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Calugi, Francesco; Robertsson, Anders; Johansson, Rolf

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Output Feedback Adaptive Control of Robot Manipulators Using Observer Backstepping

Francesco Calugi Anders Robertsson Rolf Johansson

Department of Automatic Control, Lund Institute of Technology, Lund University, PO Box 118, SE-221 00 Lund, Sweden. E-mail: fcalugi@yahoo.com, Anders.Robertsson | Rolf.Johansson@control.lth.se

Abstract

In this paper we present an observer-based adaptive control scheme for robot manipulators, for which we have both unmeasured velocity and uncertain parameters. Using the observer backstepping method, a reduced-order adaptive velocity observer can be designed independently from the state-feedback controller, which uses damping terms to compensate the presence of the estimation error in the tracking error dynamics. The resulting closed-loop system is semiglobally asymptotically stable with respect to the estimation error and tracking errors. Furthermore a simulated example shows the performance of the control scheme applied to a two-link manipulator.

1 Introduction

The problem of output-feedback adaptive control for robot manipulators is hard to solve with traditional methods. Some results have been stated for systems in output-feedback form, in which the nonlinearities depend only on the measured output, and the unmeasured states are not coupled with the unknown parameters. Krstic et al. [3] have developed some output feedback adaptive control schemes for these systems, using K-filters and tuning functions, or gradient-least square identifiers, together with the backstepping method. Unfortunately, robot systems cannot in general be written in this form, as the nonlinearities depend strongly on the velocity, that we suppose to be unmeasured, and there is further coupling between velocity and inertial parameters, which could be uncertain or completely unknown. Erlic and Lu [1] have recently presented a reduced-order adaptive velocity observer for robot manipulators that cannot be implemented directly, only a discrete-time approximation can. However this solution has been used in a closed-loop together with a PD-type controller and seemed to have a good behaviour. In this paper we want to show how the adaptive observer can be designed independently from the control law, using observer vectorial backstepping. Such a solution has been already used by Fossen and Grøvlen for control of dynamically positioned ships [2]. It allows the adaptive observer to be designed independently from the state-feedback controller. The main idea of the method is to apply the backstepping to the error between the desired velocity and the estimated velocity, and compensate the estimation error terms that appear in the tracking error dynamics with nonlinear damping, as they were disturbances. Furthermore, the proposed control scheme is applied in a simulation to a two-link manipulator, to show its performance.

2 System Model and Properties

Model equations of a n-links rigid-body motion robotic system can be written in matrix form as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (1)$$

	q	angular positions	$q \in \mathbf{R}^n$
	$\dot{q}$	angular velocities	$\dot{q} \in \mathbf{R}^n$
	$\ddot{q}$	angular accelerations	$\ddot{q} \in \mathbf{R}^n$
with	$M(q)$	moment of inertia	$M \in \mathbf{R}^{n\times n}$
	$C(q,\dot{q})\dot{q}$	Coriolis, centripetal and frictional forces	$C \in \mathbf{R}^{n\times n}$
	$G(q)$	gravitational forces	$G \in \mathbf{R}^{n\times n}$

It is assumed that only the positions q are available for measurement. The matrices in Eq. (1) have the following important properties:

Property 1 $0 < M_m < \|M(q)\| < M_M$ *where* M_m, M_M *are positive constants.*

Property 2 $C(q,\dot{q}_1)\dot{q}_2 = C(q,\dot{q}_2)\dot{q}_1$

Property 3 $\|C(q,\dot{q})\| < C_M\|\dot{q}\|$ *where* C_M *is a positive constant.*

Property 4 $M(q) - 2C(q,\dot{q})$ *is skew symmetric.*

Property 5 *$M(q)\psi + C(q,\xi)\xi + G(q) = \varphi_0(q,\xi,\psi) + \varphi(q,\xi,\psi)\theta$ where $\xi,\psi \in \mathbf{R}^n$ and $\theta \in \mathbf{R}^p$ is the unknown parameter vector.*

