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# Evolutionary Stability in Bargaining with an Asymmetric Breakdown Point

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## Abstract

We study an asymmetric two-player bargaining game with risk of breakdown and no discounting. We characterize the modified evolutionarily stable strategies (MESS) by modelling strategies as automata. Payoff and complexity considerations are taken in the automata-selection process. We show that a MESS exists in the bargaining game and that agreement is reached immediately. It turns out that in the search for evolutionary foundation, we find support for all partitions that assigns the positive breakdown utility or more to the player with the higher breakdown utility, given that it exceeds half the surplus.

*Keywords:* Modified evolutionary stable strategies; Automata; Breakdown; Asymmetric bargaining.

*JEL classification:* C72; C73; C78.

## 1 Introduction

In this paper we are studying a bargaining game similar to Rubinstein's (1982) alternating-offers bargaining game, but with the following modifications. Instead of discounting the players perceive that the bargaining might break down in a random matter. This modification shift the driving force in the model from the players' time preferences to the players' risk preferences. In the latter case it is fear of a breakdown in the negotiations and not impatience that makes the players prefer any given share of the pie sooner

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rather than later. We also make the following assumption. In case of a breakdown the players receive different payoffs. Thus, the breakdown point is asymmetric.

To simplify matters we assume that the breakdown point is either  $(0, x)$  or  $(x, 0)$ , where  $x > 0$  but less than the gains from agreement. In this case, it is straightforward to show that the game has a unique subgame perfect equilibrium outcome. Agreement is immediate and the payoffs are (when the 'pie' is of size 1):

- $\left(\frac{1+\delta x}{1+\delta}, \frac{\delta(1-x)}{1+\delta}\right)$  with  $(x, 0)$ , or
- $\left(\frac{1-x}{1+\delta}, \frac{\delta+x}{1+\delta}\right)$  with  $(0, x)$ , where  $\delta$  is the continuation probability.

The interesting question is how much of this result is maintained if we let the players' behavior be formed by an evolutionary process instead of being the result of backward induction reasoning. The evolutionary forces favor high-payoff strategies at the expense of low-payoff competitors. They also favor simplicity at the expense of complexity whenever the payoffs are the same. To find a suitable strategy we use the idea of a modified evolutionarily stable strategy (MESS), introduced by Binmore and Samuelson (1992) for the study of repeated games. Like Abreu and Rubinstein (1988), Banks and Sundaram (1990), Binmore and Samuelson (1992), Binmore *et al.* (1998) and Chatterjee and Sabourian (1999; 2000), we model strategies as automata. Complexity is measured by following the common practice of counting the number of states in an automaton, i.e. the more states an automaton uses, the more complex it is. We could also employ a broader measure of complexity by using the collapsing state condition introduced by Binmore *et al.* (1998), i.e. automaton  $M$  is less complex than  $A$  if each state used by  $M$  can be obtained by consolidating collections of states in  $A$ .

We require that the automaton is able to play the game both when it has the positive breakdown utility  $x$  and when its opponent has it. The automaton must also be able to play the game in both player-roles, i.e. as player  $I$  (when it has to start the bargaining process by making the first demand) and as player  $II$  (when it has to respond to its opponent's demand which started the bargaining process). Given this, we establish that a necessary and sufficient condition for an automaton  $A$  to be a MESS is that use of  $A$  against itself constitutes a Nash equilibrium in which an agreement is achieved immediately. It turns out that quite a few partitions can be supported by a MESS automaton; all partitions that assigns the positive breakdown utility  $x$  or more to the player with the higher breakdown utility, given that it exceeds half the surplus. For a bargaining game in which the positive breakdown

utility  $x$  is not greater than half the surplus, we establish the existence of a MESS.

The present study is based on the work of Binmore *et al.* (1998). Other related studies are Kandori *et al.* (1993), Young (1993a; 1993b), Bolton (1997), Rosenthal and Landau (1979) and Ponti and Seymour (1997). The paper proceeds as follows. In Section 2 we present the bargaining model. The evolutionary approach is applied in Section 3. Section 4 contains the results and Section 5 concludes the paper.

## 2 The Bargaining Game

We study an alternating-offers bargaining game with risk of breakdown. It is a two-player infinite-horizon game of perfect information in *discrete* time, indexed  $t = 1, 2, 3, \dots$ . Two *players*,  $I$  and  $II$ , bargain over the partition of a "*pie*" of size 1. In each period, one player is the proposer and demands a piece of the pie. A *demand*  $\theta(t)$  in period  $t$  always indicates the fraction going to the proposer, where  $\theta(t) \in [0, 1]$ . The other player is the responder, who either accepts or rejects the proposer's demand. If the demand is accepted, the bargaining ends and the agreement is implemented. If the demand is rejected, the responder makes a counteroffer in the next period, where the roles are reversed. We assume that with probability  $1 - \delta$  the bargaining break down in disagreement before the counteroffer is made, where  $\delta \in (0, 1)$ . Thus  $\delta \in (0, 1)$  is interpreted as a common *continuation probability*, which means that if a responder rejects a demand the game continues to the next period with probability  $\delta$ .

The players' utility functions  $u_i$  are von Neumann-Morgenstern utility functions that are linear in the pie. The payoff-pair obtainable through perpetual disagreement is called the *Impasse point* and gives payoff  $(I_I, I_{II})$ . The payoff-pair obtainable through breakdown is called the *Breakdown point* and gives payoff  $(b_I, b_{II})$ . The expected payoff to player  $i$  from making a demand  $\theta(t)$  that is accepted in period  $t$  is then  $\theta(t) \delta^{t-1} + (1 - \delta^{t-1}) b_i$ . The corresponding payoff to the opponent from accepting this demand is  $(1 - \theta(t)) \delta^{t-1} + (1 - \delta^{t-1}) b_{-i}$ . Finally, it follows that  $(I_I, I_{II}) = (b_I, b_{II})$  since  $\lim_{t \rightarrow \infty} (1 - \delta^{t-1}) = 1$ .

For the rest of the paper, let the breakdown point be either  $(0, x)$  or  $(x, 0)$ , where  $x \in (0, 1)$ .

## 2.1 Subgame Perfect Equilibrium

**Proposition 1** *There exists a unique subgame perfect equilibrium outcome. In this equilibrium, agreement is reached immediately and the payoffs are (always expressed  $(u_I, u_{II})$ ):*

- $\left(\frac{1+\delta x}{1+\delta}, \frac{\delta(1-x)}{1+\delta}\right)$  if  $b_I = x$ ,
- $\left(\frac{1-x}{1+\delta}, \frac{\delta+x}{1+\delta}\right)$  if  $b_{II} = x$ .

For proof, see Proposition 4.1 in Muthoo (1999, pp. 75-76).

## 3 The Evolutionary Approach

We study an automata-selection metagame in which two metaplayers choose automata to play the bargaining game for them. Strategies are modelled as automata because they are excellent instruments for measuring the complexity of strategies. For example, by counting the number of states each automata has, we can determine whether strategies are simple or not. This is vital for us when we treat the automaton-selection process as a metaphor for an evolutionary process, in which we assume that simplicity beats complexity whenever two automata perform equally well. However, automata that perform relatively poorly are overrun by those that perform relatively well, regardless of complexity.

### 3.1 The Metagame & Nature

The automaton must be able to play the bargaining game as player  $I$  and as player  $II$ , which is equally likely to happen. Likewise, they are required to play the game both when they have the positive breakdown utility and when their opponent has it. Both events are equally likely to happen. Let  $\pi(A, M) \in [0, 1]$  denote the expected payoff to a metaplayer who uses automaton  $A$  against another metaplayer who uses automaton  $M$ . Let  $\pi_x(A, M)$  denote the expected payoff to automaton  $A$  when it has the positive breakdown utility  $x$ , and let  $\pi_\emptyset(A, M)$  denote the expected payoff to  $A$  when its opponent ( $M$ ) has the positive breakdown utility  $x$ . Let

$$\pi(A, M) = \frac{1}{2}\pi_x(A, M) + \frac{1}{2}\pi_\emptyset(A, M). \quad (1)$$

Let  $\pi^i(A, M)$  denote the expected payoff to  $A$  when it plays the bargaining game as player  $i = I, II$ . Let

$$\pi(A, M) = \frac{1}{2}\pi^I(A, M) + \frac{1}{2}\pi^{II}(A, M). \quad (2)$$

We assume that Nature does two things. First, it randomly assigns the positive breakdown utility  $x$  to one of the automata and then transmits information about the outcome to both  $A$  and  $M$ , i.e. telling them who was assigned  $x$ . Nature then randomly picks one of the automata to be player  $I$  and sends a message to the automaton that is chosen. Receiving this second message is interpreted by each automaton as being chosen to be player  $I$ . Assume that  $A$  is chosen to be player  $I$ . Then the other automaton ( $M$ ) learns that play has begun by receiving a message in the form of a demand from the opponent, which is interpreted as being chosen to be player  $II$ .

### 3.2 The Automaton

An automaton  $A$  can have many states of which three are special.  $A$  can use a *pre-play* state, which is occupied before the game begins. The pre-play state makes it possible for  $A$  to condition its play on whether it was assigned the positive breakdown utility  $x$  or not by Nature. In order to start the play, each automaton has an *initial* state which is occupied before the actual bargaining begins.  $A$  has two initial states if it has a pre-play state, otherwise it has only one initial state. Each automaton must have an *acceptance* state, denoted  $Y$ , which if reached ends the game with an agreement. Features common to all other states are that they produce a demand when they are first reached, and that they determine a shift to another state when they are occupied and a demand from the opponent is received.

In order to allow each automaton to condition its play on whether it has the positive breakdown utility or not, two outputs can be attached to the pre-play state. The output is either  $x^*$  or  $\emptyset$ , which are interpreted by the automaton as having the positive breakdown utility  $x$  or not, respectively. Rather than causing a message in the form of a demand to be transmitted to the opponent,  $x^*$  and  $\emptyset$  both cause an immediate shift to one of the automaton's initial states, at which point the automaton awaits new stimuli. Using the pre-play state and its additional output ( $x^*$  or  $\emptyset$ ) is, however, optional. Further, an additional output, denoted  $I^*$ , can be attached to the initial state(s) in order to allow the automaton to condition its play on whether it is chosen to be player  $I$  or player  $II$ . This output allows the automaton to shift to another of its states before a demand is produced and transmitted to the opponent.

