Industry Diversification, Financial Development and Productivity-Enhancing Investments

Schclarek Curutchet, Alfredo

2006

Industry Diversification, Financial Development and Productivity-Enhancing Investments[*]

Alfredo Schclarek[†]

September 20, 2006

Abstract

This paper theoretically studies the role of the financial system in promoting macroeconomic stability and growth. It also explains endogenously the development of the financial system as part of the growth process. The productive sector engages in R&D activities, and finances its activities through access to the financial system. While vertical innovation spurs economic growth, horizontal innovation creates new industry sectors, and thus enhances industry diversification. Higher industry diversification deepens the financial system by improving its ability to finance the productive sector. Economies that are more diversified, and thus more financially developed, have higher growth rates and are less volatile. There is a role for the government to subsidize innovation, especially horizontal innovation.

1　Introduction

For several decades, long-run productivity growth and short-run business cycles have been investigated separately in the economics literature. Recently, however, there has been a return to the Schumpeterian view of growth and cycles as a unified phenomenon. This development is especially taken into account by the endogenous growth literature with quality-improving innovations (Aghion and Howitt, 1998). Endogenous growth theory with quality-improving innovations argues that growth is generated by a random sequence of quality improving (or "vertical") innovations that themselves result from (uncertain) R&D research activities. Technical progress makes existing technologies or products obsolete, emphasizing Schumpeter's process of "creative destruction". On the other hand, the endogenous growth literature also emphasizes the existence of expanding variety innovations ("horizontal innovation"): a discovery consists of the technical knowledge required to manufacture a new good that does not displace existing ones. Therefore, innovation takes the form of an expansion in the variety of available products (Gancia and Zilibotti, 2005). These two strands of the literature are not mutually exclusive, but complementary, and theoretical models have been developed that take into account both vertical and horizontal innovation (Howitt, 1999).

Regarding the link between financial development and economic growth, there is a vast literature that is summarized in Levine (2005). Among the papers most closely related to this paper, the following can be mentioned. Aghion et al. (2005a) study how credit constraints affect the cyclical behavior of productivity-enhancing investment. They state that a less developed financial system implies both higher aggregate volatility and a lower mean growth rate. Aghion et al. (2005b) study the effect of financial development on convergence. They predict that any country with more than some critical level of financial development converge to the growth rate of the world technology frontier, and that all other countries have a strictly lower long-run growth rate. However, unlike the micro-founded literature on financial markets and institutions (Bhattacharya et al., 2004), these two papers simply assume credit constraints in their model. Acemoglu and Zilibotti (1997) model the relationship between cross-sectional risk, diversification, and growth. They find that that the variability of growth decreases with economic development, and that productivity endogenously increases as the diversification opportunities improve. Their results are driven by the assumption that less developed countries specialize in low risk and low return sectors. However, this assumption is refuted by the results of the empirical paper by Koren and Tenreyro (2005), who find that the opposite is true. Carranza and Galdon-Sanchez (2004) build a model of financial intermediation that analyze output variability during the development process. They find that output is more volatile in middle-income economies than in both low and high-income economies.

The objective of this paper is to study the role of the financial system in promoting macroeconomic stability and growth. It develops a simple growth model where the financial system has a central role to play. The model is similar to

2

Aghion et al. (2005a), but instead of assuming exogenous credit constraints, it derives an endogenous micro-founded model of the financial system. The financial system is modelled as an imperfect capital market with informational asymmetries and moral hazard regarding agents' choices as in Holmstrom and Tirole (1998). Firms engage both in vertical and horizontal innovation, but have to finance liquidity shocks for these innovations to be successful. Successful vertical innovation is the driving force behind the growth of the economy. Horizontal innovation does not affect the growth rate directly, but generates new industrial sectors, which diversify the economy. Industry diversification deepen the financial system by improving its probability of supplying enough liquidity to firms at the aggregate level. Thus, a more diversified economy has a higher probability of successful horizontal and vertical innovations. Fluctuations across time arise because the fraction of firms fulfilling their investment projects at each period of time varies. Economies that are more diversified, and thus more financially developed, have higher mean growth rates than economies that are less diversified. The volatility of the growth rate is initially increasing, but becomes decreasing at intermediate and high levels of industry diversification. In this model, there is an important role for the government in subsidizing innovations, especially horizontal innovation, to promote financial development and economic growth. The active role of the government is especially suitable at early stages of financial development.

This paper offers several new insights with respect to the existing literature on financial development and growth. The papers by Aghion et al. (2005a) and Aghion et al. (2005b) take the level of financial development as an exogenous parameter, and do not model endogenously how the growth process affect financial development. In contrast, by combining the endogenous growth literature with an explicit micro-founded model of the financial system, this paper endogenously model the development of the financial system as a consequence of the growth process. Nonetheless, it is reassuring that the conclusions of this paper, regarding the causal effect of financial development on growth, are in line with those of Aghion et al. (2005a) and Aghion et al. (2005b). Another contribution of this paper is to explain financial development as a consequence of industry diversification. In our model, industry diversification is part of the growth process in the sense that it is a consequence of horizontal innovation. In that way, horizontal innovation has a key role to play in the development of the financial system. Although the argument that diversification helps dampen aggregate risk has been used previously (see for example Acemoglu and Zilibotti (1997)), no explicit micro-founded model has used this feature to endogenously explain financial development as part of the growth process.

Section 2 presents the basic setup of the model, which follows closely Holmstrom and Tirole (1998). The aggregate demand for liquidity and the role of the financial intermediary is introduced in section 3. The consequences of industry diversification for the financial system, and the issue of partial liquidation is presented in section 4. Section 5 analyzes the relation between industry diversification, financial development and economic growth. It also discusses the consequences of government subsidies to innovation. The conclusions are dis-

3

cussed in section 6.

2   Model

The economy is characterized by a simple, dynamic moral hazard model with overlapping generations of three-period-lived agents as in Holmstrom and Tirole (1998).[1] The economy is populated by three types of agents, firms (or entrepreneurs), investor (or consumers) and an intermediary (or bank). There is only one good that is used for both consumption and investment. All agents are risk-neutral with an additively separable utility function over undiscounted consumption streams. Each firm is indexed by i, has access to an investment project with constant returns to scale, and belongs to a certain industry j. The total number of different industries existing in the economy is J. Each generation is indexed by s, which is the moment of time when they were born.[2]

For an initial investment $T_s I$ in period 0, the investment project of firm i returns $R T_s I$ in period 2 if it succeeds, where T_s is the current available level of aggregate knowledge for generation s, I is the investment scale, and R is the gross rate of return of the project.[3][4] If the project fails, the return is 0. In period 1, all the firms belonging to industry j are hit by a random liquidity shock C_j, which has to be financed for the projects not to be abandoned (in which case the return is 0). Note that in period 1, the total number of shocks that hit the economy is J, i.e. there is one shock for each industry j. The liquidity shocks, or adjustment cost shocks, are proportional to the initial investment $T_s I$, i.e. $C_j = c_j T_s I$. In addition, the industry-level shocks c_j are independently and identically distributed with finite mean and variance. The shocks have a continuous distribution function $F(c_j)$ on $[0, \infty]$, with a probability density function $f(c_j)$. Note that the shocks are also independently and identically distributed across generations s.