It is further assumed that the robot velocity is bounded by a known constant ω_{max} such that

$$\|\dot{q}(t)\| \le \omega_{max}, \qquad \forall t \in \mathbf{R}. \tag{2}$$

We define the state vector as

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \dot{q} \\ q \end{pmatrix} \tag{3}$$

then, from Eq. (1) we can write

$$\dot{x}_1 = M^{-1}(x_2)\Big(\tau - C(x_2,x_1)x_1 - G(x_2)\Big) \tag{4}$$
$$\dot{x}_2 = x_1 \tag{5}$$

3 The reduced-order adaptive observer

Consider a reduced-order adaptive observer for estimating the angular velocity and the unknown parameter vector, when the angle is measurable. The observer equation [1] is given by

$$\dot{\hat{x}}_1 = \psi(q,\hat{x}_1,\tau,\hat{\theta}) + K\tilde{x}_1 \tag{6}$$
$$\psi(q,\hat{x}_1,\tau,\hat{\theta}) = \hat{M}(q)^{-1}\Big(\tau - \hat{C}(q,\hat{x}_1)\hat{x}_1 - \hat{G}(q)\Big)$$

where $\hat{x}_1$ is the velocity estimate, $\tilde{x}_1 = x_1 - \hat{x}_1$ is the observation error, $K > 0$ is a diagonal gain matrix. The estimated parameters used in (6) are obtained with the following adaptation law:

$$\dot{\hat{\theta}} = -\Gamma\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1 \tag{7}$$

where $\varphi^T(q,\hat{x}_1,\psi)$ is the regressor determined by property 5 and $\Gamma > 0$ is a diagonal gain matrix.

4 Stability of the observer

The following theorem establishes the stability properties for the above observer [1]:

Theorem 1 *Consider the observer (6) with the adaptation law (7). Define*

$$\underline{\sigma} = \lambda_{min}(KM(q) + M(q)K)/2, \tag{8}$$

the initial estimation error as

$$e_0 = e(0) = \begin{pmatrix} \tilde{x}_1^T(0) & \tilde{\theta}^T(0) \end{pmatrix}^T \tag{9}$$

and

$$p_l = \lambda_{min}(P), \quad p_u = \lambda_{max}(P) \tag{10}$$

with $P = diag\{\ M(q) \quad \Gamma^{-1}\ \}$. If

$$\underline{\sigma} > C_M\omega_{max} + \beta \tag{11}$$

where C_M is given in property 3, $\beta > 0$ is a fixed constant, and the initial estimation error $e(0)$ belongs to the ball B_e, defined by

$$B_e = \left\{ e_0 \in \mathbf{R}^{n+p} : \|e_0\| < \sqrt{\frac{p_l}{p_u}}\Big(\frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}\Big)\right\}$$

then

$$\lim_{t\to\infty} \tilde{x}_1 = 0 \tag{12}$$

A proof of Theorem 1 is given in Appendix A

Remark 1 *It is important to note that the observer (6), (7) is not implementable in case the velocity measurements are not available. This is because (6) and (7) involve the use of $\tilde{x}_1 = x - \hat{x}_1$. However, an implementable discrete-time approximation of it is possible and is shown in Appendix B.*

5 Observer Backstepping

Consider the robot equation (1), and suppose that the estimates of the unmeasured velocity x_1 and the unknown parameters θ are given by the adaptive observer (6) and (7). Define a smooth reference trajectory q_d satisfying

$$\ddot{q}_d, \dot{q}_d, q_d \in \mathcal{L}_\infty. \tag{13}$$

and the first error variable $z_1 = q - q_d$. We have

$$\dot{z}_1 = x_1 - \dot{q}_d. \tag{14}$$

The main idea of backstepping is to choose one of the state variables as virtual control. It turns out that

$$\xi_1 = \hat{x}_1 = z_2 + \alpha_1 \tag{15}$$

is an excellent choice for the virtual control. ξ_1 is defined as the sum of the next error variable z_2, and α_1 which can be interpreted as a stabilizing function. Hence

$$\dot{z}_1 = z_2 + \alpha_1 + \tilde{x}_1 - \dot{q}_d. \tag{16}$$

We choose the following stabilizing function

$$\alpha_1 = -C_1z_1 - D_1z_1 + \dot{q}_d \tag{17}$$

where $C_1 \in \mathbf{R}^{n\times n}$ is a strictly positive constant feedback design matrix, usually diagonal, and $D_1 \in \mathbf{R}^{n\times n}$ is a positive diagonal damping matrix defined as