Formally, the automaton consists of the following objects:

- A set of *states*  $S$ . The set  $S$  can contain a *pre-play* state  $s^*$  but this is optional. The set must contain an acceptance state  $Y$ . The automata begin either with a pre-play state or an initial state  $s \in S$ . The game ends if state  $Y$  is reached.
- An *output function*  $\vartheta : S \rightarrow [0, 1] \cup \{x^*, \emptyset, I^*\}$ . An output  $\theta$  in  $[0, 1]$  is interpreted as a demand. To ensure a well-defined game, only the pre-play state  $s^*$  can have  $x^*$  or  $\emptyset$  as its output and only an initial state can have  $I^*$  as its output.
- A *transition function*  $f$ , where

$$f : \{S \times [0, 1]\} \cup \{(s', I^*)\} \cup \{(s^*, x^*), (s^*, \emptyset)\} \rightarrow S \cup \{Y\}.$$

$f(s, m)$  is the state to which an automaton shifts when it is in state  $s$  and receives message  $m$ .  $f(s', I^*)$  identifies the state to which the automaton shifts when it is chosen to be player  $I$ , where  $s'$  is an initial state.  $f(s^*, x^*)$  and  $f(s^*, \emptyset)$  identify the states to which the automaton shifts when it is assigned the positive breakdown utility  $x$  and when it is not assigned  $x$ , respectively. We let  $f(s', I^*) \neq \{s', Y\}$  in order to ensure well-defined outcomes. For the same reason, we let  $f(s, \theta) \neq s'$  for all  $s \in S$  if  $\vartheta(s') = I^*$ ,  $f(s, \theta) \neq s^*$  for all  $s \in S$  and  $f(s^*, x^*), f(s^*, \emptyset) \neq Y$ .

### 3.3 Bargaining Protocol

The play of an automaton  $A$  is controlled by the following protocol. If  $A$  receives a message when in state  $s$ , it checks its "inray" to find out whether this messages is a demand from its opponent or not, where *inray* stand for the imaginary place where all messages are temporarily stored in the automaton.

*Case 1.* If  $A$  finds no demand in its inray, then state  $s$  is either a pre-play state or an initial state and  $A$  examines the output attached to this state  $\vartheta(s)$ .

- If the output is a demand  $\theta \in [0, 1]$ , then state  $s$  is the initial state and  $A$  transmits its demand to the opponent.  $A$  then awaits a response. If this is not the first demand of the game, then with probability  $1 - \delta$  Nature intervenes and ends the game, with the players then receiving

the payoff-pair given by the Breakdown point. With probability  $\delta$ , the demand reaches the opponent and the game proceeds.<sup>1</sup>

- If the output is  $x^*$  or  $\emptyset$ , then state  $s$  is the pre-play state and  $A$  immediately shifts to state  $f(s, x^*)$  or state  $f(s, \emptyset)$ , where it awaits new stimuli.
- If the output is  $I^*$ , then state  $s$  is an initial state and  $A$  immediately shifts to the state  $f(s, I^*)$ . In state  $f(s, I^*)$  it emits the output attached to this latter state  $\vartheta(f(s, I^*))$ , which is then transmitted to the opponent.  $A$  then awaits a response.

*Case 2.* If  $A$  finds a demand  $\tilde{\theta}$  in its intray, then state  $s$  is not a pre-play state and  $A$  immediately shifts to state  $f(s, \tilde{\theta})$ , clearing its intray in the process.

- If  $f(s, \tilde{\theta}) = Y$ , the game ends with the opponent receiving the share  $\tilde{\theta}$  of the surplus and  $A$  getting what is left.
- If  $f(s, \tilde{\theta}) \neq Y$ , then a counterdemand is produced  $\vartheta(f(s, \tilde{\theta}))$  and transmitted to the opponent.  $A$  then awaits a response from its opponent, who with probability  $\delta$  receives  $A$ 's counterdemand.

We define a period as each time a demand is made.

**Example 3.1** Let  $A$  be up against automaton  $M$ . Let  $A$  use the pre-play state as well as the additional output  $I^*$  in both its initial states. Let  $M$  only use the additional output  $I^*$  in its unique initial state  $s$ . The first message from Nature makes  $A$  (but not  $M$ ) produce an output (assume it was  $x^*$ ) which causes it to shift to one of its initial states (e.g.  $s''$ ). Then  $A$  awaits new stimuli. Assume that Nature then picks  $M$  as player  $I$  and sends him a message that play is to begin. This message causes  $M$  to produce output  $I^*$ , which makes it shift to state  $f(s, I^*)$  and emit the output attached to this latter state  $\vartheta(f(s, I^*)) = \theta$ . This demand is transmitted to  $A$ , then  $M$  awaits a response. Now,  $A$  receives a message and finds demand  $\theta$  in its intray.  $A$  immediately shifts to state  $f(s', \theta)$ , clearing its intray in the process. Only if this state is not  $Y$ ,  $A$  transmits the output attached to the state,  $\vartheta(f(s', \theta)) = \theta''$ , to  $M$  and awaits a response. With probability  $\delta$ , the demand  $\theta''$  reaches  $M$  and the game proceeds.

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<sup>1</sup>Like Binmore *et al.* (1998), we assume that the initial demand always reaches the opponent in order to follow the alternating-offers model.

To simplify matters we often use a figure to describe an automaton. Before we continue it is necessary to explain how these figures should be interpreted. From Example 3.1 it is clear that an automaton might not have a pre-play state, and its initial state may (see Figure 1) or may not have the additional output  $I^*$  (see Figure 2).<sup>2</sup>

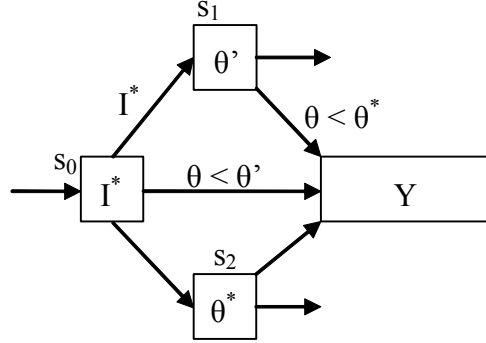


Figure 1: No pre-play state but output  $I^*$ .

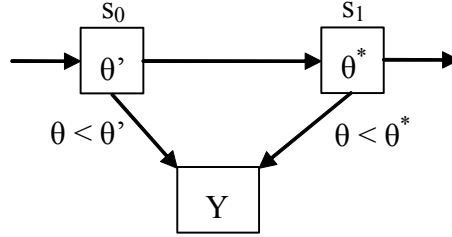


Figure 2: No pre-play state and output  $I^*$ .

Thus an initial state  $s_0 \in S$  comes in two versions, which are described in Figures 1 and 2. These and all other figures should be interpreted as follows. The output attached to a state is always written inside the box representing that state. The name of the state is written as close to the box as possible, normally above or below the box. The only exception is state  $Y$ , with its name written inside the box. For example, output  $I^*$  is attached to state  $s_0$  in Figure 1. The arrows that point from empty spaces into the pre-play state and/or the initial state(s) are messages from Nature or the first demand in the game when the opponent is player  $I$ . All other arrows show the transitions between states. Thus, an arrow pointing from state  $s$  to state  $s'$  symbolizes a transition rule. The conditions for the transition are placed as

<sup>2</sup>Even if an automaton has a pre-play state, its initial states may (see Figure 3.10) or may not have the additional output  $I^*$  (see Figure 3.5).

close to the arrows as possible, normally above or to the left of the arrow. To avoid messy figures some arrows lack the conditions for the transition, but these are always implied by the other conditions. For example, the rule for transition from state  $s_0$  to state  $s_1$  in Figure 2 is implied by the transition condition from state  $s_0$  to state  $Y$ . The demand is not accepted if  $\theta \geq \theta'$ ; thus this is the condition for shifting to state  $s_1$ . Notice that arrows can point back to their point-of-origin (see Figure 3). Output  $\emptyset$  is denoted by 0 in the figures. Moreover, the box representing the pre-play state will only have output  $x^*$  inside it. This should be interpreted as; if output  $x^*$  is not produced, then the output is  $\emptyset$ .<sup>3</sup>

The automaton in Figure 1 is interpreted as follows. Its initial state  $s_0$  has output  $I^*$  attached to it. Thus, if it is chosen to be player  $I$ , it shifts to state  $s_1$  and transmits demand  $\theta'$  to its opponent. Since it has no pre-play state it will do so whenever it is chosen to be player  $I$  and regardless on whether it was assigned the positive breakdown utility or not. Similarly, if it is chosen to be player  $II$  and thus finds a demand  $\theta$  in its intray, it shifts to  $Y$  if  $\theta < \theta'$ , otherwise it shifts to  $s_2$  where it produces and transmits demand  $\theta^*$  to its opponent. Notice that only the beginning of the automaton is displayed in Figure 1. This is the case in most figures since we are only concerned with this part of the automaton. The empty boxes representing unspecified states are arbitrary.

**Remark 3.1** It might be possible to remove the pre-play state and let Nature simultaneously assign the positive breakdown utility  $x$  and let one of the automata know that it is chosen to be Player  $I$ . The automaton then has two initial states, one if it is assigned the positive breakdown utility  $x$  and one if it is not assigned  $x$ . Nature picks one automaton and sends stimuli to one of the latter's states. This automaton is chosen to be Player  $I$  and, given the initial state Nature picked, has the positive breakdown utility or not. The important thing for our result is that when an automaton conditions its play on whether it is assigned the positive breakdown utility or not, it is costly. We choose to use the pre-play state because this makes things considerably simpler when proving our result.

### 3.4 Complexity and Preferences

The complexity measure we use is simply the number of states in each automaton. This counting-states criterion is used by Abreu and Rubinstein

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<sup>3</sup>To paraphrase the Rock n' Roll saying "a cancelled gig is also a gig"; no output is also an output.

(1988), Banks and Sundaram (1990), Binmore and Samuelson (1992) and Chatterjee and Sabourian (1999).<sup>4</sup> There are other plausible measures that can be used in this paper, e.g. the *Collapsing state condition* developed by Binmore *et al.* (1998). This condition says that automaton  $M$  is simpler than automaton  $A$  if there is one or more collections of states in  $A$ , one collection containing at least two states, each of which is collapsed into a single state in  $M$ . Whenever we collapse states in one automaton into one state in a new automaton, the newer automaton will always have fewer states than the former.

In the social context we have in mind, strategies (or automata) can be transmitted from one player to another, either by learning or imitation. The more states an automaton has, the more error-prone are these transmissions. Many of the errors will lead to dysfunctional strategies that cannot play the game or play the game badly, rendering lower payoffs. A player using a dysfunctional strategy is more willing to learn or imitate another strategy. Players using successful strategies have no incentives to learn or imitate another strategy. Some of the errors might lead to better strategies that either render a higher payoff or is less complex than their "parent" (or both). In the latter case transmissions are less error-prone compared to their parents. As a rule, high payoff strategies survive at the expense of low payoff strategies and simpler strategies will be more likely to survive than complex ones whenever payoffs are equal. Thus, from an evolutionary perspective, it is costly to use an automaton which has states for punishing deviations, monitoring an opponent's play and condition its play to circumstances that can occur in the game. Formally,

**Definition 1 (Complexity)** *Let  $A \succ M$  mean that  $A$  is more complex than  $M$  (or  $M$  is simpler than  $A$ ). Automaton  $A$  is more complex than automaton  $M$  if and only if  $A$  has more states than  $M$ .*

The metaplayers are assumed to have lexicographic preferences. Consider two automata,  $A$  and  $M$ . A metaplayer prefers  $A$  to  $M$  if  $A$  yields a higher expected payoff than  $M$  against the opponent's automaton. If and only if  $A$  and  $M$  yield the same expected payoff against the opponent's automaton, a metaplayer will prefer  $A$  to  $M$  provided that  $A$  is less complex than  $M$ .