In addition to the economic return, successful investment projects generate both vertical and horizontal technical innovation in period 2. Liquidated or abandoned projects do not produce any technological innovation. Vertical innovation improves the quality of already existing products, and increases the knowledge T of the economy. We assume, as Aghion et al. (2005a) does, that the knowledge accumulated by generation s is available to generation $s+1$, and that the creation of knowledge is proportional to the initial investment $T_s I$ of generation s. Thus, the dynamics of the knowledge T of the economy evolves

---

[1] The basic setup of this model is also used by Holmstrom and Tirole (2000).

[2] In our model, the concept of "time" refers to the evolution across generations s. At each moment of time s, there are three generations coexisting at the same time. This is a consequence of this being an overlapping generations model where agents live for three periods.

[3] We assume that all the firms and industries are of equal size, and therefore we skip the i and j indexes when using them is not essential.

[4] All the variables are expressed in proportion to T in order to guarantee a balanced growth path.

according to

$$\Delta T_{s+1} = \int_i vIT_s\ell_s^i, \tag{1}$$

where v is a vertical R&D productivity parameter and ℓ_s^i is an indicator variable equal to 1 if the liquidity shock of firm i has been financed and the project is successful, and 0 otherwise. Following the endogenous growth literature, the growth rate of the economy in this model is equal to the growth rate of knowledge T (see for example Aghion and Howitt (1998, ch. 2)). In terms of our representation of the growth process, this assumption in combination with the specific functional form of equation (1) imply that productivity growth is increasing in the level of productivity-enhancing investments. It is this characteristic that interlinks our growth model with the endogenous growth theory.[5] Note also that in this model, the economy is always at steady-state and, thus, at the balanced growth path.

Horizontal innovation creates new products (or industries), and is associated with increases in the total number of industries J over time. Thus, horizontal innovation implies an increase in the industry diversification of the economy. J evolves according to

$$\Delta J_{s+1} = \int_i hIJ_s\ell_s^i, \tag{2}$$

where h is a horizontal R&D productivity parameter and ℓ_s^i is an indicator variable equal to 1 if the liquidity shock has been financed and the project is successful, and 0 otherwise. From equations (1) and (2), it is clear that horizontal innovation does not directly affect the productivity of the economy. This assumption is based on Howitt (1999). In his model, the growth rate of the economy is not altered by the number of existing products because it is assumed that, as the number of products grow, the contribution of each vertical innovation with respect to any given product have a smaller impact on the aggregate economy. The role of horizontal innovation is to eliminate the "scale effects" generated by the growth of the population. As will become clearer in sections 4 and 5, the role of horizontal innovation in our model is to deepen the financial system. Specifically, an increased diversification (a larger J) improves the intermediary's chances to provide liquidity to firms in period 1.

The total output at time s is given by

$$Y_s = \int_i RT_{s-2}I\ell_{s-2}^i, \tag{3}$$

where ℓ_{s-2}^i is an indicator variable equal to 1 if the liquidity shock of firm i has been financed and the project is successful, and 0 otherwise. Note that the output at time s is the realized output of the investment projects undertaken by firms at time $s-2$.

---

[5]In other words, this paper assumes that economic growth is a consequence of technological progress, and does not build a fully specified endogenous growth model where the process of innovation is modelled.

Each firm has a period 0 endowment of cash, $T_s A > 0$, and no endowments in periods 1 and 2. In order to implement a project of scale $T_s I > T_s A$, the firm must borrow $T_s(I - A)$ from outside investors. In addition, it needs to finance the industry-level liquidity shock C_j in period 1. The firm uses the project's return in period 2 as collateral to obtain these loans. Investment projects are subject to moral hazard, as in Holmstrom and Tirole (1998), because each firm privately chooses the probability of success of the project after the continuation decision in period 1. The probability of success may be high (p_H) or low (p_L), conditional on the effort exerted by the firm, where $p_H - p_L \equiv \Delta p > 0$. If the firm exerts a low effort, it still enjoys a private benefit, $B T_s I > 0$, which is proportional to the initial investment.

For the investment to be profitable, the expected return of the project must exceed the initial investment plus the adjustment cost. Therefore, in period 1, the investment is continued if and only if the industry-level liquidity shock c_j is less or equal to $\tilde{c}$ $(c_j \leq \tilde{c})$, where $\tilde{c}$ is a certain threshold for which the investment has a positive net present value. We assume that the continuation condition holds only for p_H, but not for p_L, i.e. the project's net present value is positive only if the firm exerts a high effort. The positive NPV condition per unit of investment under industry-level liquidity shocks is

$$\max_{\tilde{c}}\{F(\tilde{c})p_H R - 1 - \int_0^{\tilde{c}} c_j f(c_j)dc_j\} > 0,$$

where $F(\tilde{c})p_H R$ is the expected gross return given that the firm exerts a high effort, $F(\tilde{c})$ is the probability that the industry-level liquidity shock c_j is less or equal to $\tilde{c}$, and $\int_0^{\tilde{c}} c_j f(c_j)dc_j$ is the expected value of the liquidity shock given that $c_j \leq \tilde{c}$. Note that the upper limit of integration is given by $\tilde{c}$. The reason is that projects with liquidity shocks above $\tilde{c}$ are abandoned, and thus have a liquidity demand equal to 0.

As explained above, firms need to get finance from outside investors, and therefore a contract between the parts must be set up. This loan agreement between the firm and outside investors has to specify the scale of the investment I, the payoffs to the parts and a "cutoff" threshold for the liquidity shock such that it is optimal to continue if and only if

$$c_j \leq c^*.$$

For ease of exposition, all the quantities are "detrended" from now on, i.e. they are divided by the current technology level T_s. Figure 1 presents a simplified account of the events at the firm-level and for the intermediary. The role of the intermediary is explained in section 3.

For the contract between the firm and outside investors to be optimal, it has to be designed so that the firm has incentives to exert a high effort. Further, the design must also take into account that outside investors have to break-even. Regarding the firm's incentive problem, the expected return that the firm obtains given a high effort must exceed the expected return it obtains given a low effort plus the private benefit. This implies that $p_H R_f(c_j) \geq p_L R_f(c_j) + B$,
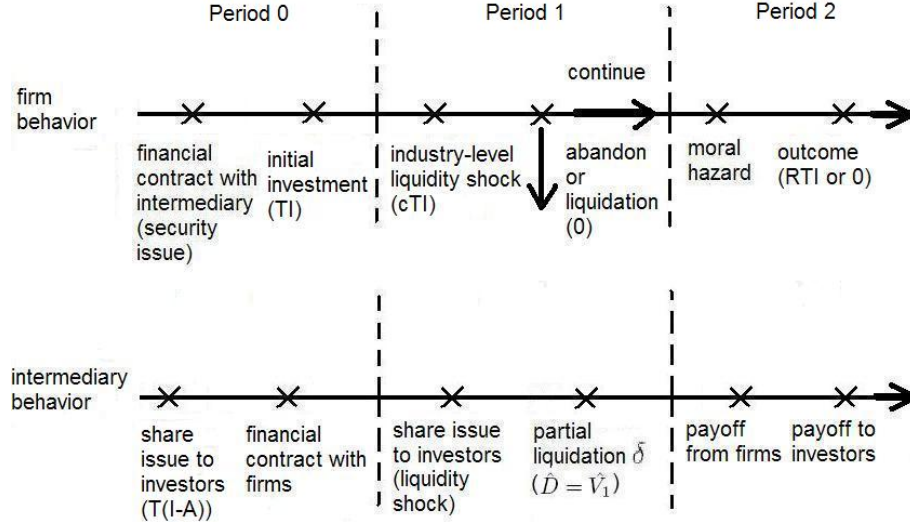
Figure 1: Firm and intermediary behavior

