\rho = 0.85$, none of the choices of $k$ is able to produce a panel data stationarity test without size distortions.

In Figure 5, we present the corresponding results for the case where the deterministic components consist of both an intercept and a time trend. The results for $k \in \{4, 8\}$ are principally unchanged when compared to the case where only an intercept was allowed for. However, for the case where k=12, the size properties of the panel data stationarity test seems to be good for values of $\mu_\rho$ as large as 0.75. For the cases where $k = 16$, $k = 20$ or $k = 24$, the panel data stationarity test has a downward size distortion, rendering a test that always fail to reject the null hypothesis 5% of the time even though this is the chosen significance level.

The results presented in this section applied to all situations where a lag window has to be chosen, regardless of what method that is applied to obtain the lag window. That is, even if we apply the data-dependent lag window selection procedure of e.g. Newey and West (1994), the panel data stationarity test will possess the same properties as presented above as long as $l$ is close to the values obtained for different choices of $k$ above.[9] Corresponding results, regarding the power of the stationarity test, have been established in the univariate setting by e.g. (Lee, 1996; Caner and Kilian, 2001). The results of this paper indicates that the problems documented in the univariate environment become more accentuated in the panel data setting, i.e. as the number of cross sections considered increases. The results also indicate that more research has to be directed towards the small-sample properties of panel data stationarity test of Hadri (2000) in order to solve the problems caused to serially correlated disturbance terms.

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[7]Simulation results for other sample sizes are available upon request.

[8]We let the autoregressive parameter vary across $i$ since the panel data stationarity test allows for heterogeneity, also in the serial correlation pattern.

[9]Methods using a pre-whitening procedure in the estimation of the long-run variance, such as the method suggested by Andrews and Monahan (1992), could be applied in this context. However, Lee (1996) has found that these procedures renders a fall in power for the univariate KPSS test. The fall in power can be so large that the test becomes biased (see Lee, 1996, p 135). There is no reason why these results cannot be extended to the panel data context. Indeed, it is likely that the bias of the univariate test accumulates across the cross sections, rendering a panel data stationarity test with an even larger bias. Hence, we focus on the estimation of the long-run variance, without considering a pre-whitening procedure.

# 5 Conclusions

In this paper, we study the small-sample properties of the panel data stationarity test of Hadri (2000) when disturbances are allowed to display serial correlation under the null hypothesis. We find that the moments that have been previously supplied for the panel data stationarity test are inappropriate when we allow for the possibility of serially correlated disturbances. More specifically, we find that the panel data stationarity test has a severe size distortion that causes the test to reject the null hypothesis too often. We supply standardizing moments that renders a test that has no size distortions. However, simulation results show that the power properties of the panel data stationarity test can be very poor in small samples under specific circumstances. When the model contains both an intercept and a time trend, and allows for a large degree of serial correlation, the panel data stationarity test is shown to be biased for situations where the time series dimension is as large as 50. The results indicates that results from empirical applications of the test should be interpreted cautiously under these circumstances.

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Table 1: Size of panel stationarity test when using asymptotic moments.