$$D_1 = \text{diag}[d_1,\ldots,d_n] \tag{18}$$

where $d_i > 0\,(i = 1,\ldots,n)$. The damping term $-D_1z_1$ has been added because $\tilde{x}_1$ in (14) can be treated as a disturbance term to be compensated. Then we can write

$$\dot{z}_1 = -\Big(C_1 + D_1\Big)z_1 + z_2 + \tilde{x}_1. \tag{19}$$

The next step is to specify the desired dynamics of z_2; from (15), we have

$$\begin{aligned}\dot{z}_2 &= \dot{\xi}_1 - \dot{\alpha}_1 = \dot{x}_1 + \Big(C_1 + D_1\Big)\dot{z}_1 - \ddot{q}_d \quad (20)\\ &= -\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)(z_2 + \tilde{x}_1) - \ddot{q}_d\\ &+ \hat{M}(q)^{-1}\Big(\tau - \hat{C}(q,\hat{x}_1)\hat{x}_1 - \hat{G}(q)\Big) + K\tilde{x}_1.\end{aligned}$$

Now we choose the control law as follows

$$\begin{aligned}\tau &= -\hat{M}(q)\Big[-\Big(C_1 + D_1\Big)^2 z_1 + \Big(C_1 + D_1\Big)z_2\\ &- \ddot{q}_d + C_2 z_2 + D_2 z_2 + z_1\Big]\\ &+ \hat{C}(q,\hat{x}_1)\hat{x}_1 + \hat{G}(q),\end{aligned} \tag{21}$$

where $C_2 \in \mathbf{R}^{n\times n}$ is a strictly positive constant feedback design matrix, usually diagonal. Substituting (21) into (20), we have

$$\begin{aligned}\dot{z}_2 &= -C_2 z_2 - D_2 z_2 - z_1 + \Omega\tilde{x}_1 \quad (22)\\ \Omega &= \Big(C_1 + D_1\Big) + K. \quad (23)\end{aligned}$$

The damping matrix $D_2 \in \mathbf{R}^{n\times n}$ is defined in terms of the rows of Ω—*i.e.*, the columns of Ω^T—as

$$D_2 = \mathrm{diag}[d_{n+1}\omega_1^T\omega_1, \ldots, d_{2n}\omega_n^T\omega_n] \tag{24}$$

where $\Omega^T = [\omega_1, \ldots, \omega_n]$ and $d_i > 0 \quad (i = n+1, \ldots, 2n)$.

Stability Analysis of the Closed-Loop System

From (19), (22) and (52), we can write the error dynamics as

$$\begin{aligned}\dot{z} &= -(C_z + D_z + E)z + W\tilde{x}_1 \quad (25)\\ M(q)\dot{\tilde{x}}_1 &= -C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1 - M(q)K\tilde{x}_1\\ &- \varphi(q,\hat{x}_1,\psi)\tilde{\theta} \quad (26)\end{aligned}$$

where

$$\begin{aligned}z &= \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad C_z = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \quad (27)\\ D_z &= \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \quad (28)\\ W &= \begin{bmatrix} I \\ \Omega \end{bmatrix}. \quad (29)\end{aligned}$$

Consider the following Lyapunov function candidate

$$V(z,\tilde{x}_1,\tilde{\theta}) = \frac{1}{2}\Big(z^Tz + \tilde{x}_1^T M(q)\tilde{x}_1 + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\Big) \tag{30}$$

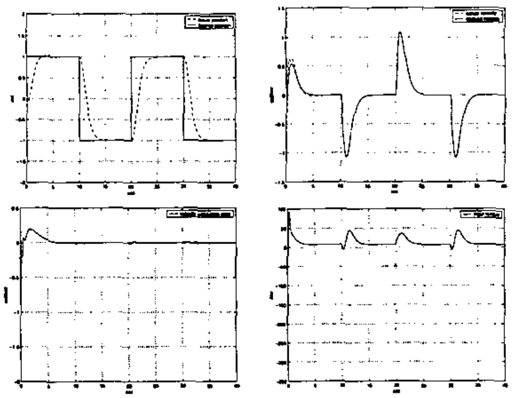

Figure 1: Simulation results for the first link

its time derivative along the solutions of (25) and (26) is

$$\begin{aligned}\dot{V} &= -z^TC_zz - z^TD_zz + z^TW\tilde{x}_1\\ &- \tilde{x}_1^T\Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1)\Big)\tilde{x}_1\\ &+ \tilde{x}_1^T\Big(\frac{1}{2}\dot{M}(q) - C(q,x_1)\Big)\tilde{x}_1\\ &- \tilde{\theta}^T\Big(\varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1 + \Gamma^{-1}\dot{\hat{\theta}}\Big)\end{aligned} \tag{31}$$