where $R_f(c_j)$ is the amount the firm earns if the project succeeds (given a liquidity shock c_j). Thus, the payoff to the firm that is consistent with its incentives to exert a high effort is

$$R_f(c_j) \geq R_b \equiv \frac{B}{\Delta p}. \tag{4}$$

Regarding outside investors, the payoff they receive if the project succeeds is $R - R_f(c_j)$, which is the return that is left after discounting the payoff to the firm. Thus, the payment to outside investors that is consistent with their break-even condition is

$$F(c^*)[p_H(R - R_f(c_j))]I \geq I - A + \int_0^{c^*} c_j f(c_j) dc_j I, \tag{5}$$

where the left hand side is the expected pledgable income, and the right hand side is the investors' period-0 outlay, $I - A$, plus the expected liquidity demand, $\int_0^{c^*} c_j f(c_j) dc_j I$. The expected pledgable income is given by the probability that the liquidity shock is equal or below c^*, $F(c^*)$, and what is left to outside investors given that the firm exerts a high effort, $[p_H(R - R_f(c_j))]I$. Note that by setting $R_f(c_j) = R_b$ in equations (4) and (5), the firm maximizes the amount that it can pay to outside investors (per unit of I), $c_p \equiv p_H(R - (B/\Delta p))$. We call c_p the "pledgable income" because we have assumed that the firm uses the return of its project in period 2 as collateral for obtaining the funds from outside investors.

Given this setup, the firm maximizes the return per unit of its own invest-
ment A by optimally choosing the amount to borrow from outside investors
and the optimal cutoff value. The amount borrowed from outside investors de-
termines the investment scale I of the project. The firm's objective function
is

$$
\begin{aligned}
U_b &= m(c^*)I \\
&= m(c^*)k(c^*)A,
\end{aligned}
\tag{6}
$$

where

$$
m(c^*) \equiv F(c^*)p_H R - 1 - \int_0^{c^*} c_j f(c_j) dc_j
$$

is the project's expected net return per unit of investment,

$$
I = k(c^*)A
\tag{7}
$$

is the investment scale, and

$$
\begin{aligned}
k(c^*) &= \frac{1}{1 + \int_0^{c^*} c_j f(c_j) dc_j - F(c^*)p_h(R - \frac{B}{\Delta p})} \\
&= \frac{1}{1 + \int_0^{c^*} c_j f(c_j) dc_j - F(c^*)c_p}
\end{aligned}
\tag{8}
$$

is the equity multiplier, which determines the maximum investment in period
0 that allows outside investors to break-even (the firm's "debt capacity"). The
debt capacity is maximal when the threshold c^* is equal to the unit expected
pledgeable income c_p, in which case $k(c_p) > 1$. This becomes clearer by integrat-
ing equation (8) by parts, which is done in in subsection A.1 of the appendix.

The maximization of the firm's objective function (6) is equivalent to mini-
mizing the expected unit cost $c(c^*)$ of effective investment:

$$
\min_{c^*} c(c^*) \equiv c^* + \frac{1 - \int_0^{c^*} F(c_j) dc_j}{F(c^*)}.
\tag{9}
$$

The formal proof of this equivalence is in subsection A.1 of the appendix. The
first order condition for (9) is

$$
\int_0^{c^*} F(c_j) dc_j = 1,
\tag{10}
$$

which implies that at the optimum, the threshold liquidity shock is equal to the
expected unit cost of effective investment:

$$
c(c^*) = c^*.
\tag{11}
$$

Thus, at the optimum the firm's net return is[6]

$$
U_b = \frac{c_r - c^*}{c^* - c_p} A,
\tag{12}
$$

---

[6]This result is easier to corroborate by considering equations (32) and (33), from subsection
A.1 in the appendix, in combination with equation (11).

where $c_r \equiv p_H R$ is the period 1 expected gross return per unit of investment, $c_p \equiv p_H(R - (B/\Delta p))$ is the period 1 pledgeable unit return from investment, and c^* is the optimal continuation threshold level. Furthermore, the optimal threshold c^* lies between the pledgeable income c_p and the expected gross return c_r:

$$c_p < c^* < c_r. \qquad (13)$$

This is a consequence of both the expected net return per unit of investment $m(c^*)$ and the equity multiplier $k(c^*)$ being decreasing above the expected gross return c_r, and increasing below the pledgeable income c_p. Condition (13) is consistent with definition (12) because if the optimal threshold c^* exceeds the expected gross return c_r, the project can not be financed profitably. Further, if the optimal threshold c^* is lower than the pledgeable income c_p, the debt capacity and the borrower's utility is infinite. Note that the optimal threshold c^* lies between the pledgeable income c_p and the expected gross return c_r, but does not depend on either of them. In addition, from equation (8) it is clear that at the optimum the investment scale I depends only on the pledgeable income c_p.

3   Intermediation and aggregate liquidity

In the preceding section, the basic setup of the model has been presented. We have characterized the aggregate behavior of the economy across time regarding technological knowledge and industry diversification. The behavior of firms regarding their incentives have been analyzed, and the optimal continuation threshold level have been established. In this section, we continue to characterize the economy by introducing the role of the intermediary in the economy. The aggregate demand and supply of liquidity is analyzed.

   We assume that there is no exogenously given storage technology so that wealth cannot be transferred from one period to the other through cash and/or private assets (such as real state). The only way to transfer wealth is through financial instruments, such as shares and/or securities.[7] In periods 0 and 1, the intermediary (a bank) issues shares to investors, which are claims on its financial position in period 2. These shares are priced so that investors break even ex ante. With the proceeds, the intermediary buys up all the external claims on firms (securities) in periods 0 and 1.[8] With the security issues, firms are able to finance their initial investment in period 0, and the industry-level liquidity shock in period 1.

   Concretely, in period 0, the intermediary issues shares to investors in order to lend $I - A$ to each firm for the initial investment. Also it agrees with firms on

---

[7]If intertemporal wealth transfers is possible through cash and/or private assets, there is no role for the intermediary, and no shortage of liquidity. We make this assumption, following Holmstrom and Tirole (1998), because we are interested in studying the endogenous supply of liquidity, and the role of the financial system in supplying liquidity.

[8]Another way to transfer wealth can be through the financial market, but as Holmstrom and Tirole (1998) demonstrate, firms are unable to finance liquidity shocks by individually issuing securities, and buying shares of a market portfolio.

an irrevocable line of credit in the amount c^*I to cover the period 1 industry-level liquidity shock. This agreement is conditional on the intermediary's ability to collect sufficient aggregate liquidity in period 1. Firms are priced so that the intermediary breaks even ex ante on each firm issue. In period 1, it sells shares to investors in the amount V_1, which reflects the total value of external claims on the aggregate conglomerate of firms, and if that amount is enough to finance the aggregate demand for liquidity $\bar{D}$, the intermediary can honor its promises to firms. Note that the intermediary pools firms risks, and subsidizes firms with a high liquidity demand by allowing them to draw on the market value of firms that experience a low liquidity demand. The ability to pool firms' risks is one of the two key features that characterize the intermediary. The second key attribute is discussed in section 4.