### 3.5 Evolutionary Stability

The metaplayers in the automaton-selection process are a metaphor for an evolutionary process, which means that we seek an automaton that satisfies

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<sup>4</sup>In Banks and Sundaram (1990) the number of transitions as well as the number of states are costly.

an evolutionary stability criterion. For this purpose, we use the concept of *modified evolutionarily stable strategy* (MESS) introduced by Binmore and Samuelson (1992).<sup>5</sup>

**Definition 2 (Modified stability)** *An automaton  $A$  is a MESS if at least one of the following conditions holds for each automaton  $M \neq A$  :*

- (i)  $\pi(A, A) > \pi(M, A)$  ;
- (ii)  $\pi(A, A) = \pi(M, A)$  and  $\pi(A, M) > \pi(M, M)$  ;
- (iii)  $\pi(A, A) = \pi(M, A)$  and  $\pi(A, M) = \pi(M, M)$  and  $M \succ A$ .

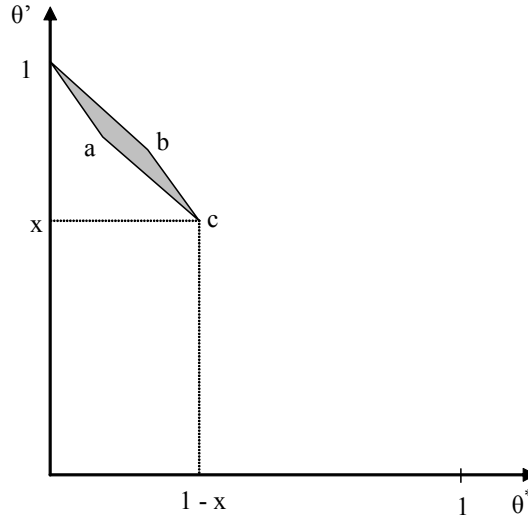
## 4 Results

Our main results are presented in two propositions.

**Proposition 2** *If  $x \in (\frac{1}{2}, 1)$  and the automaton  $A$  is a MESS, then there is immediate agreement when  $A$  plays itself and*

$$(\pi_x(A, A), \pi_\emptyset(A, A)) = \left\{ \begin{array}{ll} (\theta', 1 - \theta') & \text{if Player I has } x \\ (1 - \theta^*, \theta^*) & \text{if Player II has } x \end{array} \right\},$$

where  $(\theta', \theta^*) \in [0, 1]^2$  is a point in the rhomb  $(\theta' \neq x, 1 \text{ and } \theta^* \neq 1 - x, x)$



defined by

$$\begin{aligned} a: & \left( \frac{\delta+x}{1+\delta}, \delta \frac{1-x}{1+\delta} \right) \\ b: & \left( \frac{1+\delta x}{1+\delta}, \frac{1-x}{1+\delta} \right) \\ c: & (x, 1-x). \end{aligned}$$

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<sup>5</sup>This is a refinement of Maynard Smith's (1982) concept of a neutrally stable strategy.

The proof of Proposition 2 is given in series of lemmas below. They establish that the MESS automaton  $A$  in Proposition 2 must be constructed in the following way:

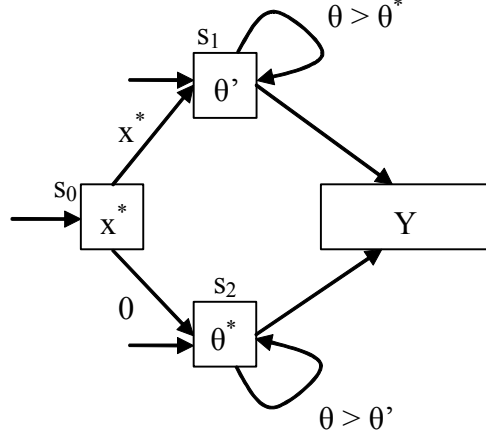


Figure 3: The MESS automaton  $A$ .

**Proposition 3** For  $x \in (0, \frac{1}{2}]$ , the automaton  $B$  described in Figure 4 is a MESS, where

$$\frac{1}{1+\delta} - \frac{1-\delta}{1+\delta}x \leq \theta^* \leq \frac{\delta}{1+\delta} + \frac{1-\delta}{1+\delta}x. \quad (3)$$

Agreement is reached immediately when  $B$  plays itself, and  $\pi^I(B, B) = \theta^*$  and  $\pi^{II}(B, B) = 1 - \theta^*$ .

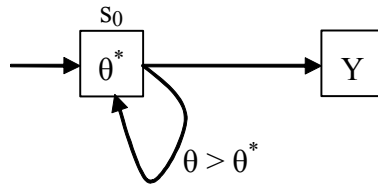


Figure 4: The MESS automaton  $B$ .

The proof of Proposition 3 is given below.

Like the result in Binmore *et al.* (1998) there is nothing in Proposition 2 and Proposition 3 that requires the pie to be infinitely divisible. There is, however, a major difference between the two propositions. Proposition 2 dictates how a MESS automaton must be constructed and which agreements it must reach given that  $x \in (\frac{1}{2}, 1)$ . In contrast, Proposition 3 only displays

a MESS automaton given that  $x \in (0, \frac{1}{2}]$ . It is possible for a MESS to be constructed in another fashion when  $x \in (0, \frac{1}{2}]$ , e.g. by using the pre-play state. But if the MESS automaton  $A$  in Proposition 2 is to be a MESS for  $x \in (0, \frac{1}{2}]$ , then the points  $(\theta', \theta^*) = (\theta^*, \theta^*)$  must be removed from the extended rhomb, where

$$\frac{1}{1+\delta} - \frac{1-\delta}{1+\delta}x \leq \theta^* \leq \frac{\delta}{1+\delta} + \frac{1-\delta}{1+\delta}x. \quad (4)$$

Otherwise, the mutant automaton  $B$  plays identically to  $A$  but is much simpler. Besides  $B$ , no automaton can be constructed to yield equal payoff and be less complex than  $A$ .

## 4.1 Proof of Proposition 2

The proof of Proposition 2 is outlined as follows. We start by substantiating a trivial conclusion. If automaton  $A$  is a MESS then it must reach an agreement when it plays itself (Lemma 1). Next, we show that if  $A$  is a MESS then the agreement it reaches must be contingent on the breakdown point (Lemma 2). This implies that  $A$  uses the additional output  $x^*$  (Lemma 3). In other words, if  $A$  is a MESS it conditions its behavior on whether it has the positive breakdown utility or not. Lemma 4 establishes that if  $A$  is a MESS that does not condition its play on whether it is chosen to be player  $I$  or player  $II$ , then it reaches an agreement in period 1 when it plays itself.

Lemma 5 establishes that  $\vartheta(s) \neq I^*$  if  $A$  is a MESS. First it shows that if  $A$  conditions its play on whether it is chosen to be player  $I$  or player  $II$ , then agreement must be reached in period 1 when  $A$  plays itself. But if agreement is reached in period 1 when  $A$  plays itself, then  $\vartheta(s) \neq I^*$  if  $A$  is a MESS. Finally, in Lemma 6, we show that  $A$  is constructed as in Figure 3 with the demands specified in Proposition 2 if  $A$  is to be a MESS.

**Lemma 1** *If automaton  $A$  is a MESS, then  $A$  always reaches an agreement when it plays itself.*

**Proof.** Suppose agreement is not reached when  $A$  plays itself. Three cases arise:

1.  $A$  always fails.
2.  $A$  fails when the automaton without the positive breakdown utility is chosen to be player  $I$ .
3.  $A$  fails when the automaton with the positive breakdown utility is chosen to be player  $I$ .

For all three cases we can construct a mutant that is a better reply to  $A$  than  $A$  itself, or a best reply to  $A$  while  $A$  is not a best reply to it.

*Case 1.* We have  $\pi(A, A) = x/2 < \frac{1}{2}$ . Let  $M$  use the pre-play state and, in addition, condition its play on the player-role. If  $M$  is assigned the positive breakdown utility and becomes player  $I$ , it demands some  $\theta^* > x$  in the first period and then it imitates  $A$ . If  $M$  is assigned the positive breakdown utility and becomes player  $II$ , it accepts  $\theta \leq 1 - \theta^*$  in the first period and then it imitates  $A$ . If  $M$  is not assigned  $x$  and becomes player  $I$ , it demands  $1 - \theta^*$  in the first period and then it imitates  $A$ . If  $M$  is not assigned  $x$  and becomes player  $II$ , it accepts  $\theta \leq \theta^*$  in the first period and then it imitates  $A$ . Obviously,  $\pi(M, M) = \frac{1}{2}$ . When  $M$  plays against  $A$ , we have  $(\theta' \geq \theta^*, \tilde{\theta} \leq \theta^*)$ :

$$\begin{array}{cc}
\pi_x(M, A) & \pi_\emptyset(A, M) \\
(a) & \frac{1}{2}\theta^* + \frac{1}{2}(1 - \theta') & \frac{1}{2}(1 - \theta^*) + \frac{1}{2}\theta' \\
(b) & \frac{1}{2}\theta^* + \frac{1}{2}x & \frac{1}{2}(1 - \theta^*) \\
(c) & \frac{1}{2}x + \frac{1}{2}(1 - \theta') & \frac{1}{2}\theta' \\
(d) & x & 0
\end{array} \tag{5}$$

and

$$\begin{array}{cc}
\pi_\emptyset(M, A) & \pi_x(A, M) \\
(e) & \frac{1}{2}(1 - \theta^*) + \frac{1}{2}(1 - \tilde{\theta}) & \frac{1}{2}\theta^* + \frac{1}{2}\tilde{\theta} \\
(f) & \frac{1}{2}(1 - \theta^*) & \frac{1}{2}\theta^* + \frac{1}{2}x \\
(g) & \frac{1}{2}(1 - \tilde{\theta}) & \frac{1}{2}\tilde{\theta} + \frac{1}{2}x \\
(h) & 0 & x
\end{array} \tag{6}$$

