Design of Optimal Low-Order Feedforward Controllers

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Published in:
2nd IFAC Conference on Advances in PID Control 2012

2012

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Abstract: Design rules for optimal feedforward controllers with lead-lag structure in the presence of measurable disturbances are presented. The design rules are based on stable first-order models with time delays, FOTD, and are optimal in the sense of minimizing the integrated-squared error. The rules are derived for an open-loop setting, considering a step disturbance. This paper also discusses a general feedforward structure, which enables decoupling in the design of feedback and feedforward controllers, and justifies the open-loop setting.

Keywords: Feedforward design, optimal control, load-disturbance rejection, lead-lag filter.

1. INTRODUCTION

Feedforward is an efficient way to reduce control errors both for reference tracking and disturbance rejection, given that the disturbances acting on the system are measurable. This paper treats the subject of disturbance rejection. Due to model uncertainties, feedforward cannot eliminate the disturbance and it is therefore often used along with feedback control.

For the design of feedback controllers a large number of design methods exists. For design of PID-controllers there exists a large number of analytical methods for choosing the control parameters, see e.g., (Åström and Hägglund, 2004), (Skogestad, 2003) or (Ziegler and Nichols, 1942). However, there seems to be a lack of simple methods for tuning feedforward controllers.

The design of low-order feedforward controllers has previously been addressed by e.g., (Isaksson et al., 2008) and (Guzmán and Hägglund, 2011). (Isaksson et al., 2008) proposes an iterative design procedure, to minimize a system norm in the frequency domain, that takes the feedback controller into account. (Guzmán and Hägglund, 2011) provides simple tuning rules for feedforward controllers, taking the feedback controller into account, in order to reduce the integrated absolute error, IAE.

This paper presents an analytic solution to the problem of designing a feedforward lead-lag filter which minimizes the integrated square error when the system is subjected to a measurable step disturbance. The design rules are derived for FOTDs. The resulting feedforward controller is optimal in an open-loop setting. In general, feedforward controllers should be designed taking the feedback controller into account since they interact.

In (Brosilow and Joseph, 2002) a feedforward structure that separates the feedback and feedforward control design, was presented. This idea has been adopted in this paper and justifies that the designed controller, while optimal in the open-loop case, gives good performance when used in conjunction with feedback control. This structure makes use of the same process models that is used for the design of the feedforward controller. The structure have similarities with Internal Model Control, IMC, see (Garcia and Morari, 1982). Robust feedforward design within the IMC framework has addressed by (Vilanova et al., 2009).

2. FEEDFORWARD STRUCTURE

This section describes different structures for feedforwarding from measurable disturbances. Firstly, the most common open-loop and closed structures are discussed. Secondly, a feedforward structure that separates the design of feedforward and feedback controllers, as presented in (Brosilow and Joseph, 2002) is discussed.

2.1 Open-Loop Behavior

Consider the open-loop structure in Fig. 1 where d is the measurable disturbance, y is the system output and u is the system input. The transfer function from d to y is given by

$$G_o(s) = P_2(s)\big(P_3(s) - P_1(s)G_{ff}(s)\big). \tag{2.1}$$

In order to eliminate the effect of the disturbance d the feedforward controller should be chosen as $G_{ff}(s) = P_3(s)P_1^{-1}(s)$. This controller is not always possible or desirable to realize, as e.g., the order of $P_1(s)$ is greater than the order of $P_3(s)$, the time delay of $P_1(s)$ is greater than the time delay of $P_3(s)$ or if $P_1(s)$ has zeros in the right-half plane.

2.2 Feedforward - Feedback Interaction

Compensating for a measurable disturbance using only an open-loop feedforward structure is seldom desirable. Due

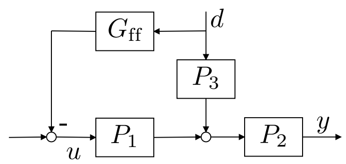

Fig. 1. Closed-loop structure.