Assuming there is a continuum of firms with unit mass, the aggregate demand for liquidity in period 1 is

$$
\begin{aligned}
\bar{D} &= (\frac{c_1 J_1(c^*) + ... + c_J J_J(c^*)}{J})I \\
&= \frac{I}{J} \sum_{j=1}^{J} c_j J_j(c^*),
\end{aligned}
\tag{14}
$$

where I is the representative firm's initial investment scale, c_j is the liquidity shock for industry sector j, $J_j(c^*)$ is an indicator variable for industry j, which equals 1 if $c_j \leq c^*$, and 0 if $c_j > c^*$, and J is the number of existing industries. Note that only those firms with liquidity shocks below c^* continue with their investment project in period 1, which is the reason we have the indicator variable $J_j(c^*)$ for each industry j. The total value of external claims on the productive sector in period 1 is

$$
\begin{aligned}
V_1 &= \frac{\sum_{j=1}^{J} J_j(c^*)}{J} c_p I \\
&= F^J(c^*) c_p I,
\end{aligned}
\tag{15}
$$

where $F^J(c^*)$ is the observed fraction of firms with liquidity shock below the optimal threshold c^*, c_p is the pledgeable unit return from investment, $J_j(c^*)$ is the indicator variable for industry j used in equation (14), and J is the number of existing industries.

The intermediary is able to finance all firms as long as the value of external claims on the productive sector V_1 is larger than the aggregate demand for liquidity $\bar{D}$, i.e. the value of the investment portfolio $S_1 \equiv V_1 - \bar{D} > 0$. Using equations (14) and (15), the value of the investment portfolio S_1 is

$$
\begin{aligned}
S_1 &= V_1 - \bar{D} \\
&= \frac{I c_p}{J} \sum_{j=1}^{J} J_j(c^*) - \frac{I}{J} \sum_{j=1}^{J} c_j J_j(c^*) \\
&= \frac{I}{J} \sum_{j=1}^{J} (c_p - c_j) J_j(c^*),
\end{aligned}
\tag{16}
$$

10

where c_j is the liquidity shock for industry sector j, c_p is the pledgeable unit return from investment, $J_j(c^*)$ is the indicator variable for industry j, I is the investment scale, and J is the number of existing industries.

As a benchmark case, consider a completely diversified economy. When this is the case, $J \to \infty$ and the value of the investment portfolio S_1 is equal to $I - A$, which is positive by assumption. This follows from

$$
\begin{aligned}
\text{plim}_{J\to\infty} S_1 &= \text{plim}_{J\to\infty} V_1 - \text{plim}_{J\to\infty} \bar{D} \\
&= F(c^*)c_p I - \int_0^{c^*} c_j f(c_j) dc_j I \qquad (17) \\
&= I - A, \qquad (18)
\end{aligned}
$$

where the total value of external claims on the productive sector V_1 has $F(c^*)c_p I$ as its limit as $J \to \infty$ because $F^J(c^*)$ tends to $F(c^*)$ as $J \to \infty$. Note that $F(c^*)$ is both the ex ante probability that a given firm faces a liquidity shock c_j equal to or below the optimal threshold c^*, and the realized fraction of firms that continue in period 1 when $J \to \infty$. Regarding the aggregate demand for liquidity in period 1 $\bar{D}$, its limit as $J \to \infty$ is equal to $\int_0^{c^*} c_j f(c_j) dc_j I$ because $\sum_{j=1}^J c_j J_j(c^*)/J$ tends to $\int_0^{c^*} c f(c) dc$ as $J \to \infty$. Equation (17) becomes equation (18) by combining the investment scale definition from equation (7) with the equity multiplier definition from equation (8).

The expected value of the aggregate demand for liquidity $\bar{D}$ conditional on the industry-level liquidity shocks c_j being equal or less than the optimal threshold c^*, $E(\bar{D}|M)$ with $M = \{c_j \leq c^*\}$, is equal to the deterministic value $\int_0^{c^*} c_j f(c_j) dc_j I$. Further, $E(V_1|M)$ is equal to the deterministic value $F(c^*)c_p I$. This analysis implies that the expected value of the value of the investment portfolio S_1 conditional on the industry-level liquidity shocks c_j being equal or less than the optimal threshold c^*, $E(S_1|M)$, is equal to the positive value $I - A$, as in equation (18). This result is important for the discussion of the next section, where we analyze the relationship between the degree of diversification and partial liquidation when there is an aggregate liquidity shortage, i.e. when $S_1 < 0$. The distribution function of S_1 has a central role in this discussion.

4 Diversification and partial liquidation

As seen in the last section, when the economy is completely diversified ($J \to \infty$), the value of the investment portfolio S_1 is positive and equal to $I - A$. Thus, there is no aggregate liquidity shortage, and all the investment projects with liquidity shocks below the optimal threshold c^* receive funding from the intermediary. When the economy is not completely diversified, it is no longer true that the value of the investment portfolio $S_1 = I - A > 0$, and S_1 may be negative. If that is the case, the intermediary needs to exercise partial liquidation because the aggregate demand for liquidity is greater than what it can collect from investors, i.e. there is an aggregate shortage of liquidity. Partial

11

liquidation implies that only the fraction δ of firms are allowed to continue in period 1. Note that we are assuming that partial liquidation is only possible at the industry level, and not at the firm level, i.e. the scale of an individual project cannot be reduced.[9] The ability to exercise partial liquidation is the second key attribute of the intermediary, beside its ability to pool firms' risks, as explained in section 3.

In terms of the concrete implementation of partial liquidation, in period 1, after the liquidity shocks and the values of V_1 and $\bar{D}$ are realized, the intermediary decides which firms to liquidate. The aggregate demand for liquidity after partial liquidation becomes

$$\hat{D} = \frac{I}{J} \sum_{j=1}^{J} c_j J_j(c^*) L_j(S_1), \tag{19}$$

where I is the investment scale, J is the number of existing industries, c_j is the liquidity shock for industry sector j, $J_j(c^*)$ is an indicator variable for industry j, which equals 1 if $c_j \leq c^*$, and 0 if $c_j > c^*$, and $L_j(S_1)$ is an indicator variable for industry j, which equals 0 if the intermediary decides that industry j should be liquidated, and 1 otherwise.[10] $L_j(S_1)$ is a variable that depends on the realized value of S_1 because as explained above partial liquidation is only relevant when there is an aggregate liquidity shortage, i.e. $S_1 < 0$. The total value of external claims on the productive sector in period 1 after partial liquidation becomes

$$
\begin{aligned}
\hat{V}_1 &= \frac{\sum_{j=1}^{J} L_j(S_1)}{J} \frac{\sum_{j=1}^{J} J_j(c^*)}{J} c_p I \\
&= \delta F^J(c^*) c_p I, \tag{20}
\end{aligned}
$$

where $\delta = \sum_{j=1}^{J} L_j(S_1)/J$ is the fraction of firms that are liquidated by the intermediary, $F^J(c^*)$ is the observed fraction of firms with liquidity shock below the optimal threshold c^*, c_p is the pledgeable unit return from investment, and I is the investment scale. Note that δ is a variable that adopts values between 0 and 1, and depends on the realized value of the investment portfolio S_1. δ is a positive function of the value of the investment portfolio S_1 because the more negative S_1 becomes, the smaller the fraction of firms that can continue. When S_1 is positive, there is no aggregate shortage of liquidity, and there is no need for partial liquidation, i.e. $\delta = 1$. As seen in section 3, this is always the case when the economy is completely diversified. Figure 2 presents graphically the relationship between S_1 and δ.

---

[9]If the scale of an individual project could be partially liquidated, there would be no role for the intermediary, and firms would be able to finance the liquidity shock by issuing securities directly to investors (Holmstrom and Tirole, 1998).

