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INSTITUT DE PHYSIQUE NUCLEAIRE D'ORSAY

THE NUCLEAR POTENTIAL ENERGY SURFACE AND ODD-PARTICLE EFFECTS
FOR SOME NUCLEI IN THE REGION $80 < A < 120$

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Abstract

The nuclear potential energy surface including the quadrupole and hexadecapole degrees of freedom has been investigated for nuclei in the $Z = 40-60$ and neighbouring regions. The estimation of the parameters of the single-particle potential in those mass regions is discussed in some detail and their influence on the properties of the potential energy surface is assessed.

Introduction

For the description of transitional nuclei static and dynamical aspects have to be considered simultaneously. As an example of a dynamic theory one should mention the well-known calculations by Baranger and Kumar [1]. In this theory both rotational and vibrational dynamics are incorporated in terms of the ellipsoidal beta and gamma coordinates.

Our contribution to this problem is limited to the static parts of the problem. We thus claim to be able to calculate an improved potential-energy surface, in which, however, we for the moment have only included the ϵ (or β_2) and the ϵ_4 degrees of freedom. A more complete calculation, also including the gamma or axial asymmetry coordinate is presently well under way in Lund as in Basel and Dubna. We thus calculate the potential energy

$$W(\epsilon, \epsilon_4, \text{ and later } \gamma).$$

Making the tacit assumption that dynamics does not upset the situation we furthermore posit that the shape of the lowest minimum is directly related to the ground state properties of the nucleide in question.

The single-particle potential and the choice of the corresponding parameters

We have studied a single-particle potential of the following type [2]

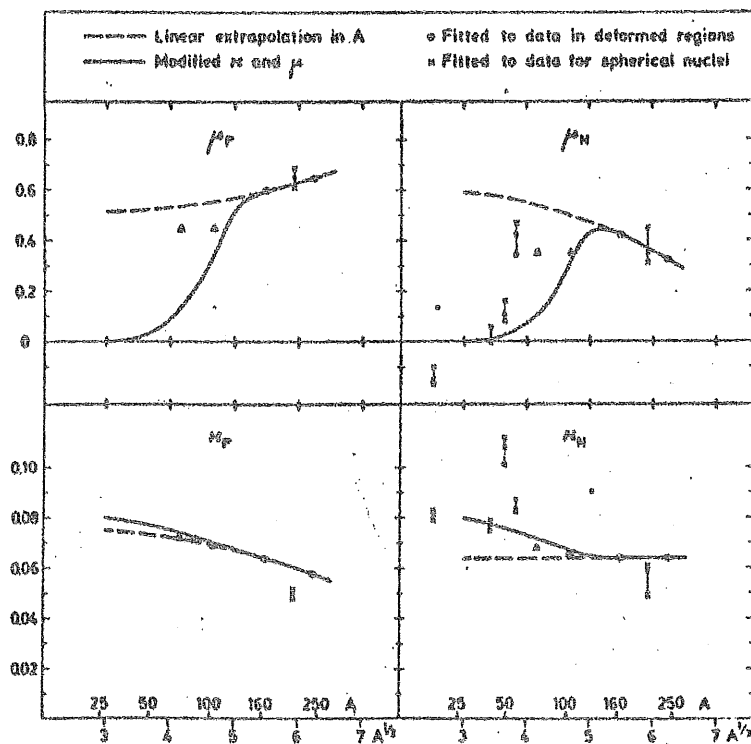
$$V_{s.p.} = \hbar \omega_0 \left(\epsilon_0 + \epsilon_4 \right) \rho^2 \left(1 - \frac{2}{3} \epsilon P_2 + 2 \epsilon_4 P_4 \right) - \kappa \hbar \omega_0 \left[2 \mathcal{L}_t \cdot \tilde{s} + \mu \left(\mathcal{L}_t^2 - \langle \mathcal{L}_t^2 \rangle \right) \right]$$

The choice of the potential parameters κ and μ is a most critical one in the region $A = 80-120$ to which region of nuclei we have confined our interest in the present investigation. These parameters are well determined for the deformed regions $A = 150-190$ and $A \geq 220$ as well as the small deformed region near $A \approx 25$. One might also in this determination of κ and μ make use of spectroscopic data [3] for closed shell ± 1 nuclei near $^{16}_0, ^{40}_{20}\text{Ca}, ^{48}_{20}\text{Ca}, ^{56}_{28}\text{Ni}$ as $^{208}_{82}\text{Pb}$ to study the variation of κ and μ over a larger region for instance as a function of A or $A^{1/3}$. (Actually one has reason also to suspect at least a dependence on isospin in addition to the dependence on A).

