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Claesson, Johan; Hellström, Göran

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LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

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# **Analytical Studies of the Influence of Regional Groundwater Flow on the Performance of Borehole Heat Exchangers**

by \*Johan Claesson<sup>1</sup> and Göran Hellström<sup>2</sup>

<sup>1</sup> Dept. of Building Physics, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden, fax +46-31-7721993, e-mail: claesson@buildphys.chalmers.se

<sup>2</sup> Dept. of Mathematical Physics, Lund University, P.O.Box 118, SE-221 00 Lund, Sweden, fax +46+46-2224416, e-mail: goran.hellstrom@swipnet.se

## **KEY-WORDS**

Borehole heat exchanger, ground-coupled heat pump, groundwater flow, analytical solution

## **Abstract**

This paper presents a new analytical solution for the influence of regional groundwater flow on the performance of borehole heat exchangers. The solution concerns a vertical borehole. The transient thermal response for the fundamental case of a step-change in heat injection rate is given. The solution is remarkably simple and the formulas quite handy considering the complexity of the convective-diffusive heat flow process. The influence on the required borehole length for a ground-coupled heat pump is discussed.

## **Introduction**

The influence of regional groundwater flow on the performance of ground-coupled heat pumps using boreholes has been a topic of discussion. In the case of net heat injection, the flowing groundwater will remove heat from the ground region near the borehole and thus reduce the temperature of the heat carrier fluid. The performance is improved. What is the effect for different flow rates? Can shorter and less expensive boreholes be used in cases with regional flow? There is a need for design tools that account for groundwater flow.

A method based on superposition of thermal response functions was developed by CLAESSON and ESKILSON (1987) for the case without groundwater flow. However, an estimate of the influence of groundwater flow using a two-dimensional, steady-state solution was given. Numerical calculations for this problem are presented in van MEURS (1985).

This paper presents the results of a study for a single, vertical borehole in a uniform regional flow that extends well below the borehole depth (CLAESSON and HELLSTRÖM 2000). The convective-diffusive heat flow process is three-dimensional and time-dependent. The mathematical analyses are quite extensive, but the final results are of a rather simple form suitable to implement in design tools.

## Continuous line heat source

The temperature in the homogeneous ground satisfies the heat conduction equation with an added term to account for the convective heat flow due to the constant regional groundwater flow  $q_w$  (m<sup>3</sup> of water per m<sup>2</sup> and s) in the  $x$ -direction:

$$\frac{1}{a} \cdot \frac{\partial T}{\partial t} = \nabla^2 T - \frac{2}{\ell} \cdot \frac{\partial T}{\partial x} \quad a = \frac{\lambda}{\rho c} \quad \frac{2}{\ell} = \frac{\rho_w c_w q_w}{\lambda} \quad (1)$$

There is a line heat source along  $0 < z < H$  with the strength  $q_0$  (W/m) which acts continuously from  $t = 0$ . The temperature field from this source in an infinite surrounding is (CARSLAW and JAEGER 1959):

$$T(x, y, z, t) = \frac{q_0}{2\pi\lambda} \cdot \int_0^{4at} \frac{1}{4u} \cdot \left[ \operatorname{erf}\left(\frac{z}{\sqrt{u}}\right) + \operatorname{erf}\left(\frac{H-z}{\sqrt{u}}\right) \right] \cdot e^{-(x^2+y^2)/u - u/(4\ell^2) + x/\ell} \cdot du \quad (2)$$

In the general formula, we have performed an integration along the line heat source, which results in the two error functions in the integrand.

## Dimensionless response functions

Our main interest is the temperature in the borehole heat carrier fluid, and in particular the temperature  $T_b(t)$  at the borehole wall, that is needed in order to sustain the constant heat injection rate  $Q_0$  from time  $t = 0$ . The corresponding dimensionless function is denoted by  $g$ :

$$T_b(t) = \frac{q_0}{2\pi\lambda} \cdot g_{total}(t, \dots) \quad q_0 = \frac{Q_0}{H} \quad (3)$$

The  $g$ -functions depend on time, thermal properties, etc. The groundwater flow will diminish the  $g$ -function. We write the total  $g$ -function in the following way:

$$g_{total}(t; q_w) = \frac{2\pi\lambda}{q_0} \cdot T_b(t) = g(t; q_w = 0) - g_{gw}(t) \quad (4)$$

The first term on the right-hand side is the ordinary  $g$ -function for the considered borehole without the effect of groundwater ( $q_w = 0$ ) (CLAESSON and ESKILSON 1987). The second term accounts for the added (or rather subtracted) effect of the groundwater flow.