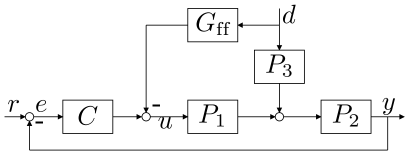

Fig. 2. Closed-loop structure.

to model errors and unmeasurable disturbances a feedback controller is needed. Connecting a feedback controller $C(s)$, see Fig. 2, renders the following transfer function from d to y,

$$G_{\text{cl}}(s) = \frac{P_2(s)\,(P_3(s) - P_1(s)G_{\text{ff}}(s))}{1 + P_2(s)P_1(s)C(s)}. \tag{2.2}$$

When it is possible to realize perfect feedforward $G_{\text{ff}} = P_3(s)P_1^{-1}(s)$ no problems will arise since (2.2) will be zero. However, when the perfect feedforward is not realizable the closed-loop behavior will differ from the open-loop behavior given by (2.1). Ways of modifying the feedforward controller in order to get a satisfying system response from the closed-loop system has been presented in (Isaksson et al., 2008) and (Guzmán and Hägglund, 2011).

2.3 Non-Interacting Feedforward Structure.

In (Brosilow and Joseph, 2002) a feedforward structure, equivalent to the one in Fig. 3, was presented. Dropping the argument s, the transfer function from d to y is given by

$$G_{\text{cl}} = \frac{P_2P_3 + P_2P_1(CH - G_{\text{ff}})}{1 + P_2P_1C}. \tag{2.3}$$

Choosing H as

$$H = P_2P_3 - P_2P_1G_{\text{ff}}, \tag{2.4}$$

the closed loop transfer function (2.3) then equals

$$G_{\text{cl}} = P_2(P_3 - P_1G_{\text{ff}}) = G_{\text{o}}.$$

The closed-loop response from a disturbance d will thus be the same as the response in the open-loop case in (2.1) and the feedback controller, C, will not interact with the feedforward controller, G_{ff}. By using the structure in Fig. 3 with H chosen as (2.4) it is possible to design the feedforward controller by just considering the open-loop response from d. If the feedback controller has integral action the steady-state response will be $y = r + H(0)d$. Therefore it is desirable to choose $H(0) = 0$.

The method of subtracting the feedforward response from the controller input is common when improving system response from reference signals, cf. (Åström and Hägglund, 2006).

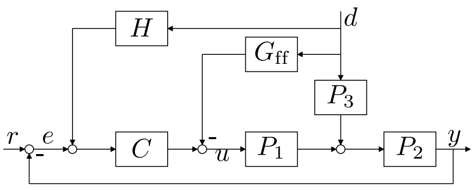

Fig. 3. Modified closed-loop feedforward structure.

3. OPTIMAL FEEDFORWARD CONTROL

In this section optimal feedforward controller parameters, based on stable FOTDs, in the case of a step disturbance d, will be derived. Using the structure in Fig. 3 with H chosen in accordance with (2.4) we consider optimization over the structure in Fig. 1. The rules are derived for the case $P_2 = 1$. In applications where this is not the case, P_2 can be incorporated into P_1 and P_3 followed by first-order approximations, cf., (Åström and Hägglund, 2006). The optimality measure is the integrated square error,

$$\text{ISE} = \|e\|_2^2 = \int_0^\infty e^2(t)\,\mathrm{d}t. \tag{3.1}$$

A vast number of other optimality criteria could be considered, cf., (Åström and Hägglund, 2006). The ISE measure is an established performance measure and was chosen since it enables analytical solutions for finding the minimal cost for the setting considered in this paper. The drawbacks with the ISE is that it may yield large control signals and prolonged time for steady state.

The processes $P_i(s)$ are assumed to be FOTDs, i.e.,

$$\begin{aligned} P_i(s) &= \frac{K_i}{1 + sT_i}e^{-L_i s}, \qquad i = 1, 3 \\ L_i &\geq 0, \quad T_i > 0 \\ P_2(s) &= 1. \end{aligned} \tag{3.2}$$

The feedforward controller has the following structure

$$G_{\text{ff}}(s) = K_{\text{ff}}\frac{1 + sT_z}{1 + sT_p}e^{-sL_{\text{ff}}}. \tag{3.3}$$

There are in total four parameters to be determined in order to minimize (3.1). We require that T_p should be non-negative since negative values of T_p would give an unstable system response. For the case of $L_1 \leq L_3$ perfect feedforward, i.e., no control error, is obtained with the following choice of parameters:

$$G_{\text{ff}}(s) = \frac{K_3}{K_1}\frac{1 + sT_1}{1 + sT_3}e^{-(L_3 - L_1)s}.$$

The following will therefore focus on the case when $L_1 > L_3$ and hence, perfect disturbance rejection is not possible. The time delays in the process models can, without loss of generality, be shifted so that $L = \hat{L}_1 = L_1 - L_3 > 0$ and $\hat{L}_3 = 0$, Furthermore the reference signal r can, without loss of generality be regarded to be zero.