[10]Note, however, that although all the firms of the same industry have the same liquidity shock and liquidity needs, it is not necessary to liquidate the whole industry. If only a fraction of the firms of industry j are liquidated, $L_j(S_1)$ may assume a value between 0 and 1, instead of 1, which represents the fraction of firms of industry j that are allowed to continue.

The value of the investment portfolio after partial liquidation $\hat{S}_1 \equiv \hat{V}_1 - \hat{D}$ must be zero. Thus, the intermediary decides which firms to liquidate so that $\hat{S}_1 = 0$, i.e. there is no aggregate liquidity shortage. Note that the intermediary's decision to liquidate industry j $(L_j(S_1) = 0)$ affects negatively both the value of the aggregate demand for liquidity after partial liquidation in equation (19) and the total value of external claims on the productive sector after partial liquidation in equation (20). Therefore, the intermediary optimally implements partial liquidation by not financing the liquidity shocks of those industries that have been hit by the largest liquidity shocks. Then, $\hat{D}$ decreases at a higher speed than $\hat{V}_1$, which eventually makes them become equal, i.e. $\hat{S}_1 = 0$. Note also that although all the firms of the same industry have the same liquidity demand, it is not necessary to liquidate the whole industry if that implies that $\hat{S}_1 > 0$. In this case, it is optimal for the intermediary to allow some of the firms in this industry to continue.[11] Figure 1 presents a simplified account of the events at the firm-level and for the intermediary.
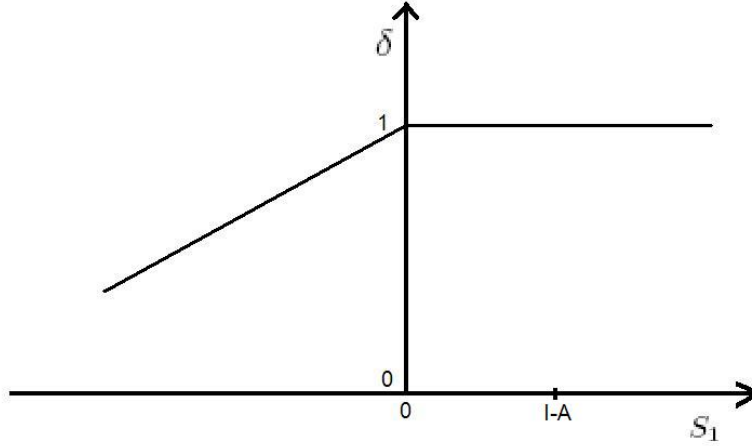

Figure 2: Relationship between S_1 and δ

Now that we have established what happens after the realization of the liquidity shocks, we go back a step and study the distribution function of S_1 before the realization of the liquidity shocks. This analysis will clarify the relationship between the value of the investment portfolio S_1 and the fraction of firms allowed to continue in period 1 δ. Concretely, we next study how the number of industries J affects the distribution function of the value of the investment portfolio S_1. To do so, we first have to analyze the expected value of the value of the investment portfolio S_1, its variance, and how the number

---

[11]It can be assumed that the firms that continue from this industry are drawn randomly.

of industries J affects its variance. Recall that the number of industries J is also equal to the number of independently and identically distributed liquidity shocks c_j hitting the economy.

As analyzed in section 3, the expected value of the value of the investment portfolio S_1 conditional on the liquidity shocks being below the optimal threshold c^* is the deterministic positive amount $I - A$. To study the conditional variance of S_1, we redefine the definition of S_1 in equation (16) as follows

$$
\begin{aligned}
S_1 &= \frac{I}{J} \sum_{j=1}^{J} (c_p - c_j) J_j(c^*) \\
&= \frac{I}{J} \sum_{j=1}^{J} w_j,
\end{aligned}
\tag{21}
$$

where $w_j = (c_p - c_j) J_j(c^*)$ is a new variable. Moreover, since c_p is a constant, $0 \le J_j(c^*) \le 1$, and c_j has a finite variance, we conclude that w_j also has a finite variance σ_*^2. Note that w_j has the same finite variance σ_*^2 for all j because we have assumed that the liquidity shocks c_j are independently and identically distributed. Using the new definition of S_1 from equation (21), its conditional variance is

$$
\begin{aligned}
Var(S_1|M) &= Var(\frac{I}{J} \sum_{j=1}^{J} w_j | M) \\
&= \frac{I^2}{J^2} \sum_{j=1}^{J} \sigma_*^2 \\
&= \frac{I^2}{J} \sigma_*^2,
\end{aligned}
\tag{22}
$$

which implies that $Var(S_1|M) \to 0$ as $J \to \infty$. Moreover, $Var(S_1|M)$ is decreasing in J, i.e. the conditional variance of the value of the investment portfolio S_1 becomes smaller, the more diversified the economy is.

The above analysis implies that the probability density function of the value of the investment portfolio S_1 is centered on $I - A > 0$. Further, the density function has a spread that is decreasing in J, i.e. the density function is more spread when the economy is less diversified. Thus, the weight in the tails increases when the economy is less diversified, which also means that variable S_1 becomes more variable or risky (Rothchild and Stiglitz, 1970). This, in turn, implies that the $Prob(S_1 < 0|M)$ is larger when J is smaller (see Figure 3). Note also that the density function collapses to $I - A > 0$ when J reaches infinity.

The expected value of the fraction of firms allowed to continue in period 1 $E(\delta|N)$, where $N = \{\hat{S}_1 = 0\}$ means that partial liquidation has eliminated
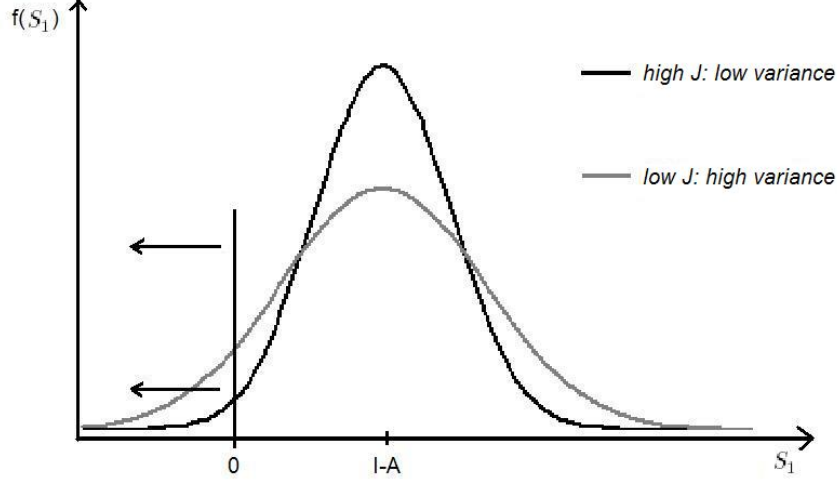
Figure 3: Probability density function of S_1

aggregate liquidity shortage, is

$$
\begin{aligned}
E(\delta|N) &= E\left(\frac{\sum_{j=1}^{J} L_j(S_1)}{J}\bigg|M\right) \\
&= E(L_j(S_1)|M) \\
&= Prob(L_j(S_1) = 1|N),
\end{aligned}
\tag{23}
$$

where $Prob(L_j(S_1) = 1|N)$ is the probability that $L_j(S_1)$ is equal to 1, i.e. the probability that industry j is not liquidated by the intermediary. Note that the probability that industry j is not liquidated by the intermediary $Prob(L_j(S_1) = 1|N)$ is negatively related with the probability that the value of the investment portfolio S_1 is negative $Prob(S_1 < 0|M)$. The reason is that the larger the probability that there be an aggregate liquidity shortage due to S_1 being negative, the smaller is the probability that a certain industry j is not liquidated by the intermediary. Combining the fact that there is a negative relationship between $Prob(L_j(S_1) = 1|N)$ and $Prob(S_1 < 0|M)$, and that $Prob(S_1 < 0|M)$ is larger when J is smaller, implies that $E(\delta|N)$ is a positive function of J. The expected fraction of firms that continue $E(\delta|N)$ is higher when the economy is more diversified because there is a higher probability that the financial intermediary is able to collect enough liquidity to meet the aggregate demand for liquidity. This is a key result for the analysis in section 5.