## Integral for groundwater $g$ -function

The groundwater  $g$ -function  $g_{gw}$  is, apart from the scale factor before  $T_b(t)$  in Eq. (4), equal to the change of the borehole temperature due to the groundwater flow. The temperature from the line heat source, Eq. (2), is infinite at

$z = 0$ , but the difference between the solution with and without the groundwater flow is finite. This difference varies somewhat along the heat source. Therefore we consider the average along the borehole. With this rather small approximation, we get an explicit integral for the decrease of the borehole temperature caused by the groundwater flow. We have:

$$g_{gw}(t) = \frac{2\pi\lambda}{q_0} \cdot \int_0^H [T(0,0,z,t;q_w=0) - T(0,0,z,t;q_w)] \cdot dz \quad (5)$$

Expression (2) for  $x = 0$ ,  $y = 0$ ,  $\ell = \infty$  ( $q_w = 0$ ) and the corresponding expression for the actual  $\ell$  are inserted in Eq. (5). We get after some calculations (CLAESSON and HELLSTRÖM 2000):

$$g_{gw}(\tau, h) = \int_0^\tau \frac{1}{2s} \cdot (1 - e^{-h^2 s/4}) \cdot \operatorname{erfm}(1/\sqrt{s}) \cdot ds \quad (6)$$

Here, we have performed an integration along the line heat source. From this we get in the integrand the following mean of the error function:

$$\operatorname{erfm}(x) = \frac{1}{x} \cdot \int_0^x \operatorname{erf}(s) ds = \operatorname{erf}(x) - \frac{1 - e^{-x^2}}{\sqrt{\pi} \cdot x} \quad (7)$$

It is noteworthy that the groundwater g-function depends on two parameters only, a dimensionless time  $\tau$  and a dimensionless groundwater flow rate  $h$ :

$$\tau = \frac{4at}{H^2} \quad h = \frac{H}{\ell} = \frac{H\rho_w c_w q_w}{2\lambda} \quad (8)$$

## Groundwater g-function and approximate formulas

The integral (6) for  $g_{gw}(\tau, h)$  may be evaluated numerically. Figure 1 shows the result as a function of  $\tau$  for moderate and high values of  $h$ .

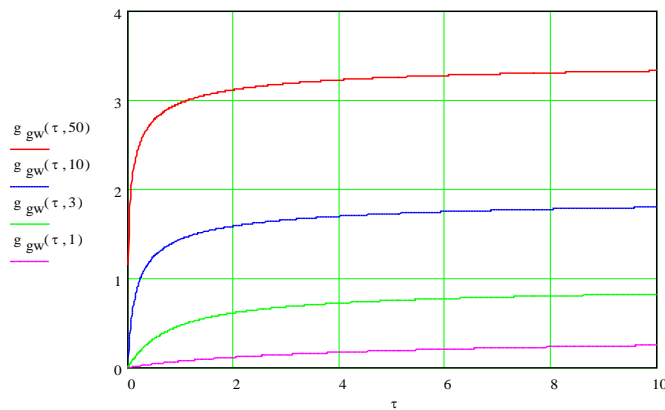


Figure 1. The groundwater g-function  $g_{gw}(\tau, h)$  as function of the dimensionless time  $\tau$  and the dimensionless groundwater flow rate  $h$ .

For  $\tau < 1$ , we have the following approximation, which is valid for any  $h$ , (CLAESSON and HELLSTRÖM 2000):

$$g_{gw}(\tau, h) \cong \frac{1}{2} \cdot \text{Ein}\left(\frac{h^2 \tau}{4}\right) - \frac{1}{h} \cdot \left(\frac{h\sqrt{\tau}}{\sqrt{\pi}} - \text{erf}\left(\frac{h\sqrt{\tau}}{2}\right)\right) \quad \tau < 1 \quad (9)$$

Here, a modified exponential integral is introduced:

$$\text{Ein}(x) = \int_0^x \frac{1 - e^{-s}}{s} ds \cong \ln(x + e^{-x}) + 0.577 \cdot (1 - e^{-7x/4}) \quad (10)$$

The third expression is an approximation with a quite small error. The largest error in Eq. (9) with the approximation (10) is 4 % for any  $h$ . The error is below 1% for  $h > 6$ . The value of  $h$  is smaller than 1 in many important applications. Then we have the following simple expression to estimate the effect of groundwater flow (CLAESSON and HELLSTRÖM 2000):

$$g_{gw}(\tau, h) \cong \frac{h^2 \tau}{8} \cdot \left(1 - \sqrt{\frac{4\tau}{9\pi}}\right) \quad \tau < 1, \quad h < 1 \quad (11)$$

The error in the indicated region is less than 3%.

