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From Features via Frames to Spaces: Modeling Scientific Conceptual Change without Incommensurability or Aprioricity

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The (dynamic) frame model, originating in artificial intelligence and cognitive psychology, has recently been applied to change-phenomena traditionally studied within history and philosophy of science. Its application purpose is to account for episodes of conceptual dynamics in the empirical sciences (allegedly) suggestive of incommensurability as evidenced by "ruptures" in the symbolic forms of historically successive empirical theories with similar classes of applications. This article reviews the frame model and traces its development from the feature list model. Drawing on extant literature, examples of frame-reconstructed taxonomic change are presented. This occurs for purposes of comparison with an alternative tool, conceptual spaces. The main claim is that conceptual spaces save the merits of the frame model and provide a powerful model for conceptual change in scientific knowledge, since distinctions arising in measurement theory are native to the model. It is suggested how incommensurability as incomparability of theoretical frameworks might be avoided (thus coming *on par* with a key-result of applying frames). Moreover, as non(inter-)translatability of world-views, it need not to be treated as a genuine problem of conceptual representation. The status of laws *vis à vis* their dimensional bases as well as diachronic similarity measures are (inconclusively) discussed.

Keywords: dimension, measurement, natural law, scientific change, symbolic representation.

1. Introduction

Starting with Minsky (1975) and more widely recognized since Barsalou's (1992) work, (dynamic) frames are a rather well-accepted tool for modeling conceptual knowledge. Beginning with the predecessor model, the feature list (section 2), we trace its development into the (dynamic) frame model (2.1) by summarizing frame-reconstructed episodes of taxonomic change (2.2) as paradigmatic examples of recent application within history and philosophy of science (3) addressing the 'incommensurability of frameworks/world-views' from a cognitive historical perspective (3.1). We hold that a frame is a sophisticated feature list, serving to support the claim that historically successive taxonomies are comparable, but criticize that the frame model seems to yield little insight beyond taxonomic change. Introducing conceptual spaces as an alternative model (4), dimensions (4.1), their combinations (4.2), how to recover frames (5) by analogue expressions (5.1), the notion of 'similarity as geometric distance' across diachronically varying spaces (5.2) and the status of scientific laws (6) are discussed.

2. Feature Lists

Its origins in Aristotelian philosophy (Taylor 2003), the feature list model may count as the most entrenched and, at the same time, the most outdated tool for reconstructing conceptual knowledge. Paradigmatically instantiated by taxonomic knowledge (e.g. in biology), Kuukkanen usefully summarizes the classical view by three assumptions:

(1) [T]he representation of a concept is a summary description of an entire class of instances that fall under it; (2) the features that represent a concept are singly necessary and jointly sufficient to define that concept; and (3) features are nested in subset relations, i.e. if a concept C is a subset of

concept Y, the defining features of Y are nested in those of C. For this reason, features are sometimes referred to as defining or essential. (Kuukkanen 2006: 88)

On the classical view, combinations of binary features (or attributes) define a thing which falls under (or instantiates) a concept, *if and only if* the features are present in (or true of) the thing. Features are rendered in natural language, normally by nouns or adjectives. As a classical example: MAN may be analyzed as [+ biped, + rational, + animal]. As a discrimination issue, problems arise upon observing that a Para-Olympic athlete may fail to instantiate MAN—which is somewhat absurd. However, short of throwing individually necessary and jointly sufficient features over board, the problem is not easily remedied.

The model's distinct merit is found in worlds cut along patterns generated by features. The choice of features may always be viewed as a matter of convention, particular conventions as contingent upon contexts. For instance, to categorize *champagne*, *vodka*, *fruit juice* and *soda water*, why not borrow from chemistry and use: [+/-C₆] alcohol, [+/-CO₂] carbon-dioxide.

Barsalou and Hale (1993) demonstrate that, as representations of conceptual knowledge, feature lists contain rich relational information, primarily with respect to *truth* (attributes count as true or false of a thing). Secondly, whatever a feature names, if true of the thing, will name one of its *aspects*. Thirdly, as set-members, a concept's defining features obey the logical relation of *conjunction*, just as several concepts obey *exclusive disjunction*. Fourthly, *contingent* relational information may be read off the feature list, allowing strict or probabilistic predictions, e.g., "Consumers of items in the +C6 category (likely) need a designated driver". Finally, nesting of concepts accounts for the *analytic* character of "A bachelor is a man," because BACHELOR, when analyzed as [+ man], [- married], is subordinate to MAN.

Exemplar and (weighed) prototype models are mathematical refinements of the feature list model, seeking to remedy the absurdity of the Para-Olympic example, above. At least in part, they are also motivated by empirical investigation into human categorization (Labov 1973, Rosch et al. 1976), strongly suggesting that we do not, invariably across contexts, categorize *via* necessary and sufficient features. Whether all models operate at the *symbolic* level, i.e., presuppose an explicit language, may be debated. At any rate, refined models "remain grounded" in feature lists, but abandon the strictness by which (possibly weighed) presence projects into category-membership. Thus, in principle, considerations of similarity (rather than identity) may govern concept boundaries (see Barsalou & Hale 1993: 103-124).