15

5   Diversification and growth

The aggregate return in period 2 for generation s is $\delta_s F_s^J(c^*) p_H R T_s I$, where δ_s is the fraction of firms that are not liquidated by the intermediary, $F_s^J(c^*)$ is the fraction of firms that are hit by liquidity shocks below the optimal threshold c^*, p_H is the fraction of successful projects, R is the gross return, and $T_s I$ is the initial investment. Note that the aggregate return in period 2 with partial liquidation becomes $\delta_s F_s^J(c^*) p_H R T_s I$, which is at most equal to the aggregate return without partial liquidation, $F_s^J(c^*) p_H R T_s I$, because $0 \leq \delta \leq 1$. Note also that we have included the s subscript to emphasize the fact that the realizations of δ and $F^J(c^*)$ differ across generations. The ultimate reason being that each generation s suffers different realizations of the liquidity shocks c_j. Recall also that each generation s is hit by a total of J liquidity shocks in period 1.

From equation (1), the growth rate of the economy due to vertical innovation is

$$\frac{\Delta T_{s+1}}{T_s} = \delta_s F_s^J(c^*) p_H v I, \tag{24}$$

where the integral and the indicator variable ℓ_s^i in equation (1) have been replaced by $\delta_s F_s^J(c^*) p_H$ in equation (24), which is the fraction of firms that have financed the liquidity shocks c_j and have finished successfully the investment projects, i.e. the fraction of firms for which the indicator variable ℓ_s^i equal 1. Note that this result implies that fluctuations across generations arise because the fraction of firms fulfilling their investment projects varies for each generation s. As noted above this is a consequence of different realizations of δ and $F^J(c^*)$ for each generation s. Interestingly, as our explanation of fluctuations is not dependent on the assumption made regarding the return of the projects, and their riskiness, it is consistent with the findings of Koren and Tenreyro (2005). Recall from the introduction that Koren and Tenreyro (2005) finds that underdeveloped countries invest in highly risky projects, which contradicts the assumption made by Acemoglu and Zilibotti (1997), which is essential for their explanation of fluctuations. Note also that in our model, the economy is always on the steady-state, and therefore on a balanced growth path, i.e. fluctuations are not a consequence of departures from the steady-state.

In order to study how the expected growth rate of the economy due to vertical innovation, and its variance, is affected by the degree of industry diversification (the total number of industries J), we redefine equation (24) as follows

$$\begin{aligned}
\frac{\Delta T_{s+1}}{T_s} &= \frac{\sum_{j=1}^{J} L_j(S_1)}{J} \frac{\sum_{j=1}^{J} J_j(c^*)}{J} p_H v I \\
&= \frac{\sum_{j=1}^{J} H_j(c^*, S_1)}{J} p_H v I,
\end{aligned} \tag{25}$$

where $H_j(c^*, S_1) = L_j(S_1) J_j(c^*)$ is a new indicator variable for industry j that assumes the value 1 if $c_j \leq c^*$ and industry j is not liquidated by the intermediary, and 0 otherwise. Note that the expected value of $H_j(c^*, S_1)$, $E(H_j(c^*, S_1)|N)$, is equal to $Prob(H_j(c^*, S_1) = 1|N)$ for all j, which, as $Prob(L_j(S_1) = $

16

$1|N)$, has a negative relationship with $Prob(S_1 < 0|M)$ (see section 4). Therefore, $Prob(H_j(c^*, S_1) = 1|N)$ is a positive function of J, i.e. $Prob(H_j(c^*, S_1) = 1|N)$ is larger, the more diversified the economy is.

Using equation (25), the expected growth rate of the economy due to vertical innovation is

$$E(\frac{\Delta T_{s+1}}{T_s}|N) = \frac{p_H vI}{J} \sum_{j=1}^{J} E(H_j(c^*, S_1)|N)$$
$$= p_H vI Prob(H_j(c^*, S_1) = 1|N), \qquad (26)$$

where $N = \{\hat{S}_1 = 0\}$ means that partial liquidation has eliminated aggregate liquidity shortage, and $Prob(H_j(c^*, S_1) = 1|N)$ is increasing in the number of industries J. Therefore, $E(\frac{\Delta T_{s+1}}{T_s}|N)$ is also increasing in the number of industries J, i.e. the expected growth rate of the economy is higher when industry diversification is higher. The reason is that a higher industry diversification implies a higher probability that industry j is not liquidated by the intermediary due to a shortage of aggregate liquidity. Thus, a larger fraction of firms are able to finish successfully with their investment projects, and there is more vertical innovation in the economy.

In other words, economies that have higher industry diversification, have also deeper financial systems, and thus have a higher probability of being able to finance investment projects when there are shocks in the economy. The higher probability of successfully financing investment projects implies that more investment projects produce vertical innovation, and thus the expected growth rate of the economy is higher. The result that the expected growth rate is increasing with the degree of financial development is in line with the conclusions of Aghion et al. (2005a) and Acemoglu and Zilibotti (1997). Note also that when the economy is perfectly diversified, and there is no aggregate liquidity shortage, the growth rate of the economy due to vertical innovation tends to the deterministic balanced-growth equilibrium

$$\text{plim}_{J \to \infty} \frac{\Delta T_{s+1}}{T_s} = F(c^*) p_H vI, \qquad (27)$$

because $\delta_s F_s^J(c^*) \to F(c^*)$ as $J \to \infty$.

The variance of the growth rate of the economy is

$$Var(\frac{\Delta T_{s+1}}{T_s}|N) = (p_H vI)^2 Var(\delta_s F_s^J(c^*)|N)$$
$$= (\frac{p_H vI}{J})^2 \sum_{j=1}^{J} Var(H_j(c^*, S_1)|N)$$
$$= \frac{(p_H vI)^2}{J} Prob(H_j(c^*, S_1) = 1|N)(1 -$$
$$Prob(H_j(c^*, S_1) = 1|N)), \qquad (28)$$

where $Prob(H_j(c^*, S_1) = 1|N)(1 - Prob(H_j(c^*, S_1) = 1|N))$ is the variance of $H_j(c^*, S_1)$ for all j. From equation (28), there are two forces that have to be considered in order to analyze how the variance of the growth rate is related to the total number of industries J (or the degree of industry diversification). On one side, the variance of the growth rate is linearly decreasing in J due to the direct effect of J being in the denominator. On the other side, the variance of the growth rate is a quadratic concave function of J due to the effect of the variance of $H_j(c^*, S_1)$, which for lower values of J is increasing in J, and for higher values of J is decreasing in J. The reason is that $Prob(H_j(c^*, S_1) = 1|N)$ is increasing in J, and therefore the variance of $H_j(c^*, S_1)$, $Prob(H_j(c^*, S_1) = 1|N)(1 - Prob(H_j(c^*, S_1) = 1|N))$, is a quadratic concave function of J. When $Prob(H_j(c^*, S_1) = 1|N) < 0, 5$, which is the case for lower values of J, the variance of $H_j(c^*, S_1)$ is increasing in J. However, when $Prob(H_j(c^*, S_1) = 1|N) > 0, 5$, which is the case for higher values of J, it is decreasing in J.