Scenario (a) occurs if  $A$  accepts  $M$ 's first demand ( $\theta^*$ ) as player  $I$  and if  $M$  accepts  $A$ 's first demand ( $\theta' \leq 1 - \theta^*$ ) as player  $I$ , when  $M$  has  $x$ . Scenario (b) occurs if  $A$  accepts  $M$ 's first demand ( $\theta^*$ ) as player  $I$  and if  $M$  rejects  $A$ 's first demand ( $\theta' > 1 - \theta^*$ ) as player  $I$ , when  $M$  has  $x$ . Scenario (c) occurs if  $A$  rejects  $M$ 's first demand ( $\theta^*$ ) as player  $I$  and if  $M$  accepts  $A$ 's first demand ( $\theta' \leq 1 - \theta^*$ ) as player  $I$ , when  $M$  has  $x$ . Scenario (d) occurs if  $A$  rejects  $M$ 's first demand ( $\theta^*$ ) as player  $I$  and if  $M$  rejects  $A$ 's first demand ( $\theta' > 1 - \theta^*$ ) as player  $I$ , when  $M$  has  $x$ . Scenario (e) occurs if  $A$  accepts  $M$ 's first demand ( $1 - \theta^*$ ) as player  $I$  and if  $M$  accepts  $A$ 's first demand ( $\tilde{\theta} \leq \theta^*$ ) as player  $I$ , when  $A$  has  $x$ . Scenario (f) occurs if  $A$  accepts  $M$ 's first demand ( $1 - \theta^*$ ) as player  $I$  and if  $M$  rejects  $A$ 's first demand ( $\tilde{\theta} > \theta^*$ ) as player  $I$ , when  $A$  has  $x$ . Scenario (g) occurs if  $A$  rejects  $M$ 's first demand ( $1 - \theta^*$ ) as player  $I$  and if  $M$  accepts  $A$ 's first demand ( $\tilde{\theta} \leq \theta^*$ ) as player  $I$ , when  $A$  has

$x$ . Scenario (h) occurs if  $A$  rejects  $M$ 's first demand  $(1 - \theta^*)$  as player  $I$  and if  $M$  rejects  $A$ 's first demand  $(\tilde{\theta} > \theta^*)$  as player  $I$ , when  $A$  has  $x$ .

Now,  $\pi_x(M, A) = x$  and  $\pi_\emptyset(M, A) = 0$  are required to satisfy  $\pi(M, A) = \pi(A, A)$ , but then  $\pi_\emptyset(A, M) = 0$  and  $\pi_x(A, M) = x$ , i.e.  $\pi(A, M) = x/2 < \pi(M, M)$ . Hence, at least one of conditions (i)-(ii) in Definition 2 is violated.

*Case 2.* Let  $M'$  use the pre-play state and, in addition, condition its play on the player-role. Let  $M'$  imitate  $A$  if it is assigned the positive breakdown utility  $x$  and becomes player  $I$  and if it is not assigned  $x$  and it becomes player  $II$ . If  $M'$  is assigned the positive breakdown utility  $x$  and becomes player  $II$ , it accepts  $\theta \leq 1 - \theta^*$  in the first period and then it imitates  $A$ . If  $M'$  is not assigned  $x$  and becomes player  $I$ , it demands  $1 - \theta^*$  in the first period and then it imitates  $A$ , where  $\theta^* \in (x, 1)$ . Let  $\pi_x(A, A) = \pi^x$  and  $\pi_\emptyset(A, A) = \pi^\emptyset$ . We have

$$\pi_x(M', M') = \pi^x + \frac{1}{2}\theta^* - \frac{1}{2}x \quad (7)$$

and

$$\pi_\emptyset(M', M') = \pi^\emptyset + \frac{1}{2}(1 - \theta^*). \quad (8)$$

When  $M$  plays against  $A$ , we have  $(\tilde{\theta} \geq \theta^*)$ :

$$\begin{array}{cc} \pi_x(M, A) & \pi_\emptyset(A, M) \\ (a) & \pi^x + \frac{1}{2}\tilde{\theta} - \frac{1}{2}x \quad \pi^\emptyset + \frac{1}{2}(1 - \tilde{\theta}) \\ (b) & \pi^x \quad \pi^\emptyset \end{array} \quad (9)$$

and

$$\begin{array}{cc} \pi_\emptyset(M, A) & \pi_x(A, M) \\ (c) & \pi^\emptyset + \frac{1}{2}(1 - \theta^*) \quad \pi^x + \frac{1}{2}\theta^* - \frac{1}{2}x \\ (d) & \pi^\emptyset \quad \pi^x \end{array} \quad (10)$$

As in the previous case, (a)-(d) are the possible scenarios. Now,  $\pi_x(M, A) = \pi^x$  and  $\pi_\emptyset(M, A) = \pi^\emptyset$  are required to satisfy  $\pi(M, A) = \pi(A, A)$ , but then  $\pi_\emptyset(A, M) = \pi^\emptyset$  and  $\pi_x(A, M) = \pi^x$ , i.e.  $\pi(A, M) < \pi(M, M)$ . Hence, at least one of the conditions (i)-(ii) in Definition 2 is violated.

*Case 3.* An argument analogous to that in the previous case applies. ■

**Lemma 2** *If automaton  $A$  is a MESS, then the agreement(s) it reaches against itself must assign a share larger than or equal to  $x$  to the player with the positive breakdown utility  $x$ .*

**Proof.** Suppose that  $A$  reaches agreement  $(z, 1 - z)$  in period  $t$  against itself if the automaton with  $x$  is chosen to be player  $I$ . Suppose that  $A$  reaches agreement  $(w, 1 - w)$  in period  $\tau$  against itself if the automaton with  $x$  is chosen to be player  $II$ . Further, let  $z < x$  and  $w > 1 - x$ , so that the share assigned to the player with the positive breakdown utility is less than  $x$ . Then we can construct a mutant  $M$  that consists of two altered versions of  $A$  and that is a better reply to  $A$  than  $A$  itself.

*The mutant automaton has the positive breakdown utility  $x$ .*

Then we can construct a mutant  $M'$  as follows.

*Case 1.* If  $M'$  is chosen to be player  $I$ , then it plays as automaton  $A$  does when it is player  $I$  until period  $t$ , at which point it starts to demand  $\theta = x$  and only accepts demands  $\theta \leq 1 - x$  from the opponent. Now, if  $A$  makes the demand in period  $t$ , then  $M'$  rejects because  $\theta > 1 - x$ . In the continued bargaining, either  $A$  accepts  $M'$ 's demand of  $x$  (or, it demands  $\theta \leq 1 - x$ ) in some period  $t^*$  or the bargaining breaks down. In any event,  $M'$ 's payoff is at least  $x$  which is larger than the share  $A$  achieves against itself when it has the positive breakdown utility. If  $M'$  makes the demand in period  $t$ , then either  $A$  eventually accepts  $M'$ 's demand of  $x$  (or, it demands  $\theta \leq 1 - x$ ) in some period  $t^*$  or the bargaining breaks down. In any event,  $M'$ 's payoff is at least  $x$ , which is larger than the share  $A$  achieves against itself when it has the positive breakdown utility.

*Case 2.* If  $M'$  is chosen to be player  $II$ , then it plays as automaton  $A$  does when it is player  $II$  until period  $\tau$ , at which point it starts to demand  $\theta = x$  and only accepts demands if  $\theta \leq 1 - x$ . Now, if  $A$  makes the demand in period  $\tau$ , then  $M'$  rejects because  $\theta > 1 - x$ . In the continued bargaining, either  $A$  accepts  $M'$ 's demand of  $x$  (or, it demands  $\theta \leq 1 - x$ ) in some period  $t^*$  or the bargaining breaks down. In any event,  $M'$ 's payoff is at least  $x$  which is larger than the share  $A$  achieves against itself when it has the positive breakdown utility. If  $M'$  makes the demand in period  $\tau$ , then either  $A$  eventually accepts  $M'$ 's demand of  $x$  (or, it demands  $\theta \leq 1 - x$ ) in some period  $t^*$  or the bargaining breaks down. In any event,  $M'$ 's payoff is at least  $x$ , which is larger than the share  $A$  achieves against itself when it has the positive breakdown utility.

*The mutant automaton has the breakdown utility 0.*

Then we can construct a mutant  $M''$  as follows.

*Case 3.* If  $M''$  is chosen to be player  $II$ , then it plays as automaton  $A$  does when it is player  $II$  until period  $t$ , at which point it only accepts demands if  $\theta \leq x$ , and until period  $t + 1$  when it starts to always demand  $\theta = 1 - x$ . Now, if  $A$  makes the demand in period  $t$ , then  $M''$  accepts. If  $M''$  makes the demand in period  $t$ , then  $A$  accepts. Hence, it reaches the same agreement as would two  $A$  automata. What happens when it plays against  $M'$ ? If  $M'$  makes the demand in period  $t$ , then  $M''$  accepts. If  $M''$  makes the demand in period  $t$ , then  $M'$  rejects, but  $M''$  accepts  $M'$ 's demand in the next period.

*Case 4.* If  $M''$  is chosen to be player  $I$ , then it plays as automaton  $A$  does when it is player  $I$  until period  $\tau$ , at which point it only accepts demands if  $\theta \leq x$ , and until period  $\tau + 1$  when it starts to always demand  $\theta = 1 - x$ . Now, if  $A$  makes the demand in period  $\tau$ , then  $M''$  accepts. If  $M''$  makes the demand in period  $\tau$ , then  $A$  accepts. Hence, it reaches the same agreement as would two  $A$  automata. What happens when it plays against  $M'$ ? If  $M'$  makes the demand in period  $t$ , then  $M''$  accepts. If  $M''$  makes the demand in period  $t$ , then  $M'$  rejects, but  $M''$  accepts  $M'$ 's demand in the next period.