[A] few principled components underlie the feature list representation across a wide variety of categorization models. All of these representations use binary features, with some allowing continuous values under a binary interpretation. Relationally, all of these representations integrate features with various relations, including 'aspect', 'truth', 'and', 'or', 'compensates', 'implies', and 'predicts'. All of the feature list representations that we have considered are built up from this small set of components. (Barsalou & Hale 1993: 123)

The above-cited relations give rise to the *frame-account of concepts* which will be introduced below. Generally, one may say that the frame-model also qualifies as an *extension* of the traditional feature list. This extension is reached by allowing non-binary features (e.g., large, medium, small) and relations of constraint and invariance.

2.1 From Feature Lists to Simple and Recursive Frames

In support of the claim that a frame model is a sophisticated extension of a feature lists, consider that, when (i) suspending the additional functions introduced by frames and (ii) constraining attribute-values to binary options, the frame model *collapses* into the feature list model, rather than some model analogous to feature lists. This should become clear when appreciating that, step-wise, frames may be generated from feature lists.

The first step beyond feature lists requires understanding a feature as the *value* of some *attribute*. For example, [+ blue], [+ green] are values of the attribute 'color' and [+ long], [+ round] are values of the attribute 'shape'. The additional structure (over that of feature lists) consists in a *set* of values being used to define an attribute. The second step is taken by minding that the values of a particular attribute may be non-binary. Thus, an additional relation (which a feature list model does not allow to represent) is that between an attribute and its value(s), called the 'type'-relation (informally: the 'is-a relation'), e.g., 'square' is a type of shape, 'blue' is a color, etc. The third step is to understand attributes as exhibiting *structural invariants* which "specify relations between attributes that *do not vary often* across instances of a concept" (Barsalou & Hale 1993: 125, italics added), while *constraints* form relations between attribute values "which instead *vary widely* across the instances of a concept" (ibid. 125, italics added). In sum, we reach the notion of a *simple frame*, defined as "a co-occurring set of multi-valued attributes that are integrated by structural invariants" (ibid. 126).

Constraints hold across values and "produce systematic variability in attribute values" (Barsalou 1992: 37), e.g., a comparatively massive person (relative to height) will likely not be skinny. Together with invariants, constraints generate structure for the purpose of representing a concept(-instance)—giving rise to the notion 'frame-pattern'—and play an important role in reconstructing scientific conceptual change (see section 5.1).

The advantage of frames over feature lists is that "the addition of 'attribute-value relations' and 'structural invariants' increases their expressiveness substantially" (ibid. 127), because we are provided with means by which to model both stable and variable relations across attributes and values. One may then regard the representation of a concept to proceed *primarily* via structural invariants and constraints. Structural invariants tell you which attributes (are likely to) "collect" or "bind" into a concept, constrained values identify concept instances.

In a final step, by recursion, one allows the components used in conceptual representation (attributes, values, structural invariants and constraints) to be represented not by words, but by frames. "[T]his recursive process can continue indefinitely, with the components of these more specific frames being represented in turn by frames themselves" (ibid. 133). Where conceptual knowledge includes not just things, but also relations (e.g., 'is a part of' or 'requires'), again, frames are employed recursively. Generally, "[a]t any level of analysis, for any frame component, there is always the potential to note new variability across exemplars of the component and capture it in a still more specific frame" (ibid. 134). Thus, there is no principled limit to finding new attributes, "simply by noting variance across the component's exemplars and representing this variance with a new attribute-value structure" (ibid. 133f.).

Which attributes to include in a frame will normally be a result of querying subjects. It is assumed that the choice of attributes is always influenced by "goals, experience and intuitive theories" (Barsalou 1992: 34). Hence, the examples of frame-representations discussed in the literature count as *partial* representations. This also holds for event frames (aka. *scripts*), which are sequential adaptations of the object-frames discussed here. In the scientific case, the identification of attributes, values, etc. will be based on the (historical) material under study.

2.2 Motivated Conceptual Change

To appreciate the frame model, a simple example may be helpful. Based on Chen (2002), it comes from ornithology, does without iteration, and employs binary features.

In the late 18th century, ornithologists discovered a strange creature from South America by the common name of 'screamer' A peculiar feature of screamers is that they have webbed feet like ducks but a pointed beak like chickens. The combination of these two features, which were supposed to be incompatible according to the Ray taxonomy, caused confusion. The constraint between *foot* and *beak* in the Ray taxonomy required that these two attributes be used together in

classification. Thus, the discovery of screamers immediately generated problems, because ornithologists did not know how screamers should be classified according to the cluster of *foot* and *beak*. Eventually, this anomaly forced them to alter the frame of *bird* and the associated taxonomy, because it made a very important constraint relation between *foot* and *beak* invalid. (Chen 2002: 7)

The diagrams below are partial frame representations of the earlier taxonomy by Ray (1678) in Fig.1 and of the revised taxonomy by Sundevall (1889) in Fig. 2. Ray uses the attributes *beak* (values: round or pointed) and *foot* (webbed or clawed), connected by a structural invariant (double-headed arrow), to distinguish WATER and LAND-BIRD (Chen 2002: 5).