| | | Intercept | | | Intercept and trend | | |
|---|---|---|---|---|---|---|---|
| | | N=10 | N=25 | N=50 | N=10 | N=25 | N=50 |
| k=24 | T=10 | - | - | - | - | - | - |
| | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 40 | 0.971 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 50 | 0.644 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 75 | 0.168 | 0.573 | 0.934 | 1.000 | 1.000 | 1.000 |
| | 100 | 0.090 | 0.262 | 0.587 | 1.000 | 1.000 | 1.000 |
| k=20 | T=10 | - | - | - | - | - | - |
| | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 30 | 0.966 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 40 | 0.529 | 0.985 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 50 | 0.267 | 0.780 | 0.994 | 1.000 | 1.000 | 1.000 |
| | 75 | 0.093 | 0.274 | 0.600 | 1.000 | 1.000 | 1.000 |
| | 100 | 0.063 | 0.153 | 0.332 | 0.996 | 1.000 | 1.000 |
| k=16 | T=10 | - | - | - | - | - | - |
| | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 30 | 0.535 | 0.986 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 40 | 0.214 | 0.689 | 0.981 | 1.000 | 1.000 | 1.000 |
| | 50 | 0.125 | 0.405 | 0.803 | 1.000 | 1.000 | 1.000 |
| | 75 | 0.061 | 0.133 | 0.282 | 0.982 | 1.000 | 1.000 |
| | 100 | 0.054 | 0.092 | 0.175 | 0.755 | 0.999 | 1.000 |
| k=12 | T=10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 20 | 0.808 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 30 | 0.166 | 0.525 | 0.915 | 1.000 | 1.000 | 1.000 |
| | 40 | 0.092 | 0.266 | 0.590 | 1.000 | 1.000 | 1.000 |
| | 50 | 0.072 | 0.176 | 0.391 | 1.000 | 1.000 | 1.000 |
| | 75 | 0.047 | 0.081 | 0.148 | 0.651 | 0.993 | 1.000 |
| | 100 | 0.050 | 0.065 | 0.097 | 0.334 | 0.806 | 0.991 |
| k=8 | T=10 | 0.963 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 20 | 0.162 | 0.528 | 0.911 | 1.000 | 1.000 | 1.000 |
| | 30 | 0.060 | 0.139 | 0.288 | 0.975 | 1.000 | 1.000 |
| | 40 | 0.055 | 0.097 | 0.195 | 0.813 | 1.000 | 1.000 |
| | 50 | 0.053 | 0.077 | 0.114 | 0.423 | 0.903 | 0.999 |
| | 75 | 0.049 | 0.059 | 0.078 | 0.205 | 0.505 | 0.859 |
| | 100 | 0.051 | 0.057 | 0.071 | 0.147 | 0.326 | 0.620 |
| k=4 | T=10 | 0.170 | 0.540 | 0.917 | 1.000 | 1.000 | 1.000 |
| | 20 | 0.057 | 0.089 | 0.152 | 0.520 | 0.961 | 1.000 |
| | 30 | 0.053 | 0.063 | 0.083 | 0.175 | 0.441 | 0.777 |
| | 40 | 0.045 | 0.060 | 0.078 | 0.172 | 0.439 | 0.768 |
| | 50 | 0.052 | 0.055 | 0.067 | 0.124 | 0.260 | 0.493 |
| | 75 | 0.055 | 0.058 | 0.058 | 0.085 | 0.133 | 0.223 |
| | 100 | 0.055 | 0.057 | 0.061 | 0.084 | 0.129 | 0.198 |

Table 2: Size of panel stationarity test when using the moments of Hadri and Larsson (2003).

|  |  | Intercept | | | Intercept and trend | | |
|---|---|---|---|---|---|---|---|
|  |  | N=10 | N=25 | N=50 | N=10 | N=25 | N=50 |
| k=24 | T=10 | - | - | - | - | - | - |
|  | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 30 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 40 | 0.963 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 50 | 0.618 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 75 | 0.162 | 0.539 | 0.913 | 1.000 | 1.000 | 1.000 |
|  | 100 | 0.086 | 0.243 | 0.548 | 1.000 | 1.000 | 1.000 |
| k=20 | T=10 | - | - | - | - | - | - |
|  | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 30 | 0.956 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 40 | 0.500 | 0.977 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 50 | 0.248 | 0.736 | 0.987 | 1.000 | 1.000 | 1.000 |
|  | 75 | 0.089 | 0.250 | 0.547 | 1.000 | 1.000 | 1.000 |
|  | 100 | 0.061 | 0.141 | 0.300 | 0.993 | 1.000 | 1.000 |
| k=16 | T=10 | - | - | - | - | - | - |
|  | 20 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 30 | 0.493 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 40 | 0.197 | 0.628 | 0.962 | 1.000 | 1.000 | 1.000 |
|  | 50 | 0.115 | 0.362 | 0.740 | 1.000 | 1.000 | 1.000 |
|  | 75 | 0.058 | 0.118 | 0.247 | 0.968 | 1.000 | 1.000 |
|  | 100 | 0.052 | 0.085 | 0.156 | 0.714 | 0.998 | 1.000 |
| k=12 | T=10 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 20 | 0.763 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 30 | 0.149 | 0.439 | 0.840 | 1.000 | 1.000 | 1.000 |
|  | 40 | 0.084 | 0.223 | 0.492 | 1.000 | 1.000 | 1.000 |
|  | 50 | 0.067 | 0.149 | 0.329 | 0.997 | 1.000 | 1.000 |
|  | 75 | 0.046 | 0.075 | 0.124 | 0.579 | 0.983 | 1.000 |
|  | 100 | 0.048 | 0.061 | 0.086 | 0.296 | 0.739 | 0.980 |
| k=8 | T=10 | 0.926 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|  | 20 | 0.137 | 0.405 | 0.787 | 1.000 | 1.000 | 1.000 |
|  | 30 | 0.053 | 0.106 | 0.197 | 0.923 | 1.000 | 1.000 |
|  | 40 | 0.050 | 0.080 | 0.146 | 0.706 | 0.998 | 1.000 |
|  | 50 | 0.050 | 0.066 | 0.087 | 0.330 | 0.802 | 0.991 |
|  | 75 | 0.047 | 0.053 | 0.067 | 0.170 | 0.401 | 0.752 |
|  | 100 | 0.049 | 0.053 | 0.063 | 0.127 | 0.265 | 0.518 |
| k=4 | T=10 | 0.121 | 0.312 | 0.643 | 1.000 | 1.000 | 1.000 |
|  | 20 | 0.048 | 0.060 | 0.081 | 0.300 | 0.730 | 0.975 |
|  | 30 | 0.049 | 0.049 | 0.052 | 0.111 | 0.232 | 0.431 |
|  | 40 | 0.042 | 0.049 | 0.057 | 0.117 | 0.274 | 0.507 |
|  | 50 | 0.049 | 0.049 | 0.050 | 0.096 | 0.169 | 0.300 |
|  | 75 | 0.052 | 0.054 | 0.050 | 0.072 | 0.099 | 0.146 |
|  | 100 | 0.053 | 0.053 | 0.054 | 0.073 | 0.100 | 0.142 |