*The mutant automaton  $M$ .*

Let  $M$  be like automaton  $M'$  when it has the positive breakdown utility  $x$  and like automaton  $M''$  when its opponent has  $x$ . That is,  $M$  uses the pre-play state and shifts to the initial state of  $M'$  if the output is  $x^*$ , otherwise it shifts to the initial state of  $M''$ . We have  $\pi_\emptyset(M, A) = \pi_\emptyset(A, A)$  and  $\pi_x(M, A) > \pi_x(A, A)$ , i.e.  $\pi(M, A) > \pi(A, A)$  which contradicts condition (i) in Definition 2. ■

**Lemma 3** *If  $x > \frac{1}{2}$ , then  $A$  uses the pre-play state and its additional output  $x^*$  (i.e.  $\vartheta(s_0, x) = x^*$ ) if  $A$  is a MESS.*

**Proof.** Suppose that  $x > \frac{1}{2}$  and automaton  $A$  is a MESS with no pre-play state. From Lemma 1 we know that  $A$  reaches an agreement against itself. Suppose it is in period  $t$ , where player  $I$  demands  $\theta^*$  which player  $II$  accepts. From Lemma 2 we have  $\theta^* \geq x$  if player  $I$  is assigned  $x$  and that  $\theta^* \leq 1 - x$  if player  $II$  is assigned  $x$ . Since both events are equally likely to happen and  $\theta^*$  cannot satisfy both conditions for  $x > \frac{1}{2}$ , there is a contradiction. An analogous argument applies when it is player  $II$  who demands  $\theta^*$  which player  $I$  accepts in period  $t$ . ■

**Lemma 4** *If  $x > \frac{1}{2}$  and automaton  $A$  is a MESS which does not condition its play on the player-role (i.e.  $\vartheta(s) \neq I^*$ ), then it reaches an agreement in the first period when it plays itself.*

**Proof.** Suppose that  $A$  does not reach an agreement in the first period when it plays itself.

*Case 1.*  $A$  reaches agreement in period  $t > 1$  when the automaton with the positive breakdown utility  $x$  is chosen to be player  $I$  and in period  $\tau > 1$  when the automaton with the positive breakdown utility  $x$  is chosen to be player  $II$ .

Let  $\tau - t = k$ . Finally, let  $\theta_x$  be the share assigned to the automaton with the positive breakdown utility  $x$  in the agreement reached in period  $t$  and let  $\tilde{\theta}_x$  be the share assigned to the automaton with the positive breakdown utility  $x$  in the agreement reached in period  $\tau$ .

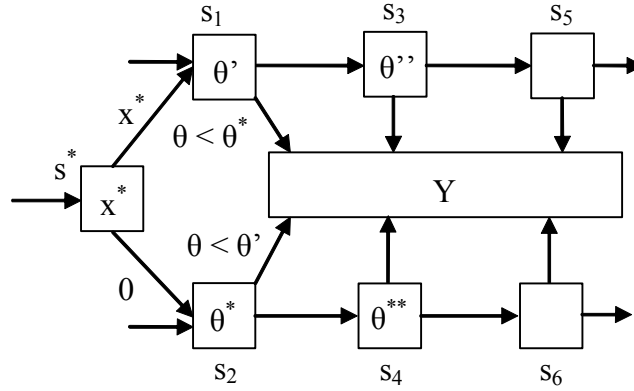


Figure 5:  $A$  with delayed agreements.

The beginning of  $A$  is described in Figure 5. Notice that no state, e.g. state  $s_p$ , can be used both when  $A$  is assigned the positive breakdown utility  $x$  and when it is not assigned  $x$ . If this is the case, then a mutant without the positive breakdown utility  $x$  can play as  $A$  until  $A$  occupies state  $s_p$ , at which point the mutant plays as would  $A$  with the positive breakdown utility  $x$ . By this the mutant gets  $\theta_x$  or  $\tilde{\theta}_x$  instead of  $1 - \theta_x$  or  $1 - \tilde{\theta}_x$ , and becomes a better reply to  $A$  than  $A$  itself.

Let  $M'$  be a mutant that, when it has (the positive breakdown utility)  $x$  and is chosen to be player  $I$ , makes the demand that is  $A$ 's first demand as player  $II$  with  $x$ , and thereafter continues to play as would  $A$  as player  $II$  with  $x$ . When  $M'$  plays against  $A$  as player  $I$  with  $x$ ,  $M'$ 's first demand makes  $A$  play as it would as player  $I$  and responding to player  $II$ 's counter-demand. Hence, the roles are reversed and agreement is delayed by  $k - 1$  periods, i.e.  $M'$  gets  $\tilde{\theta}_x$  in period  $\tau - 1$ .

Let  $M''$  be a mutant that, when it has  $x$  and is chosen to be player  $II$ ,

makes the demand that is  $A$ 's first demand as player  $I$  with  $x$ , and thereafter continues to play as would  $A$  as player  $I$  with  $x$ . When  $M''$  plays against  $A$  as player  $II$  with  $x$ ,  $M''$ 's first demand makes  $A$  play as it would as player  $II$  and responding to player  $I$ 's counter-demand. Hence, the roles are reversed and agreement is accelerated by  $k - 1$  periods, i.e.  $M''$  gets  $\theta_x$  in period  $t + 1$ .

Let  $M'''$  be a mutant that, when it is chosen to be player  $I$  and its opponent has  $x$ , makes the demand that is  $A$ 's first demand as player  $II$  without  $x$ , and thereafter continues to play as would  $A$  as player  $II$  without  $x$ . When  $M'''$  plays against  $A$  as player  $I$  without  $x$ , the roles are reversed and agreement is accelerated by  $k + 1$  periods, i.e.  $M'''$  gets  $1 - \theta_x$  in period  $t - 1$ .

Let  $M''''$  be a mutant that, when it is chosen to be player  $II$  and its opponent has  $x$ , makes the demand that is  $A$ 's first demand as player  $I$  without  $x$ , and thereafter continues to play as would  $A$  as player  $I$  without  $x$ . When  $M''''$  plays against  $A$  as player  $II$  without  $x$ , the roles are reversed and agreement is delayed by  $k + 1$  periods, i.e.  $M''''$  gets  $1 - \tilde{\theta}_x$  in period  $\tau + 1$ .

If  $A$  is to be a MESS, then none of these must be a superior reply to  $A$ . This requires:

$$\delta^{t-1}\theta_x + (1 - \delta^{t-1})x \geq \delta^{\tau-2}\tilde{\theta}_x + (1 - \delta^{\tau-2})x, \quad (11)$$

$$\delta^t\theta_x + (1 - \delta^t)x \leq \delta^{\tau-1}\tilde{\theta}_x + (1 - \delta^{\tau-1})x, \quad (12)$$

$$\delta^{\tau-1}(1 - \tilde{\theta}_x) \geq \delta^{t-2}(1 - \theta_x), \quad (13)$$

$$\delta^\tau(1 - \tilde{\theta}_x) \leq \delta^{t-1}(1 - \theta_x). \quad (14)$$

Together, (11) and (12) give

$$\theta_x - x = \delta^{k-1}(\tilde{\theta}_x - x), \quad (15)$$

and (13) and (14) give

$$1 - \theta_x = \delta^{k+1}(1 - \tilde{\theta}_x). \quad (16)$$

We now construct the following mutant  $M$ . Let  $f^M(s^*, x^*) = s_0^x$  and  $f^M(s^*, \emptyset) = s_0^\emptyset$ , i.e.  $M$  uses the pre-play state and switches to initial state  $s_0^x$  or  $s_0^\emptyset$  depending on the output Nature made it produce. Let  $M$  use the additional output  $I^*$  in its initial states and let  $f^M(s_0^x, I^*) = s_3$ ,  $f^M(s_0^x, \theta = \theta^*) = s_1$ ,  $f^M(s_0^x, \theta \neq \theta^*) = s_3$ ,  $f^M(s_0^\emptyset, I^*) = s_4$ ,  $f^M(s_0^\emptyset, \theta = \theta') = s_2$  and  $f^M(s_0^\emptyset, \theta \neq \theta') = s_4$ . The states  $s_1$ ,  $s_3$ ,  $s_2$ ,  $s_4$ , and so on, and the associated output and transition functions are identical to those in  $A$  (see Figure 5). Against itself,  $M$  switches player-roles but accelerates the agreements by one

period. When the automaton with  $x$  is chosen to be player  $I$ , agreement is reached in period  $\tau - 1$  where it is assigned  $\tilde{\theta}_x$ . When the automaton with  $x$  is chosen to be player  $II$ , agreement is reached in period  $t - 1$  where it is assigned  $\theta_x$ . Hence,

$$\begin{aligned}\pi(M, M) = & \frac{1}{4} \left( \delta^{\tau-2} \tilde{\theta}_x + (1 - \delta^{\tau-2}) x \right) + \\ & + \frac{1}{4} \left( \delta^{t-2} \theta_x + (1 - \delta^{t-2}) x \right) + \\ & + \frac{1}{4} \delta^{\tau-2} (1 - \tilde{\theta}_x) + \frac{1}{4} \delta^{t-2} (1 - \theta_x). \end{aligned} \quad (17)$$

This can be compared to

$$\begin{aligned}\pi(A, A) = & \frac{1}{4} \left( \delta^{\tau-1} \tilde{\theta}_x + (1 - \delta^{\tau-1}) x \right) + \\ & + \frac{1}{4} \left( \delta^{t-1} \theta_x + (1 - \delta^{t-1}) x \right) + \\ & + \frac{1}{4} \delta^{\tau-1} (1 - \tilde{\theta}_x) + \frac{1}{4} \delta^{t-1} (1 - \theta_x). \end{aligned} \quad (18)$$

When the mutant  $M$  plays  $A$ , we have

$$\begin{aligned}\pi(M, A) = & \frac{1}{4} \left( \delta^{\tau-2} \tilde{\theta}_x + (1 - \delta^{\tau-2}) x \right) + \\ & + \frac{1}{4} \left( \delta^t \theta_x + (1 - \delta^t) x \right) + \\ & + \frac{1}{4} \delta^\tau (1 - \tilde{\theta}_x) + \frac{1}{4} \delta^{t-2} (1 - \theta_x), \end{aligned} \quad (19)$$

and

$$\begin{aligned}\pi(A, M) = & \frac{1}{4} \left( \delta^\tau \tilde{\theta}_x + (1 - \delta^\tau) x \right) + \\ & + \frac{1}{4} \left( \delta^{t-2} \theta_x + (1 - \delta^{t-2}) x \right) + \\ & + \frac{1}{4} \delta^{\tau-2} (1 - \tilde{\theta}_x) + \frac{1}{4} \delta^t (1 - \theta_x). \end{aligned} \quad (20)$$

Using (15) and (16) in these expressions gives

$$\begin{aligned}\pi(M, A) &= \pi(A, A), \\ \pi(A, M) &< \pi(M, M). \end{aligned} \quad (21)$$

Hence,  $A$  is not a MESS.

An analogous argument can be made for  $t - \tau = k$ .

*Case 2.*  $A$  reaches agreement in period 1 when the automaton with  $x$  is chosen to be player  $I$  and in period  $\tau > 1$  when the automaton with  $x$  is chosen to be player  $II$ .

Hence,  $A$  is constructed as in Figure 5 except for the fact that it accepts demand  $\theta'$  when in state  $s_2$ . Thus,  $\theta'$  is the share assigned to the automaton with  $x$  in the agreement reached in period 1. Let  $\pi_x$  be the share assigned

to the automaton with  $x$  in the agreement reached in period  $\tau$ . Let  $\theta''$  be  $A$ 's first demand as player  $II$  with  $x$ . Let  $\theta^*$  be  $A$ 's first demand as player  $I$  without  $x$  and let  $\theta^{**}$  be  $A$ 's first demand as player  $II$  without  $x$ .

