Behaviour of Elasto-Plastic Point Set Structures

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by

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PAPERS COMPRISED IN PRESENT THESIS

The present thesis comprises

Paper I. Plem, E, 1968, Design of Point Set Structures. (National Swedish Institute for Building Research) Report 21/68, Stockholm.

Paper II. Plem, E, 1971, Punktkonstruktioners verkningssätt vid upprepade lastcykler /Behaviour of Point Set Structures under Influence of Repeated Cycles of Loading/. (Lund Institute of Technology, Division of Structural Mechanics and Concrete Construction) Bulletin 17, Lund.

INTRODUCTION

The term "point set structure" is used in what follows to designate a plane structure that can transmit a load from one loaded structural member to another through a finite number of discrete points.

Examples of structures of this kind are riveted and bolted joints, nailed joints, as well as groups of vertical piles subjected to torsional loads.

The present-day methods used for theoretical studies dealing with the behaviour of point set structures are characterised by the assumption that the behaviour of these structures is purely elastic, or by an approximate assumption concerning the position of the centre of twist in connection with total plastification.

The distribution of forces in a point set structure under perfectly elastic conditions is conventionally calculated in such a way that the external force is supposed to be applied at the centre of gravity of the point set, and is then uniformly distributed among the points of the set so that the component force at each point acts in the direction of the external force. After that, the moment of displacement is balanced by a second component force at each point. The magnitude of this component force is proportional to the radius drawn from the centre of gravity of the point set to the point in question, and its line of action is at right angles to this radius. Finally, the resultant force at each point is obtained by vector addition of the above-mentioned two component forces.

This method of analysis implies that the displacements associated with the points are considered to be caused, first, by a translation of the point set as a whole, and second, by a rotation about the centre of gravity as centre. However, a motion that is combined in this way can be described as pure rotation about an instantaneous centre. By using this instantaneous centre as a point of reference, the method of analysis summarised in the above can be simplified. This simplification leads to a modified method of analysis, in which the unknown forces at the points are obtained directly, without any vector addition of the components. Of course, neither of the above-mentioned two methods of analysis furnishes any information on the ultimate strength of the structure.

OBJECT AND SCOPE OF THESIS

The object of the present thesis is to contribute to deepening our knowledge of the mode of action of a point set structure submitted to a load that exceeds the limit at which its behaviour ceases to be elastic. The scope of this thesis covers different loading situations, which are stated in what follows. The first loading situation is met with during the first application of load to a point set structure which is initially free from stresses. The behaviour of the structure under these conditions is followed throughout the elasto-plastic stage up to total plastification. The second loading situation covers the range that extends from the subsequent removal of load to a no-load state which is characterised by a system of residual forces in internal equilibrium. Finally, in the third loading situation, the behaviour of the structure is investigated, first, during a single load cycle, and then during repeated load cycles whose limits are comprised within the elasto-plastic region. The effect produced by strain hardening on the variation in internal forces in structures made of metallic materials is approximately taken into account in the last-mentioned investigation.

The methods of analysis evolved in this thesis are independent of the numerical data relating to properties of materials. After programming for automatic data processing, these methods are applied to phenomenological studies of the above-mentioned changes in state, and the mechanisms of these changes are examined in this connection.

All materials engineering aspects of the problems under consideration lie beyond the scope of the present thesis.

ABSTRACT OF PAPER I

This publication comprises in the main a study dealing with the behaviour of a point set structure during the first application of load to the structure which is initially free from stresses.

In this connection, the point set structure is characterised by perfectly elasto-plastic properties at each individual point. It is assumed that the line of action of the external force is fixed with reference to the point set structure. The yield limit of the point set structure is defined as the magnitude of that external load which causes yield to occur at the most heavily stressed point of the structure. So long as the increase in load remains below the yield limit, the displacement of the point set is characterised by rotation about the above-mentioned instantaneous centre. This centre of twist is common to all points of the set, and its position can be determined by simple manual calculations. It has a fixed position in the plane of the point set structure during all changes of state within the load range under consideration.

The ultimate strength of the point set structure is defined as the magnitude of that external load which causes total plastification of the point set structure. When the external load exceeds the yield limit, passes through the elasto-plastic region, and continues to increase up to the ultimate strength of the point set structure, the centre of twist which is common to all points of the set, becomes movable in the plane of the structure within this load range.

The state of stress, φ, is introduced in order to characterise a certain definite state in the elasto-plastic region. The state of stress is defined as the ratio of the radius, R_o, of that boundary circle which - really or fictitiously - divides the point set into an elastic zone and a plastic zone to the radius, R_M, which is drawn to the outermost point of the set. Both these radii are drawn from that instantaneous centre which corresponds to the state φ.

In reality, the description of the state of stress in terms of the parameter φ implies that this state is expressed in terms of imposed deformation. The reason why the latter has been chosen as the primary action is bound up with the circumstance that the extent of the elasto-plastic region is great in respect of deformation, whereas it is in certain cases very limited in respect of force. Accordingly, the choice of φ as the determining parameter makes it possible to approximate much more closely the state that is aimed at.