Table 3: Standardizing moments[a], $l = int[k(\frac{T}{100})^{0.25}]$.

Intercept

| | k=4 | | k=8 | | k=12 | | k=16 | | k=20 | | k=24 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ |
| 10 | 0.218311 | 0.086760 | 0.281648 | 0.067939 | 0.359700 | 0.047595 | - | - | - | - | - | - |
| 20 | 0.185031 | 0.109906 | 0.217360 | 0.084965 | 0.263467 | 0.069086 | 0.299864 | 0.062260 | 0.359675 | 0.045999 | 0.426461 | 0.020182 |
| 30 | 0.177165 | 0.120379 | 0.193629 | 0.099521 | 0.217307 | 0.084755 | 0.246692 | 0.073216 | 0.281179 | 0.065284 | 0.319040 | 0.057403 |
| 40 | 0.176893 | 0.119688 | 0.188572 | 0.103653 | 0.204755 | 0.091488 | 0.224071 | 0.081373 | 0.246724 | 0.073175 | 0.280989 | 0.065098 |
| 50 | 0.174154 | 0.124513 | 0.182566 | 0.110587 | 0.197609 | 0.095926 | 0.211843 | 0.087214 | 0.228210 | 0.079534 | 0.253153 | 0.071169 |
| 75 | 0.171065 | 0.131384 | 0.177351 | 0.117529 | 0.185731 | 0.106363 | 0.193334 | 0.099203 | 0.205312 | 0.090909 | 0.218844 | 0.083579 |
| 100 | 0.170922 | 0.132832 | 0.175009 | 0.121358 | 0.180554 | 0.112327 | 0.187537 | 0.104443 | 0.195338 | 0.097383 | 0.204523 | 0.091372 |

Intercept and time trend

| | k=4 | | k=8 | | k=12 | | k=16 | | k=20 | | k=24 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ | $\mu^{T,k}$ | $\sigma^{T,k}$ |
| 10 | 0.132497 | 0.027909 | 0.223728 | 0.048591 | 0.337232 | 0.047923 | - | - | - | - | - | - |
| 20 | 0.089191 | 0.025877 | 0.133065 | 0.022950 | 0.198014 | 0.038755 | 0.249508 | 0.043947 | 0.335723 | 0.043431 | 0.422428 | 0.027016 |
| 30 | 0.079609 | 0.030394 | 0.101351 | 0.021576 | 0.133099 | 0.021868 | 0.174348 | 0.031902 | 0.222928 | 0.040880 | 0.277030 | 0.043374 |
| 40 | 0.079668 | 0.030212 | 0.095035 | 0.023105 | 0.115982 | 0.019883 | 0.142575 | 0.023325 | 0.174343 | 0.031405 | 0.222768 | 0.040293 |
| 50 | 0.076310 | 0.032223 | 0.087086 | 0.026030 | 0.106914 | 0.020314 | 0.125989 | 0.020312 | 0.148567 | 0.024502 | 0.183512 | 0.033401 |
| 75 | 0.072602 | 0.035140 | 0.080750 | 0.029252 | 0.091535 | 0.024157 | 0.101419 | 0.021219 | 0.117093 | 0.019597 | 0.135588 | 0.021557 |
| 100 | 0.072150 | 0.035497 | 0.077886 | 0.030925 | 0.085079 | 0.026822 | 0.093831 | 0.023279 | 0.104113 | 0.020597 | 0.116019 | 0.019540 |

Notes: [a] $\mu^{T,k}$ and $\sigma^{T,k}$ is used to denote $E(LM^{T,k})$ and $\sqrt{Var(LM^{T,k})}$, respectively.