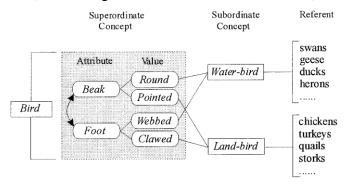


Fig. 1: Partial Frame for Ray's (1678) concept of bird (Chen 2002)

Here, constraints are thought to be *imposed by* structural invariants, functioning as follows:

[D]ue to the constraints between the value sets, some (...) property combinations are conceptually impossible, such as *round beak* with *clawed foot* and *point beak* with *webbed foot*. The results are only two property combinations (*round beak* with *webbed foot* and *pointed beak* with *clawed foot*), which form two subordinate concepts—*water-bird* and *land-bird*. In this way, the frame specifies the contrastive relations between the two subordinate concepts. (Chen 2002: 6)

Compare Fig. 1, then, with Sundevall's taxonomy (Fig. 2).

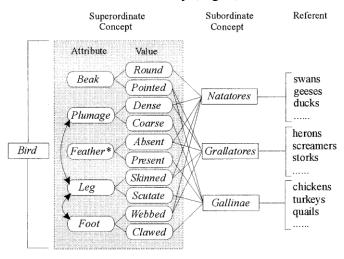


Fig. 2: Partial frame for Sundevall's (1889) concept of bird (Chen 2002)

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¹ "In the Ray taxonomy, for example, the attributes *beak* and *foot* are not independent. There are correlations between the value of *beak* and that of *foot*: webbed feet are usually associated with a round beak, and clawed feet with a pointed beak. These are physical constraints imposed by nature: webbed feet and round beaks are adapted to the environment in which water-birds live, but clawed feet and pointed beaks would be a hindrance in water. Because of these constraint relations, the attributes *beak* and *foot* must be used together as a cluster in classification" (Chen 2002: 6).

The transition between the representations in Fig. 1 and Fig. 2 amounts to a redefinition of the concept BIRD. To a frame-theorist, the point of this example is that it allows reconstructing change to a scientific taxonomy as a *motivated revision*.

Sundevall's *bird* no longer entails a constraint relation between *beak* and *foot*; instead, new constraint relations are formed between *foot* and *plumage*, as well as between *foot* and *leg covering*. [T]hese are physical constraints imposed by nature, resulting from the adaptation to the environment. The new superordinate concept inevitably alters the taxonomy by expanding the conceptual field at the subordinate level. (Chen 2002: 8)

Now contrast Fig. 2 with the yet later taxonomy by Gadow (1893) in Fig. 3, the transition to which might be seen to instantiate a *more radical* shift than the transition from Ray's to Sundevall's, because "Darwin discovered that species are not constant, and therefore affinity among species must be founded on their common origin" (Chen 2002: 12). Add to this that Gadow's taxonomy was developed in response to Sundevall's having "emphasized the dissimilarities between screamers and waterfowl" (ibid. 12), rather than their similarities.

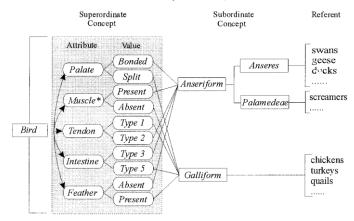


Fig. 3: Partial frame for Gadow's (1893) concept of bird (Chen 2002)

Except for the attribute 'feathering arrangement', Gadow's taxonomy employs attributes designating radically different morphological features from those shown in Fig. 1 and Fig. 2. Moreover, all attributes are connected by "Darwin-motivated" constraints. What remains constant over the three taxonomies is the use of body parts. These can be motivated by their cognitive salience (Tversky and Hemenway 1984, see Chen 2002: 16f.).

3. Frames in the History and Philosophy of Science

As an exercise in history and philosophy of science, reconstructing taxonomies as partial frames, then contrasting them, is carried out with regard to Kuhn's (1970) incommensurability of taxonomies. A frame-reconstruction is said to provide some form of comparability.

We have (...) seen taxonomic change like the one from the Sundevall system to the Gadow system, where the two taxonomies were incommensurable and no compatible lexical structures existed. But with the help of frame representations that expose the internal structures of the superordinate concepts involved in the taxonomic change, we find that the attribute lists embedded in these two incommensurable taxonomies remained compatible. (Chen 2002: 18)

Chen's claim is two-fold: Firstly, the frame method facilitates a representation by which one may explain why the Sundevall and the Gadow taxonomy are incommensurable, in the sense

² "Influenced by Darwin's evolutionary theory, ornithologists realized that many morphological characters used as classification standards in previous taxonomies were arbitrary, and they began to search for new classification criteria that could display the origins of birds" (Chen 2002: 12).

that this pair violates Kuhn's 'no overlap principle' for kind terms (Kuhn 1993, Chen 1997). The principle is rendered as: "[C]oncepts belonging to the same subordinate group cannot overlap in their referents" by Barker et al. (2003: 226).

Secondly, a more important claim is raised: In developing a consensus on the superiority of Gadow's taxonomy (which reportedly relied on more than 40 classification criteria and was based on rich empirical evidence) over that of Sundevall, the community of ornithologist *could* chose rationally, because—or so the reconstructive method is said to support—both Gadov's and Sundevall's criteria were spatial features (body parts). Contrary to the incomparability-interpretation of 'incommensurable'—which the mature Kuhn rejected (Chen 1997, Kuhn 1983, Hoyningen-Huene 1993)—, criteria could have been rationally compared.