This A -dependence is shown in fig. 1. The set of points in these figures represent alternative fits to experiments. The alternatives correspond to different weightings being given to the excited states in the single-particle spectra.

Fig. 1. The single-particle model parameters κ and μ as functions of $A^{1/3}$. In the deformed regions ($A = 165$ and $A = 242$) the values have been chosen to fit the experimental energy levels as well as possible [2]. The dashed

lines indicate a linear extrapolation in A based on the heavy deformed regions while when we have drawn the continuous lines we have somewhat arbitrarily taken the data for $A \approx 25$ into account. The crosses indicate least-square fits to spectroscopic data for closed shell ± 1 nuclei near $^{16}_0, ^{40}_{20}\text{Ca}, ^{48}_{20}\text{Ca}, ^{56}_{28}\text{Ni}$ and $^{208}_{82}\text{Pb}$. The three different crosses above each other indicate that alternative weights have been given to the different states in the single-particle spectra. The triangles represent values used in the calculations (see figs 6, 8, 9 and 10).



Generally the lowest excitations are given most weight. It is not a straight-forward problem to obtain reliable smooth functions for κ and μ from these graphs. Particularly uncertain appear the regions $50 \lesssim A \lesssim 120$, where the μ -functions bend over dramatically as functions of $A^{1/3}$.

The extrapolations to the superheavy element region, recently of interest, depend critically on these empirical determinations of κ and μ . In fact whether the main emphasis is being put on the data from the deformed region or on the data of the Pb and Ca regions leads to somewhat different gap predictions connected with $Z = 114$, $N = 184$. And the different predictions made by the different groups working on the problem of the prediction of super-heavy element half-lives can be traced back largely to the corresponding parameter fits. Particular critical is the spin-orbit splitting parameter κ .

The total nuclear energy and the shell correction method

The total energy has been evaluated on the basis of the Strutinsky shell correction method [4]. One thus calculates

$$\hat{E} = \sum \epsilon_\nu - \langle \sum \epsilon_\nu \rangle + E_{\text{PAIR}} + E_{\text{SURF}} + E_{\text{Coul}}$$

The averaged energy function denoted $\langle \sum \epsilon_\nu \rangle$ based on a smeared level density is obtained by the use of a smearing function containing a sixth order polynomial correcting for unwanted longrange smearing effects. The pairing energy is evaluated using the pairing matrix elements

$$G_p \cdot A = g_p^0 + g_p^1 \frac{N-Z}{A}$$

$$G_n \cdot A = g_n^0 + g_n^1 \frac{N-Z}{A}$$

In the region $Z > 50$ and the region $40 \leq Z \leq 50$ together with "extrapolated" shell model parameters we have used $g_p^0 = 26.2$, $g_p^1 = -23.5$, $g_n^0 = 26.2$, $g_n^1 = -38.5$ while for "modified parameters" in the region $40 \leq Z \leq 50$ we have instead used the following values 22.5, 0, 22.5, and -18.0, respectively for the listed parameters. The number of levels included in the pairing sum has in all cases been taken as $\sqrt{10Z}$, and $\sqrt{10N}$. The latter description has been extended with a trivial modification to the few cases treated with $Z < 40$. The Coulomb and surface energies have been calculated as in ref. [2].

Results of calculations for even-even nuclei

We employ estimates of κ - and μ -values for the even-even surveys with $Z > 40$ according to two variants exhibited by the solid and dashed lines of fig. 1 respectively. One corresponds to a linear extrapolation in A on the basis of data of the two heavy deformed regions only. This is called "linearly extrapolated". The other variant corresponds to values of κ and μ with a position relative to the data as exhibited in fig. 1. This variant is denoted "modified". For the special cases given by figs 6, 8, 9 and 10 we have chosen parameters as given by the triangles of fig. 1.

We give in figs. 2 and 3 a cut through the potential-energy surface for isotopes of ${}_{62}\text{Sm}$ and of ${}_{58}\text{Ce}$ using linearly extrapolated κ and μ parameters.

The onset of prolate distortions for $N > 88$ is obvious for $Z = 62$ while the $N = 88$ isotope appears non-deformed in view of the additional zero-point energy that should be associated with the beta and gamma degrees of freedom. One should probably allow for 1-1.5 MeV of such zero-point energy in this region.

In the case of Ce isotope with $N < 82$ we can by the same token expect deformed ground states only for neutron values equal to 70 and smaller.

In the $40 \leq Z \leq 50$ region some neutron rich isotopes with $N > 60$ have recently been observed [5] and a prediction of the potential energy surface is there of great interest.

