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Receivers for Faster-than-Nyquist Signaling with and without Turbo Equalization

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Abstract—Faster-than-Nyquist (FTN) signaling is a trellis coding method that maintains the error rate while reducing signal bandwidth. The combined effect is to move closer to capacity. We study some basic receiver issues: How to model the signaling efficiently in discrete time, how much the Viterbi receiver can be truncated, and how to combine the method with an outer code. The methods are modeling for minimum phase, minimum distance calculation and receiver tests. Concatenated FTN in a turbo equalization scenario proves to be a strong coding method.

I. INTRODUCTION

This paper investigates the design and complexity of receivers when the transmission method is faster-than-Nyquist (FTN) signaling, both by itself and in a turbo combination with convolutional coding. The FTN method sends ordinary linear modulation signals whose baseband form is

$$s(t) = \sqrt{E_s/T} \sum_n a_n h(t - n\tau T), \quad \tau \leq 1. \quad (1)$$

Here a_n are M -ary independent and identically distributed data symbol values with zero mean and unit variance, E_s is the average symbol energy, and $h(t)$ is a unit-energy baseband pulse, which for this paper we will assume is orthogonal to shifts by T . This form underlies QAM, TCM, and the subcarriers in orthogonal frequency division multiplex (OFDM). In 1975 Mazo pointed out that binary sinc(t/T) pulses in (1) could be sent “faster”, with symbol time $\tau T < T$, without loss of signal minimum distance. The asymptotic error probability is thus unaffected. This he called faster than Nyquist signaling, because the pulses appear faster than allowed by Nyquist’s orthogonality limit. The receiver thus encounters intersymbol interference (ISI), and FTN can be viewed as one of many ways to reduce bandwidth consumption by means of intentional ISI.

FTN signaling has since been extended in many ways. The modulation can be nonbinary, the pulses need not be sinc(\cdot), and in fact they need not be orthogonal at any T . An earlier study of receivers is [1]. Furthermore, the FTN concept can be applied simultaneously in time and frequency: Many signals of form (1) can be stacked in frequency more closely than the orthogonal limit, to form an inphase and quadrature array that still has the isolated-pulse asymptotic error rate. More details may be found in [2]. In every case there will be a closest packing (a smallest τ and/or a closest

subcarrier spacing) at which the minimum distance first falls below the isolated pulse value. This is called the *Mazo limit* to signaling with this $h(t)$ and alphabet. In this paper we are concerned with time compression, binary $\{a_n\}$, and the AWGN channel with noise density $N_0/2$, and so we are interested in the smallest τ that yields asymptotic probability $Q(\sqrt{2E_b/N_0})$. The significance of the Mazo limit is that it defines the narrowest linear modulation bandwidth that attains the isolated-pulse probability of h ; if receiver complexity is not a concern, no wider bandwidth need to be used. Note that the limit is not set by orthogonality.

The paper first sets up a finite, discrete-time model for the FTN signaling in Section II, and then estimates the error performance of a truncated Viterbi receiver in Section III. Modeling has some extra difficulties because the usual whitened matched filter approach leads to an unstable whitener. Section IV then reports some receiver tests for ordinary FTN signaling. Section V investigates a turbo equalization scenario consisting of a convolutional encoder, interleaver and FTN encoder in serial concatenation. It turns out that there is a strong synergy among these three elements, in which the convolutional coder/interleaver repairs the ISI caused by the FTN’s bandwidth reduction and the FTN reduces bandwidth. The result is a scheme that significantly reduces *both* bandwidth and energy. Receiver tests are given.

II. DISCRETE-TIME RECEIVER IMPLEMENTATION

FTN signaling is essentially a coded modulation that manipulates analog signals, but it is of course useful to reduce the transmitter–receiver system to a system with discrete-time samples, at the symbol rate τT . At least three such methods can be employed. (i) The *whitened matched filter* (WMF) receiver: A receive filter matched to $h(t)$ is sampled each τT , followed by a noise-whitening filter and then a Viterbi algorithm (VA). Since the most interesting FTN systems have infinite impulse response, the VA states must be truncated in some way. (ii) $h(t)$ is expressed as a superposition of narrower orthogonal pulses, one each τT (fractional sampling may be necessary). A receive filter matched to the narrowband pulse needs no whitening filter and its samples directly feed the VA. (iii) The matched filter to $h(t)$ is followed by the so-called Ungerboeck receiver that accepts colored noise. We

have constructed all these receivers. What is essential is that the VA work with a minimum-phase discrete model of the channel: VA truncation and minimum phase are intimately related. The first two receivers easily adapt to this and reduce to a similar physical implementation. Method (ii) is probably easier to design and we study it in a forthcoming paper. Here we treat method (i).