[T]he compatible attribute lists, rooted in the preference for body parts, or more general, the preference for spatial features in attribute selection, could have functioned as a cognitive platform for the rational comparison of the Sundevall and the Gadow systems and resulted in the quick and smooth taxonomic change. (Chen 2002: 18)

The frame model shows: Allegedly incommensurable taxonomies may cut nature along different, but spatial features. Such cuts need not result in rationally incomparable taxonomies, although violating Kuhn's no-overlap principle. Frame analysis thus has a potential use in "making good on history." This is meant as follows: Historical transitions that *prima facie* support the incommensurability thesis (because the comparison of taxonomies appears to undercut choice-rationality) *may*—namely upon comparing them as frame reconstructions—be reconciled with standard maxims of choice rational action, e.g., the mini-max principle. This result, Chen suggests, draws on a distinctly *cognitive* platform for rational comparison.

[T]axonomic change is rooted deeply in the cognitive mechanisms behind the processes of classification and concept representation. These cognitive mechanisms determine the process of mutual understanding and rational comparison during taxonomic change. In fact, the cognitive platforms for rational comparison identified in our historical cases, that is, compatible contrast sets and attribute lists, were the products of such cognitive mechanisms as the relational assumptions adopted in classification and the preference for body parts developed in concept representation. (Chen 2002: 19)

Further applications of the frame model to scientific change are found, amongst others, in Andersen, Barker and Chen (1996), Andersen and Nersessian (2000), Chen, Anderson and Barker (1998), Chen (2003, 2005), Chen and Barker (2000), and the book-length Andersen, Barker and Chen (2006). Next to taxonomic change, cases range from the wave *vs.* the particle theory of light over astronomy and nuclear physics to the transition from Maxwell's to Einstein's conception of electro-dynamic action.

The last example is briefly discussed in section 6. First, we turn to the critical reception of applications of the frame account within the history and philosophy of science.

3.1 Incomparability, Non-Translatability and the Cognitive Historical Approach

In a recent review of Andersen, Barker and Chen (2006), who prefer the term 'dynamic frame', Thagard (2009: 844) points out: "[A]lthough the attribute-value account of represent-

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[&]quot;Consequently, communication obstacles were bound to occur between the followers of the two systems. The followers of the Ray taxonomy, for example, would regard 'grallatores' from the Sundevall taxonomy as incommensurable, because they could not find an equivalent native term with referents that do not overlap those of the foreign one. Both 'water-bird' and 'land-bird' from the old taxonomy overlap 'grallatores,' which includes water-birds like herons as well as landbirds like storks. On the other hand, the followers of the Sundevall taxonomy would regard 'water-bird' from the Ray taxonomy as confusing, because they could not find an equivalent native term without violating the non-overlap principle. Sundevall's 'natatores' overlaps Ray's 'waterbird'; specifically, the former is included by the latter, but they are not in species-genus relations' (Chen 2002: 9).

ation continues to be influential, there are several other approaches that suggest that the dynamic frame account of concepts used by Andersen, Barker and Chen is at best incomplete and at worst seriously inaccurate". Exclusive use of the frame-model risks recycling an outdated model. To what extent computer-implemented connectionist models, e.g., Thagard's (1999) ECHO, have "broken free" from feature lists is a matter of debate. As computing power has increased exponentially, such models easily appear to have become more powerful. At any rate, Thagard (2009: 845) favors *multi-modal* conceptual representation over frames.

In an earlier review, Stanford (2008) observes that—although (presumably) fine for descriptive purposes—frames do in no good sense improve our understanding of incommensurable world-views as cognitive phenomena.

[T]he tools imported from contemporary cognitive science prove more effective in describing central aspects of Kuhn's account of science than in explaining how they arise or how we respond to them. The feeling one gets is of being engaged in something of an extended translation exercise. If we are willing to follow the authors in embracing Barsalou's 'dynamic-frames' theory of concepts, our reward is a description in the terms of that theory of what a Kuhnian anomaly would be, what incommensurability would be, what revolutionary science would be, and so on. This may indeed show that contemporary cognitive science is capable of countenancing Kuhnian cognitive phenomena, but it does not do much to deepen our understanding of their causes or consequences. (Stanford 2008: 116)

A slightly more drastic consequence might obtain. By employing frames, incommensurability as incomparability of world-views is seemingly reconstructed away, while incommensurability as non-(inter)translatability of world-views is reconstructively confirmed. The undeniable fact seems to be: Using a frame-model, one inevitably reaches a state of representation at which a rational comparison of the conceptual structure of two (or more) 'views of the world' consists in nothing but a comparison of two (partial) frames. This allows tracing the requisite constraint violation and observing if/how anomalies are resolved in a different frame (or not).

One may use this fact in at least two ways. One option, presumably preferred by Stanford, is to object that applying the frame-model does not yield insight into the genesis and the effects of incommensurable world views: Frames merely facilitate a different view on the non-translatability-side of the problem. This much then would speak against what Nersessian (1995) dubbed the 'cognitive historical approach'.

[S]tarting from the Kuhnian idea that a particular phenomenon is an anomaly because its existence is not permitted by a given scientific concept, the further information that, in dynamic frames terms, anomaly is a matter of a phenomenon's properties violating a concept's constraints on the assignment of values to attributes, or that the anomaly might be resolved by revising such constraints, seems to add little explanatory insight or power to Kuhn's original proposal. (Stanford 2008: 116)

A second option is to undercut Stanford's conclusion and argue: Because the cognitive historical reconstruction renders allegedly incommensurable transitions between taxonomies rationally comparable, incommensurability as incomparability is *false* as a claim on the cognitive representation of concepts. And one might continue: *If* insights into causes and effects of the incommensurability of world-views are needed, then—as far as a cognitive account of conceptual representation is concerned—, such insights might just as well lie outside of it.