Table 4: Size of panel data stationarity tests when using simulated moments.

| | | Intercept | | | Intercept and trend | | |
|---|---|---|---|---|---|---|---|
| | | N=10 | N=25 | N=50 | N=10 | N=25 | N=50 |
| k=24 | T=10 | - | - | - | - | - | - |
| | 20 | 0.056 | 0.057 | 0.055 | 0.049 | 0.057 | 0.049 |
| | 30 | 0.060 | 0.057 | 0.056 | 0.054 | 0.057 | 0.052 |
| | 40 | 0.056 | 0.057 | 0.052 | 0.057 | 0.056 | 0.053 |
| | 50 | 0.057 | 0.052 | 0.051 | 0.063 | 0.063 | 0.058 |
| | 75 | 0.060 | 0.055 | 0.054 | 0.058 | 0.056 | 0.053 |
| | 100 | 0.057 | 0.053 | 0.053 | 0.052 | 0.055 | 0.052 |
| k=20 | T=10 | - | - | - | - | - | - |
| | 20 | 0.059 | 0.055 | 0.053 | 0.054 | 0.051 | 0.053 |
| | 30 | 0.058 | 0.051 | 0.050 | 0.057 | 0.057 | 0.056 |
| | 40 | 0.054 | 0.053 | 0.052 | 0.057 | 0.054 | 0.055 |
| | 50 | 0.057 | 0.057 | 0.054 | 0.060 | 0.054 | 0.052 |
| | 75 | 0.055 | 0.055 | 0.052 | 0.051 | 0.055 | 0.052 |
| | 100 | 0.056 | 0.057 | 0.053 | 0.058 | 0.052 | 0.054 |
| k=16 | T=10 | - | - | - | - | - | - |
| | 20 | 0.054 | 0.054 | 0.054 | 0.057 | 0.056 | 0.051 |
| | 30 | 0.056 | 0.056 | 0.051 | 0.062 | 0.056 | 0.057 |
| | 40 | 0.053 | 0.057 | 0.053 | 0.059 | 0.060 | 0.057 |
| | 50 | 0.057 | 0.057 | 0.052 | 0.059 | 0.055 | 0.056 |
| | 75 | 0.058 | 0.054 | 0.056 | 0.057 | 0.054 | 0.051 |
| | 100 | 0.060 | 0.054 | 0.056 | 0.056 | 0.056 | 0.052 |
| k=12 | T=10 | 0.055 | 0.056 | 0.050 | 0.053 | 0.058 | 0.055 |
| | 20 | 0.054 | 0.053 | 0.053 | 0.066 | 0.055 | 0.053 |
| | 30 | 0.062 | 0.057 | 0.052 | 0.055 | 0.056 | 0.058 |
| | 40 | 0.057 | 0.055 | 0.053 | 0.053 | 0.054 | 0.049 |
| | 50 | 0.059 | 0.056 | 0.060 | 0.058 | 0.056 | 0.055 |
| | 75 | 0.056 | 0.056 | 0.050 | 0.063 | 0.057 | 0.053 |
| | 100 | 0.062 | 0.058 | 0.055 | 0.059 | 0.056 | 0.056 |
| k=8 | T=10 | 0.054 | 0.057 | 0.053 | 0.062 | 0.054 | 0.052 |
| | 20 | 0.055 | 0.052 | 0.054 | 0.060 | 0.055 | 0.050 |
| | 30 | 0.056 | 0.056 | 0.053 | 0.058 | 0.054 | 0.054 |
| | 40 | 0.061 | 0.055 | 0.055 | 0.060 | 0.059 | 0.050 |
| | 50 | 0.062 | 0.061 | 0.058 | 0.057 | 0.057 | 0.055 |
| | 75 | 0.061 | 0.058 | 0.056 | 0.060 | 0.057 | 0.054 |
| | 100 | 0.064 | 0.060 | 0.059 | 0.061 | 0.052 | 0.057 |
| k=4 | T=10 | 0.058 | 0.054 | 0.055 | 0.062 | 0.062 | 0.051 |
| | 20 | 0.062 | 0.059 | 0.058 | 0.057 | 0.051 | 0.053 |
| | 30 | 0.063 | 0.059 | 0.054 | 0.061 | 0.063 | 0.058 |
| | 40 | 0.058 | 0.057 | 0.055 | 0.057 | 0.056 | 0.054 |
| | 50 | 0.062 | 0.058 | 0.054 | 0.065 | 0.062 | 0.058 |
| | 75 | 0.064 | 0.063 | 0.057 | 0.060 | 0.057 | 0.057 |
| | 100 | 0.064 | 0.060 | 0.057 | 0.062 | 0.059 | 0.057 |