The WMF receiver scenario comprises the following elements: Transmit Filter $h(t)$ —AWGN—Receive Filter $h(t)$ —Sample at $n\tau T$ —Whitening Filter—Reverse frame—VA. Data symbols $\{a_n\}$ enter the transmit filter as in (1). Both transmit and receive filters are analog and matched to $h(t)$ (assume h is symmetric and centered at time 0); the whitening filter is discrete-time. The sampler creates a discrete time model of the channel and the FTN and its outputs \mathbf{r} are sufficient statistics for estimating $\{a_n\}$. They satisfy $\mathbf{r} = \mathbf{a} \star \mathbf{g} + \boldsymbol{\eta}$; as z-transforms this is $R(z) = A(z)G(z) + N(z)$. Here \mathbf{g} is the sampled autocorrelation function of $h(t)$,

$$g_k = \int h(t)h(t + k\tau T) dt \quad (2)$$

and $\boldsymbol{\eta}$ is colored Gaussian with correlation sequence \mathbf{g} . The whitening filter decorrelates $\boldsymbol{\eta}$ and is constructed from \mathbf{g} by spectral factorization of its all-zero z-transform $G(z)$ into $V(z)V(1/z^*)$; for details see [3], [4]. After whitening by the filter $1/V(1/z^*)$, what remains can be expressed as

$$\tilde{\mathbf{r}} = \mathbf{a} \star \mathbf{v} + \mathbf{w}, \quad (3)$$

or $\tilde{R}(z) = A(z)V(z) + W(z)$, where \mathbf{w} is white Gaussian noise with variance $N_0/2$. The so-called WMF model of the channel is $V(z)$, and \mathbf{v} represents causal ISI with the property $\mathbf{v}[n] \star \mathbf{v}[-n] = \mathbf{g}$.

Many spectral factorizations are possible. Because \mathbf{g} is a correlation, the factorization can take place such that $V(1/z^*)$ has zeros strictly within the unit circle; the whitener $1/V(1/z^*)$ is thus stable¹ and the channel model becomes $V(z)$ with all zeros *outside* the unit circle. This is in fact the maximum phase model for \mathbf{g} , which is a strong inconvenience for truncated decoders. However, it can effectively be converted to a minimum phase model by decoding the signal blocks backwards, and we assume this is done.

We thus can construct a practical whitener and minimum phase discrete model provided that there exists $V(z)$ with all zeros outside the circle, but this is often not directly possible with FTN signaling for a fundamental reason. Important practical pulses $h(t)$, such as the root raised cosine (root RC), have spectrum equal to zero outside a certain bandwidth; the root RC pulse with excess bandwidth factor β , for example, is zero outside $(1+\beta)/2T$ Hz. Under FTN signaling at the higher rate $1/\tau T$, this value shrinks in comparison to the folding frequency $1/\tau T$ of the whitener, and there will eventually be a null zone in the range $((1+\beta)/2T, 1/2\tau T)$ Hz. We have that the spectrum $|H(j2\pi f)|^2$ is $|G(e^{j2\pi f})|^2$, and thus a finite order

¹There are mathematical solutions to the WMF receiver when the zeros lie on the unit circle, but we take as a practical requirement the whitener to be strictly stable, that is, all its zeros must be inside.

$G(z)$ can place spectral zeros at only finitely many frequencies and this only by violating stability.

Many practical cases fall into this difficulty. How can a model be constructed? In fact, $G(z)$ need only produce a whitener and model that is reasonably close to the spectrum of $h(t)$. The test is that the Euclidean minimum distance and ultimately the receiver error rate should not be affected, and this has proven possible to achieve. One method is to find a finite $G(z)$ approximation with quartets of zeros on the unit circle, using e.g. the Matlab routine `roots`. The zeros must occur in quartets because $V(z)$ and $V(1/z^*)$ each require a conjugate pair. The model may then be refined by splitting the quartet of zeros so that one conjugate pair is slightly inside the circle and one is outside. The positions can be chosen to reduce the stopband spectrum of h . A second method constructs an all-zero filter $V(z)$ whose spectrum lies within an ϵ of the required root RC spectrum. There exist, e.g., convex programming routines that compute this quickly.