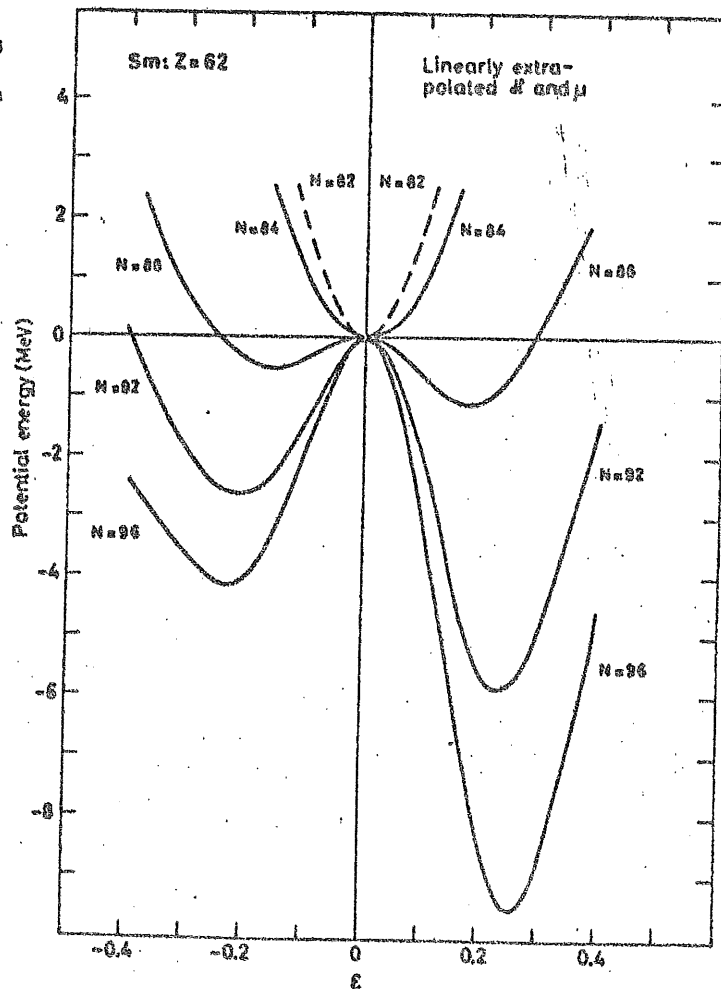


Fig. 2. A cut through the potential energy surface for some Sm-isotopes with $N \geq 82$. For each ϵ -value the energy has been minimized with respect to ϵ_4 .

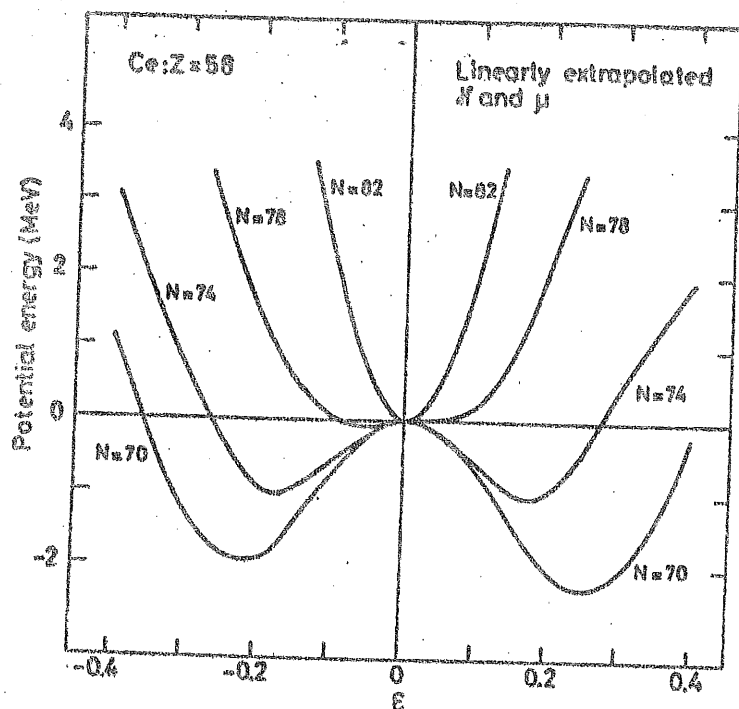


Fig. 3. Same as fig. 2. for some Ce isotopes with $N \leq 82$.

We give results of calculations for isotopes of ${}_{44}\text{Ru}$ for the two variants of κ and μ as indicated in fig. 4. Generally distorted shapes are predicted for $N > 60$ in the linearly extrapolated case. However, the neglected gamma degree of freedom there appears to play a major role. For "modified" parameters, on the other hand, oblates appear clearly favoured for $N > 64$. No stable deformation are on the whole expected for $N < 64$ for calculations based on "modified parameters". We are thus unable to give reliable predictions in this whole region of nuclei ($40 \leq Z < 50$)

in view of our insufficient knowledge of κ and μ . For predictions as to ground state shapes in these entire regions of nuclei see ref. [6].