Given a unit step disturbance d the output of the system is given by

$$Y(s) = (P_3(s) - P_1(s)G_{\text{ff}}(s))D(s) \tag{3.4}$$

where $D(s)$ is the Laplace transform of a unit step. Denote the output response by inverse Laplace transform of (3.4), $y(t) = \mathcal{L}^{-1}(Y(s))$. The optimization problem can be formulated as

$$\begin{aligned} &\text{minimize } J = \int_L^\infty y^2(t)\,\mathrm{d}t && (3.5\text{a}) \\ &\text{s.t.} \quad T_p \geq 0 && (3.5\text{b}) \\ &\qquad\;\; L_{\text{ff}} \geq 0. && (3.5\text{c}) \end{aligned}$$

(3.5b) and (3.5c) are included in the optimization formulation to ensure a stable and causal feedforward controller.

3.1 Optimal Feedforward Time Delay

Assume that the time delays are such that perfect disturbance rejection is not possible. Adding time delay in the feedforward controller would increase the time in which there is no control action and thus increase the ISE. The time delay should therefore be chosen as

$$L_{\mathrm{ff}} = \max(0, L_3 - L_1). \tag{3.6}$$

3.2 Optimal Stationary Gain

In order to ensure that $H(0) = 0$ and for the integral (3.5a) to converge the gain in the feedforward controller has to be chosen as

$$K_{\mathrm{ff}} = \frac{K_3}{K_1}. \tag{3.7}$$

3.3 Optimal T_z

Evaluating (3.5a) yields an expression with the following structure

$$J(T_p, T_z) = q_1 T_z^2 + q_2 T_z + q_3. \tag{3.8}$$

Introducing

$$a = \frac{T_1}{T_3} \tag{3.9a}$$

$$b = a\,(a+1)\,\mathrm{e}^{\frac{L}{T_3}}, \tag{3.9b}$$

the expressions for q_1, q_2 and q_3 can be seen in Appendix, (A.1). Since (A.1a) is positive, by the assumptions in (3.2), (3.8) has a unique minimum with respect to T_z which can be determined by completion of squares:

$$J(T_p, T_z) = q_1 (T_z + \frac{q_2}{2q_1})^2 - \frac{q_2^2}{4q_1} + q_3$$

for which the minimum occurs at

$$T_z(T_p) = -\frac{q_2}{2q_1} = \frac{(b-2a)T_3 + bT_p}{b(T_3 + T_p)}(T_p + aT_3). \tag{3.10}$$

The optimal T_z can also be expressed as

$$T_z(T_p) = (T_p + T_1)\left(1 - \frac{2\,T_3^2}{(T_1+T_3)(T_3+T_p)\,\mathrm{e}^{\frac{L}{T_3}}}\right).$$

By using the optimal T_z, (3.8) reduces to

$$J(T_p, T_z(T_p)) = \hat{J}(T_p) = q_3 - \frac{q_2^2}{4q_1}, \tag{3.11}$$

see (A.2) for complete expression, from which the last controller parameter, T_p, is to be determined. Since T_z is dependent of T_p it is not clear at this moment that $T_z > 0$, i.e., that the controller will be minimum-phase. This will be shown in Sec. 3.8.

3.4 Optimal T_p

Differentiating (3.11) yields

$$\begin{aligned}\frac{\mathrm{d}\hat{J}}{\mathrm{d}T_p} &= \frac{K_3^2 a^2 T_3^2}{2\,b^2 (T_3+T_p)^3 (aT_3+T_p)^2} \\ &\times \left((4a^2 - 2a - b)T_3^2 + 2\,T_p T_3 (3a - 1 - b) - (b-2)T_p^2\right) \\ &\times \left((2a+b)T_3 - (b-2)T_p\right). \end{aligned}\tag{3.12}$$

Equating (3.12) to zero to find the stationary points yields the following three:

$$T_{p_1}^* = \frac{3a - 1 - b + \sqrt{(a-1)^2\,(1+4b)}}{b-2} T_3 \tag{3.13a}$$

$$T_{p_2}^* = \frac{3a - 1 - b - \sqrt{(a-1)^2\,(1+4b)}}{b-2} T_3 \tag{3.13b}$$

$$T_{p_3}^* = \frac{2a - b}{b-2} T_3. \tag{3.13c}$$

The optimal choice for T_p will either be one of the three stationary points or the boundary, $T_p = 0$.