As examples, here are models for root RC with $\beta = .3$ when τ is respectively 0.703 and 0.5. Both these $h(t)$ have null spectral regions. The first is at the Mazo limit, with $d_{\min}^2 = 2$, and was derived by hand with the root quartet method. The second has the much smaller distance 1.016 and is found by convex programming. They play a role in the next sections:

$$\mathbf{v} = \{.750, .625, -.190, -.040, .085, -.049, .015, -.006\} \quad (4)$$

$$\mathbf{v} = \{.130, .484, .706, .368, -.178, -.228, .083, .125, -.057, -.056, .043\} \quad (5)$$

III. EUCLIDEAN MINIMUM DISTANCE AND THE TRUNCATED VA

Many useful FTN methods use a pulse with an infinite time support. Since the whitener and model are approximate and the VA is truncated, it is important to verify that the Euclidean minimum distance of the signal set has not significantly changed from the theoretical value with the analog $h(t)$. Algorithms that estimate d_{\min} both for signals of form (1) and for the discrete time form $\mathbf{s} = \mathbf{a} \star \mathbf{h}$ are well known (see [5]). We will not describe them here except to say that the problem is of size 3^L for length- L binary signals and that distance depends only on the difference $\Delta \mathbf{a} = \Delta a_{n-K}, \dots, \Delta a_n$ between transmitted and erroneous symbols through the formula

$$d^2 = \sum |q_i|^2, \quad q_i = \Delta \mathbf{a} \star \mathbf{v} \text{ at } i \quad (6)$$

For a signaling system working at the Mazo limit we require $d_{\min}^2 = 2$, but FTN signaling with lower minimum distance is also of interest.

A truncated VA of memory m works with only the most recent $m+1$ path symbols, that is, with model taps v_0, \dots, v_m . One must distinguish two kinds of truncated VA. If the branch labels at stage n are constructed by $\sum_{k=0}^m v_k a_{n-k}$, from only these taps, the VA is more properly called a mismatched receiver, because it constructs labels from a different model than the transmitter uses. Finding mismatched minimum distances has been explored for some years (see e.g. [5], Section

5.5). The receiver can be much disturbed by the symbols it cannot “see”; our calculations of the mismatched distance show that this first kind of truncated receiver has much inferior performance for the kind of pulses in (4)–(5).

A better receiver is one that *knows* the full model, even though it does not use it in the VA state description. Consider stage n branch labels each generated from some \mathbf{a} by

$$s_n = \sum_{k=0}^m a_{n-k} v_k + \sum_{m+1}^{m_{\text{tot}}} a_{n-k} v_k \quad (7)$$

where m_{tot} is the total model memory. The first term stems from the VA state symbols while the second is an offset created by the earlier symbol history; an offset is associated with each survivor state in the VA memory but is itself not the state. This sort of trellis search was proposed in the 1970s (see [6]) and applied by several authors to channel decoding in the 1980s; perhaps the best known paper is Duel-Hallen and Heegard [7]. They calculate a minimum distance, which we call d_{DH} , for the offset VA receiver²; more precisely, they derive an asymptotic error rate of the form $Q(\sqrt{d_{\text{DH}}^2 E_b/N_0})$.

We find that d_{DH} closely predicts the behavior of practical FTN receivers and it gives a theoretical indication that the VA receiver with proper design can endure severe truncation. A modern approach to finding d_{DH} is as follows. The key observation is that a VA forces at each stage a choice of survivor into each state. The standard VA analysis, whether full or truncated, finds the choice with the highest probability of error and computes the probability in terms of a distance. At a given state, the transmitted path and a neighbor path can only merge after their state symbols a_n have been the same for m stages. As an example, the paths with symbols $S, \dots, S, +1, -1$ and $S, \dots, S, -1, +1$ have difference $0, \dots, 0, 2, -2$ (S, \dots, S denotes same symbols), and cannot merge under state memory $m = 2$ until the difference is 0 for two more stages. It is the distance at this point that makes the decision, and this is the first $m + \ell$ square convolution outcomes in eq. (6) with $\Delta \mathbf{a} = \{2, -2\}$, where $\ell = 2$ is the length of $\Delta \mathbf{a}$. A full VA will not force the merge until later, and the square distance (6) is carried out to more terms, which is a larger number. Estimating the worst case distance consists of trying out all suspect error difference events, with (6) carried out $m + \ell$ terms. An efficient search is to take as candidates those differences that have distances in the untruncated case less than, say, $2d_{\text{min}}^2$; these are found by the ordinary minimum distance algorithm. Only these candidates are explored for their truncation properties. The search is efficient because very few differences in this sort of code structure have distance near the minimum.