We know that  $f^A(s_1, \theta^*)$  and  $f^A(s_2, \theta'')$  embark on the path towards the agreement in period  $\tau$  and that  $f^A(s_2, \theta') = Y$ . Now, by only changing the first demand to  $\theta''$  instead of  $\theta'$  when chosen to be player  $I$ , a mutant with  $x$  can get  $\pi_x$  in period  $\tau - 1$  instead of  $\theta'$  in period 1. Likewise, by only changing the first demand to  $\theta'$  instead of  $\theta''$  when chosen to be player  $II$ , a mutant with  $x$  can get  $\theta'$  in period 2 instead of  $\pi_x$  in period  $\tau$ . (Besides this the mutants play like  $A$  does). If  $A$  is to be a MESS, neither can be a superior reply to  $A$ . This requires

$$\theta' \geq \delta^{\tau-2}\pi_x + (1 - \delta^{\tau-2})x, \quad (22)$$

$$\delta\theta' + (1 - \delta)x \leq \delta^{\tau-1}\pi_x + (1 - \delta^{\tau-1})x. \quad (23)$$

That is,

$$\theta' = \delta^{\tau-2}\pi_x + (1 - \delta^{\tau-2})x. \quad (24)$$

Consider mutant  $M$ , with  $\vartheta^M(f^M(s^*, x^*)) = \theta'$  and  $\vartheta^M(f^M(s^*, \emptyset)) = 1 - \theta'$ . States  $s_2, s_4, s_6, \dots$  are identical to those in  $A$ . Let  $M$  have transition functions:

- $f^M(f^M(s^*, x^*), \theta^*) = f^M(s^*, x^*),$
- $f^M(f^M(s^*, x^*), 1 - \theta') = Y,$
- $f^M(f^M(s^*, \emptyset), \theta') = Y,$
- $f^M(f^M(s^*, \emptyset), \theta'') = s_2, f^M(f^M(s^*, \emptyset), \theta''') = s_4, \text{ etcetera.}$

Notice that, when  $M$  plays against  $A$  as player  $I$  without  $x$ ,  $M$ 's first demand of  $1 - \theta'$  makes  $A$  to shift to one of its states, i.e.  $s_1, s_3, s_5, \dots, Y$ . (It cannot be  $s_2, s_4, s_6, \dots$ , nor can it be some state that is not used by  $A$  when it plays itself.) The following transition rules ensure that if  $A$  shifts to  $s_3$  ( $s_5$ ) then  $M$  shifts to  $s_4$  ( $s_6$ ) and thereby secures  $1 - \pi_x$  in period  $\tau$  ( $\tau - 2$ ).

The payoffs are as follows:

$$\pi_x(A, A) = \frac{1}{2}\theta' + \frac{1}{2}[\delta^{\tau-1}\pi_x + (1 - \delta^{\tau-1})x] \quad (25)$$

$$\pi_\emptyset(A, A) = \frac{1}{2}(1 - \theta') + \frac{1}{2}\delta^{\tau-1}(1 - \pi_x) \quad (26)$$

$$\pi_x(M, M) = \theta' \quad (27)$$

$$\pi_{\emptyset}(M, M) = 1 - \theta'. \quad (28)$$

When  $M$  plays against  $A$ , the payoffs are:

$$\pi_x(M, A) = \frac{1}{2}\theta' + \frac{1}{2}[\delta\theta' + (1 - \delta)x] \quad (29)$$

$$\pi_{\emptyset}(M, A) \left\{ \begin{array}{ll} = 1 - \theta' & \text{if } f^A(s_1, 1 - \theta') = Y \\ = \frac{1}{2}(1 - \theta') + \frac{1}{2}\delta(1 - \theta') & \text{if } f^A(s_1, 1 - \theta') = s_1 \\ \geq \frac{1}{2}(1 - \theta') + \frac{1}{2}\delta^{\tau-1}(1 - \pi_x) & \text{otherwise} \end{array} \right\} \quad (30)$$

$$\pi_x(A, M) \left\{ \begin{array}{ll} = \theta' & \text{if } f^A(s_1, 1 - \theta') = Y \\ = \frac{1}{2}\theta' + \frac{1}{2}(\delta\theta' + (1 - \delta)x) & \text{if } f^A(s_1, 1 - \theta') = s_1 \\ \geq \frac{1}{2}\theta' + \frac{1}{2}(\delta^{\tau-1}\pi_x + (1 - \delta^{\tau-1})x) & \text{otherwise} \end{array} \right\} \quad (31)$$

$$\pi_{\emptyset}(A, M) = \frac{1}{2}(1 - \theta') + \frac{1}{2}\delta(1 - \theta'). \quad (32)$$

Now we compare the payoffs. First,  $\pi_x(M, A) = \pi_x(A, A)$  because

$$\delta\theta' + (1 - \delta)x = \delta^{\tau-1}\pi_x + (1 - \delta^{\tau-1})x. \quad (33)$$

Simplification yields

$$\theta' - x = \delta^{\tau-2}(\pi_x - x), \quad (34)$$

and this equality holds by condition (24). From (34) we conclude that  $\theta' < \pi_x$  and  $1 - \theta' > 1 - \pi_x$ . Second,  $\pi_{\emptyset}(A, M) < \pi_{\emptyset}(M, M)$ . Third, we have:

(i)  $\pi_{\emptyset}(M, A) > \pi_{\emptyset}(A, A)$  if  $f^A(s_1, 1 - \theta') = Y$ , because  $1 - \theta' > \delta^{\tau-1}(1 - \pi_x)$ .

(ii)  $\pi_{\emptyset}(M, A) > \pi_{\emptyset}(A, A)$  if  $f^A(s_1, 1 - \theta') = s_1$ , because

$$\delta(1 - \theta') > \delta^{\tau-1}(1 - \pi_x).$$

(iiia)  $\pi_{\emptyset}(M, A) > \pi_{\emptyset}(A, A)$  if  $f^A(s_1, 1 - \theta') \neq \{s_1, s_3, Y\}$ , because

$$\delta^{\tau-n}(1 - \theta') > \delta^{\tau-1}(1 - \pi_x)$$

where  $n \geq 3$ .

(iiib)  $\pi_{\emptyset}(M, A) = \pi_{\emptyset}(A, A)$  if  $f^A(s_1, 1 - \theta') = s_3$ .

Finally,

(i)  $\pi_x(A, M) = \pi_x(M, M)$  if  $f^A(s_1, 1 - \theta') = Y$ .

- (ii)  $\pi_x(A, M) \leq \pi_x(M, M)$  if  $f^A(s_1, 1 - \theta') = s_1$ , because  $\delta\theta' + (1 - \delta)x \leq \theta'$  when  $\theta' \geq x$  by Lemma 2.
- (iii)  $\pi_x(A, M) < \pi_x(M, M)$  if  $f^A(s_1, 1 - \theta') = s_3$ , because  $\delta^{\tau-1}\pi_x + (1 - \delta^{\tau-1})x < \theta' = \delta^{\tau-2}\pi_x + (1 - \delta^{\tau-2})x$ .

Calculating the payoffs for the different scenarios yields  $\pi(M, A) > \pi(A, A)$  in (i), (ii) and (iiia), and  $\pi(M, A) = \pi(A, A)$  and  $\pi(A, M) < \pi(M, M)$  in (iiib), thus  $A$  is not a MESS.

*Case 3.*  $A$  reaches agreement in period 1 when the automaton with  $x$  is chosen to be player  $II$  and in period  $\tau > 1$  when the automaton with  $x$  is chosen to be player  $I$ . Then an argument analogous to that of the previous case applies. ■

**Lemma 5** *If automaton  $A$  is a MESS, then  $A$  does not condition its play on the player-role and agreement is reached in the first period when it plays itself.*

**Proof.** First, we show that if  $\vartheta(s) = I^*$  in  $A$ , then agreement is reached in period 1 if  $A$  is to be a MESS. Suppose automaton  $A$  with  $\vartheta(s) = I^*$  does not reach agreement in the first period when it plays itself. Notice that  $A$  can have three types of delay and it can use output  $I^*$  in three different ways, which are described in Table 1.

	$\vartheta(s) = I^*$ when $x$	$\vartheta(s) = I^*$ when $\emptyset$	$\vartheta(s) = I^*$ in both
Delay if:			
$I$ has $x$	Case A	Case D	Case G
$II$ has $x$	Case B	Case E	Case H
Always	Case C	Case F	Case J

Table 1: Different cases of  $A$  and types of delay.

For all cases we can construct a simpler mutant  $M$  yielding identical play, by collapsing the initial state(s) and the state to which  $A$  shifts and makes its first demand from if  $\vartheta(s) = I^*$ . Thus,  $A$  is not a MESS according to condition (iii) in Definition 2. We show this for Cases B and J, but the same argument applies in all cases.

*Case B.*  $A$  conditions its play on whether it is chosen to be player  $I$  or player  $II$  only when it is assigned the positive breakdown utility  $x$ . The delayed agreement occurs if the automaton without the positive breakdown

utility  $x$  is chosen to be player  $I$  (see Figure 6). The remaining transitions are arbitrary.

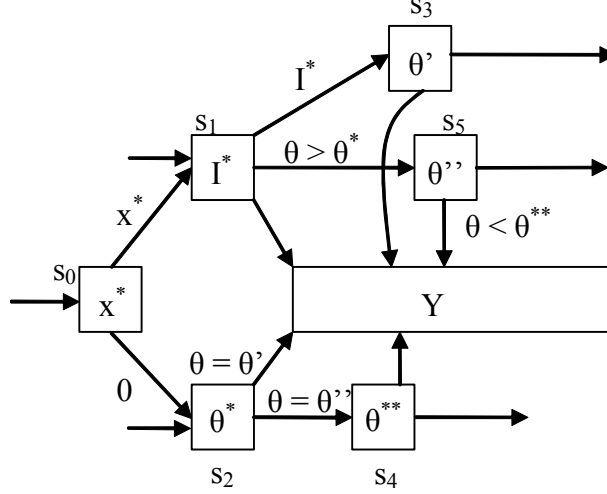


Figure 6: Case B automaton.

By collapsing states  $s_1$  and  $s_3$ , mutant  $M$  is less complex than  $A$  but yields identical play (see Figure 7).

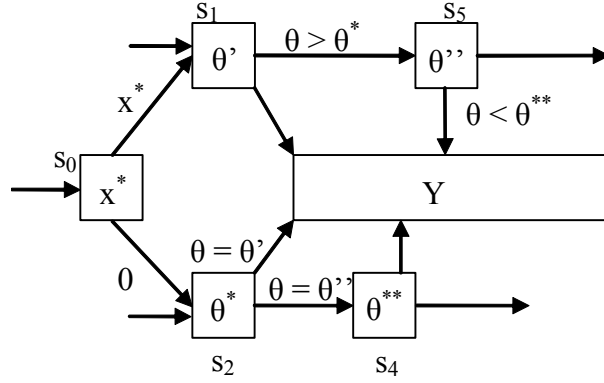


Figure 7: Case B mutant.