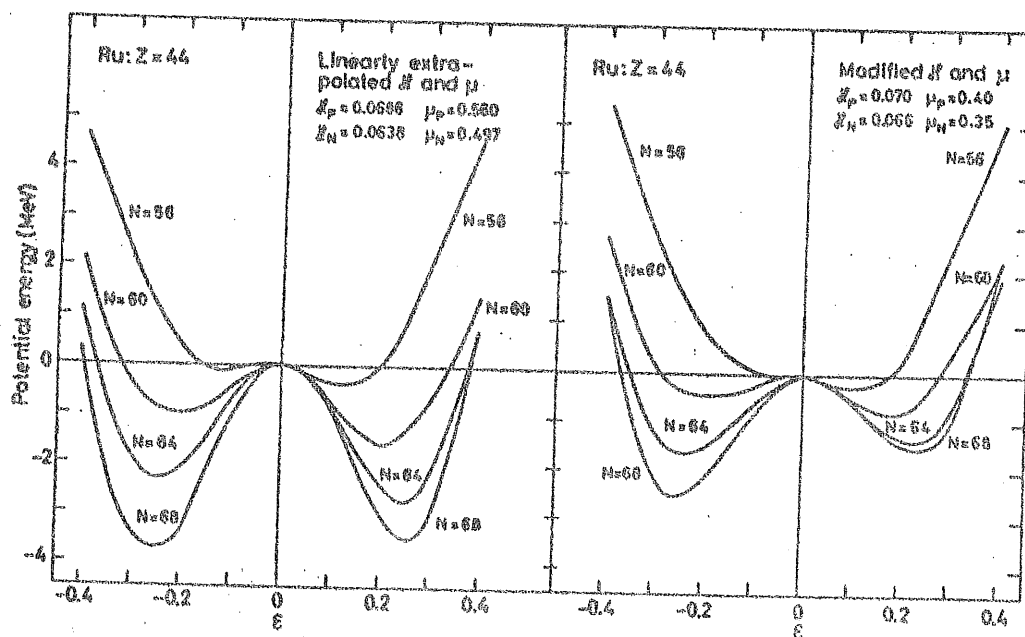


Fig. 4. Same as fig. 2 for some neutronrich Ru-isotopes. A comparison is made for the results obtained based on the "linear extrapolated" and "modified" κ - and μ -values, respectively.

The plot of masses based on the modified oscillator potential published in ref. [2] has been extended below $A = 150$ for both of the two parameter variants described. It turns out that the so-called "modified" variant gives by far the better masses as seen from fig. 5.

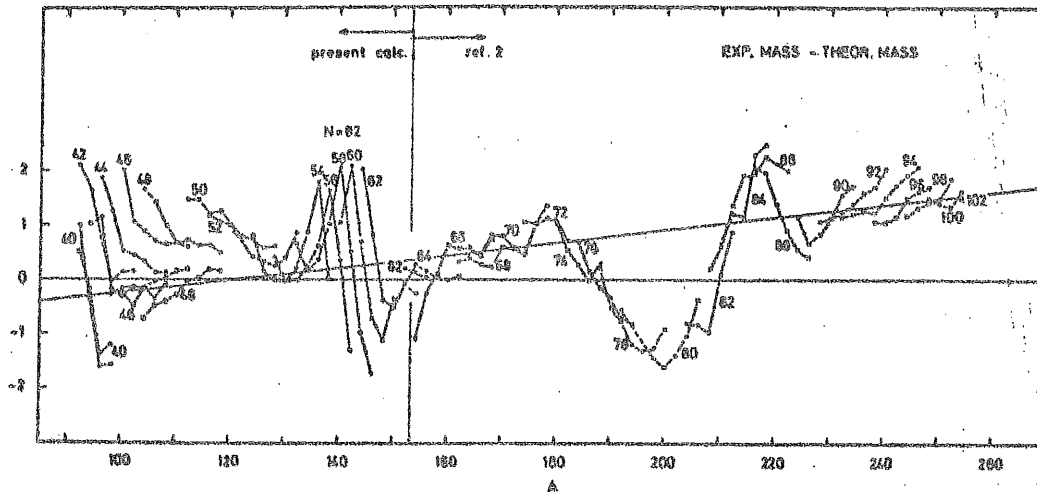


Fig. 5. The difference between the experimental and theoretical mass values. For $Z \geq 62$ the results of ref. 2 have been taken over. For $Z \leq 62$ the crosses correspond to "linearly extrapolated" κ - and μ - values while the points correspond to the "modified" values.