where we have used the fact that $z^TEz = 0$, and property 2. Now, using (7), property 4 and adding the zero term

$$\frac{1}{4}\Big(\tilde{x}_1^TP\tilde{x}_1 - \tilde{x}_1^TP\tilde{x}_1\Big) = 0 \tag{32}$$

Eq. (31) becomes

$$\begin{aligned}\dot{V} &= -z^TC_zz - z^TD_zz + z^TW\tilde{x}_1 - \frac{1}{4}\tilde{x}_1^TP\tilde{x}_1\\ &- \tilde{x}_1^T\Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1) - \frac{1}{4}P\Big)\tilde{x}_1.\end{aligned}$$

Defining the matrix P as

$$P = pI, \quad p = \sum_{i=1}^{6}\frac{1}{d_i} \tag{33}$$

we have, as shown in the Appendix C,

$$-z^TD_zz + z^TW\tilde{x}_1 - \frac{1}{4}\tilde{x}_1^TP\tilde{x}_1 \le 0. \tag{34}$$

Hence we can write

$$\begin{aligned}\dot{V} &\le -z^TC_zz - \tilde{x}_1^T\Big(M(q)K + C(q,x_1)\\ &- C(q,\tilde{x}_1) - \frac{1}{4}P\Big)\tilde{x}_1\end{aligned} \tag{35}$$

Using properties 1, 3, and assumption (2), we have

$$\begin{aligned} \dot{V} &\leq -(\underline{\sigma} - C_M \omega_{max} - C_M \|\tilde{x}_1\| - \frac{1}{4}p)\|\tilde{x}_1\|^2 \\ &- z^T C_z z \end{aligned} \tag{36}$$

where $\underline{\sigma} = \lambda_{min}(KM(q) + M(q)K)/2$. Hence $\dot{V} \leq 0$ if

$$\underline{\sigma} > C_M \omega_{max} + C_M \|\tilde{x}_1\| + \frac{1}{4}p. \tag{37}$$

As the region of attraction can be arbitrarily increased by the gain K, we have semi-global exponential stability.

Remark 2 *Using again (64) and (65) for the implementation of the adaptive observer, the implementation of controller (21) involves simply the calculation of $\tau(t)$ at time instant $t = i\Delta$.*

6 A Simulated Example

We consider a two-link manipulator, with masses m_1, m_2 [Kg], lengths l_1, l_2 [m], angles q_1, q_2 [rad], and torques τ_1, τ_2 [Nm]. The end-effector load m_2 is assumed to be unknown but constant with equations

$$\begin{aligned} \tau &= M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q), \qquad \theta = m_2 \\ M(q) &= \begin{pmatrix} m_2 l_2^2 + 2m_2 l_1 l_2 c_2 + (m_1+m_2)l_1^2 & m_2 l_2^2 + m_2 l_1 l_2 c_2 \\ m_2 l_2^2 + m_2 l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \\ C(q,\dot{q}) &= \begin{pmatrix} -2m_2 l_1 l_2 s_2 \dot{q}_2 & -m_2 l_1 l_2 s_2 \dot{q}_2 \\ m_2 l_1 l_2 s_2 \dot{q}_1 & 0 \end{pmatrix} \\ G(q) &= \begin{pmatrix} m_2 l_2 g c_{12} + (m_1+m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{pmatrix} \end{aligned} \tag{38}$$

with the short notation $c_2 = cos(q_2), c_{12} = cos(q_1 + q_2)$, etc. The model parameters are $m_1 = 1$ [Kg], $m_2 = 1.5$ [Kg], $l_1 = 1$ [m], $l_2 = 1$ [m]. Furthermore the regressor $\varphi = \varphi(q, \dot{q}, \ddot{q})$ associated with the unknown parameter m_2 is