In pursuing this option, one suggests that causes and effects of this phenomenon (which, on the view ascribed to Stanford, is not captured by the frame-model in an enlightening way) are located altogether beyond issues of conceptual representation. Instead, incommensurability as non-(inter)translatability of world-views (and communication breakdown) may be straightforwardly explained by human imperfection. One might cite psychological deficits, in the sense of having remained, or become, unable to adopt (and switch between) different

views, or as strong, perhaps quasi-religious biases, in the sense of no longer considering, e.g., that claims to *one* ultimate ontology ('final description of the world') may be dogmatic, or group-sociological/institutional, in the sense that actors are rationally uncompelled to consider alternatives while investing in, or after having profited from, a particular research program.

This option may not sit well with everybody. *Vis à vis* the comparability claim, which can be supported by the frame-model, I find it hard to resist. *If* comparability can be secured, translatability is a less pressing issue. I take the frame model to support that, as a thesis on the rational incomparability of conceptual structures, incommensurability is a false claim. As a claim on the non-(inter-)translatability of world-views, the plausibility of incommensurability can—largely, though perhaps not entirely—be accounted for by drawing on factors other than those pertaining to conceptual representation.

For a more upbeat review of Andersen Barker and Chen (2006), see Botteril (2007).

4. Conceptual Spaces

The expressive power that frames gain over feature lists, while notable, remains meager. In support of this claim, frames will now be compared to *conceptual spaces* (Gärdenfors 2000). The latter appears (to me) to be more useful in application to scientific concepts, as it incorporates the measurement theoretic considerations underlying *nominal*, *ordinal*, *interval*, and *ratio scales*. It should therefore sit much better with the intuitions of working scientists. From the point of view of conceptual spaces, reference to *only one* empirical world is of lesser importance; ontological finality is not implied, nor precluded. If a measurement structure "picks out" a 'real structure' is rather not a pressing question (see section 6).

Conceptual spaces provide a *geometric* and *topological* account of concept representation. An assumption which seems basic to the frame model—namely: concepts must be represented in symbols—, is discarded. Rather, information is modeled at a level *between* the symbolic and the subconceptual one. So, symbolic forms such as the laws of mathematical physics are not seen as representing concepts, but as specifying 'mathematical relational structures'.

Past Stevens' (1946) influential work (to which the above classification of differentially informative measurement scales goes back), in "mature" measurement theory, mathematical relational structures are normally understood as being embeddable into empirical relational structures, i.e., principally projectable into an ultimate ontology (structures may therefore be called *real*). Stevens did not, in any detail, treat conditions that empirical structures should satisfy (Diez 1997a: 180). However, from a conceptual spaces point of view, this is fine. After all, the dimensions postulated in a conceptual space aren't "out there" either.

Importantly, some mathematical relational structures are claimed to be *constitutive* of empirical relational structures or (methodologically) *a priori* (Friedman 2001). This Neo-Kantian aspect is briefly taken up in section 5. Now follows a non-technical summary of conceptual spaces. Rigorous treatments are Aisbett & Gibbon (2001) or Adams & Raubal (2009).

4.1 Dimensions

A conceptual space is built up from a number of quality dimensions. Examples include *temperature*, *weight*, *brightness*, *pitch*, as well as the three ordinary spatial dimensions (*height*, *width*, *depth*). Moreover, we find quality dimensions of an abstract non-sensory character, e.g., *mass*, *force*, *energy*, introduced by science. The notion of a dimension may be taken literally. Each quality dimension is assumed to be endowed with *geometrical* structures.

Fig. 4 illustrates the weight-dimension (one-dimensional with a zero point). It is isomorphic to the half-line of the non-negative numbers. That there are no negative weights is a basic constraint commonly made in science.

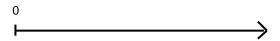


Fig. 4: The weight dimension

Far from trivial, the non-negativity of the weight dimension is a historical contingency. As an *ad hoc* assumption, the fire-substance *phlogiston* (a theoretical entity) was assumed to have negative weight, in the late 18th century giving way to the *oxygen* account (McCann 1978).

As a second example, following Munsell (1915), the cognitive representation of color can be described by three dimensions (Fig. 5). The first is *hue*, represented by the familiar color circle (red via yellow to green, blue and back to red). The topological structure of this dimension is different from the dimensions representing time or weight (both isomorphic to the real line). The second psychological dimension of color is *saturation* (or chromaticity), ranging from grey (zero color intensity) to increasingly greater intensities. This dimension is isomorphic to an interval of the real line. The third dimension is *brightness* that varies from white to black and is thus a linear dimension with end points.

Together, these three dimensions, one with circular and two with linear structure, constitute the color domain which is a subspace of our perceptual conceptual space. This domain is often illustrated by the so-called *color spindle* (two cones attached at their bases). Brightness is shown on the vertical axis. Saturation is represented as the distance from the centre of the spindle. Finally, hue is represented by the positions along the perimeter of the central circle. The circle at the center of the spindle is tilted so that the distance between yellow and white is smaller than the distance between blue and white.