In conformity with the above definition, the yield limit of the point set structure corresponds to $\varphi = 1$, and its ultimate strength corresponds to $\varphi = 0$. For a given value of φ within the interval from unity to zero, the position of the instantaneous centre of twist can not be determined in the same simple manner as in the purely elastic region. Generally speaking, the state of the structure is determined by the three equations of equilibrium in the plane for the total system of forces, so that the two equations of projection give the position of the instantaneous centre, and the equation of moments gives that magnitude of the external load which corresponds to the value of φ under consideration. However, the equations of projection assume here such a form that it is not possible to solve them in the general case. Therefore, by studying the internal work of the system, these equations are re-interpreted so as to be converted into residuals of the gradient of internal work, and then the state of equilibrium is determined by satisfying a minimum condition.

When the position of the instantaneous centre has been determined, the remaining problems is statically determinate, and therefore the equation of moments gives a unique magnitude of the external force, and hence the internal force pattern.

It is demonstrated that the limiting position of the instantaneous centre at the ultimate strength of a point set structure subjected to a load which consists of a pure moment coincides with the Torricellian point of the point set. This is the point characterised by the fact that the sum of the distances from this point to all points of the set is a minimum. A geometrical construction of the position of this point in the special case of a point set that consists of three points is described at the same time.

Finally, it is shown how the elasto-plastic distribution of internal forces can be calculated in the case of a point set structure submitted to a temperature difference between the two structural members which are connected together by means of the point set structure. In this case, the relative displacement of the points of the set is a displacement that is affine with respect to a pole which is shown to coincide with the centre of twist of a point set structure subjected to a moment alone. Accordingly, in the stage of failure, the pole of affinity coincides with the Torricellian point of the set.

The digital computer programme that has been prepared on the basis of the method of analysis outlined in the above is reproduced in this paper, commented upon, and demonstrated in its applications to several examples.

ABSTRACT OF PAPER II

To begin with, this publication presents a method of analysis which makes it possible to follow the changes in the internal state of a point set structure during a single load cycle whose limits are comprised within the elasto-plastic region on the assumption that the properties of each individual point are perfectly elasto-plastic. In this connection, the load cycle is supposed to start from that state which exists after a first application of load to a point set structure that is initially free from stresses. In this case too, the method of analysis is based on a re-interpretation of the three equations of equilibrium in the plane, which are converted into residuals of the gradient of internal work, and this enables the state of equilibrium to be determined by satisfying a maximum or minimum condition. The three unknown quantities involved in the problem in this case are the co-ordinates of the centre of twist that belongs to an additional rotation, as well as the magnitude of this rotation.

By using the method epitomised in the above, the force at an individual point of a point set structure can be determined in magnitude as well as in direction as a function of the magnitude of the external force. In this case, the line of action of the external force is likewise assumed to be fixed with reference to the point set structure.

It is shown that the instantaneous centre, which was fixed during the elastic stage of loading, and movable during the elasto-plastic stage, but was the whole time common to all points of the point set, is replaced during the following load cycle by individual instantaneous centres for each point. It is furthermore shown that these instantaneous centres are movable in the plane of the point set structure during the elasto-plastic stages of the load cycles as well as during its elastic stages.

In particular, this method of analysis makes it possible to determine the distribution of residual forces in internal equilibrium, that is to say, the internal forces which remain in the structure when the external load is removed after the increase in load to an elasto-plastic stage.

It is demonstrated that the final state of the structure, after it has been submitted to a load cycle with elasto-plastic cycle limits, is different from its initial state except when the load cycle is symmetrical, that is to say, when the lower limit of the cycle is numerically equal to its upper limit.

At the outset, the paper under review was based on the limiting assumption that the load-deformation curve for the individual points of the set is perfectly elasto-plastic. In a separate chapter, this assumption is abandoned, and, for the sake of generalisation, an approximate method is evolved in order that any load-deformation curve which is not concave upwards, but which is otherwise arbitrarily chosen for the individual points of the set, may be taken into account more realistically. The load-deformation curve adopted in each case is approximated to any arbitrarily chosen degree of accuracy by a load-deformation curve which is built up of a number of yield levels and intermediate elastic curve portions situated between these levels.

This refinement of the method is important especially in the analysis of the mechanical behaviour of joints using high-strength friction-grip bolts. The individual elements of these joints are characterised by a load-deformation curve which comprises two distinct yield levels, viz., one which corresponds to the force that is applied to overcome friction, and the other which initiates yield in the material. The method that has been generalised in this way is used to study some point set structures characterised by load-deformation curves which differ from the perfectly elasto-plastic curve, first, in respect of the distribution of residual forces after the removal of load, and second, in respect of the variation in the force at a point during a single load cycle. This study is based on two simplified assumptions. The first is that a decrease in the force at a point during the removal of load takes place in accordance with a linear force-deformation curve which is parallel to the initial tangent to the load-deformation curve under consideration. The second assumption is that an increase in the force at a point during the application of load is determined by the assumed load-deformation curve.