$$\varphi = \begin{pmatrix} (l_2^2 + 2l_1 l_2 c_2 + l_1^2)\ddot{q}_1 + (l_2^2 + l_1 l_2 c_2)\ddot{q}_2 \\ -(2l_1 l_2 s_2 \dot{q}_1 \dot{q}_2 + l_1 l_2 s_2 \dot{q}_2^2) + (l_2 g c_{12} + l_1 g c_1) \\ (l_2^2 + l_1 l_2 c_2)\ddot{q}_1 + l_2^2 \ddot{q}_2 + l_1 l_2 s_2 \dot{q}_1^2 + l_2 g c_{12} \end{pmatrix}$$

The velocity estimate provided by the reduced-order adaptive observer at the ith time instant is calculated with (64) and (65) where

$$\begin{aligned} \psi(i-1) &= \hat{M}(q(i-1))^{-1}\Big(\tau(i-1) \\ &- \hat{C}(q(i-1), \hat{x}_1(i-1))\hat{x}_1(i-1) \\ &- \hat{G}(q(i-1))\Big) \end{aligned} \tag{39}$$

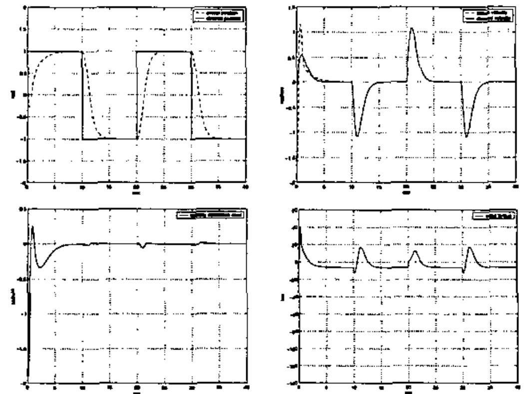

Figure 2: *Simulation results for the second link*

$$\hat{M}(q) = \begin{pmatrix} \hat{\theta} l_2^2 + 2\hat{\theta} l_1 l_2 c_2 + (m_1 + \hat{\theta}) l_1^2 & \hat{\theta} l_2^2 + \hat{\theta} l_1 l_2 c_2 \\ \hat{\theta} l_2^2 + \hat{\theta} l_1 l_2 c_2 & m_2 l_2^2 \end{pmatrix} \tag{40}$$

$$\hat{C}(q, \hat{x}_1) = \begin{pmatrix} -2\hat{\theta} l_1 l_2 s_2 \hat{x}_{12} & -\hat{\theta} l_1 l_2 s_2 \hat{x}_{12} \\ \hat{\theta} l_1 l_2 s_2 \hat{x}_{11} & 0 \end{pmatrix} \tag{41}$$

$$\hat{G}(q) = \begin{pmatrix} \hat{\theta} l_2 g c_{12} + (m_1 + \hat{\theta}) l_1 g c_1 \\ \hat{\theta} l_2 g c_{12} \end{pmatrix} \tag{42}$$

$$x_1 = \begin{pmatrix} x_{11} \\ x_{12} \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{43}$$

with $\varphi = \varphi(q, \hat{x}_1, \psi)$ as

$$\varphi = \begin{pmatrix} (l_2^2 + 2l_1 l_2 c_2 + l_1^2)\psi_1 + (l_2^2 + l_1 l_2 c_2)\psi_2 \\ -(2l_1 l_2 s_2 \hat{x}_{11}\hat{x}_{12} + l_1 l_2 s_2 \hat{x}_{12}^2) + (l_2 g c_{12} + l_1 g c_1) \\ (l_2^2 + l_1 l_2 c_2)\psi_1 + l_2^2 \psi_2 + l_1 l_2 s_2 \hat{x}_{11}^2 + l_2 g c_{12} \end{pmatrix}$$