The boundary point as $T_p \to \infty$ is in practice the same as no feedforward and will therefore be discarded as a possible solution since

$$\lim_{T_p \to +\infty} J = +\infty.$$

The following subsections are devoted to finding which of the solutions that is optimal. A summary of the resulting, optimal, algorithm can be found in Sec. 4.

3.5 Conditions for Positive Stationary Points

To fulfill (3.5b) we only consider a stationary point (3.13) as a candidate for optimality if it is positive.

Case I: $T_{p_1}^* > 0$. From (3.13a), we can conclude that the denominator is positive if $b > 2$. Note that $\mathrm{e}^{\frac{L}{T_3}} > 1 \Leftrightarrow b > a(a+1)$. Denote the numerator of (3.13a) by n_1 i.e.,

$$n_1 = 3a - 1 - b + \sqrt{(a-1)^2(1+4b)}.$$

In order to determine the sign of $T_{p_1}^*$ we first examine when n_1 changes its sign.

$$\begin{aligned}&n_1 = 0 \Leftrightarrow \\ &b + 1 - 3a = \sqrt{(a-1)^2(1+4b)} \Leftrightarrow \\ &b^2 - 2\left(2a^2 - a + 1\right) b + 4a\,(2a-1) = 0 \Rightarrow \\ &b_1 = 2 \\ &b_2 = a\,(4a-2).\end{aligned}$$

Assume $a < 1$. Then $a(4a-2) < a(a+1)$. Hence, both numerator and denominator of $T_{p_1}^*$ can only change signs at $b = 2$. By evaluating n_1 for $a < 1$ and for arbitrary $b \neq 2$ we can conclude that $T_{p_1}^*$ is negative for $a < 1$.

Assume instead $a > 1$. Then $a(a+1) > b_1$ and n_1 can only change its sign for $b = b_2$. Furthermore, the denominator is positive for $a > 1$ since $b > 2$. By evaluation of n_1 for arbitrary $a > 1$ and $b < a\,(4a-2)$ we can conclude that $n_1 > 0$. Since the denominator is positive for $a > 1$, $T_{p_1}^*$ is positive if

$$b < a\,(4a-2) \Leftrightarrow \mathrm{e}^{\frac{L}{T_3}} < \frac{4a-2}{a+1}.$$

This means that $T_{p_1}^*$ is positive when

$$T_1 > T_3 \text{ and } L < T_3 \ln\left(\frac{4a-2}{a+1}\right). \tag{3.14}$$

Case II: $T_{p_2}^* > 0$. Denote the numerator and denominator in (3.13b) by n_2 and d_2 respectively. The numerator is negative since

$$\begin{aligned}n_2 = 3a - 1 - b - \sqrt{(a-1)^2(1+4b)} &< \\ 3a - 1 - a(a+1) - \sqrt{(a-1)^2(1+4b)} &= \\ -(a-1)^2 - \sqrt{(a-1)^2(1+4b)} &< 0.\end{aligned}$$

The sign of $T^*_{p_2}$ is thus only dependent on d_2. Since $b > a(a+1)$, d_2 will be positive for $a > 1$. For $a < 1$,

$$T^*_{p_2} > 0 \Leftrightarrow d < 0 \Leftrightarrow b < 2 \Leftrightarrow \mathrm{e}^{\frac{L}{T_3}} < \frac{2}{a(a+1)}.$$

To summarize, $T^*_{p_2}$ is positive when

$$T_1 < T_3 \text{ and } L < T_3 \ln\left(\frac{2}{a(a+1)}\right). \tag{3.15}$$

Case III: $T^*_{p_3} > 0$. By inspection of (3.13c) we can conclude that $T^*_{p_3}$ is positive if and only if $a < 1$ and

$$\frac{2}{a+1} < \mathrm{e}^{\frac{L}{T_3}} < \frac{2}{a(a+1)}.$$

$T^*_{p_3}$ is positive when

$$\begin{gathered} T_1 < T_3 \quad \text{and} \\ T_3 \ln\left(\frac{2}{a+1}\right) < L < T_3 \ln\left(\frac{2}{a\,(a+1)}\right). \end{gathered} \tag{3.16}$$

3.6 Conditions for Optimal T_p

A stationary point, $T^*_{p_i}$, is a local minimizer if and only if the second derivative of (3.11) with respect to T_p is positive, i.e., $\frac{\mathrm{d}^2\hat{J}}{\mathrm{d}T_p^2} > 0$. Since the cost function (3.11) has three stationary points and approaches infinity when T_p approaches infinity, the cost function can have no more than two local minima.