The "modification" seems to bring about an enlargement of the $Z = 40$ gap (see fig. 7.)

Some preliminary calculations have also been performed for nuclei lighter than $Z = 40$. It turns out that, based on a set of single-particle potential parameters intermediate between "extrapolated" and "modified" (cf. fig. 1), oblate distortions are expected in the region, around $Z \approx N \approx 36$. We give as an example in fig. 6 the potential energy surface for some isotopes of Kr ($Z = 36$). Of these the interesting isotopes with $N = 38$ or 36 should be experimentally accessible with present technique. The energy minima have distortions that correspond to ϵ between -0.2 and -0.35 . The corresponding ϵ_4 -values are small. The effects of the zero-point fluctuations on the shallow minima remain to be investigated as the entire effect of the gamma degree of freedom.

The same observation as to the predictions of a preponderance of oblate shapes connected with Z and N near 36 have earlier been made by Dickman et al. [7].

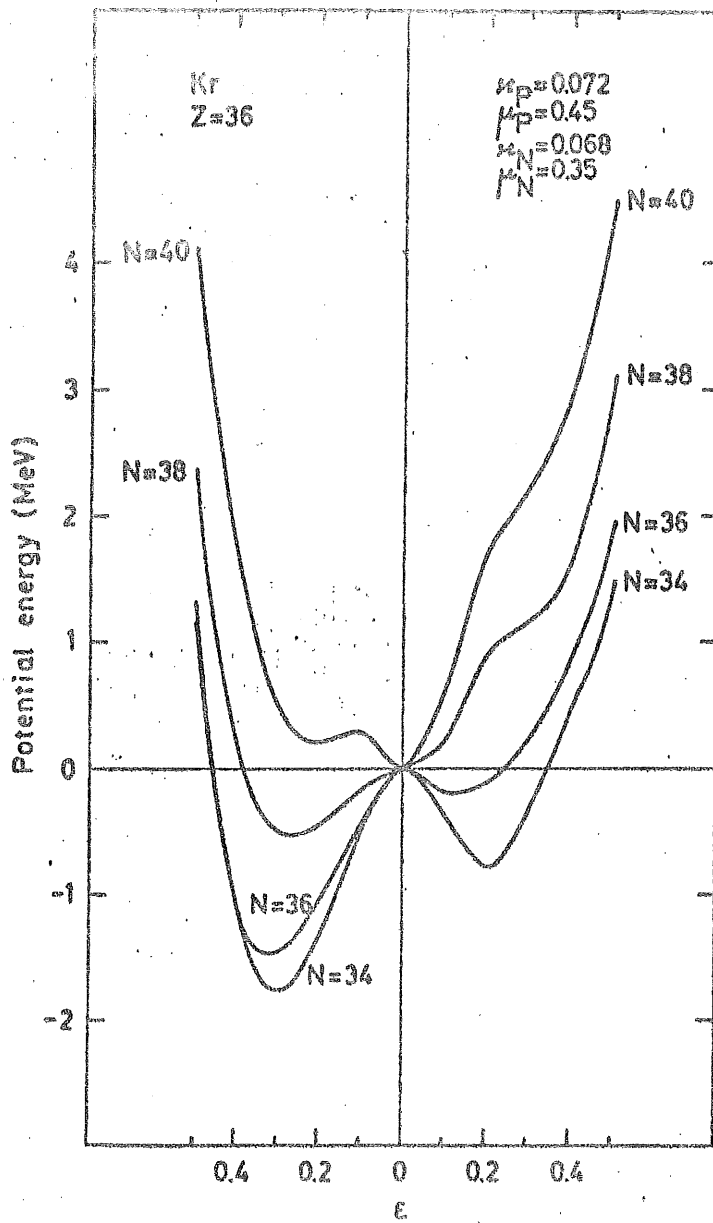


Fig. 6. Same as fig. 2 for some neutron-deficient Kr-isotopes. The κ - and μ -values are given according to the triangles of fig. 1.

