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A Genie-Aided Detector with a Probabilistic Description of the Side Information

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Abstract— **Building on Forney's concept of the genie [4], [5], and introducing the idea of an *explicit statistical description* of the side information provided to the genie-aided detector, we develop a generic tool for derivation of lower bounds on the bit-error rate of any actual receiver [3]. With this approach, the side information statistics become *design parameters*, which may be chosen to give the resulting bound a desired structure. To illustrate this, we choose statistics in order to obtain a special case: the lower bound derived by Mazo [6]. The statistical description of the side information makes the lower bounding a transparent application of Bayesian theory.**

I. Introduction

The idea of a *good genie* with a corresponding *genie-aided detector (GAD)* has, in particular, often been used to determine a lower performance bound for the probability of bit-error [1], [2], [3], [4], [5]. The GAD has access to more information than any actual detector: it has access to the side information supplied by the genie and is expected to handle all information *optimally.* Because of this, it is argued, it cannot have a worse performance than any detector working without the side information. However, in order that optimal processing of the side information be well-defined in the sense of Bayesian detection theory, an explicit (statistical) description of the side information is required. This paper introduces such a representation of the genie, augmenting the foundational ideas of Forney's work in [4], [5]. Our aim is to introduce the side information supplied by the genie as the output of a "side information channel" parallel to the original channel and governed by a *probabilistic rule* with free parameters.

II. The Side Information Channel

Consider a transmission system where binary data is sent through a discrete-time, additive Gaussian channel with intersymbol interference, and where additional side information is carried to the detector through a parallel channel (representing the genie).

We discuss the detection of bit number k in the important special case when the side information consists of a pair of sequences and one of the sequences is equal to the transmitted sequence, *cf.* [4], [5]. Define $\mathcal{C}_k^+$ and $\mathcal{C}_k^-$ as the sets of sequences with the bit in position k as $+1$ and -1, respectively, and denote the side information with z and its outcome with ζ. Let ζ consist of pairs in $\mathcal{C}_k^+ \times \mathcal{C}_k^-$ of the form $\zeta_{i,j} \triangleq (\boldsymbol{\beta}_i^+, \boldsymbol{\beta}_j^-)$, for $1 \leq i, j \leq 2^{N-1}$. With the transmitted sequence being $\boldsymbol{\beta}$, let the additional sequence be chosen at random among the sequences differing from $\boldsymbol{\beta}$ in bit k, according to the *known, probabilistic* transition rule:

$$\Pr\{z = \zeta_{i,j} | \mathbf{b} = \boldsymbol{\beta}\} = \begin{cases} p(j|i) & \text{if } \boldsymbol{\beta} = \boldsymbol{\beta}_i^+ \in \mathcal{C}_k^+ \\ q(i|j) & \text{if } \boldsymbol{\beta} = \boldsymbol{\beta}_j^- \in \mathcal{C}_k^- \\ 0 & \text{otherwise.} \end{cases}$$

Hence, the properties of the genie, or equivalently, the properties of the output of the side information channel, are defined by the statistics (or transition probabilities) $p(j|i)$ and $q(i|j)$.

III. The Genie-Aided Detector

With the complete statistical description of the transmission system, including a set of transition probabilities, the GAD with minimum bit-error probability is derived in terms of a binary Bayesian hypothesis test. By evaluating the performance of this GAD, a lower bound on the probability of bit-error of any detector, with or without access to the side information, is obtained as

$$\begin{aligned} P_{\text{BER},k} \geq\ & \textstyle\sum_{i,j} Q\left(\frac{d_{i,j}}{2} + \frac{\ln \gamma_{i,j}}{d_{i,j}}\right) q(i|j) \Pr\{\mathbf{b} = \boldsymbol{\beta}_j^-\} + \\ & \textstyle\sum_{i,j} Q\left(\frac{d_{i,j}}{2} - \frac{\ln \gamma_{i,j}}{d_{i,j}}\right) p(j|i) \Pr\{\mathbf{b} = \boldsymbol{\beta}_i^+\}, \end{aligned}$$

where $d_{i,j}$ is the Euclidian distance between $\boldsymbol{\beta}_i^+$ and $\boldsymbol{\beta}_j^-$,

$$\gamma_{i,j} = \frac{q(i|j) \Pr\{\mathbf{b} = \boldsymbol{\beta}_j^-\}}{p(j|i) \Pr\{\mathbf{b} = \boldsymbol{\beta}_i^+\}}$$

and $Q(x) \triangleq (1/\sqrt{2\pi}) \int_x^{+\infty} e^{-t^2/2} dt$.

The transition probabilities $\{p(j|i), q(i,j)\}$ are free parameters which can be chosen to optimize the performance of the GAD. They might for example be chosen to make the corresponding bound tight, or to give the bound a simple structure. We choose several sets of transition probabilities as examples in order to discuss the properties of their respective performance bounds. In this, we also discuss the relation of the attainable performance bounds to the works by Forney [4], [5] and Mazo [6].

References

[1] T. Aulin, "Genie-aided sequence detection under mismatch," In *Proc. ISITA '94*, pp. 869–873, Sydney, Australia, Nov. 1994.

[2] R.E. Blahut, *Digital Transmission of Information.* New York: Addison-Wesley Publ. Comp., 1990.

[3] H.B. Eriksson, P. Ödling, T. Koski and P.O. Börjesson "A genie-aided detector based on a probabilistic description of the side information," Subm. to *IEEE Trans. Inf. Th.*, May 1995.

[4] G.D. Forney, "Lower bounds on error probability in the presence of large intersymbol interference," *IEEE Trans. Commun.*, vol. 20, no. 1, pp. 76–77, Feb. 1972.

[5] G.D. Forney, "Maximum likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inf. Th.*, vol. 18, no. 3, pp. 363–378, May 1972.

[6] J.E. Mazo, "Faster-than-Nyquist-signaling," *The Bell System Technical Journal*, vol. 54, Oct. 1975, pp. 1451–1462.