For the FTN case in (4), which is root RC at the Mazo limit, the procedure yields $d_{\text{DH}}^2 = 1.90$ and 1.98 under $m = 1$ and 2 , that is, under VA truncation to 2 and 4 states. Since the full d_{min}^2 is 2, this shows that a truncated VA with only 2 states nearly reaches the Mazo limit, which carries with it a bandwidth reduction of 30%. For the case in (5), which is

a 50% bandwidth reduction, $d_{\text{DH}}^2 = .60, .83, .86, .93, .95, .98$ under $m = 2, \dots, 7$. About 32 states are thus enough for the 50% reduction.

Of course, the procedure here is only an estimate, and real decoders have other dynamics, notably error propagation. It is necessary to construct and test a receiver.

IV. TRUNCATED VA RECEIVER TESTS

We have constructed an offset VA receiver of the form in (7) and tested it with the straightforward FTN systems in (4)–(5) over the simulated AWGN channel. Regardless of the $h(t)$, the sequence reaching the VA is always minimum phase. Size 800 frames of random ± 1 symbols were encoded, and enough frames were taken to give 20–100 error events. The frames were terminated before and after by m_{tot} ‘+1’ symbols. An error event is taken to begin when the receiver output state splits from the transmitter state path, and it ends when the output rejoins in the sense of the full model, i.e., after m_{tot} symbols are the same.

Figure 1 plots the observed error event rate³ against E_b/N_0 in dB for the severe FTN case with $m = 2, 3, 7$. For the 3, 7 cases, the d_{DH} -causing difference sequence is $2, -2, 2$ which has a multiplicity factor of 1/4; consequently, the error event rate estimate is $Q(\sqrt{d_{\text{DH}}^2 E_b/N_0})/4$. This with the respective d_{DH} is shown as two solid lines. The most common event was indeed observed to be $\{2, -2, 2\}$. At $m = 2$ the error event situation is more confused, and the estimate line is $Q(\sqrt{d_{\text{DH}}^2 E_b/N_0})$. On the average, an error event contained 3–5 symbol errors, with the higher numbers corresponding to smaller VA state memory. The bit error rate is thus 3–5 times the event rate.

The plot for the Mazo limit case (4) for the same m is similar, but less exciting because all the m lead to about the same event rate. For either FTN case, the VA state size needs to be very much larger if the VA input is converted to some phase other than the minimum one. Taken together, the results show that the d_{DH} procedure is accurate, the truncated state size can be small, and error propagation and other event difficulties are not a threat.

V. TURBO EQUALIZATION BASED ON FTN SIGNALING

In this section we investigate FTN signaling as part of a turbo equalization system. The transmitter consists of the sequence Rate 1/2 Convolutional Encoder—Interleaver—FTN Encoder. A block of K information bits is first encoded by a rate 1/2 convolutional code; this produces $2K$ code symbols. These feed a size $2K$ interleaver. The symbol vector \mathbf{u} is formed by mapping the interleaver output onto a 2PAM

³Error event rates must be carefully computed. The rate is the number of distinct events divided by the number of “healthy” stages where events are free to start. The latter is the number of events plus the number of stages where the transmitter and receiver outputs agree in state. Studies with many trellis codes have shown that this rate accurately predicts $\mu Q(\sqrt{d_{\text{min}}^2 E_b/N_0})$ over a wide range of E_b/N_0 , where μ is the multiplicity for the d_{min} -causing difference event.

²In [7], the receiver is called a DDFSE receiver.