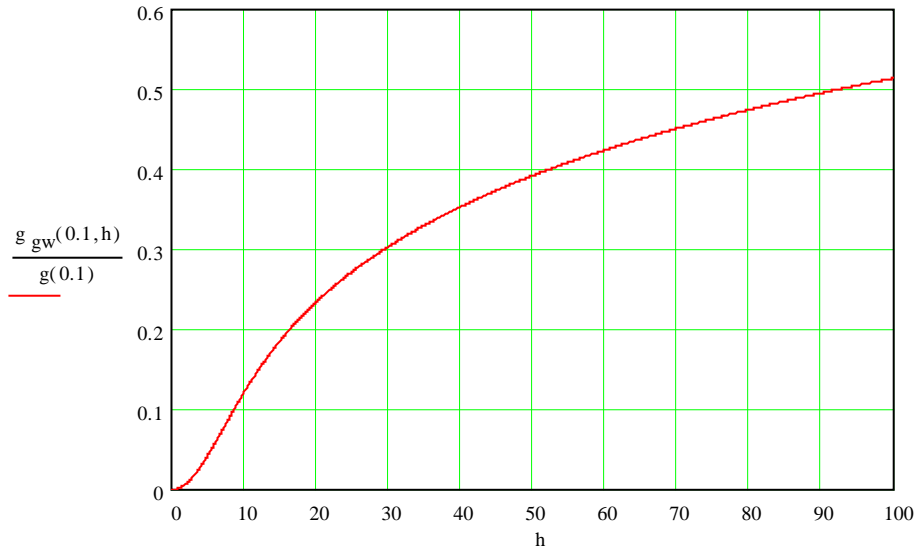


Figure 2. Ratio between the groundwater g-function for varying  $h$  and the undisturbed g-function for  $\tau = 0.1$ .

## A first comparison

The undisturbed g-function (the term  $g(t; q_w = 0)$  in (4)) assumes values in the range 2.5 to 7, except for the first few days where it is smaller. The value of  $\tau$  is smaller than 1 for most applications of technical interest. An example illustrates this:

$$a = 1 \cdot 10^{-6} \text{ m}^2/\text{s}, \quad H = 100 \text{ m}, \quad \Rightarrow \quad \frac{H^2}{4a} = 79 \text{ years}$$

For  $\tau = 0.1$  we have from CLAEISSON and ESKILSON (1987) an undisturbed value of the g-function of  $g(0.1)=5.6$ . Figure 2 shows for  $\tau = 0.1$  the ratio between the groundwater g-function for varying  $h$  and the undisturbed g-function. There is a 10% influence for  $h=8$ , 30% for  $h=30$ , and 50% for  $h=90$ . For  $h=1$  we get from (11) the ratio  $0.0123/5.6=0.002$ .

We see that the effect of the groundwater is *completely negligible* for  $h < 1$ . The value of  $h$  must exceed, say, 5 in order to be of any importance.

## Effect of the ground surface

In the above considerations, we have neglected the effect of the ground surface. The temperature at the ground surface must be zero for our considered excess temperature (above undisturbed conditions). This is readily achieved by a negative mirror line heat source above the ground surface. In this way we may keep the solution (2) which is valid in an infinite ground. We get an influence on the temperature along the original line heat source from the mirror source. By integration we may calculate the total average temperature for the line source. It turns that the new integrals may be expressed in terms of the function (6). We have (CLAEISSON and HELLSTRÖM 2000):

$$g_{total}(t; q_w) = g(t; q_w = 0) - g_{gw}(\tau, h) + (1+d) \cdot g_{gw}(\tau/(4(1+d)^2), 2(1+d)h) + \\ + d \cdot g_{gw}(\tau/(4d^2), 2dh) - (1+2d) \cdot g_{gw}(\tau/(1+2d)^2, (1+2d)h) \quad d = D/H \quad (12)$$