Solution 1. From the inequalities (3.14), (3.15) and (3.16) we can conclude that if $a > 1$, $T^*_{p_1}$ is the only positive stationary point. Since (3.14) is the only stationary point for $a > 1$, this stationary point cannot be a maximum since (3.5a) approaches infinity when T_p approaches infinity. Furthermore,

$$\frac{\mathrm{d}\hat{J}}{\mathrm{d}T_p}(0) = K_3^2 \frac{(b-2a)(b+2a-4a^2)}{2b^2}. \tag{3.17}$$

If $a > 1$, then $b > 2a$ and subsequently

$$\begin{gathered} \frac{\mathrm{d}\hat{J}}{\mathrm{d}T_p}(0) < 0 \Leftrightarrow b + 2a - 4a^2 < 0 \Leftrightarrow \\ \mathrm{e}^{\frac{L}{T_3}} < \frac{4a-2}{a+1}. \end{gathered} \tag{3.18}$$

From (3.14) and (3.18) we therefore conclude that $T^*_{p_1}$, given by (3.13a), is optimal when it is positive.

Solution 2 From (3.14) and (3.15) we can conclude that $T^*_{p_1}$ and $T^*_{p_2}$ cannot simultaneously be positive. Furthermore, when $T^*_{p_2}$ is positive, $T^*_{p_3}$ is either negative or corresponds to a maximum, see the next section.

In order to determine when $T_p = T^*_{p_2}$ is a better solution than $T_p = 0$, take the difference between the corresponding costs as

$$\begin{gathered} \hat{J}(T^*_{p_2}) - \hat{J}(0) = 2aK_3^2 \frac{n}{d} \\ \hat{J}(T^*_{p_2}) < \hat{J}(0) \Leftrightarrow 2aK_3^2 \frac{n}{d} < 0 \end{gathered}$$

where expressions for n and d can be found in the Appendix, (A.3). From these expressions we conclude that $d > 0$. Since $T^*_{p_2}$ is negative for $a > 1$, consider only the case $a < 1$. Whether $T^*_{p_2}$ is better than $T_p = 0$ or not is determined by the sign of n. Solving the equation $n = 0$ gives the following solutions for b

$$b^* = a + \sqrt{a}. \tag{3.19}$$

Hence, n can only change its sign for $b = b^*$. By evaluation of n with $a < 1$ and both $b < b^*$ and $b > b^*$ we can conclude that $J(T^*_{p_2}) < J(0)$ if $a < 1$ and $b < a + \sqrt{a}$. Hence, $T_p = T^*_{p_2}$ is the optimal solution when

$$\begin{gathered} a < 1 \\ \text{and } \mathrm{e}^{\frac{L}{T_3}} < \frac{\sqrt{a}+a}{a\,(a+1)} \Leftrightarrow \\ L < T_3 \ln \frac{\sqrt{a}+a}{a\,(a+1)}. \end{gathered}$$

Solution 3 Inserting $T^*_{p_3}$ given by (3.13c) into (3.10) yields $T_p = T_z$ i.e., the static feedforward controller

$$G_{\mathrm{ff}}(s) = \frac{K_3}{K_1}. \tag{3.20}$$

The second derivative of (3.11) with respect to T_p evaluated in $T_p = T^*_{p_3}$ is

$$\frac{\mathrm{d}^2\hat{J}}{\mathrm{d}T_p^2}(T^*_{p_3}) = -\frac{K_3^2\,(b-2)^5\,a^2}{4\,(a-1)^3\,b^3}. \tag{3.21}$$

$T^*_{p_3}$ is a minimum point if (3.21) is greater than zero. For $a > 1$ this is equivalent to

$$\begin{gathered} a\,(a+1)\,\mathrm{e}^{\frac{L}{T_3}} - 2 < 0 \Leftrightarrow \\ \mathrm{e}^{\frac{L}{T_3}} < \frac{2}{a(a+1)} < 1. \end{gathered}$$

Since both L and T_3 are positive this condition is never fulfilled.