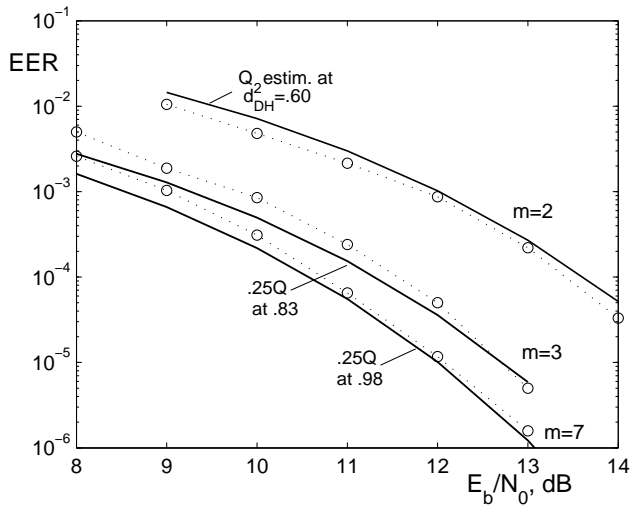


Fig. 1. Error event rates for FTN system (5) vs. E_b/N_0 in dB (dotted lines), showing VA truncation to 4, 8 and 128 states ($m = 2, 3, 7$). Performance predicted by d_{DH}^2 shown for comparison (solid lines).

alphabet but in principle any PAM alphabet can be used. Finally, the transmitted signal $s(t)$ is constructed according to (1). We investigate only the (7,5) convolutional code and we set the block size $K = 5000$.

Decoding is done via standard turbo equalization [8]. In [9] and [10] it has been shown that recursive precoding leads to additional gains in turbo equalization but such a precoder has not been employed here. The performance of the considered system can therefore never be better than the performance of the underlying convolutional code. However, for FTN this performance is obtained at a considerably higher bit rate. By studying the EXIT charts [11] of the system the convergence threshold can be determined: The system will converge to the outer code performance as soon as there is an open convergence tunnel between the EXIT curves for the FTN system and the outer code. Then the error performance can be measured by actual receiver tests. The pulse shape $h(t)$ used here is root RC with excess bandwidth $0 \leq \beta \leq 1$; if $\beta = 0$ a sinc pulse is obtained. The one sided baseband bandwidth is $(1 + \beta)/2T$. One aim of the section is to establish the best β .

We have observed an open convergence tunnel between the EXIT curves for all τ above a certain threshold. Above it the error performance of the concatenated system is virtually identical to the that of the outer convolutional code. The threshold depends on the SNR; in this paper we use the SNR where the (7,5) code alone achieves BER 10^{-5} , that is, $E_b/N_0 = 5.85$ dB. The EXIT chart in Figure 2 shows a case near the threshold τ , where the convergence tunnel is narrow.

In Figure 3 turbo equalization receiver tests are shown for $\beta = .1, .2, .3, .4$. The component decoder for the FTN signaling is a BCJR algorithm that truncates the ISI response to memory 6 (64 states); 10 iterations in the turbo equalization have been performed. We plot the BER versus τ . The critical thresholds where the error rate departs from $\approx 10^{-5}$ are clearly seen and lie in the range .30–.43 for the different β .

In order to compare different β we must take the bandwidth

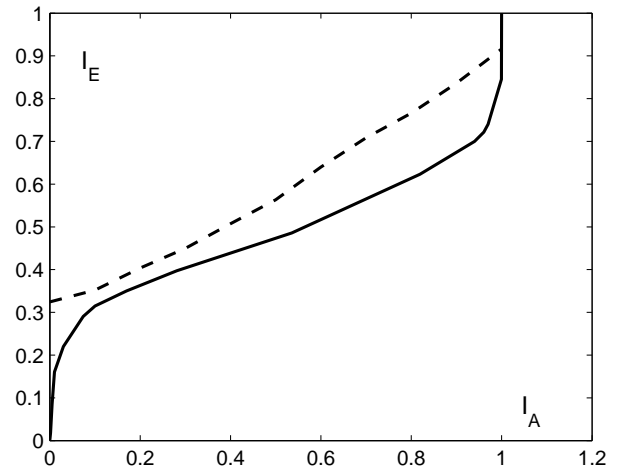


Fig. 2. An EXIT chart at $E_b/N_0 = 5.85$ dB, showing extrinsic vs. a priori information for block length 5000. Dashed curve is from root RC pulse with $\beta = .3$ and $\tau = .32$; solid curve is from (7,5) outer convolutional code.

