Why Use Adaptive Techniques for Steering Large Tankers?

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Why use adaptive techniques for steering large tankers?

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The purpose of this paper is to analyse the problem of ship steering and to discuss qualitatively the problems associated with variations of the process dynamics and the disturbances. The paper is based on simple models which can be dealt with analytially. It serves as a complement to computer studies using more elaborate models. Models for ship steering dynamics and for disturbances due to wind and waves are reviewed. Criteria for ship steering and autopilot design are discussed. The performance of fixed gain autopilots under different operating conditions is analysed. It is shown that there is a good incentive for using adaptive autopilots.

1. Introduction

Control problems associated with steering of large tankers are discussed in this paper. Many of the arguments can be applied to ship-steering in general. A particular problem with large tankers is, however, that they may be directionally unstable in certain operating conditions. This gives rise to special problems.

The paper is organized as follows. A review of ship-dynamics is given in § 2. The characterization of disturbances due to winds, waves and currents is the topic of § 3. Section 4 deals with criteria. It is shown that normal course-keeping fits nicely into the linear-quadratic–gaussian formulation of the control problem. The control design is thus straightforward. For turning it is, however, important to take the non-linear effects into account. The process dynamics and the characteristics of the disturbances will change with the operating conditions. The consequences of this for the control system design are discussed in § 5. Analysis based on a simple model gives the order of magnitudes involved. It is shown that it is indeed possible to find a constant gain controller which gives a closed loop system which is reasonably damped. There is, however, a substantial loss in performance with a controller having fixed regulator gains and consequently it is beneficial to use an adaptive controller.

2. Ship-steering dynamics

The equations describing the motion of a ship are well known. They are obtained from conservation of momentum and angular momentum (see Norrbin (1960) and Abkowitz (1964)). It is customary to write the equations using a coordinate frame fixed to the ship (see Fig. 1). If the ship is considered as a rigid body it has six degrees of freedom. The translational motions are called surge, sway and heave, and the rotational motions are called roll, pitch and yaw. For a ship like a tanker there is little coupling between the different modes and the steering dynamics can therefore be described by considering the

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surge, sway and yaw motions separately. Introduce coordinates and variables as shown in Fig. 1. The equations of motion can be written as

$$\left.\begin{aligned} m(\dot{u}-vr-x_G r^2)&=X\\ m(\dot{v}+ur+x_G\dot{r})&=Y\\ I_z\dot{r}+mx_G(\dot{v}+ru)&=N \end{aligned}\right\} \quad (2.1)$$

where u and v are the x- and y-coordinates of the velocity $r=d\psi/dt$ and x_G denotes the x-coordinate of the centre of mass. The mass of the ship is m and its moment of inertia with respect to the z-axis is I_z. The right-hand sides of (2.1) are the hydrodynamic forces and moments. These are complicated functions of the motion which are often expressed as functions of acceleration, velocity and helm angle. The main difficulty in modelling ship-dynamics is to find suitable expressions for the hydrodynamic forces. The first equation in (2.1) is often neglected when analysing steering because the forward speed u can often be regarded as being a constant.

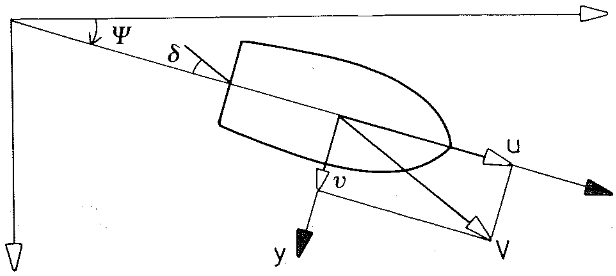

Figure 1. Coordinates and variables used for the equation of motion.

Stationary motions

Assuming that the rudder is kept in a fixed position the steady state solutions to the equations of motion are obtained by setting time derivatives in (2.1) equal to zero. For slender ships which are directionally stable there is only one stationary solution. For large tankers it can frequently happen that there are three steady state solutions. For example when the rudder is held at centre position a directionally stable ship will have a solution corresponding to a straight line motion. For a large tanker this motion can be unstable. Instead there are two stable stationary motions corresponding to turning port or starboard with constant yaw rate. It is common practice to represent the stationary motions with a graph of yaw rate against rudder angle as shown by the curve in Fig. 2. The curve can be determined experimentally by setting a constant rudder angle and observing the yaw rate or for unstable ships by controlling the ship for a constant turning rate and measuring the mean value of the rudder angle.