For $a < 1$ we can conclude that in order for $T^*_{p_3}$ to be a minimum point the following condition must hold

$$L > T_3 \ln\left(\frac{2}{a\,(a+1)}\right).$$

The feedforward strategy given by (3.20) does not give a lower cost than the strategy given by the controller with $T_p = 0$ since

$$\hat{J}(T^*_{p_3}) - \hat{J}(0) = \frac{K_3^2(b-2a)^2}{2b^2} a T_3 \geq 0.$$

3.7 Special Cases

Two cases have been disregarded in the analysis above. Firstly, the case when $a = 1$, i.e., the process time constants are equal, and secondly, the case where $b = 2$, i.e., when the denominators of (3.13) are zero.

Case I: Equal time constants, $T_1 = T_3$. In the case of equal time constants in the processes, $a = 1$ and (3.11) simplifies to

$$\hat{J}(T_p) = K_3^2 \frac{T_p\,T_3\left(\mathrm{e}^{\frac{L}{T_3}} - 1\right)^2}{2\,(T_p + T_3)\,\mathrm{e}^{\frac{2L}{T_3}}}$$

from which we conclude that $T_p = 0$ is the optimal solution since $L > 0$ by assumption.

Case II: $b=2$. If $b=2$, (3.12) reduces to

$$\frac{\partial \hat{J}}{\partial T_p} = K_3^2 T_3^4 \frac{\left((2a+1)T_3 + 3\,T_p\right)(a-1)^2}{2(T_3+T_p)^3(aT_3+T_p)^2} a^2$$

for which there is only one stationary point,

$$T_p^* = -\frac{(2\,a+1)}{3} T_3,$$

which is less than zero. Hence, if $b=2$, $T_p=0$ is the optimal solution.

3.8 Optimal T_z Revisited

Since the optimal T_z, given by (3.10), depends on T_p it is unclear whether T_z for some set of parameters can be negative or not. We here set out to prove that it for all process parameters will be positive. Introducing (3.9) in (3.10) yields

$$T_z = \frac{(b-2a)T_3 + bT_p}{b(T_3+T_p)}(aT_3+T_p). \tag{3.22}$$

Since $T_p \geq 0$ and $b > a(a+1)$ we can conclude that $T_z > 0$ if $a > 1$.

If $a < 1$, T_z will be positive when $e^{\frac{L}{T_3}} > \frac{2}{a+1}$. When $e^{\frac{L}{T_3}} < \frac{2}{a+1}$, $T_{p_2}^*$ is the optimal solution since

$$\frac{2}{a+1} < \frac{\sqrt{a}+a}{a\,(a+1)}.$$

The sign of T_z is determined by the sign of

$$(b-2a)T_3 + bT_p \tag{3.23}$$

Inserting $T_p = T_{p_2}^*$ in (3.23) yields

$$\frac{T_3}{b-2}\left(-(3-a)\,b - 4a - b\sqrt{(a-1)^2\,(1+4b)}\right).$$

Recalling that

$$e^{\frac{L}{T_3}} < \frac{a+\sqrt{a}}{a(a+1)} \Rightarrow b-2 < a+\sqrt{a}-2 < 0$$

we can conclude that when $T_{p_2}^*$ is the optimal solution T_z will be positive and thus T_z will be positive for all values on the process parameter.

4. DESIGN SUMMARY

Below follows a summary of how to choose the parameters in the feedforward controller in order to minimize the integrated square error (3.5a).

(1) $K_{\text{ff}} = \dfrac{K_3}{K_1}$.

(2) $L_{\text{ff}} = \max(0, -L)$, $L = L_1 - L_3$.

(3)
- Introduce $a = \dfrac{T_1}{T_3}$ and $b = a\,(a+1)\,e^{\frac{L}{T_3}}$
- If $a > 1$ and $b < 4a^2 - 2a$
  $$T_p = \frac{3a - 1 - b + \sqrt{(a-1)^2\,(1+4b)}}{b-2} T_3.$$
- If $a < 1$ and $b < \sqrt{a} + a$
  $$T_p = \frac{3a - 1 - b - \sqrt{(a-1)^2\,(1+4b)}}{b-2} T_3.$$
- Else, $T_p = 0$.