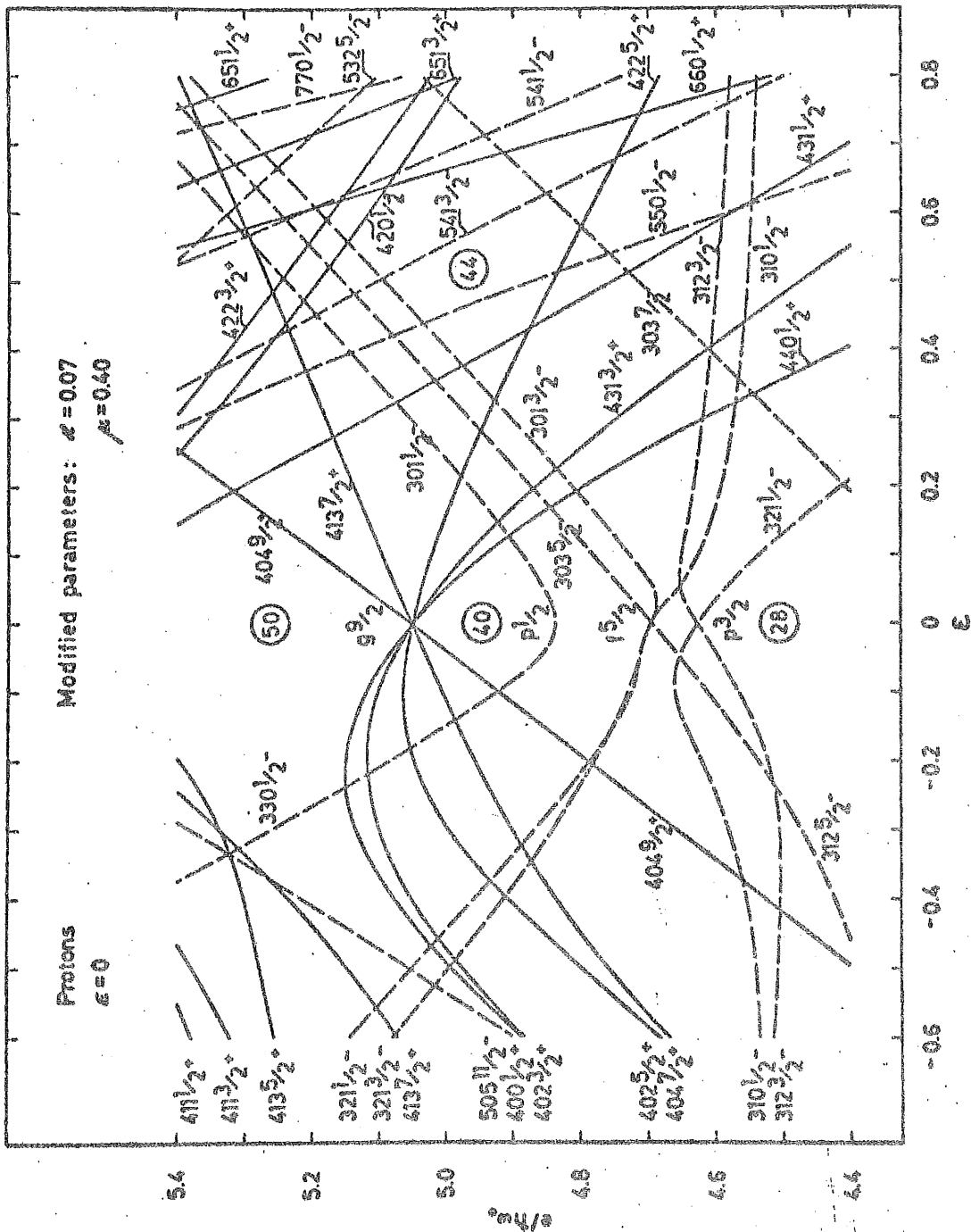


Fig. 7. The single-proton scheme for the "modified" parameters and $A \approx 110$. Note the $[404\ 9/2^+]$ and $[505\ 11/2^-]$ levels going steeply downwards on the oblate side.

Odd-particle effects

Theoretical studies indicate that more stably deformed shapes can be obtained in this mass region for odd-A or odd-odd nuclei due to the association of the odd particle with particular orbitals of certain, usually high, spins. Particularly favourable in this respect are the orbitals f $7/2$ $7/2$, g $9/2$ $9/2$ and h $11/2$ $11/2$ associated with N and Z values near 15, 35 and 55-60 respectively. The first orbital, when occupied by the odd neutron in ^{29}Si , is found to be associated experimentally with a $K = 7/2$ band, of which two members have been observed. This shape isomeric state is also borne out in theoretical calculations published by Ragnarsson and Nilsson [8].

We give in fig. 7 the single-proton level scheme around $Z = 40$ ("modified" κ and μ) exhibiting both of the mentioned g $9/2$ $9/2$ or $[404$ $9/2]$ and the h $11/2$ $11/2$ or $[505$ $11/2]$ orbitals, the latter only partially.

In Fig. 8 the low-lying energy-surfaces of ^{73}Br are exhibited. These should be compared to that of ^{74}Kr (fig. 6).