The reference signals $\ddot{q}_d, \dot{q}_d, q_d$ are obtained with a second-order filter with poles in $-a$, that is

$$F(s) = \frac{a^2}{(s+a)^2}. \tag{44}$$

Furthermore the observer-controller parameters and the initial conditions are

$$\begin{gathered} K = 5I, \quad \Delta = 0.01\,[\mathrm{s}], \quad \Gamma = 0.1, \quad a = 2 \\ d_i = 1\,(i = 1, \ldots, 4), \quad C_1 = 2I, \quad C_2 = 2I \\ q(0) = \{0, 0\}\,[\mathrm{rad}], \quad \dot{q}(0) = \{0, 0\}\,[\mathrm{rad/s}] \\ \hat{x}_1(0) = \{1, 1\}\,[\mathrm{rad/s}], \quad \hat{\theta}(0) = 0.7\,[\mathrm{Kg}]. \end{gathered}$$

Results in figures 1, 2 and 3 show a good behaviour of the proposed adaptive observer-controller, even if the input torques have high peaks in the very first seconds, due to the initial velocity estimation error.

7 Discussion

In 1996 Lim et al. [6] have presented an output feedback control scheme for robot manipulators, based on an observed integrator backstepping procedure, which achieves semiglobal exponential stability for tracking errors. The adaptive observer-controller proposed in this paper is an extension of that result, covering also parameter uncertainties and smooth time-varying parameters, thanks to the adaptation law. Furthermore, it allows us to eliminate the need of tachometers, that are required by adaptive controllers [4] and introduce some noise anyway. With sensor noise, controller gains are not allowed to be high, so it results in larger tracking errors, and velocity filtering can be only partially a solution because of the introduced time delay that can not be accepted in high performance tracking. A passivity-based approach for designing observer-based adaptive robot control is shown by Berghuis in [5], by using a bounded adaptation law, but achieving only stability for the tracking error dynamics. Instead, as pointed out above, the control scheme presented in this paper achieves asymptotic stability both for the estimation error and the tracking errors.

8 Conclusion

An output feedback adaptive control scheme for robot manipulators has been presented, that allows the separate design of the adaptive observer from the state-feedback controller. By applying Lyapunov stability theory, for the closed-loop system semiglobal asymptotic stability has been proven, with respect to position and velocity tracking errors and velocity estimation error. Using this approach, the behaviour of the closed-loop system seems to be good even for small observer gains, that means low sensitivity to noise and smooth control signals.

Appendix A—Proof of Theorem 1

Consider the Lyapunov function

$$V(e(t)) = \frac{1}{2}e^T(t)Pe(t), \tag{45}$$

where

$$e(t) = \begin{pmatrix} \tilde{x}_1^T(t) & \tilde{\theta}^T(t) \end{pmatrix}^T \tag{46}$$

it follows that

$$\frac{1}{2}p_l\|e(t)\|^2 \le V(e(t)) \le \frac{1}{2}p_u\|e(t)\|^2. \tag{47}$$

The time derivative of $V(e(t))$ along trajectories of $\tilde{x}_1$ and $\tilde{\theta}$ is

$$\dot{V} = \tilde{x}_1^T M(q)\dot{\tilde{x}}_1 + \frac{1}{2}\tilde{x}_1^T \dot{M}(q)\tilde{x}_1 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}. \tag{48}$$

Subtracting (6) from (4), we have

$$\begin{aligned} M(q)\dot{\tilde{x}}_1 &= -C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1 \\ &- M(q)K\tilde{x}_1 - \Theta \end{aligned} \tag{49}$$

where

$$\begin{aligned} \Theta &= \Big(M(q)\psi + C(q,\hat{x}_1)\hat{x}_1 + G(q)\Big) \\ &- \Big(\hat{M}(q)\psi + \hat{C}(q,\hat{x}_1)\hat{x}_1 + \hat{G}(q)\Big) \end{aligned} \tag{50}$$

By applying property 5, we have

$$\begin{aligned} \Theta &= \varphi_0(q,\hat{x}_1,\psi) + \varphi(q,\hat{x}_1,\psi)\theta - \varphi_0(q,\hat{x}_1,\psi) \\ &- \varphi(q,\hat{x}_1,\psi)\hat{\theta} = \varphi(q,\hat{x}_1,\psi)\tilde{\theta} \end{aligned} \tag{51}$$

and (49) becomes

$$\begin{aligned} M(q)\dot{\tilde{x}}_1 &= -C(q,x_1)x_1 + C(q,\hat{x}_1)\hat{x}_1 \\ &- M(q)K\tilde{x}_1 - \varphi(q,\hat{x}_1,\psi)\tilde{\theta}. \end{aligned} \tag{52}$$