(4) $T_z(T_p) = (T_p + T_1)\left(1 - \dfrac{2\,T_3^2}{(T_1+T_3)(T_3+T_p)\,e^{\frac{L}{T_3}}}\right)$.

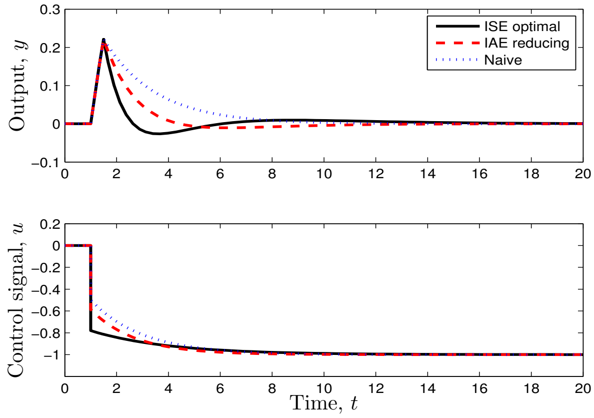

Fig. 4. Output and control signals for Example 1.

Note that even though a small T_p can be optimal, it is not necessarily practical or possible to realize such a controller. The high-frequency gain is given by

$$K_{\text{ff}} \frac{T_z}{T_p}.$$

If the high-frequency gain is too large, choose a larger T_p and recalculate T_z until the high-frequency gain is satisfying.

5. DESIGN EXAMPLES

Example 1. Optimal Open-Loop Feedforward Control. Consider the open-loop in Fig. 1 with

$$P_1(s) = \frac{1}{1+s}e^{-0.5s},\; P_2(s) = 1,\; P_3(s) = \frac{1}{1+2s} \tag{5.1}$$

and unit step d disturbing the system at $t=1$. Using the design rule from Sec. 4 gives the following optimal feedforward controller

$$G_{\text{ff}}^{\text{ISE}}(s) = \frac{1+2.35s}{1+3.02s}. \tag{5.2}$$

For comparison, two other feedforward controllers are simulated. The second controller is tuned in accordance with the rule presented in (Guzmán and Hägglund, 2011). This rule sets $T_z = T_1$ and tunes T_p in order to reduce the IAE. The IAE-reducing feedforward controller is given by

$$G_{\text{ff}}^{\text{IAE}}(s) = \frac{1+s}{1+1.71s}. \tag{5.3}$$

The third controller is given by

$$G_{\text{ff}}^{\text{naive}}(s) = \frac{1+T_1 s}{1+T_3 s} = \frac{1+s}{1+2s}, \tag{5.4}$$

which is the optimal controller if the time delay is disregarded. The output signals along with the control signals can be seen in Fig. 4. The performance measures from the simulation can be seen in Table 1. The ISE-minimizing feedforward controller out-performs the two other controllers, not only in terms of ISE but also in IAE.

Table 1. Performance measures. Ex. 1.

Strategy	ISE	IAE
$G_{\text{ff}}^{\text{ISE}}$	0.022	0.267
$G_{\text{ff}}^{\text{IAE}}$	0.034	0.313
$G_{\text{ff}}^{\text{naive}}$	0.058	0.502

Table 2. Performance measures. Ex. 2.

Strategy	ISE	IAE
$G_{\text{ff}}^{\text{ISE}}$	0.013	0.260
$G_{\text{ff}}^{\text{IAE}}$	0.021	0.346
$G_{\text{ff}}^{\text{naive}}$	0.037	0.452
No ff	1.13	3.158

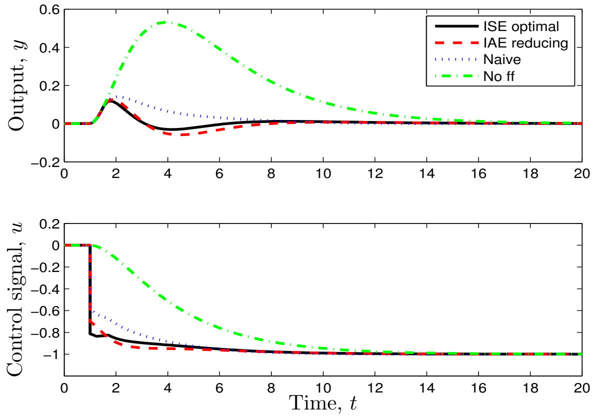

Fig. 5. Output and control signals for Example 2.