The overall effect of the number of industries J (industry diversification) on the variance of the growth rate is that it is increasing in J for low levels of J, but strictly decreasing for higher levels of J. In other words, the variance of the growth rate is initially, for low levels of industry diversification (or financial development), increasing with industry diversification. For intermediate and high levels of industry diversification (or financial development), the variance is strictly decreasing with industry diversification. Figure 4 present a graphic example of the relationship between the variance of the growth rate of the economy and the total number of industries J (industry diversification). This ambiguous effect of financial development on the variance of the growth rate is in line with the results of Aghion et al. (2005a), Acemoglu and Zilibotti (1997), and Carranza and Galdon-Sanchez (2004). Note also that when the economy is perfectly diversified, and there is no aggregate liquidity shortage, the variance of the growth rate of the economy due to vertical innovation tends to zero.

The growth rate of the number of industries J in the economy due to horizontal innovation is given by equation (2), which becomes

$$\frac{\Delta J_{s+1}}{J_s} = \delta_s F_s^J(c^*) p_H h I. \tag{29}$$

Combining equation (29) with the new indicator variable $H_j(c^*, S_1)$, as in equation (25), the expected growth rate of the number of industries J is

$$\begin{aligned} E(\frac{\Delta J_{s+1}}{J_s}|N) &= \frac{p_H h I}{J} \sum_{j=1}^{J} E(H_j(c^*, S_1)|N) \\ &= p_H h I Prob(H_j(c^*, S_1) = 1|N). \tag{30} \end{aligned}$$

Clearly, an increase in $Prob(H_j(c^*, S_1) = 1|N)$, increases the growth rate of industries J, which makes the economy more diversified. Again, and as discussed in this section, $Prob(H_j(c^*, S_1) = 1|N)$ depends positively on J, which implies that economies that are more diversified have higher expected growth rate of industries J. The reason is that higher levels of industry diversification imply
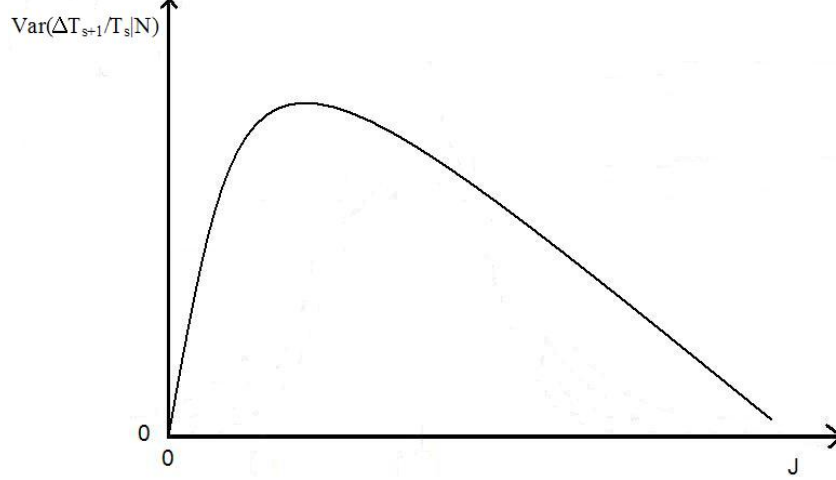
Figure 4: Variance of the growth rate of the economy

that the financial system is more developed, and have higher chances of successfully providing liquidity to firms when shocks occur. This, in turn, imply that more firms are able to complete their investment projects and produce horizontal innovation. Note also that when the economy is perfectly diversified, and there is no aggregate liquidity shortage, the growth rate of horizontal innovation tends to the deterministic balanced-growth equilibrium

$$\text{plim}_{J\to\infty}\frac{\Delta J_{s+1}}{J_s} = F(c^*)p_H hI, \tag{31}$$

because $\delta_s F_s^J(c^*) \to F(c^*)$ as $J \to \infty$.

From equation (30), it is clear that horizontal innovation has a reinforcing effect on itself. The reason is that the higher J is at present, the more horizontal innovation there will be in the future due to a higher $Prob(H_j(c^*, S_1) = 1|N)$. A higher $Prob(H_j(c^*, S_1) = 1|N)$, in turn, implies that J will have an even higher growth rate in the future. Thus, a high initial J imply that the growth rate of J in the future is higher than it would be if the initial J is low. Clearly, countries that are more diversified become even more diversified at higher speeds than countries that are less diversified. In other words, countries with initially high levels of industry diversification, and thus high levels of financial development, enjoy faster industry diversification and financial development than countries with initially low levels of industry diversification (and thus low levels of financial development).

The reinforcing effect of horizontal innovation does not only increase the future expected growth rate of horizontal innovation (industry diversification), but

19

also increases the speed at which the financial system develops, i.e it improves the probability of the financial system to finance liquidity shocks. This effect on the financial system implies that horizontal innovation enhances, indirectly, the future growth rate of vertical innovation. This follows directly from the positive relationship between $Prob(H_j(c^*, S_1) = 1|N)$ and J, and equation (26). Thus, a high initial level of industry diversification does not only imply that current expected growth rates of vertical and horizontal innovation are higher than if the initial level of industry diversification was low, but also that the future expected growth rates of vertical and horizontal innovation will be even higher than the current ones. Note that although the expected growth rates of vertical and horizontal innovation tends to increase as the economy becomes more diversified, the expected growth rates tend to the balanced-growth equilibriums $\Delta T_{s+1}/T_s = F(c^*)p_H vI$ (equation (27)) and $\Delta J_{s+1}/J_s = F(c^*)p_H hI$ (equation (31)), respectively, which are the growth rates of a perfectly diversified economy.

In this model, horizontal innovation produces an externality through its reinforcing effect on itself, and its effects on the financial system. A high level of industry diversification implies that the current growth rate of the economy is higher, and less volatile, than it would be if industry diversification was low, but also that the future growth rates are going to be higher than the current one. This externality implies that current lucky countries, in terms of getting low liquidity shocks, and thus less aggregate liquidity shortage, will benefit even in the future by having higher, and less volatile, growth rates of the economy. Consider, for example, two countries that have the same level of industry diversification J. One of the countries, however, is more lucky than the other in terms of getting lower liquidity shocks for a number of periods. Then, the lucky country ends up having a higher, and less volatile, growth rate of the economy than the unlucky country even in the future. This result is in line with the theoretical model developed by Acemoglu and Zilibotti (1997).

Regarding government intervention, it is clear from the model that there is a role for the government to subsidize vertical and horizontal innovation. This result is in line with Aghion and Howitt (1998) and Howitt (1999) among others. In our model, a government subsidy means that the government provides additional liquidity to firms in period 1. Government intervention is especially relevant when there is an aggregate shortage of liquidity, i.e. when the intermediary cannot collect enough liquidity to finance all the profitable projects. As seen in section 4, an aggregate shortage of liquidity leads to partial liquidation. In this case, the provision of additional liquidity by the government in period 1 lowers the need for partial liquidation. Thus, the fraction of liquidated firms becomes lower than would be the case without intervention. Note that if there is no aggregate liquidity shortage, government intervention does not lead to a better outcome relative to the pure market outcome. Holmstrom and Tirole (1998) analyzes thoroughly the demand for and supply of government-supplied liquidity when there is a shortage of aggregate liquidity.