Linearization

To linearize the equations of motion it is necessary to introduce the partial derivatives of the hydrodynamic forces and moments. The partial derivatives are called ' hydrodynamic derivatives '. They are denoted as follows

$$Y_v=\frac{\partial Y}{\partial v}$$

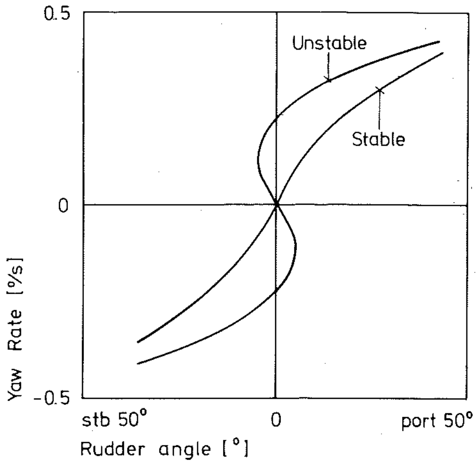

Figure 2. Steady state yaw rate as a function of rudder angle.

Using this notation the equations of motion (2.1) can be linearized around the stationary solution $v=0$, $r=0$ and $u=u_0$ to give

$$\begin{bmatrix} m-Y_{\dot v} & mx_G - Y_{\dot r} \\ mx_G - N_{\dot v} & I_z - N_{\dot r} \end{bmatrix} \begin{bmatrix} \dot v \\ \dot r \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - mu_0 \\ N_v & N_r - mx_G u_0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \end{bmatrix} \delta \tag{2.2}$$

The velocity is constant up to terms of second order. The derivatives $Y_{\dot v}$ and $N_{\dot r}$ are negative. They appear in the linearized equations in the same way as mass and inertia, and are therefore called ' added mass ' and ' added inertia '. In more accurate representations these terms will depend on the frequency of excitation. The equations of motion are customarily rewritten using dimension-free variables by introducing the length of a ship as unit of length and the time it takes to travel a shiplength as the time unit.

The hydrodynamic derivatives can be determined approximately from hydrodynamic theory (see Comstock (1967) and Norrbin (1970)). The derivatives will depend on many factors, among others loading, trim and water depth. The derivatives will thus depend on the operating conditions of the ship. The hydrodynamic derivatives can also be determined from experiments using scale models (see Motora (1972), Comstock (1967) and Strøm-Tejsen and Chislett (1966)). The hydrodynamic derivatives have recently been determined from experiments on ships using system identification methods (see Åström and Källström (1973, 1976) and Åström *et al.* (1974)).

State equations

The linearized equations of motion (2.2) are easily converted to state-space form by solving for the derivatives $\dot v$ and $\dot r$. This gives the following model for the yaw motion of the ship

$$\frac{d}{dt}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix} \delta \tag{2.3}$$

where the heading ψ has been introduced as a state variable.

The linearized ship-steering dynamics can thus be described as a third-order dynamical system. It was shown in Åström and Källström (1976) that the numerical values of the parameters of the model (2.3) are remarkably similar for many different ships if normalized units are used. The parameters will, however, depend on loading, trim, and depth. Table 1 shows the parameters of different tankers under different loading conditions. The parameters are normalized by choosing the length unit as the length of the ship l and the time unit as l/u_0.

Tanker	Loading condition	a_{11}	a_{12}	a_{21}	a_{22}	b_{11}	b_{21}
1	Fully loaded	−0·44	−0·28	−2·67	−2·04	0·07	−0·53
	Ballast	−0·41	−0·38	−0·30	−1·07	0·11	−1·07
2	Fully loaded	−0·42	−0·26	−2·59	−2·29	0·09	−0·75
	Ballast	−0·34	−0·35	−0·18	−1·56	0·14	−1·17
3	Fully loaded	−0·46	−0·28	−1·95	−1·90	0·11	−0·88
	Ballast	−0·41	−0·31	−1·39	−1·57	0·11	−0·89
4	Fully loaded	−0·48	−0·30	−2·90	−1·93	0·06	−0·48
	Ballast	−0·42	−0·35	−1·19	−1·50	0·09	−0·79
5	Fully loaded	−0·51	−0·25	−2·30	−2·30	0·12	−0·95

Table 1. Normalized parameters of state variable model (2.3) for different tankers.

Transfer function models

It follows from (2.3) that the input–output relation between the rudder angle δ and the heading ψ can be represented by the transfer function

$$G(s)=\frac{b_1 s+b_2}{s(s^2+a_1 s+a_2)}=\frac{K(1+sT_3)}{s(1+sT_1)(1+sT_2)} \tag{2.4}$$

where

$$\begin{aligned} a_1 &= -a_{11}-a_{22} \\ a_2 &= a_{11}a_{22}-a_{12}a_{21} \\ b_1 &= b_{21} \\ b_2 &= a_{21}b_{11}-a_{11}b_{21} \end{aligned}$$

The model (2.4) is commonly used for analysing steering and autopilots. The form (2.4) suggests the approximation

$$G(s)\approx\frac{K}{s(1+sT_N)} \tag{2.5}$$

which was originally proposed by Nomoto (1957).

