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An overview of sum rules and physical limitations for passive metamaterial structures

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Abstract— Metamaterials offer many new possibilities to design structures and devices with improved performance. In this paper, we present an overview of physical limitations for passive structures. The physical limitations relate the product between performance and bandwidth with the size of the structure. They are derived from sum rules that relate a weighted all spectrum integral of the performance with the low- and high-frequency responses. We present results for extinction cross sections, radar absorbers, high-impedance surfaces, extraordinary transmission, antennas, and temporal dispersion of metamaterials. The results provide a tradeoff between possible performance and bandwidth. As they are derived solely using passivity and linearity, the results also state when it is necessary to use active or non-linear media.

Passivity is inherent in many physical processes. Here, we consider an input output system that can be written on convolution form $u = R * v$, that follow from the assumptions of linearity, time-translational invariance, and continuity [14]. The time-domain characterizations for admittance passivity and scattering passivity are

$$\mathcal{E}_{\text{adm}}(T) = \int_{-\infty}^{T} u(t)v(t)\,\mathrm{d}t \geq 0 \quad \text{and } \mathcal{E}_{\text{scatt}}(T) = \int_{-\infty}^{T} |v(t)|^2 - |u(t)|^2\,\mathrm{d}t \geq 0, \tag{1}$$

respectively, where the inequality should hold for all T and v [14]. It is clear that time-domain passivity implies causality. The Laplace (or Fourier) transform of a passive system introduces a transfer function that is related to a positive real function or Herglotz function [1].

Herglotz functions, $h(z)$, are holomorphic in the upper half plane $\operatorname{Im} z > 0$ and map the upper half plane into itself, *i.e.*, $\operatorname{Im} h(z) \geq 0$, see [1, 11]. Here, we also restrict the analysis to symmetric Herglotz function $h(z) = -h^*(-z^*)$. Their asymptotic expansions are of the form

$$h(z) = \sum_{n=0}^{N_0} a_{2n-1} z^{2n-1} + \mathrm{o}(z^{2N_0-1}) \quad \text{as } z\hat{\to}0 \quad \text{and } h(z) = \sum_{n=0}^{N_\infty} b_{1-2n} z^{1-2n} + \mathrm{o}(z^{1-2N_\infty}) \quad \text{as } z\hat{\to}\infty \tag{2}$$

for some $N_0 \geq 0$ and $N_\infty \geq 0$, see [1], where $\hat{\to}$ denotes limits for $0 < \alpha \leq \arg(z) \leq \pi - \alpha$. The expansions (2) guarantee that $\operatorname{Im}(x)$ satisfy the integral identities

$$\frac{2}{\pi}\int_0^\infty \frac{\operatorname{Im}\{h(x)\}}{x^{2n}}\,\mathrm{d}x = a_{2n-1} - b_{2n-1} = \begin{cases} -b_{2n-1} & n < 0 \\ a_{-1} - b_{-1} & n = 0 \\ a_1 - b_1 & n = 1 \\ a_{2n-1} & n > 1, \end{cases} \tag{3}$$

where $n = 1 - N_\infty, ..., N_0$ and $a_{2n-1} = b_{1-2n} = 0$ for $n < 0$, see [1] for details. Note that a simplified notation is used here where the limits in (3) are dropped, *i.e.*, the integrand is the limit $h(x + \mathrm{i}y)$ as $y \to 0$. The non-negative constraint, $\operatorname{Im} h \geq 0$, imply the inequity $\frac{2}{\pi}\int_{x_1}^{x_2} \operatorname{Im}\{h(x)\}x^{-2n}\,\mathrm{d}x \leq a_{2n-1} - b_{2n-1}$.

The identity (3) is the foundation for deriving sum rules and limitations on a many physical systems [1], see also [3,12,13]. In short; it is sufficient to identify a passive system and to determine its low- and/or high-frequency asymptotics. In the presentation, we illustrate the use of sum rules and physical bounds for various passive electromagnetic systems. We present an overview of results that answer questions such as

- What limits the scattering and absorption of electromagnetic waves by finite objects over a frequency interval [13]?
- How does the ultimate bandwidth depend on the size and volume of antennas [5, 6]?
- What are the tradeoffs between thickness and bandwidth for; absorbers [12], high-impedance surfaces [2, 7], total cross sections [10], and blockage of EM waves [9]?
- How does the aperture size and shape related to the bandwidth for extraordinary transmission of electromagnetic waves through apertures in thin sheets [8]?
- What is the ultimate bandwidth for negative refraction [4]?
- How is the bandwidth related to the permeability in artificial magnetism [4]?

The derived sum rules and physical bounds are useful as they relate the dynamic response to the often much simpler low- or high frequency properties. The asymptotic values can also often be determined analytically and are related to simple parameters of the structure, such as the thickness [7, 12] or polarizability [13]. The derived bounds offer simple tradeoffs between performance and bandwidth for many cases [4–7, 12]. The results are also useful as they show fundamental limits for linear and passive structures and hence indicate when it is necessary to utilize active or non-linear devices.

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