The reason that the government can provide additional liquidity, when the intermediary is unable to obtain this liquidity, is that the government can use its future tax revenues as collateral (see for example Holmstrom and Tirole (1998)).

20

The intermediary can only collect liquidity if it has an asset to put as collateral. In our model, this was the case when the value of the investment portfolio S_1 is positive. The government, instead, can always commit future tax revenues because it has the legal right to collect taxes, and can physically punish (jail, bankruptcy, etc) those that do not pay taxes.

The consequence of government intervention, when there is an aggregate shortage of liquidity, is that a lower fraction of firms are liquidated, and more investment projects are completed. This means that a government subsidy to vertical innovation implies a higher growth rate of the economy. Moreover, a subsidy to vertical innovation in this setting reduces the fluctuation of the growth rate across generations s. Thus, subsidies to vertical innovation can be used as a policy instrument in a stabilization strategy. A subsidy to horizontal innovation produces a higher industry diversification than the pure market outcome. Further, through the effect of industry diversification on the financial system, a horizontal subsidy leads to higher, and less volatile, growth rates of the economy in the future. Thus, subsidies to horizontal innovation can be used as a policy instrument to avoid future fluctuations in the economy. Due to the externality produced by industry diversification, subsidizing horizontal innovation is particularly beneficial for countries at initial and intermediate stages of financial development compared with subsidizing vertical innovation. The reason is that a subsidy to horizontal innovation increases permanently the expected growth rate of the economy through its effect on the financial system. In contrast, a subsidy to vertical innovation produces only a temporary increase in the growth rate of the economy. Note also that government intervention is especially suited for countries at initial and intermediate stages of financial development because in these stages there is a larger probability of getting an aggregate shortage of liquidity.

6   Concluding remarks

This paper presents a theoretical model where the financial system develops endogenously and has a central role in determining the growth rate of the economy, and its volatility. In the model, the productive sector is engaged in both vertical and horizontal innovation, but has to finance liquidity shocks for these innovations to be successful. Economic growth is determined by vertical innovation, which improves the quality of already existing goods. Horizontal innovation, on the other hand, does not affect economic growth directly, but produces new goods, which increases industry diversification. Industry diversification deepens the financial system because it improves the probability of the financial system in providing liquidity to the productive sector. Fluctuations across time arise because the fraction of firms fulfilling their investment projects at each period of time varies. The financial system has two key attributes that makes it especially suited for providing liquidity to the productive sector. The first is its ability to pool firms' risks, and the second is its ability to exercise partial liquidation at the industry level.

The main results of this paper are summarized as follows. Industry diversification is the main factor behind financial development. Thus, horizontal innovation has a central role in explaining financial development as part of the growth process. The expected growth rate of the economy due to vertical innovation is positively related to the level of industry diversification, and thus to the level of financial development. The volatility of the growth rate of the economy is initially increasing with the level of industry diversification, but becomes decreasing at intermediate and high stages of industry diversification. The growth rate of industry diversification due to horizontal innovation is positively associated with the level of industry diversification, and thus to the level of financial development. Industry diversification produces an externality through its effect on the financial system in the sense that a high initial level of industry diversification does not only imply high current growth rates of the economy and industry diversification, but also increasing growth rates in the future due to a deeper financial system. The implication of this externality is that, given the same initial level of industry diversification and financial development, current lucky countries, in terms of getting low liquidity shocks, benefit even in the future by having higher growth rates than the unlucky countries.

In this model, there is a role for the government to subsidize vertical and horizontal innovation when the financial system is unable to provide liquidity to all firms. Government subsidies to vertical innovation lead to a higher growth rate of the economy than would be possible without government intervention. They also mitigate fluctuations in the growth rate across time, serving as a policy instrument in a stabilization strategy. Subsidies to horizontal innovation entail a higher industry diversification, and thus financial development. Thus, they lead to higher, and less volatile, growth rates of the economy in the future. They may be used as policy instruments to avoid future fluctuations in the economy. Due to the externality generated by industry diversification on the financial system, subsidizing horizontal innovation, in contrast to vertical innovation, is particularly beneficial for countries at initial and intermediate stages of industry diversification and financial development. Furthermore, government intervention is especially suited for countries at initial and intermediate stages of financial development because in these stages there is a larger probability of getting an aggregate shortage of liquidity.

A   Appendix

A.1   Proof of equivalence of equations (6) and (9)

From equation (6), we have that

$$U_b = \frac{F(c^*)p_H R - 1 - \int_0^{c^*} cf(c)dc}{1 + \int_0^{c^*} cf(c)dc - F(c^*)c_p} A.$$

Multiplying equation (6) by $F(c^*)/F(c^*)$ and rearranging, we get

$$U_b = \frac{p_H R - \frac{1+\int_0^{c^*} c f(c) dc}{F(c^*)}}{\frac{1+\int_0^{c^*} c f(c) dc}{F(c^*)} - c_p} A. \tag{32}$$

Maximizing equation (32) is clearly equivalent to minimizing

$$c(c^*) = \frac{1 + \int_0^{c^*} c f(c) dc}{F(c^*)}, \tag{33}$$

which is the expected unit cost of effective investment. Moreover, if equation (33) is integrated by parts, we obtain

$$
\begin{aligned}
c(c^*) &= \frac{1 + c^* F(c^*) - \int_0^{c^*} F(c) dc}{F(c^*)} \\
&= c^* + \frac{1 - \int_0^{c^*} F(c) dc}{F(c^*)},
\end{aligned}
$$

which is what is minimized in equation (9). Q.E.D.

References

Acemoglu, D. and Zilibotti, F. (1997). Was prometheus unbound by chance? risk, diversification, and growth. *Journal of Political Economy*, 105(4):709–51.

Aghion, P., Angeletos, G.-M., Banerjee, A., and Manova, K. (2005a). Volatility and growth: Credit constraints and productivity-enhancing investment. NBER Working Papers 11349.

Aghion, P. and Howitt, P. (1998). *Endogenous growth theory.* MIT Press, Cambridge, MA.

Aghion, P., Howitt, P., and Mayer-Foulkes, D. (2005b). The effect of financial development on convergence: Theory and evidence. *The Quarterly Journal of Economics*, 120(1):173–222.

Bhattacharya, S., Bout, A., and Thakor, A. (2004). *Credit, intermediation, and the macroeconomy.* Oxford University Press.

Carranza, L. and Galdon-Sanchez, J. (2004). Financial intermediation, variability and the development process. *Journal of Development Economics*, 73:27–54.

Gancia, G. and Zilibotti, F. (2005). Horizontal innovation in the theory of growth and development. In P. Aghion and S. Durlauf, eds., *Handbook of Economic Growth*, North-Holland, 2005.

Holmstrom, B. and Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, 106(1):1–40.

Holmstrom, B. and Tirole, J. (2000). Liquidity and risk management. *Journal of Money, Credit and Banking*, 32(3):295–319.

Howitt, P. (1999). Steady endogenous growth with population and r&d inputs growing. *Journal of Political Economy*, 107:715–730.

Koren, M. and Tenreyro, S. (2005). Volatility and development. CEP Discussion Paper No. 706, LSE.

Levine, R. (2005). Finance and growth: Theory and evidence. In P. Aghion and S. Durlauf, eds., *Handbook of Economic Growth*, North-Holland, 2005.

Rothchild, M. and Stiglitz, J. E. (1970). Increasing risk i: A definition. *Journal of Economic Theory*, 2(3):225–43.