Thus,  $A$  is not a MESS.

*Case J.* Figures 8 and 9 show that we can construct a simpler automaton by collapsing states. The remaining transitions are arbitrary. In Figure 9 we have collapsed states  $s_1$  and  $s_3$ , and states  $s_2$  and  $s_4$ . As a consequence, the mutant  $M$  has two less states than  $A$  and this disqualifies  $A$  from being a MESS.

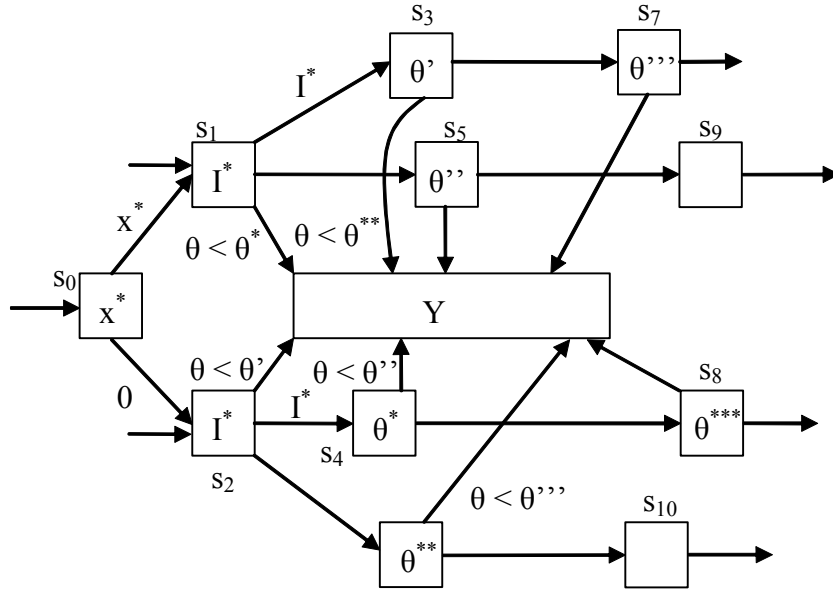


Figure 8: Case J automaton.

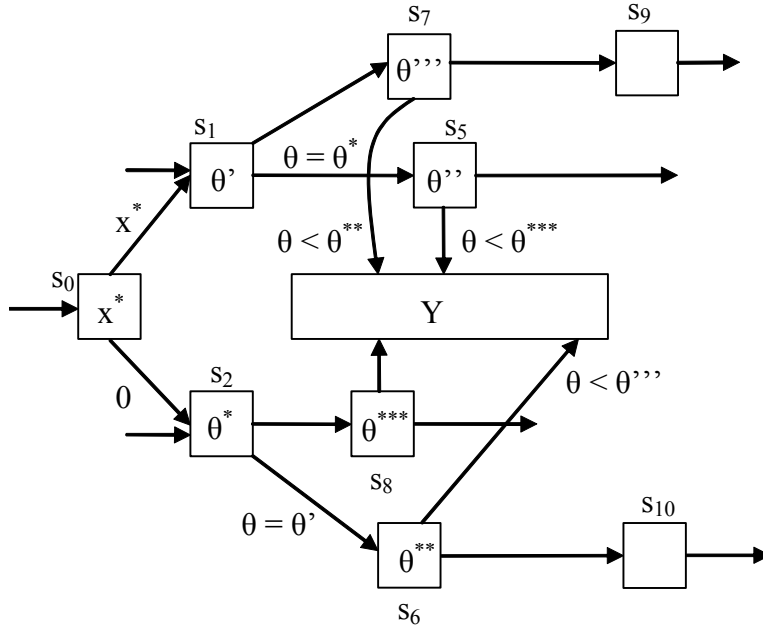


Figure 9: Case J mutant.

Finally, we end the proof of Lemma 5 by showing that if agreement is reached in the first period, then  $\vartheta(s) \neq I^*$  if  $A$  is to be a MESS. Suppose automaton  $A$  is a MESS with  $\vartheta(s) = I^*$  that reaches agreement in period 1 against itself. Then we can construct a less complex mutant  $M$  yielding identical play, where  $M$  does not use  $\vartheta(s) = I^*$ . This is illustrated in Figures 10 and 11. Automaton  $A$  described in Figure 10 achieves immediate agreement against itself. If the automaton with  $x$  is chosen to be player  $I$ , then player  $I$  demands  $\theta'$  which player  $II$  accepts. If the automaton without  $x$  is chosen to be player  $I$ , then player  $I$  demands  $\theta^*$  which player  $II$  accepts.

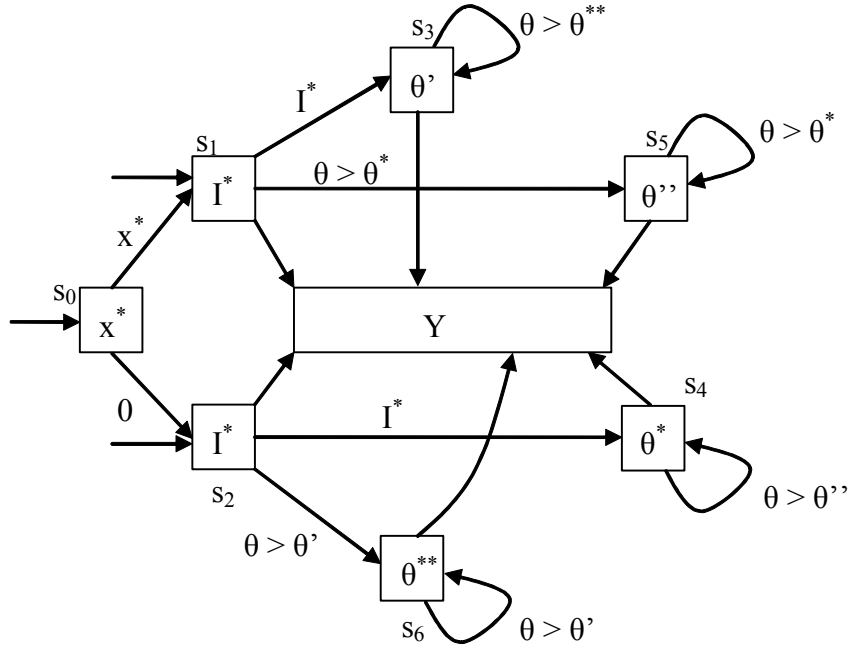


Figure 10:  $A$  with  $\vartheta(s) = I^*$  and immediate agreement.

Now, a simpler automaton shifts from  $s_1$  to  $s_5$  if  $\theta > \theta^*$  and from  $s_2$  to  $s_6$  if  $\theta > \theta'$ , and collapses states  $s_1$  and  $s_3$  and states  $s_2$  and  $s_4$  (see Figure 11).

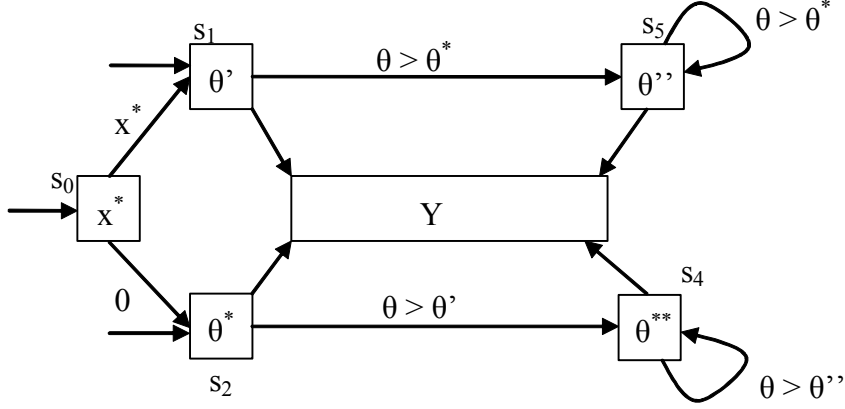


Figure 11:  $M$  with collapsed states.

However, since states  $s_5$  and  $s_6$  are never used when  $A$  or  $M$  play themselves or each other, the simplest automaton  $M'$  does not have these states. This is illustrated in Figure 12.

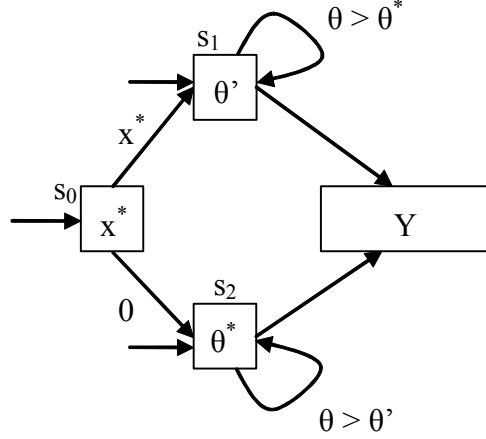


Figure 12: The simplest  $M'$ .

Again,  $A$  is not a MESS. The payoffs are identical, but  $A$  fails condition (iii) in Definition 2. Notice that nothing changes if  $A$  only conditions its play on a player-role when it is assigned the positive breakdown utility (or when it is not assigned the positive breakdown utility). We can still collapse  $s_1$  and  $s_3$  (or  $s_2$  and  $s_4$ ) and thus not use  $\vartheta(s_1) = I^*$  (or  $\vartheta(s_2) = I^*$ ). ■

**Lemma 6** *If  $x > \frac{1}{2}$  and automaton  $A$  is a MESS, then it has the four states described in Figure 12, in which  $\theta' \in (x, 1)$  and  $\theta^* \in (0, 1 - x)$  satisfy*

$$1 - \theta' \geq \delta\theta^* \quad (35)$$

$$\theta' \geq \delta(1 - \theta^*) + (1 - \delta)x \quad (36)$$

$$\theta^* \geq \delta(1 - \theta') \quad (37)$$

$$1 - \theta^* \geq \delta\theta' + (1 - \delta)x \quad (38)$$

**Proof.** If  $A$  is a MESS, then it reaches agreement against itself in period 1. Suppose  $A$  has more than four states. Then a mutant that is constructed as above yields identical play, but is less complex than  $A$ . Thus,  $A$  is not a MESS. If  $A$  is a MESS it cannot have less than four states. It has to use the pre-play state (Corollary 3), and thus shift to different initial states depending on whether it is assigned the positive breakdown utility  $x$  or not, and it must have an acceptance state  $Y$  (Lemma 1).