Now, substituting (52) into (48) and using property 2, we have

$$\begin{aligned} \dot{V} &= -\tilde{x}_1^T\Big(M(q)K + C(q,x_1) - C(q,\tilde{x}_1)\Big)\tilde{x}_1 \\ &+ \tilde{x}_1^T\Big(\frac{1}{2}\dot{M}(q) - C(q,x_1)\Big)\tilde{x}_1 \\ &+ \tilde{\theta}^T\Big(-\Gamma^{-1}\dot{\hat{\theta}} - \varphi^T(q,\hat{x}_1,\psi)\tilde{x}_1\Big) \end{aligned} \tag{53}$$

which in conjunction with (7), properties 1, 3 and 4, and assumption (2), gives

$$\dot{V} \le -(\underline{\sigma} - C_M\omega_{max} - C_M\|\tilde{x}_1\|)\|\tilde{x}_1\|^2. \tag{54}$$

Hence $\dot{V} \le -\beta\|\tilde{x}_1\|^2$ if $\underline{\sigma} > C_M\omega_{max} + C_M\|\tilde{x}_1\| + \beta$.. Since $\|e\| > \|\tilde{x}_1\|$, this condition holds if

$$\|e\| < \frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}. \tag{55}$$

From (54), (55) and (47), if follows that if

$$\|e(0)\| < \sqrt{\frac{p_l}{p_u}}\Big(\frac{1}{C_M}(\underline{\sigma} - \beta) - \omega_{max}\Big) \tag{56}$$

then

$$\dot{V} \le -\beta\|\tilde{x}_1(t)\|^2 \quad \forall t \ge 0 \tag{57}$$

and the desired result follows.

Appendix B—An implementable approximation of the observer

Integrating (6) and (7) over the time interval $[t_0, t]$ and using the estimated initial conditions $\hat{x}_1(t_0)$ and $\hat{\theta}(t_0)$ we obtain

$$\hat{x}_1(t) = f(t) + \int_{t_0}^{t}[\psi(q,\hat{x}_1,\tau,\hat{\theta}) - K\hat{x}_1]\mathrm{d}t \tag{58}$$

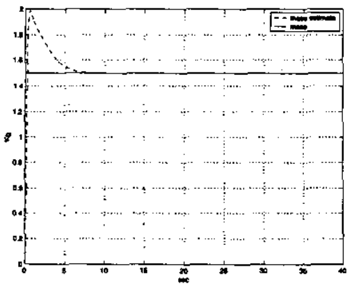

Figure 3: *Parameter estimate*

where

$$f(t) = \hat{x}_1(t_0) + K[q(t) - q(t_0)] \quad (59)$$

is known, and

$$\hat{\theta}(t) = \hat{\theta}(t_0) - \Gamma \int_{t_0}^{t} \varphi^T(q, \hat{x}_1, \psi)\tilde{x}_1 \mathrm{d}t. \quad (60)$$

Using $\tilde{x}_1 = x - \hat{x}_1$ with $x = dq/dt$, (60) becomes

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t_0) - \Gamma \int_{q(t_0)}^{q(t)} \varphi^T(q, \hat{x}_1, \psi)\mathrm{d}q \\ &+ \Gamma \int_{t_0}^{t} \varphi^T(q, \hat{x}_1, \psi)\hat{x}_1 \mathrm{d}t. \end{aligned} \quad (61)$$

The system of integral (58) and (61), together with initial conditions $\hat{x}_1(t_0)$ and $\hat{\theta}(t_0)$, provides an equivalent version of the adaptive observer (6) and (7), but is also not implementable since the evaluation of $\theta\hat{(t)}$ in (61) involves the differentiation of the joint position $q(t)$. From (58) and (61), it follows that for an arbitrary fixed time interval $\Delta > 0$, we have

$$\begin{aligned} \hat{x}_1(t) &= \hat{x}_1(t-\Delta) + K[q(t) - q(t-\Delta)] \\ &+ \int_{t-\Delta}^{t} [\psi(q, \hat{x}_1, \tau, \hat{\theta}) - K\hat{x}_1]\mathrm{d}t \end{aligned} \quad (62)$$

and

$$\begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t-\Delta) - \Gamma \int_{q(t-\Delta)}^{q(t)} \varphi^T(q, \hat{x}_1, \psi)\mathrm{d}q \\ &+ \Gamma \int_{t-\Delta}^{t} \varphi^T(q, \hat{x}_1, \psi)\hat{x}_1 \mathrm{d}t. \end{aligned} \quad (63)$$

