Information Flow and Active Social Influence in Social Networks

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2011

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March 10, 2011

When individuals in a social network exchange information, beliefs or opinions through their immediate connections, the following questions naturally emerge:

1. *What are the social networks which most likely form* when individuals are concerned with the efficient and effective dissemination of *endogenous* information through the network?

2. Given a network of connections, *what is the optimal targeting policy* for which an *exogenous* belief can be adopted to the largest extent by the network?

The above questions, although different, overlap to a large degree. On the one hand, we recognize that individuals are dynamically changing their links to search for efficient information flow through the network. In this case, we are interested to know what are the networks which most likely are going to form. On the other hand, when information or beliefs are exogenously implanted to the network, adoption of these beliefs will highly depend on which individuals are initially targeted and what is their influence to the network (i.e., their centrality measure). In the following discussion, we analyze these two questions independently.

The first part of this discussion is motivated by the current research on social network formation [1, 2] and how social networks form when individuals have discretion over the links they establish or sever. We model the problem as a noncooperative game, where each individual makes decisions based on myopic considerations, i.e., so that its own utility is maximized. Links are assumed unidirectional, which model phenomena such as web links, observations of others, citations, etc. [2]. The utility considered for each individual reflects the ability to disseminate information efficiently through the network similarly to [3, 4].

Several models for endogenous network formation have been proposed that are based on game theoretic formulations. These include *static models*, [3], where agents play an one-stage game, with actions corresponding to network links. These studies characterize networks in terms of the Nash equilibria of the associated game, called *Nash networks*. The processes under which such equilibria emerge are proposed via *dynamic* or *evolutionary* models [4, 5, 6]. In these models, players adaptively form and sever links in reaction to an evolving network, and in some models, their decisions are subject to small random perturbations.

Our approach is also concerned with dynamic or evolutionary models, and is mostly related to the papers of [4, 5]. Our contributions are the following: i) We discuss the case where nodes can form links only with a subset of the other nodes (i.e., neighborhood structures), as opposed to the entire network; ii) We introduce utility functions that are distance-dependent variations of the *connections model* of [3] and guarantee that Nash networks exist; iii) We introduce state-dependent utility functions that can model dynamic phenomena such as *establishment costs*; iv) We derive a learning process that guarantees convergence to Nash equilibria for the state-based extension of weakly acyclic games; and v) We employ *payoff-based* dynamics for convergence to Nash networks based on a reinforcement learning scheme and drop the typical assumptions that nodes have knowledge of the full network structure and can compute optimal link decisions.

The second part of this discussion is concerned with the derivation of optimal targeting policies for the diffusion of beliefs in a social network. Equivalently, we may think of the targeting policies as advertising strategies and the

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individuals as customers. Contrary to the first part of the discussion, here the network is assumed constant and the customers' preferences are affected by both their neighbors and the incentives provided through advertising. Our contribution lies in the inclusion of three important factors in the derivation of an optimal advertising strategy: i) dynamic network effects in the formation of preferences, ii) possible misspecifications/uncertainties in the assumed model of evolution of preferences, and iii) uncertainty in the intentions of a competitive firm that also tries to influence the network.

Prior work has focused on i) the derivation of dynamic models which capture the sales response to advertising, and ii) the computation of an optimal policy of advertising as a function of the sales. Those models which capture the effect of advertising on sales, usually assume the following behavior: i) advertising effects persist over the current period but diminish with time [7], ii) marginal advertising effects diminish or remain constant with the size of advertising [8], iii) advertising effects diminish with the size of sales [7], iv) advertising effects diminish with the size of competitive advertising [9], and v) advertising effects are affected by word-of-mouth communication (or excess advertising) [10].

Our model is related to the sales response models [7] (which capture the evolution of the rate of sales) and diffusion models [11] (which capture the market growth). It exhibits diminishing returns with time in the absence of advertising effort, constant marginal returns with the size of advertising, and diminishing returns with the size of competitive advertising. It emanates from traditional advertising models by also considering the effect of word-of-mouth communication through a network of interactions similarly to [12]. The difference here is that the dynamics of preferences become part of the optimization. We derive analytically optimal advertising strategies and relate them to centrality measures usually considered in sociology [13]. This result also establishes a connection with the first part of our discussion, since nodes of high centrality measure can be provided through an analysis of endogenous network formation.

We also consider the possibility that we are uncertain of the accuracy of the model of preferences' update, instead of assuming a deterministic update. This form of uncertainty is usually neglected in prior work on optimal advertising. We derive optimal policies which are robust to a norm-bounded uncertainty. We show that the model exhibits a certainty equivalence property, since the optimal policy for the perturbed model coincides with the optimal policy for the unperturbed model. Finally, we consider the possibility that a competitive firm also tries to influence the network, introducing a second form of uncertainty. In this case, we compute robust optimal policies through the notion of Stackelberg and Nash solutions.

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