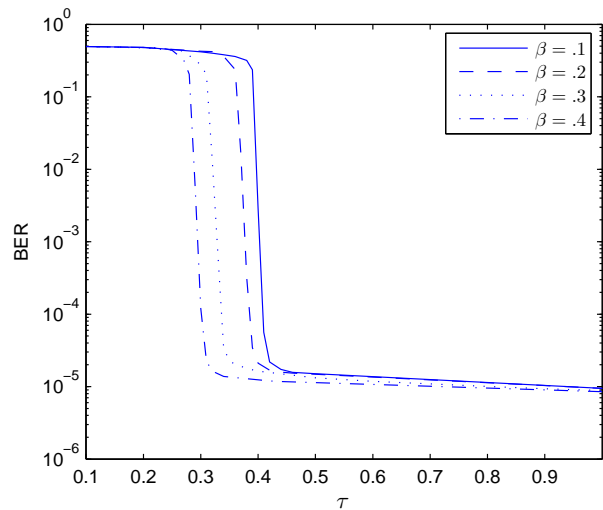


Fig. 3. Receiver tests for systems based on root RC pulses with excess bandwidth β . All systems operate at $E_b/N_0 = 5.85$ dB.

consumption into account. If system based on $\beta = .4$ can have more compression than one based on $\beta = .2$, it cannot necessarily be claimed that $\beta = .4$ is better, since .4 uses more bandwidth. We must plot the BER against the normalized bandwidth, which is W/R , where W is the one-sided baseband bandwidth and R the data bit rate. We have $W/R = ((1 + \beta)/2T)/(1/2\tau T) = (1 + \beta)\tau$. In Figure 4 we show the same plot as in Figure 3 but now against the normalized bandwidth. As can be seen, the best β are $\beta = .4$ and $.3$, which are slightly better than $\beta = .2$ and $.1$. This has significant practical importance since larger β are easier to implement.

Although it is not reproduced here, we have obtained a similar outcome to the above when $h(t)$ is a short, finite-support pulse, such as a triangle. Discrete-time modeling is easy with such pulses, but their bandwidth is relatively wide.

If a full complexity BCJR decoder is used as component

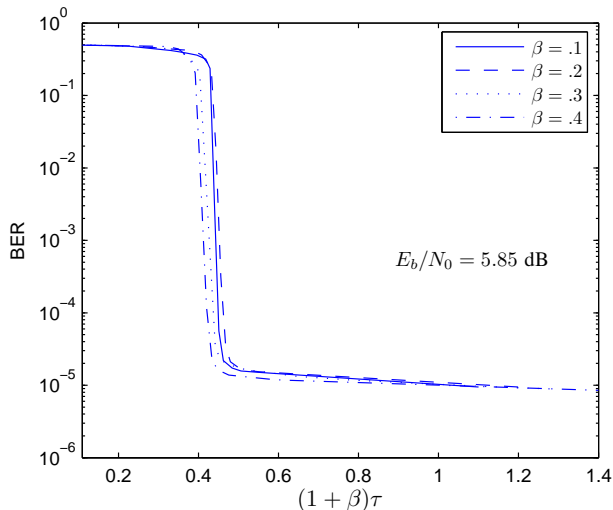


Fig. 4. Receiver tests for systems based on root RC pulses with β , plotted against the normalized bandwidth. All systems operate at $E_b/N_0 = 5.85$ dB.

decoder for the inner code (the ISI mechanism), we are limited to a rather small algorithm, that is, truncation to a rather short length (which prohibits the sinc pulse). We are therefore in the early stages of testing a reduced complexity MAP equalizer called the M^* -algorithm.⁴ This recently proposed [12] algorithm has shown very good performance on ISI channels. The algorithm retains only M out of the S states at each trellis depth, but rather than eliminating the other states they are merged into the M survivor states. This keeps the number of +1s and -1s on the remaining trellis branches in balance.

VI. CONCLUSION

We have investigated a number of issues that arise in the construction of a receiver for FTN signals, when the FTN is employed alone and when it is part of a turbo equalization system. The emphasis throughout was systems based on practical narrowband root RC pulses. First, a workable discrete time model was derived for pure FTN signaling. Our model was based on a whitened matched filter; several other approaches exist, and these should be investigated in future work, since they may lead to a lower complexity at the same error rate. Next we resurrected an older truncated VA receiver and performed an FTN distance analysis. The receiver and its distance were closely verified by actual tests. These show that strong truncation is possible if the VA input is minimum phase, and that the FTN bandwidth reduction can be purchased with little receiver complexity. Finally, we constructed a turbo equalization system based on FTN. Its receiver is more complex, but there is a strong synergy between the convolutional and FTN elements of the signaling. With the simple (7,5) convolutional code, energy savings of 4 dB and bandwidth reduction of 30% *both* can be achieved.

⁴Note that the alphabet size M and the M^* in the algorithm are unrelated.

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