Assuming that $\psi(q, \hat{x}_1, \tau, \hat{\theta})$, $\varphi(q, \hat{x}_1, \psi)$ and $q(t)$ are continuous time functions and that Δ is sufficiently small, (62) and (63) suggest a discrete implementation of the proposed observer as follows

$$\begin{aligned} \hat{x}_1(i) &= (I - \Delta K)\hat{x}_1(i-1) + \Delta\psi(i-1) \\ &+ K[q(i) - q(i-1)] \end{aligned} \quad (64)$$

$$\begin{aligned} \hat{\theta}(i) &= \hat{\theta}(i-1) + \Gamma\varphi^T(i-1)[\Delta\hat{x}_1(i-1) \\ &- q(i) + q(i-1)]. \end{aligned} \quad (65)$$

Remark 3 *Although (64) and (65) are only an approximation of the proposed observer (58) and (61), anyway they are implementable and approach to (58) and (61) as Δ approaches to zero. Therefore (64) and (65) stand for a good representative of the observer if the sampling interval Δ is sufficiently small.*

Appendix C—Proof of (34)

The left side of Eq. (34) can be expanded as

$$\begin{aligned} &- z^T D_z z + z^T W\tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P\tilde{x}_1 + \\ &= -z_1^T D_1 z_1 - z_2^T D_2 z_2 + z_1^T\tilde{x}_1 + z_2^T\Omega\tilde{x}_1 - \frac{p}{4}\tilde{x}_1^T\tilde{x}_1. \end{aligned} \quad (66)$$

Using definitions (18), (23), and (24) together with $z_1 = [\bar{z}_1, \bar{z}_2, \bar{z}_3]^T$ and $z_2 = [\bar{z}_4, \bar{z}_5, \bar{z}_6]^T$, (66) can be rewritten as follows

$$\begin{aligned} &- z^T D_z z + z^T W\tilde{x}_1 - \frac{1}{4}\tilde{x}_1^T P\tilde{x}_1 \\ &= -\sum_{i=1}^{3}\Big[d_i\Big(\bar{z}_i - \frac{1}{2d_i}\tilde{x}_1\Big)^T\Big(\bar{z}_i - \frac{1}{2d_i}\tilde{x}_1\Big) \\ &+ d_{i+3}\Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{x}_1\Big)^T \\ &\Big(\bar{z}_{i+3}\omega_{i+3} - \frac{1}{2d_{i+3}}\tilde{x}_1\Big)\Big] \le 0 \end{aligned} \quad (67)$$

because all the quadratic terms in (67) are less than or equal to zero.

References

[1] M. Erlic, W.-S. Lu. A Reduced-Order Adaptive Velocity Observer for Manipulator Control (IEEE Transactions on Robotics and Automation, Vol. 11, No. 2, April 1995)

[2] T.I. Fossen, Å. Grøvlen. Nonlinear Output Feedback Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping (IEEE Transactions on Control Systems Technology , Vol. 6, No. 1, January 1998)

[3] M. Krstic, I. Kanellakopoulos, P. Kokotovic: Nonlinear and Adaptive Control Design (Wiley, 1995)

[4] R. Johansson: Adaptive Control of Robot Manipulator Motion (IEEE Transactions on Robotics and Automation, Vol. 6, No. 4, August 1990)

[5] H. Berghuis: Model-Based Robot Control: From Theory to Practice (PhD thesis, University of Twente, The Netherlands, 1993)

[6] S.Y. Lim, D.M. Dawson, K. Anderson: Re-Examining the Nicosia-Tomei Robot Observer-Controller from a Backstepping Perspective (IEEE Transactions on Robotics and Automation, Vol. 4, No. 3, May 1996)