## Table 5: Small-sample power, intercept only

**k=4**

| N | Ψ | T=10 | 20 | 30 | 40 | 50 | 75 | 100 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.14 | 0.40 | 0.61 | 0.65 | 0.76 | 0.88 | 0.90 |
| | 0.40 | 0.30 | 0.76 | 0.94 | 0.94 | 0.98 | 1.00 | 1.00 |
| | 0.60 | 0.47 | 0.94 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.64 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.79 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=25 | 0.20 | 0.23 | 0.65 | 0.88 | 0.90 | 0.96 | 0.99 | 1.00 |
| | 0.40 | 0.49 | 0.97 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.76 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.33 | 0.86 | 0.99 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 0.40 | 0.74 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.95 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

**k=8**

| N | Ψ | T=10 | 20 | 30 | 40 | 50 | 75 | 100 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.08 | 0.19 | 0.33 | 0.39 | 0.50 | 0.64 | 0.73 |
| | 0.40 | 0.12 | 0.41 | 0.70 | 0.78 | 0.88 | 0.95 | 0.97 |
| | 0.60 | 0.17 | 0.65 | 0.90 | 0.95 | 0.98 | 0.99 | 1.00 |
| | 0.80 | 0.23 | 0.83 | 0.97 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.29 | 0.93 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=25 | 0.20 | 0.10 | 0.31 | 0.55 | 0.65 | 0.78 | 0.90 | 0.95 |
| | 0.40 | 0.17 | 0.69 | 0.94 | 0.97 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.27 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.40 | 0.98 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.54 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.13 | 0.47 | 0.78 | 0.88 | 0.95 | 0.99 | 1.00 |
| | 0.40 | 0.25 | 0.90 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.43 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.63 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.79 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

**k=12**

| N | Ψ | T=10 | 20 | 30 | 40 | 50 | 75 | 100 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.06 | 0.11 | 0.19 | 0.26 | 0.31 | 0.45 | 0.56 |
| | 0.40 | 0.05 | 0.20 | 0.44 | 0.59 | 0.66 | 0.83 | 0.91 |
| | 0.60 | 0.05 | 0.32 | 0.70 | 0.82 | 0.88 | 0.97 | 0.99 |
| | 0.80 | 0.04 | 0.46 | 0.86 | 0.94 | 0.97 | 1.00 | 1.00 |
| | 1.00 | 0.06 | 0.61 | 0.94 | 0.98 | 0.99 | 1.00 | 1.00 |
| N=25 | 0.20 | 0.06 | 0.15 | 0.33 | 0.44 | 0.52 | 0.72 | 0.84 |
| | 0.40 | 0.06 | 0.34 | 0.73 | 0.87 | 0.93 | 0.99 | 1.00 |
| | 0.60 | 0.06 | 0.56 | 0.95 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.06 | 0.77 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.06 | 0.89 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.06 | 0.22 | 0.51 | 0.65 | 0.75 | 0.92 | 0.97 |
| | 0.40 | 0.06 | 0.53 | 0.93 | 0.98 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.07 | 0.81 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.07 | 0.95 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.08 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

**k=16**

| N | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.08 | 0.14 | 0.19 | 0.22 | 0.34 | 0.43 |
| | 0.40 | - | 0.12 | 0.29 | 0.42 | 0.52 | 0.73 | 0.81 |
| | 0.60 | - | 0.16 | 0.46 | 0.65 | 0.77 | 0.92 | 0.96 |
| | 0.80 | - | 0.22 | 0.63 | 0.83 | 0.90 | 0.98 | 0.99 |
| | 1.00 | - | 0.29 | 0.78 | 0.92 | 0.97 | 1.00 | 1.00 |
| N=25 | 0.20 | - | 0.10 | 0.20 | 0.30 | 0.38 | 0.60 | 0.69 |
| | 0.40 | - | 0.17 | 0.48 | 0.69 | 0.81 | 0.96 | 0.98 |
| | 0.60 | - | 0.26 | 0.75 | 0.92 | 0.97 | 1.00 | 1.00 |
| | 0.80 | - | 0.38 | 0.91 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 1.00 | - | 0.54 | 0.97 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | - | 0.13 | 0.32 | 0.48 | 0.59 | 0.83 | 0.90 |
| | 0.40 | - | 0.26 | 0.72 | 0.91 | 0.97 | 1.00 | 1.00 |
| | 0.60 | - | 0.42 | 0.94 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | - | 0.63 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | - | 0.79 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