The ^{73}Br energy surface has been obtained under the condition that the odd particle is restricted to orbitals with prescribed spin and parity. Of special interest is the state with $\Omega = 9/2+$. As $[404$ $9/2]$ is the only available orbital with $\Omega = 9/2+$, this gives rise to very large "specialisation energies" away from equilibrium, resulting in a very deep potential energy minimum, which should imply an associated, well deformed nuclear shape, as other

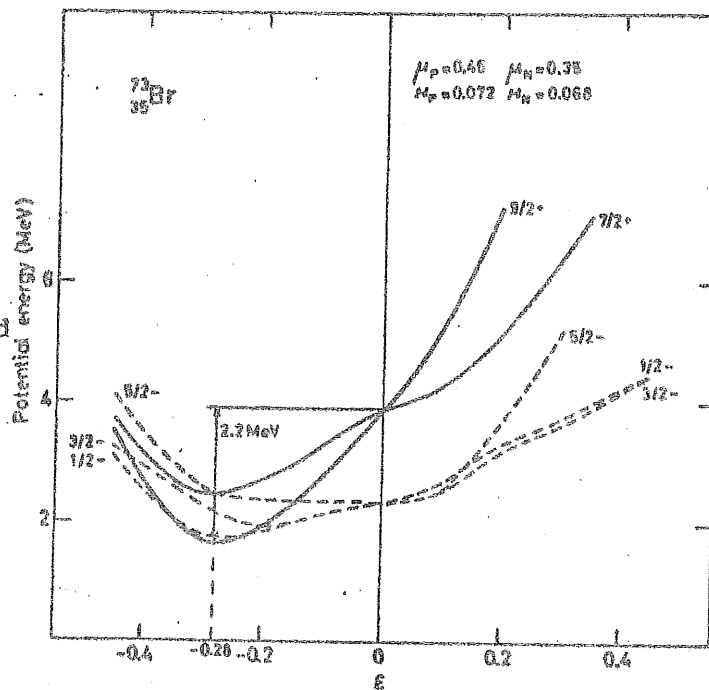


Fig. 8. The potential energy surface for ^{73}Br when the odd particle is associated with different values of Ω and parity. Only the lowest states are indicated. The κ - and μ -values are the same as those in fig. 6. For each ϵ -value the energy is minimized with respect to ϵ_4 .

states with $\Omega = 9/2^+$ can only be obtained through the breaking of pairs and should be rather distant in energy. These excitations of higher seniority have been neglected in these calculations.

Recent experimental results [9] may also indicate that a low-lying $11/2$ isomer in ^{103}Ru corresponds to $\ell = 5$ and is indeed the $[505\ 11/2]$ orbital being occupied by the 59th neutron for oblate shapes. An experimental identification of the $I = 13/2$ member of the rotational band should obviously be of very great interest.

In fig. 9 we give projected potential-energy surfaces associated with $N = 59$. The $11/2$ -orbital is found to give a relatively low-lying minima for all the isotones plotted. Even though some other low-spin states may be associated with soft spherical shapes, the $11/2$ -state should occur as a shape isomer in the spectrum.

The extra hindrance due to the particular oblate shape associated with the $11/2$ -state has not been quantitatively estimated. A factor of 100 or less may be expected from experience in other regions of nuclei.

Similar plots are shown for $Z = 51, 53$ and 55 in fig. 10 associated with the same orbital $[505\ 11/2]$.