Example 2. Closed-Loop with FOTD Approximations. To examine how the design-rules handle high-order dynamics, consider the same P_1 and P_3 as in the previous example but with

$$P_2(s) = \frac{1}{0.5s+1}.$$

Incorporating P_2 into P_1 and P_3 with subsequently FOTD approximations, (Åström and Hägglund, 2006), renders the following approximations

$$\hat{P}_1 = \frac{1}{1+1.31s}e^{-0.69s},\ \hat{P}_2 = 1,\ \hat{P}_3 = \frac{1}{1+2.25s}e^{-0.25s}. \tag{5.5}$$

Based on these approximations a feedback controller, $C(s)$, has been tuned using the AMIGO method. The resulting PI controller is

$$C(s) = 0.38\,(1+\frac{1}{1.21s}).$$

The optimal feedforward controller for the process approximations (5.5) is given by

$$G_{\text{ff}}^{\text{ISE}}(s) = \frac{1+2.82s}{1+3.46s}. \tag{5.6}$$

H was based on the first-order approximations, i.e.,

$$H = \hat{P}_2\hat{P}_3 - \hat{P}_2\hat{P}_1 G_{\text{ff}}^{\text{ISE}}$$

As before, for comparison, the two other feedforward controllers given by

$$G_{\text{ff}}^{\text{IAE}}(s) = 0.99\frac{1+1.31s}{1+1.84s} \tag{5.7}$$

and

$$G_{\text{ff}}^{\text{naive}}(s) = \frac{1+1.31s}{1+2.25s}, \tag{5.8}$$

where (5.8) was used with the same structure as (5.6) with H as

$$H = \hat{P}_2\hat{P}_3 - \hat{P}_2\hat{P}_1 G_{\text{ff}}^{\text{naive}}.$$

For simulation of (5.7) the structure given in Fig. 2 was used. The result from simulations can be seen in Fig. 5 and the performance measures in Table 2.

6. CONCLUSIONS

In this paper we present design rules for a lead-lag feedforward controller that minimizes the integrated squared error in the case of stable first-order process models with time delay, affected by a measurable step disturbance in an open-loop setting. A control structure that separates feedback and feedforward design has been discussed.

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Appendix A. MISCELLANEOUS EQUATIONS

$$q_1 = \frac{1}{2}\frac{K_3^2}{T_p + aT_3} \tag{A.1a}$$

$$q_2 = K_3^2\frac{(2a-b)T_3 - bT_p}{b(T_3+T_p)} \tag{A.1b}$$

$$q_3 = \frac{K_3^2}{2b^2(T_3+T_p)(T_p+aT_3)}\cdot\Big(\big(a(a+1)^2 + b(b-4a)\big)a^2T_3^3 + \big(a(a+1)^3 + b^2(a+3) - 4ab(a+2)\big)aT_pT_3^2 + \big((a(a+1)-2b)^2 + 3b^2(a-1)\big)T_p^2T_3 + b^2T_p^3\Big) \tag{A.1c}$$

$$\hat{J}(T_p) = \frac{K_3^2T_3a}{2\,b^2(T_3+T_p)^2(T_p+aT_3)} \times\Big(\big(a(a+1)^2 + b(b-4a)\big)T_p^3 + a^2(a-1)^2T_3^3 + \big(a(a+2)(a+1)^2 - 4ba^2 - 4a\,(b+1) + 2\,b^2\big)T_3T_p^2 + \big((2a^2-b)^2 - a(a-1)^2(2a-1)\big)T_3^2T_p\Big) \tag{A.2}$$

$$\hat{J}(T_{p_2}^*) - \hat{J}(0) = \frac{2\,K_3^2\,a\,T_3}{(1-a)(b+1+\sqrt{1+4b})(3+\sqrt{1+4b})^2b^2} \times\Big[\big(-(16+10b)a^3 + (16+26b+10b^2)a^2 - (4+17b^2+10b+2b^3)a + 4\,b^2 + 5/2b^3\big)\sqrt{1+4b} - (4b^2+38b+16)a^3 + (42b^2+54b+16+4b^3)a^2 - (33b^2+4+18b+19b^3)a + (b^2+19/2b+4)b^2\Big] \tag{A.3}$$