**k=20**

| N | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.06 | 0.10 | 0.14 | 0.18 | 0.26 | 0.33 |
| | 0.40 | - | 0.06 | 0.16 | 0.28 | 0.39 | 0.58 | 0.70 |
| | 0.60 | - | 0.06 | 0.25 | 0.48 | 0.61 | 0.83 | 0.91 |
| | 0.80 | - | 0.07 | 0.36 | 0.65 | 0.80 | 0.95 | 0.98 |
| | 1.00 | - | 0.06 | 0.48 | 0.79 | 0.91 | 0.98 | 1.00 |
| N=25 | 0.20 | - | 0.06 | 0.12 | 0.21 | 0.28 | 0.44 | 0.56 |
| | 0.40 | - | 0.07 | 0.25 | 0.49 | 0.65 | 0.87 | 0.95 |
| | 0.60 | - | 0.08 | 0.44 | 0.77 | 0.90 | 0.99 | 1.00 |
| | 0.80 | - | 0.09 | 0.62 | 0.92 | 0.98 | 1.00 | 1.00 |
| | 1.00 | - | 0.09 | 0.78 | 0.98 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | - | 0.07 | 0.17 | 0.32 | 0.44 | 0.67 | 0.79 |
| | 0.40 | - | 0.08 | 0.40 | 0.74 | 0.89 | 0.99 | 1.00 |
| | 0.60 | - | 0.10 | 0.66 | 0.95 | 0.99 | 1.00 | 1.00 |
| | 0.80 | - | 0.12 | 0.86 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 1.00 | - | 0.15 | 0.96 | 1.00 | 1.00 | 1.00 | 1.00 |

**k=24**

| N | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.05 | 0.07 | 0.10 | 0.13 | 0.21 | 0.26 |
| | 0.40 | - | 0.03 | 0.09 | 0.17 | 0.26 | 0.46 | 0.59 |
| | 0.60 | - | 0.03 | 0.12 | 0.26 | 0.43 | 0.71 | 0.84 |
| | 0.80 | - | 0.02 | 0.15 | 0.37 | 0.60 | 0.87 | 0.95 |
| | 1.00 | - | 0.01 | 0.20 | 0.49 | 0.75 | 0.95 | 0.99 |
| N=25 | 0.20 | - | 0.04 | 0.08 | 0.14 | 0.20 | 0.35 | 0.46 |
| | 0.40 | - | 0.03 | 0.13 | 0.28 | 0.45 | 0.75 | 0.88 |
| | 0.60 | - | 0.03 | 0.19 | 0.46 | 0.71 | 0.95 | 0.99 |
| | 0.80 | - | 0.01 | 0.27 | 0.64 | 0.89 | 0.99 | 1.00 |
| | 1.00 | - | 0.00 | 0.38 | 0.80 | 0.96 | 1.00 | 1.00 |
| N=50 | 0.20 | - | 0.04 | 0.10 | 0.18 | 0.29 | 0.52 | 0.68 |
| | 0.40 | - | 0.02 | 0.18 | 0.41 | 0.68 | 0.94 | 0.99 |
| | 0.60 | - | 0.01 | 0.30 | 0.68 | 0.92 | 1.00 | 1.00 |
| | 0.80 | - | 0.00 | 0.45 | 0.88 | 0.99 | 1.00 | 1.00 |
| | 1.00 | - | 0.00 | 0.61 | 0.96 | 1.00 | 1.00 | 1.00 |

## Table 6: Small-sample power, intercept and trend

### k=4

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.0 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.05 | 0.19 | 0.37 | 0.41 | 0.55 | 0.78 | 0.82 |
| | 0.40 | 0.04 | 0.38 | 0.72 | 0.79 | 0.91 | 0.99 | 0.99 |
| | 0.60 | 0.03 | 0.60 | 0.92 | 0.95 | 0.99 | 1.00 | 1.00 |
| | 0.80 | 0.02 | 0.76 | 0.98 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.02 | 0.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=25 | 0.20 | 0.04 | 0.29 | 0.61 | 0.67 | 0.84 | 0.97 | 0.99 |
| | 0.40 | 0.03 | 0.64 | 0.96 | 0.98 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.02 | 0.88 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.01 | 0.97 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.04 | 0.43 | 0.84 | 0.89 | 0.97 | 1.00 | 1.00 |
| | 0.40 | 0.02 | 0.87 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.60 | 0.02 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