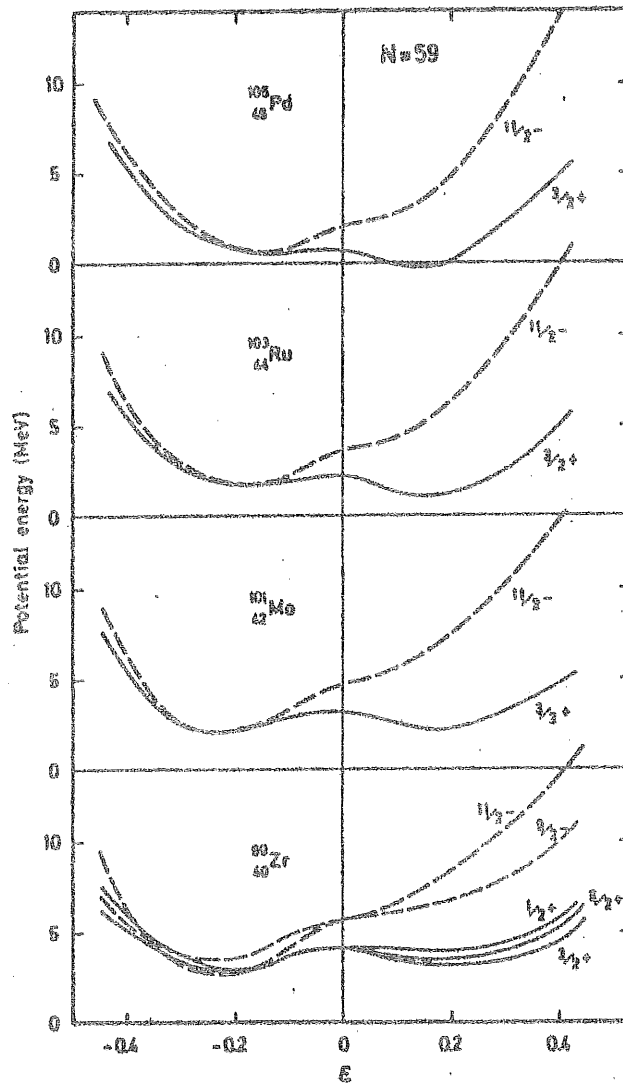


Fig. 9. Same as fig. 8, for some nuclei with $N=59$. The low-lying states are in all cases similar to those of ^{99}Zr and therefore just the lowest ones are shown for the other nuclei. Only the ϵ degree of freedom has been included. The κ^- and μ^- values have been chosen as indicated by the triangles in fig. 1. ($\kappa^+ = 0.069$, $\mu^+ = 0.45$, $\kappa^- = 0.066$, $\mu^- = 0.35$).

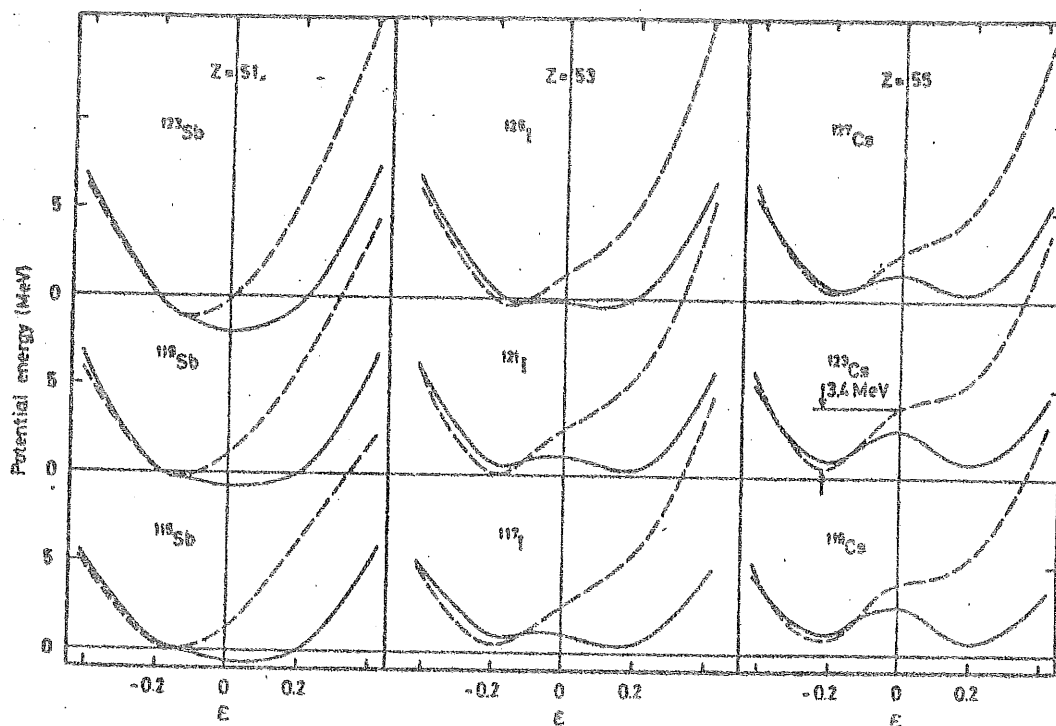


Fig. 10. Same as fig. 8 for some neutron deficient nuclei with $Z = 51, 53$ and 55 . Only the lowest state and the $11/2$ -state have been drawn and just the ϵ degree of freedom has been studied. The parameters are the same as in fig. 9, i. e. for these heavier nuclei the μ -values have probably been chosen somewhat small (see fig. 1.).

The mentioned orbitals $[303\ 7/2]$, $[404\ 9/2]$ and $[505\ 11/2]$ are thus highly favourable in the generation of oblate shape isomers provided the energy surface due to the even-even core is already associated with a weak oblate minimum or generally very soft to distortions.

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