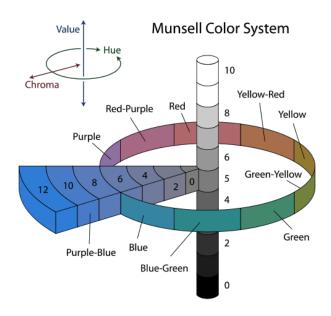


Fig. 5: The Munsell color system⁴

4.2 Combinations

A *conceptual space* can now be defined as a collection of quality dimensions. However, the dimensions of a conceptual space should not be seen as totally independent. Rather, they are *correlated* in various ways since the properties of the objects modeled in the space co-vary. For example, in the domain of fruits the ripeness and the color dimensions co-vary.

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⁴ Image reference: http://en.wikipedia.org/wiki/File:Munsell-system.svg (public domain).

It is not possible to give a complete list of the quality dimensions that make up the conceptual spaces of humans. Some of these dimensions seem to be *innate* and to some extent "hardwired" (e.g. color, pitch, force, and probably ordinary space). Others are presumably *learned*, yet others are introduced by science.

In modeling a scientific concept, the requisite dimensions have to be identified and the respective values, i.e., a *metric* (see Berka 1983: 93), must be assigned. If it is not possible to assign a value on one dimension without also assigning a value on another, then the dimensions are said to be *integral*; otherwise they are called *separable*. For instance, an object cannot be given a brightness value without also giving it a hue; the pitch of a sound always goes along with its loudness. In Newtonian mechanics, an object is fully described only when it is assigned values on eight dimensions: 3-D space, 1-D time, 3-D force, 1-D mass.⁵

On this distinction, the notion of a *domain* can be defined as a set of integral dimensions separable from all other dimensions. More precisely, domains C and D are separable in a theory, if the transformations of the dimensions in C do not involve dimensions from D. For example, until the rise of relativity theories in physics, the three spatial dimensions were separable from the time dimension. So, the spatial coordinates x, y, z are separable from t (the time coordinate) in Galilean, but not in Lorentz transformations. Moreover, mass is separable from everything else in Newton's theory, but no longer separable from energy in special relativity.

As the criterion for identifying a domain, we propose the independence of the respective measurement procedures (Diez 1997a: 183f.). For example, in classical mechanics, the measurement of distance and duration (trigonometry and chronometry) are independent, as light signals are tacitly assumed to propagate instantaneously rather than at finite speed.

For criticism, primarily as to the necessity of positing such spaces, see Decock (2006).

5. Frames Recovered in Conceptual Spaces

A comparison between frames and conceptual spaces for cases of taxonomic knowledge is straightforward. It consists of transposing the terminology of the former into that of the latter model. The notions *attribute*, *value*, *structural invariance* and *constraint* (see section 2) can be provided with analogues. Our claim is that frames can be recovered rather easily within the conceptual spaces model. In particular, the structural invariants and constraints of a frame arise naturally from the geometry of the conceptual space (e.g., category membership is principally a matter of occupying regions of a space).

Using one or the other modeling tool may be regarded a matter of convenience and thus related to the complexity of a representation. To model taxonomic knowledge, conceptual spaces appear over-powerful. Alternatively, representing knowledge with binary features is undercomplex. In section 6, we discuss limits of frame representations.

5.1 Analogue Expressions

An 'attribute' corresponds to a single 'dimension' or to combinations thereof. E.g., each color can be represented as a sub-region of the space spanned by the three dimensions *hue*, *saturation* and *brightness*, rather than by natural language color terms (see section 4).

The 'value' of an attribute corresponds to a point or to an interval on one or several dimensions. The metric of the dimensions will mimic the attributes' values. E.g., on the assumption of being an equal distance apart, the values 'big', 'medium' and 'small' of the attribute 'size' will yield an interval (else an ordinal) scale. Unlike the conceptual spaces model – where "being-in-between" is meaningful by virtue of the dimensions' geometric properties –, nothing in

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⁵ Since F = ma holds, some values can be inferred, e.g., for the three force dimensions.

the frame model *represents*, in a motivated way, that 'medium' is between 'big' and 'small'. Model users know as much, the model does not.

The purpose of a 'constraint' is to rule out (or make unlikely) some among logically many attribute-value combinations. Constraints result from the particular selection of attribute values that define a subordinate category. To mimic this in conceptual spaces, where instances of a concept are represented as points or vectors in an *n*-dimensional space, one may speak of sub-regions of a conceptual space being empty (or comparatively unpopulated).

The notion 'structural invariance' corresponds to a correlation of dimensions. This means no more than that a number of dimensions represent jointly. In the frame-model, structural invariants have been interpreted to represent *synthetic a priori* knowledge, i.e., knowledge about the empirical world which originates in a (taxonomic) structure not based in experience, but constitutive of it. For example, structural invariants are claimed to account for such *synthetic a priori* knowledge claims as: "There are no [normal] birds with legs that attach to their necks" (Barker, Chen & Anderson 2003: 225f.). Denial of this claim may lead a hearer to the assumption that a speaker does not understand the concept BIRD. The *synthetic a priori* status of such knowledge can, in principle, be saved in conceptual spaces, assuming that one has somehow identified it. At the same time, it is unclear (to me) if singling out some (and not other) elements as *synthetic a priori* is helpful or necessary.