### k=8

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.0 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.04 | 0.04 | 0.13 | 0.18 | 0.29 | 0.43 | 0.53 |
| | 0.40 | 0.03 | 0.03 | 0.25 | 0.39 | 0.60 | 0.82 | 0.90 |
| | 0.60 | 0.02 | 0.02 | 0.39 | 0.60 | 0.83 | 0.96 | 0.99 |
| | 0.80 | 0.01 | 0.01 | 0.54 | 0.77 | 0.95 | 1.00 | 1.00 |
| | 1.00 | 0.03 | 0.03 | 0.67 | 0.88 | 0.99 | 1.00 | 1.00 |
| N=25 | 0.20 | 0.02 | 0.02 | 0.19 | 0.29 | 0.47 | 0.69 | 0.81 |
| | 0.40 | 0.01 | 0.01 | 0.41 | 0.63 | 0.89 | 0.99 | 1.00 |
| | 0.60 | 0.01 | 0.01 | 0.65 | 0.88 | 0.99 | 1.00 | 1.00 |
| | 0.80 | 0.00 | 0.00 | 0.81 | 0.97 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.03 | 0.02 | 0.92 | 0.99 | 1.00 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.03 | 0.02 | 0.26 | 0.43 | 0.70 | 0.91 | 0.97 |
| | 0.40 | 0.01 | 0.01 | 0.61 | 0.87 | 0.99 | 1.00 | 1.00 |
| | 0.60 | 0.00 | 0.00 | 0.86 | 0.99 | 1.00 | 1.00 | 1.00 |
| | 0.80 | 0.00 | 0.00 | 0.97 | 1.00 | 1.00 | 1.00 | 1.00 |
| | 1.00 | 0.00 | 0.00 | 0.99 | 1.00 | 1.00 | 1.00 | 1.00 |

### k=12

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | 0.05 | 0.03 | 0.04 | 0.07 | 0.11 | 0.24 | 0.34 |
| | 0.40 | 0.04 | 0.02 | 0.02 | 0.09 | 0.19 | 0.51 | 0.70 |
| | 0.60 | 0.03 | 0.01 | 0.01 | 0.10 | 0.28 | 0.75 | 0.90 |
| | 0.80 | 0.03 | 0.00 | 0.01 | 0.12 | 0.39 | 0.89 | 0.97 |
| | 1.00 | 0.02 | 0.00 | 0.00 | 0.14 | 0.49 | 0.96 | 1.00 |
| N=25 | 0.20 | 0.04 | 0.02 | 0.03 | 0.07 | 0.15 | 0.40 | 0.57 |
| | 0.40 | 0.03 | 0.01 | 0.01 | 0.10 | 0.29 | 0.80 | 0.95 |
| | 0.60 | 0.02 | 0.00 | 0.00 | 0.12 | 0.47 | 0.97 | 1.00 |
| | 0.80 | 0.02 | 0.00 | 0.00 | 0.16 | 0.65 | 1.00 | 1.00 |
| | 1.00 | 0.01 | 0.01 | 0.02 | 0.19 | 0.78 | 1.00 | 1.00 |
| N=50 | 0.20 | 0.04 | 0.01 | 0.02 | 0.08 | 0.20 | 0.58 | 0.80 |
| | 0.40 | 0.03 | 0.00 | 0.01 | 0.12 | 0.44 | 0.96 | 1.00 |
| | 0.60 | 0.02 | 0.00 | 0.00 | 0.16 | 0.68 | 1.00 | 1.00 |
| | 0.80 | 0.01 | 0.00 | 0.00 | 0.22 | 0.86 | 1.00 | 1.00 |
| | 1.00 | 0.01 | 0.00 | 0.00 | 0.28 | 0.95 | 1.00 | 1.00 |

### k=16

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.04 | 0.03 | 0.03 | 0.04 | 0.15 | 0.21 |
| | 0.40 | - | 0.02 | 0.02 | 0.02 | 0.04 | 0.29 | 0.44 |
| | 0.60 | - | 0.01 | 0.01 | 0.01 | 0.03 | 0.46 | 0.70 |
| | 0.80 | - | 0.00 | 0.00 | 0.00 | 0.02 | 0.60 | 0.84 |
| | 1.00 | - | 0.00 | 0.00 | 0.02 | 0.01 | 0.74 | 0.93 |
| N=25 | 0.20 | - | 0.01 | 0.00 | 0.01 | 0.03 | 0.21 | 0.35 |
| | 0.40 | - | 0.00 | 0.00 | 0.00 | 0.01 | 0.48 | 0.74 |
| | 0.60 | - | 0.00 | 0.00 | 0.00 | 0.01 | 0.72 | 0.94 |
| | 0.80 | - | 0.00 | 0.00 | 0.00 | 0.01 | 0.87 | 0.99 |
| | 1.00 | - | 0.01 | 0.01 | 0.01 | 0.01 | 0.96 | 1.00 |
| N=50 | 0.20 | - | 0.00 | 0.00 | 0.01 | 0.03 | 0.31 | 0.52 |
| | 0.40 | - | 0.00 | 0.00 | 0.00 | 0.02 | 0.70 | 0.93 |
| | 0.60 | - | 0.00 | 0.00 | 0.00 | 0.01 | 0.92 | 1.00 |
| | 0.80 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.99 | 1.00 |
| | 1.00 | - | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 1.00 |