The demands  $A$  makes must be of such nature that  $A$  is immune to mutants that delay the agreement in order to get another share, e.g. reject  $\theta'$  in order to get  $1 - \theta^*$  in the next period. Condition (35) must be satisfied, otherwise it is optimal for a mutant to reject  $\theta'$  when it is player  $II$  without the positive breakdown utility  $x$  and delay the agreement by one period. Condition (36) must be satisfied, otherwise it is optimal for a mutant to demand 1 when it is player  $I$  with the positive breakdown utility and delay the agreement by one period. Condition (37) must be satisfied, otherwise it is optimal for a mutant to demand 1 when it is player  $I$  without the positive breakdown utility and delay the agreement by one period. Condition (38) must be satisfied, otherwise it is optimal for a mutant to reject  $\theta^*$  when it is player  $II$  with the positive breakdown utility and delay the agreement by one period.

Conditions (35)-(38) define a rhomb in space  $[0, 1]^2$ , i.e.

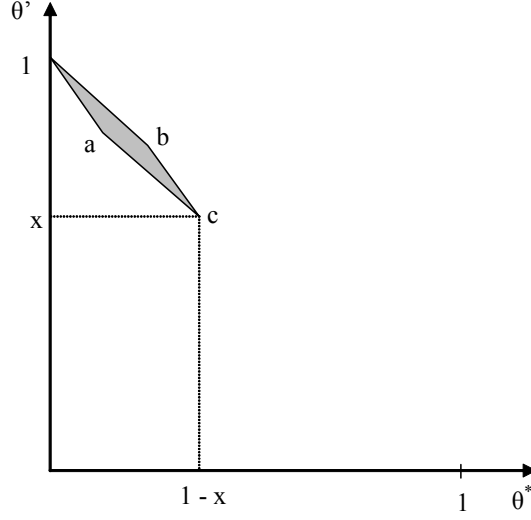


Figure 13: The MESS demands.

where

$$\begin{aligned} \text{a: } & \left( \frac{\delta+x}{1+\delta}, \delta \frac{1-x}{1+\delta} \right) \\ \text{b: } & \left( \frac{1+\delta x}{1+\delta}, \frac{1-x}{1+\delta} \right) . \\ \text{c: } & (x, 1-x) \end{aligned} \quad (39)$$

Any pair of demands  $(\theta', \theta^*)$  that satisfies conditions (35)-(38) is a point in the rhomb. However, we have that  $\theta' \in (x, 1)$  and  $\theta^* \in (0, 1-x)$  if  $A$  is a MESS. That is,  $(\theta', \theta^*) \neq (1, 0)$  and  $(\theta', \theta^*) \neq (x, 1-x)$ . First, suppose that  $(\theta', \theta^*) = (1, 0)$ . Then let a mutant have  $\theta^* = \epsilon > 0$  and  $\theta' = 1$ , but accept the same demands as  $A$ . We have  $\pi_x(M, A) = 1$ ,  $\pi_\emptyset(A, M) = 0$  and  $\pi_\emptyset(A, M) \geq 0$ , where the last payoff is positive if and only if  $A$  accepts  $\epsilon$  when  $M$  is player  $II$ . Thus,

$$\pi_x(M, A) \left\{ \begin{array}{l} = \frac{1}{2} + \frac{1}{2}(\delta + (1-\delta)x) \\ = \frac{1}{2} + \frac{1}{2}(1-\epsilon) \end{array} \right\}, \quad (40)$$

which both are less than 1. This either makes  $M$  a better reply to  $A$  than  $A$  itself, or it makes  $M$  a best reply to  $A$  and a better reply to itself than is  $A$ . Now, if  $\theta^* > 0$  then  $\theta' < 1$  by conditions (35)-(38). Second, suppose that  $(\theta', \theta^*) = (x, 1-x)$ . Then let a mutant have  $\theta' = x + \epsilon > x$  and  $\theta^* = 1-x$ , but accept the same demands as  $A$ . We have  $\pi_\emptyset(M, A) = 1-x$ ,  $\pi_x(A, M) = x$  and  $\pi_x(M, A) \geq x$ , where the last payoff is larger than  $x$  if and only if  $A$  accepts  $x + \epsilon$  when  $M$  is player  $I$ . Thus,

$$\pi_\emptyset(A, M) \left\{ \begin{array}{l} = \frac{1}{2}(1-x) + \frac{1}{2}\delta(1-x) \\ = \frac{1}{2}(1-x) + \frac{1}{2}(1-x-\epsilon) \end{array} \right\}, \quad (41)$$

which are both less than  $1 - x$ . This either makes  $M$  a better reply to  $A$  than  $A$  itself, or it makes  $M$  a best reply to  $A$  and a better reply to itself than is  $A$ . Now, if  $\theta' > x$  then  $\theta^* < 1 - x$  by conditions (35)-(38). ■

This completes the proof of Proposition 2.

## 4.2 Proof of Proposition 3

When  $x \in (0, \frac{1}{2}]$ , no MESS automaton can be constructed less complex than  $B$  in Figure 4. By Lemma 1 we know that a MESS reaches an agreement when it plays itself. Hence, it must have a state in which a demand is produced and an acceptance state. Lemma 2 establishes that if  $B$  is a MESS then the agreement it reaches must be contingent on the positive breakdown utility. But unlike the MESS automaton  $A$  in Proposition 2,  $B$  does not have to condition its behavior on whether it has the positive breakdown utility or not. Suppose  $B$  reaches agreement in period  $t$ , where player  $I$  demands  $\theta^*$  which player  $II$  accepts. From Lemma 2 we have that  $\theta^* \geq x$  if player  $I$  is assigned  $x$  and that  $\theta^* \leq 1 - x$  if player  $II$  is assigned  $x$ . There exists  $\theta^*$  that satisfies both conditions because  $x \leq \frac{1}{2}$ . The demand  $B$  makes must be of such nature that  $B$  is immune to mutants that delay the agreement in order to get another share, i.e. reject  $\theta^*$  in order to get  $1 - \theta^*$  in the next period. The conditions are:

1.  $1 - \theta^* \geq \delta\theta^*$ , otherwise a mutant without the positive breakdown utility acting as player  $II$  would reject  $B$ 's demand  $\theta^*$ ,
2.  $\theta^* \geq \delta(1 - \theta^*)$ , otherwise a mutant without the positive breakdown utility acting as player  $I$  would demand more than  $\theta^*$  and delay the agreement,
3.  $1 - \theta^* \geq \delta\theta^* + (1 - \delta)x$ , otherwise a mutant with the positive breakdown utility acting as player  $II$  would reject  $B$ 's demand  $\theta^*$ ,
4.  $\theta^* \geq \delta(1 - \theta^*) + (1 - \delta)x$ , otherwise a mutant with the positive breakdown utility acting as player  $I$  would demand more than  $\theta^*$  and delay the agreement.

It follows that

$$\frac{1}{1 + \delta} - \frac{1 - \delta}{1 + \delta}x \leq \theta^* \leq \frac{\delta}{1 + \delta} + \frac{1 - \delta}{1 + \delta}x. \quad (42)$$

We have that

$$\frac{1}{1 + \delta} - \frac{1 - \delta}{1 + \delta}x \geq x \quad \text{and} \quad \frac{\delta}{1 + \delta} + \frac{1 - \delta}{1 + \delta}x \geq x \quad (43)$$

for  $x \leq \frac{1}{2}$ .

Since a mutant only causes a delay (or a breakdown) if it does not accept  $\theta^*$  or demands more than  $\theta^*$ , it can only "beat"  $B$  by being simpler than  $B$  - but that is impossible.

This completes the proof of Proposition 3.

## 5 Conclusions and Remarks

We have shown that if one imposes evolutionary stability on the strategies used in a bargaining game with risk of breakdown and an asymmetric breakdown point, then quite a few partitions remains possible. If the positive breakdown utility  $x$  is greater than half the surplus, we can construct a MESS automaton to promote any partition where  $x$  or more of the pie is assigned to the player with the positive breakdown utility. In particular, we can construct a MESS supporting the split-the-difference partition. The associated demands  $(\frac{1+x}{2}, \frac{1-x}{2})$  are the middle point in the rhomb, or to be exact, the intersection of two lines; the first between points a and b and the second between points c and (1,0) in Figure 13.<sup>6</sup> Notice that this is the subgame perfect equilibrium agreement in Proposition 1.

Our finding still implies that only one automaton can be the incumbent strategy in a population. For example, if half of the population uses an automaton ( $A$ ) supporting the split-the-difference partition and the other half uses an automaton ( $B$ ) supporting a partition arbitrarily close to the partition  $(x, 1 - x)$ , then a mutant can be constructed to invade the population. Since  $A$  and  $B$  fail to reach agreement whenever they are matched to play each other, a mutant that reaches agreement against both  $A$  and  $B$  as well as against itself, has a higher payoff. The mutant  $M$  always needs to accept both  $A$ 's and  $B$ 's demands and make  $A$ 's demands when it is assigned the positive breakdown utility  $x$  and  $B$ 's demands when it is not assigned  $x$ . In the worse case, agreement is delayed one period when  $M$  plays against  $A$  or  $B$ , while  $A$  and  $B$  always fail when playing each other.

For games in which the positive breakdown utility  $x$  is smaller than half the surplus, our result shows that almost all partitions can be supported by a MESS automaton due to the use of the pre-play state.<sup>7</sup> By not allowing

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<sup>6</sup>Given that  $\delta \rightarrow 1$ .

<sup>7</sup>The player with the positive breakdown utility still has to be assigned more than  $x$  in the agreement.

automata to have a pre-play state, the MESS automaton is most likely automaton  $B$  in Proposition 3.<sup>8</sup>

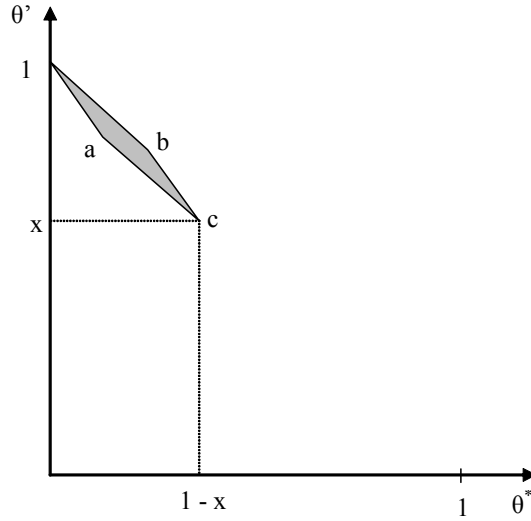


Figure 13: The MESS demands.

Our result can fail if we have a polymorphic populations with different attitude towards risk (this corresponds to individual discount factors in Rubinstein, 1982).

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<sup>8</sup>One might be able to use the same line of proof as Binmore *et al.* (1998) to establish this fact, however, this is just an assessment from our part.

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