An influential attempt at securing an important sense of 'rational' across scientific changes is Friedman's (2002, 2008) Neo-Kantian account, where theoretical principles may be identified as *methodological a priori* propositions, e.g., 'Space is (not) Euclidian'. Such principles are said to enable the measurement-experiences expressible within a given theoretical framework by means of laws (e.g., the law of gravitation). His tri-partition separates (i) empirical laws, (ii) constitutively *a priori* principles making these laws possible, and (iii) philosophical meta-paradigms which "provide a basis for mutual communication (...) between otherwise incommensurable (and therefore non-intertranslatable) scientific paradigms" (2002: 189).

The meta-paradigmatic level seems to primarily serve the purpose of leaving the historical dynamics of *a priori* principles rationally discussable. This level in hand, Friedman can easily accept symbolic disruptiveness (aka. symbolic non-continuity) at level (ii). However, it is unclear what besides avoiding, in a principled way, a possible communication breakdown between scientists applying different frameworks (world-views) may be cited in support of Friedman's conception. See also Howard (2009) and the brief discussion in section 3.1.

5.2 Similarity as Distance across Space

Reconsider Sundevall's taxonomy (Fig. 2). The attributes and their values in brackets are:

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beak<br/>plumage(round, pointed)feather(absent, present)leg(skinned, scutate)foot(webbed, clawed)
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We treat each attribute as a dimension. All values are binary, so each dimension gives rise to a "scale with two ranks". In table 1, capital letters abbreviate attributes, lower case letters values. This yields five integral dimensions at ordinal level.

Natarotes	BE-ro, PL-de, FE-ab, LE-sk, FO-we	swans, geese, ducks
Grallatores	BE-po, PL-de, FE-ab, LE-sk, FO-we	herons, screamers, storks
Gallinae	BE-po, PL-co, FE-pr, LE-sc, FO-cl	chickens, turkeys, quails

Table 1. Comparison of dimensions in Sundevall's taxonomy

Note that *Natarotes* and *Grallatores* are *similar* up to the beak-dimension (BE-ro *vs.* BE-po). This similarity remains rather hidden in the frame-model, but is immediate in a feature list or a conceptual space. Moreover, in a frame and a feature list model, it is not clear how to measure—by virtue of the tool—the comparative distance between *Natarotes*, *Grallatores* and *Gallinae*. In the idiom of conceptual spaces, the *Gallinae* region is maximally distant from the *Natarotes* region, as it differs on four dimensions from *Grallatores*. That this distance cannot be expressed more informatively is a result of employing binary features. Note that, when expressing taxonomic difference as distance, conceptual spaces have implicitly been applied.

In Gadow's taxonomy (Fig. 3), since the attribute (dimension) *feather* is retained with identical values, one may describe the change from Sundevall's to Gadow's taxonomy as a replacement or revision of four dimensions (*cum* invariants and constraints). This yields a *trivial*, but correct reconstruction of conceptual change. Such is easier to accept when incommensurability of world-views is not seen as a problem of representation (see section 3.1).

The partial frame of Gadow's new taxonomy features five dimensions, not all of which take binary values. One may therefore say that complexity (as measured by the number and scale-strength of dimensions) is not constant. Gadow uses four *new* dimensions. Featuring also one region less, in this respect, his taxonomy is simpler than Sundevall's. On the other hand, the types of *intestines* (Type 3 and 5) suggest that complexity increased. The same seems to hold for the *tendon* dimension. *Prima facie*, these still constitute ordinal scales.

Generally, by defining change-operations on the dimensions and their mode of combination, the conceptual spaces model may also be applied dynamically. In increasing order of severity of revision, these are: (i) addition/deletion of laws, (ii) change in scale, (iii) change in integrality/separation of dimensions, (iv) addition/deletion of dimensions, (v) change in importance (or salience) of dimensions (see Gärdenfors & Zenker 2010 for examples).

A more informative reconstruction might employ the comparative distance between taxonomic items (pre- vs. post-change). Thus, relative distance between reconstitutions of dimensional points within (regions of) spaces would measure if, e.g., screamers have become *more* similar to ducks (or not). *Severity of scientific change* then comes out as 'distance between spaces', i.e., as a function of the above change operations and a distance measure.

Below, we exploit this idea, offering it as a promising mode of addressing the (alleged) incommensurability of theoretical frameworks. We view scientific laws as symbolic expression of constraints on conceptual spaces. On this view, historical transitions to new spaces are in principle always *continuously* reconstructable, leaving no room for incommensurability in a cognitive account of scientific conceptual knowledge representation.

6. Scientific Laws as Constraints on Dimensions

Frame representations almost exclusively use natural language. Attributes and values are ultimately linguistic entities. This may be fine when representing changes in taxonomic knowledge. That taxonomies mostly employ binary features may be motivated historically: "better tools" were not available. The foundations for a theory of measurement (in the modern sense) arise only with Helmholtz (1887), are provided with (some say, insufficient) systematizion by Stevens (1946), and developed by Krantz, Luce, Suppes & Tversky (1971, 1989, 1990). For an overview and the caveat in Steven's work, see Diez (1997a,b), Hand (2004).