### k=20

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.04 | 0.03 | 0.03 | 0.03 | 0.07 | 0.13 |
| | 0.40 | - | 0.03 | 0.02 | 0.01 | 0.01 | 0.08 | 0.24 |
| | 0.60 | - | 0.02 | 0.01 | 0.01 | 0.01 | 0.09 | 0.39 |
| | 0.80 | - | 0.02 | 0.00 | 0.00 | 0.00 | 0.11 | 0.52 |
| | 1.00 | - | 0.03 | 0.02 | 0.01 | 0.02 | 0.12 | 0.65 |
| N=25 | 0.20 | - | 0.02 | 0.01 | 0.00 | 0.00 | 0.07 | 0.17 |
| | 0.40 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.09 | 0.40 |
| | 0.60 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.10 | 0.62 |
| | 0.80 | - | 0.00 | 0.00 | 0.01 | 0.00 | 0.14 | 0.79 |
| | 1.00 | - | 0.03 | 0.01 | 0.01 | 0.01 | 0.16 | 0.90 |
| N=50 | 0.20 | - | 0.03 | 0.01 | 0.00 | 0.00 | 0.08 | 0.26 |
| | 0.40 | - | 0.02 | 0.00 | 0.00 | 0.00 | 0.10 | 0.59 |
| | 0.60 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.13 | 0.84 |
| | 0.80 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.95 |
| | 1.00 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.99 |

### k=24

| | Ψ | T=10 | 20.00 | 30.00 | 40.00 | 50.00 | 75.00 | 100.00 |
|---|---|---|---|---|---|---|---|---|
| N=10 | 0.20 | - | 0.05 | 0.03 | 0.03 | 0.03 | 0.03 | 0.08 |
| | 0.40 | - | 0.04 | 0.02 | 0.02 | 0.01 | 0.02 | 0.09 |
| | 0.60 | - | 0.03 | 0.01 | 0.01 | 0.01 | 0.01 | 0.11 |
| | 0.80 | - | 0.02 | 0.00 | 0.00 | 0.00 | 0.01 | 0.14 |
| | 1.00 | - | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | 0.16 |
| N=25 | 0.20 | - | 0.03 | 0.01 | 0.01 | 0.01 | 0.01 | 0.08 |
| | 0.40 | - | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 |
| | 0.60 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 |
| | 0.80 | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 |
| | 1.00 | - | 0.03 | 0.01 | 0.01 | 0.00 | 0.01 | 0.22 |
| N=50 | 0.20 | - | 0.04 | 0.01 | 0.01 | 0.01 | 0.01 | 0.09 |
| | 0.40 | - | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 |
| | 0.60 | - | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 |
| | 0.80 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.24 |
| | 1.00 | - | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 |

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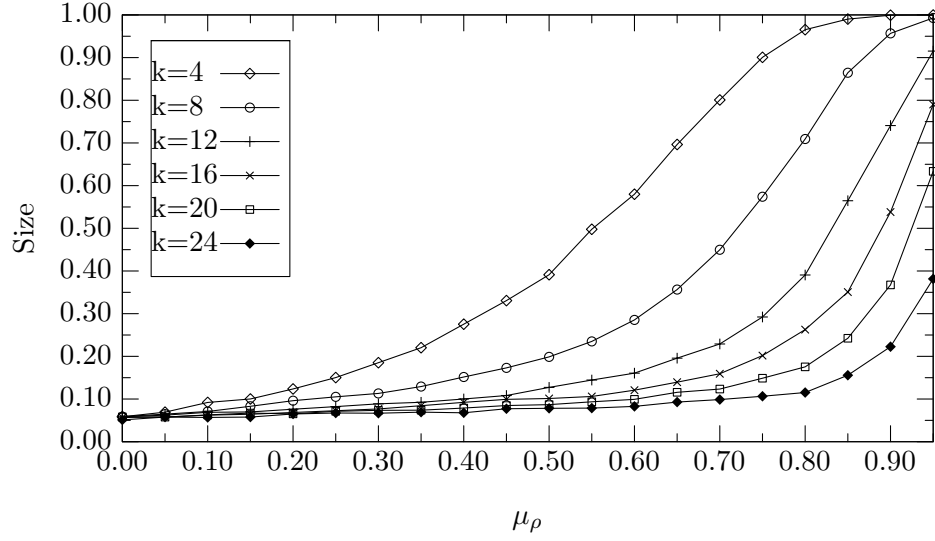Figure 1: Size when disturbances are autocorrelated, intercept only.



Figure 2: Size when disturbances are autocorrelated, intercept and trend.