Finally, the method of analysis summarised in the above is applied to a phenomenological study of the behaviour of a point set structure under the action of repeated load cycles. In this connection, the method of analysis is modified so as to take account of the effect produced by strain hardening on the mechanical properties of the individual point.

In this connection, it was found difficult to deduce an analytical expression for the increase in the yield limit during the process of loading, since the force at a point changes in direction, and its magnitude varies at the same time. As an approximation that is not too rough, the yield force at a point was calculated as a function of the number of occasions on which yield occurs at the point in question. In this investigation, the interest was primarily centred, first, in those changes in the distribution of internal forces at the upper limit of a load cycle which take place from one load cycle to another, and second, in the successive increase in the upper yield limit and in the upper ultimate strength of the point set structure.

In the analysis, use is made of fixed limits of the load cycle in the elasto-plastic region. However, these limits have been moved towards the ultimate strength during the process of loading when the increasing yield limit tends to exceed the limit of the load cycle, in which case the continued behaviour of the structure becomes purely elastic, and hence trivial.

The main results of this investigation are summarised in what follows.

(1) The extent of the yield zone becomes successively smaller.

(2) A redistribution of the internal forces takes place in such a way that those points which have been slightly stressed from the outset are gradually unloaded, whereas those points which have been heavily stressed are subjected to increased loads.

(3) The Bauschinger effect reduces the upper yield limit during the first load cycle.

(4) In the course of repeated load cycles, the yield limit increa-

ses at a higher rate than the ultimate strength, and this implies that the brittleness of the point set structure successively increases.

(5) After a certain definite number of load cycles, the yield limit reaches the limit of the cycle, and then the continued behaviour of the point set structure becomes purely elastic. After that, no further changes in properties take place in the point set structure, unless the numerical value of the cycle limits are increased.

(6) The residual forces are successively removed by repeated load cycles.

The method of analysis evolved in this paper, with its alternative modifications referred to in the above, has been embodied in a digital computer programme, which is written in Algol 60, and is adapted to the UNIVAC 1108 automatic data processing system. This programme, which is reproduced and described in the paper, makes it possible to choose between three types of analysis in conformity with the above.

The first type of analysis consists in the determination of the elasto-plastic state after the first application of load to a point set structure that is initially free from stresses. Thus, the function of this programme is the same as that of the programme described in Paper I. However, it is recommended to use the new version of the programme, first, because it permits the application of a more generalised load-deformation curve, and second, because it is based on a more effective strategy of computation than the previous version.

The second type of analysis deals with the changes which occur in the elasto-plastic state of a point set structure while it is subjected to a single cycle of load. In this case too, it is possible to use a generalised load-deformation curve for any point of the point set structure. In principle, the method of analysis described in the above enables the position of the instantaneous centre of twist associated with each point of the structure to be determined at any instant of a load cycle. In practice, when this method is used in the digital computer programme, the load cycle is divided into an arbitrarily chosen number of equidistant stages of calculation.

Finally, the third type of analysis is intended for the calculation of the variations in state at the upper limit of the load cycle, as well as at the yield limit and at the ultimate strength of the point set structure, under the action of repeated load cycles. This analysis takes account of the strain hardening effect, and the process of strain hardening can be varied within wide limits by an appropriate choice of three parameters.

CONCLUDING REMARKS

The two papers summarised in the above deal with the behaviour of a point set structure under elasto-plastic conditions on the assumption that the structure is submitted to a variable external load, and that the line of action of the external load is fixed.

An imaginable variant of the method of analysis presented in these papers would be characterised by a cyclic parallel displacement of an external force which is constant in magnitude. This loading situation is met with when a beam subjected to a movable load is fastened by means of riveted or bolted joints. In principle, it should be possible to solve this problem on the same lines as that which is dealt with in the present thesis, that is to say, by studying the variations in the internal work of the system. In fact, only minor modifications of the programme reproduced in this thesis would be required in order that it might be used for dealing with this problem.

The two types of variation in force referred to in the above are characterised by the fact that these variations are unambiguously determined from a mechanical point of view by the state of external force at the two end points of the interval of variation.

A different situation has to be dealt with if these two types of variation are combined. This can be required, for instance, in the analysis of bolted joints used as constructional components in machines which comprise oscillating or rotating masses. In such cases, the variation in forces is not unambiguously determined by the states at the limits of the interval of variation. In principle, however, such an action of forces, which is more complicated, could probably also be analysed with the help of the method evolved in these papers. This could be achieved by dividing the interval of variation into appropriate sub-intervals, and by alternately using the above-mentioned two simple formulae for the variation in forces.

These possibilities of development of the method of analysis in question relate to different forms of variation in the external load, but the internal mechanical properties of a point set structure can also be subject to variation. Only one cause of such variations in properties will be touched upon here, namely the fact that the load-deformation curve for an individual point of a point structure can vary with the temperature. The effect of this variation has to be taken into account, for example, in the design of joints using high-strength friction-grip bolts which can be exposed to fire. However, a rational design that takes this problem into consideration presupposes that the behaviour of the constructional component in question at a temperature which varies with the time is known, and this knowledge is not available at present. Consequently, this field is virgin soil for research dealing with point set structures.

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