When dimensions are fine grained, we approach scientifically exact measurement. Here, shortcomings in the information conveyed by the frame model's attribute-value structure may be observed, suggesting a revision of this model. If attribute values are not bi-, but *n*-ary, any attempt at modeling ordering relations with frames presumably incurs a revision towards conceptual spaces. When representing a scientific concept (and, eventually, the conceptual space spanned by an empirical theory), e.g., in physics, dimensions tend to be *ratio-scales*. One will want to make sense of the fact that empirical theories and their (mathematical) laws depend on and give rise to measurement results at this level of scale. In brief, frames seem under equipped to represent conceptual knowledge beyond the taxonomic level.

Moreover, it is easily overlooked that one may attempt to motivate the *symbolic* character of scientific laws by virtue of the representational tool. Thus, Andersen and Nersessian (2000) clearly state they "believe that [frame] analysis can be extended to represent the similarity class of problem situations for *nomic* concepts" (ibid. 230), i.e., those obeying law-like generalizations. In their electromagnetism example (Fig. 6), the Lorentz force-treatment is distinguished from the electromotive force-treatment; "frame-style", the attributes *conductor*, *ether* and *magnet* (values: *moving* or *at rest*) are coordinated to the respective force laws. Their symbolic forms differ strikingly—and implausibly so, as the application situation is identical. (In modern terms, applications pertain to the *relative* motion of a magnet *vis* à *vis* that of a conductor.) Recall that, "in Maxwellian electrodynamics, although the resultant electromagnetic induction is the same whether it is the magnet or the conductor that is moving and the other at rest, these are interpreted as two *different kinds* of problem situations" (ibid. 237, italics added). The point of their example is: Suspending the attribute *ether*, Einstein's revision of Maxwell's electrodynamics removed a "total overlap" (ibid.) between the two treatments.

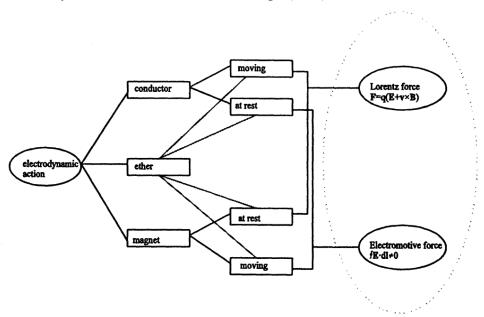


Fig. 6: Partial Frame for Maxwellian 'electrodynamic action' Andersen & Nersessian (2000: S238)

In Andersen and Nersessian's use of frames, laws are appended, rather than motivated by the frame structure. It therefore seems (to me) that frames apply to scientific laws *without* providing insight into their status as symbolic generalizations. Strikingly different formulae, which evidence the "symbolic rupture" (allegedly incurred) in *radical* scientific change, can also be viewed as the symbolic expressions of constraints holding over different conceptual spaces. In fact, scientific laws may be viewed as *nothing but* the symbolic forms of constraints on some

space. Note that ontological qualms in theory change may also be explicated with respect to the dimensions of an empirical theory—one need not pin this to the laws or the axioms.

This move no doubt demotes the importance of laws in scientific change *vis* à *vis* the dominant view (e.g., Dorato 2005). On the dominant view, for instance, any continuity of mathematical structure achieved by limiting case reduction (see Batterman 2003)—which, following Worrall (1989), structural realists tend to cite as strong evidence in disfavor of incommensurability claims—, would no longer be exclusively a matter of laws. Instead, once one characterizes empirical theories *primarily* through identifying the scale-type of the dimensions—or, more contemporaneously, the admissible transformation of a scale (see Diez 1997b)—and their modes of combinations (integral vs. separable), 'continuity in scientific change' denotes the continuous generation of one conceptual space into another.

Questioning the assumption that the rationality of a scientific change is inherently a symbolic matter (i.e., has to be demonstrated *in symbols*), then, one may motivate the claim that conceptual spaces provide a model for scientific change (across various disciplines), without yielding incommensurability or incurring *a priori* notions. In this sense, the assumption that a conceptual space is not (an) intrinsically symbolic (model) is indispensable.

To represent scientific concepts, theories in which they occur as well as their dynamics, similarity measures over *diachronically* related spaces appear promising. How such measures are to be defined, is open to discussion. Extant treatments of conceptual dynamics project (or transform) conceptual spaces according to contexts which, in the widest sense, vary *synchronically*, e.g., spatial environmental features under day and night conditions (Raubal 2004). The change operations (section 5.2) and the definition of a domain may serve in providing the building blocks for such diachronic similarity measures.

7. Conclusion

It should be stressed that, with the exception of saying something meaningful on the status of symbolic generalizations, the frame model is presumably applicable whenever the conceptual spaces model it. Alas, the latter gains in applicability to concepts which are based on and give rise to exact measurement. Having reviewed the development of feature lists into the frame account of conceptual representation, and having moreover shown how to recover frames in conceptual spaces, one may conclude that the latter model gains its advantages, because key notions of modern measurement theory are *native* to it. Conversely, any attempt to achieve this within the frame model will (very likely) look just like a conceptual space.

Distinct correspondences between frames and conceptual spaces were pointed out. Moreover, it was suggested that using one or the other model is also a matter of convenience. For taxonomic knowledge, for example, conceptual spaces appear over-complex. Importantly, whenever the question is raised if—through a change in taxonomy—items have become more (or less) similar, it should be admitted that one implicitly uses the conceptual spaces model. After all, neither frames nor feature lists provide a notion of difference as geometric distance.

Consequently, future work should concern definitions of distance measures across diachronically varying spaces. No measure was defined, but the building blocks pointed out.

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