## Influence of groundwater flow on the design

The influence of the groundwater flow on the design of a ground-coupled heat pump is illustrated by an example typical for a single-family house in Sweden. The total heat demand is taken to be 21400 kWh/year with a typical load variation over the year. The heat pump, which has a heating capacity of 6 kW and an assumed seasonal performance factor of 3, uses a single vertical borehole with a diameter of 0.115 m. The borehole heat exchanger is a single U-pipe of 40 mm polyethylene tubing resulting in a borehole thermal resistance of 0.1 K/(W/m). The ground is assumed to be permeable with a hydraulic gradient of 0.01 m/m. The thermal conductivity is 2.5 W/(m,K) and the volumetric heat capacity is 2.5 J/(m<sup>3</sup>,K). The undisturbed ground temperature is assumed to be 10 °C at the ground surface with vertical gradient of 24 °C/km. The borehole length is chosen so that the minimum entering fluid temperature (EWT) to the heat pump becomes -0.5 °C for a 15 year operation of the system. In the case of no groundwater flow the required borehole length is 110 m. The hydraulic conductivity of the ground is then varied from impermeable up to 10<sup>-4</sup> m/s. The resulting groundwater flow rate  $q_w$  varies between 0 and 10<sup>-6</sup> m/s. The

groundwater flow velocity is obtained by dividing the groundwater flow rate with the porosity of the ground. The porosity is taken to be 0.25. The design software Earth Energy Designer (EED) (SANNER and HELLSTRÖM 1997) was modified to include the groundwater g-function. The required borehole length as function of the flow velocity is shown in fig.3.

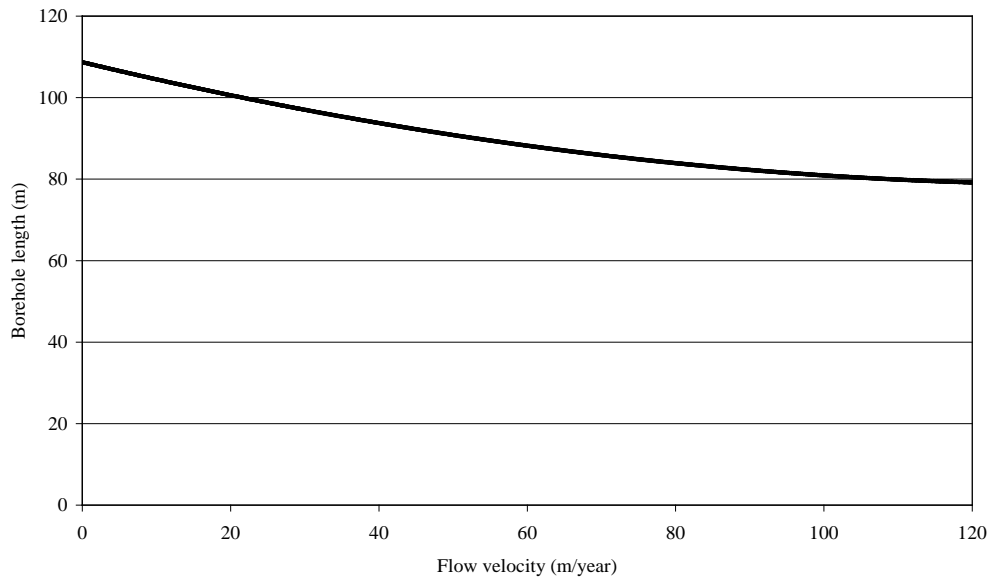


Figure 3. The required borehole length (m) as function of the flow velocity (m/year) (for a porosity of 0.25). Design data and criterion are given in the text.

It should be noted that a hydraulic gradient of 0.01 m/m is very large, in particular for a hydraulic conductivity of  $10^{-4}$  m/s. The upper limit (120 m/year) in the above example is quite extreme. This extreme groundwater flow results in a reduction of borehole length by 25%. The upper limit gives in this example  $h=67$ . This is quite consistent with figure 3, which gives 44% for this value of  $h$ . The main reason for the lower influence is due to the effect of the borehole thermal resistance. The temperature difference over the resistance is to be added, but that part is independent of the groundwater flow

### Acknowledgement

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