Non-linear models

The linearized models can adequately describe the motion of a ship on a straight line course. For tankers the linearized models are, however, inadequate for turning. The non-linear terms in the equations of motion become important even at moderate yaw rates. Compare Fig. 2. The non-linear models are obtained by Taylor series expansions of the hydrodynamic forces and moments. Norrbin (1963) has proposed the following approximate model.

$$T_1 T_2 \frac{d^2 r}{dt^2} + (T_1 + T_2)\frac{dr}{dt} + KH(r) = K\left(\delta + T_3 \frac{d\delta}{dt}\right) \tag{2.6}$$

where H is the non-linear function which gives the steady-state relation between δ and r (see Fig. 2). This model has also been investigated by Bech and Smitt (1969).

3. Disturbances

The motion of a ship is influenced by wind, waves and current. Since the purpose of the autopilot is to counteract the influences of the disturbances it is of interest to characterize the disturbances. The currents will not influence the hydrodynamic forces. They will, however, influence the inertial velocity of the ship. There will be a considerable influence of the motion due to wind and waves. In a simplified analysis this is handled by introducing the forces and the moments generated by wind and waves. The equations of motion (2.1) are then changed to

$$\left.\begin{aligned} m[\dot{u} - (v + v_c)r - x_G r^2] &= X + X_{\text{wind}} + X_{\text{waves}} \\ m[\dot{v} + (u + u_c)r + x_G \dot{r}] &= Y + Y_{\text{wind}} + Y_{\text{waves}} \\ I_z \dot{r} + m x_G[\dot{v} + r(u + u_c)] &= N + N_{\text{wind}} + N_{\text{waves}} \end{aligned}\right\} \tag{3.1}$$

where v_c and u_c are the velocity components of the current. X_{wind}, Y_{wind} and N_{wind} are the forces and moments generated by the wind and X_{waves}, Y_{waves} and N_{waves} are the forces and moments generated by the waves. Equation (3.1) is only an approximation because the superposition principle is not necessarily valid for large motion. There may also be couplings to the other motions due to wind and waves. For example a pitching motion can change the area exposed to the wind considerably. The moment N_{waves} may thus depend on the pitch angle. Similarly the airflow around the ship may be significantly influenced by the waves which also indicates that it is not always correct to separate the forces due to wind and waves. Under many operating conditions eqn. (3.1) is, however, a reasonable approximation.

Currents

The influence of currents will now be discussed. In the X-equation v_c appears in the product $v_c r$ which is neglected in the linearized equation. In the Y- and N-equations the current u_c appears in the combination $u + u_c$. The influence of u_c will not be very large unless the currents are comparable to the forward velocity. Since the components of the current u_c and v_c will depend on the heading the currents will also introduce a coupling between yaw angle and the Y- and N-equations.

Wind-generated disturbances

The wind forces X_{wind}, Y_{wind} and the wind moment N_{wind} depend on the shape of the ship above the water-line and the relative wind force as seen from the ship. The wind-generated disturbances will thus also depend on the motion of the ship because this will influence the relative wind speed. The wind forces have been investigated both theoretically and experimentally by Wagner (1967) and van Berlekom *et al.* (1975). Wagner (1967) gives the following models for the wind forces.

$$\left.\begin{aligned} X_{\text{wind}} &= \tfrac{1}{2} C_X(v) \rho V^2 A_l \\ Y_{\text{wind}} &= \tfrac{1}{2} C_Y(v) \rho V^2 A_l \\ N_{\text{wind}} &= \tfrac{1}{2} C_N(v) \rho V^2 A_l \end{aligned}\right\} \tag{3.2}$$

where ρ and V are air-density and relative wind velocity and A_l is a reference area. The angle v is the angle between the relative air velocity and the x-axis (see Fig. 3).

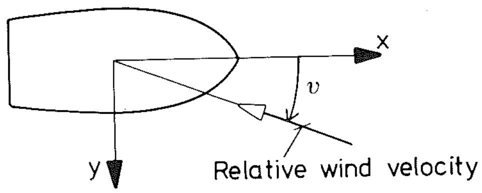

Figure 3.

For a symmetrical ship the functions C_X, C_Y and C_N have the following form

$$\begin{aligned} C_X(v) &= C_X \cos v \\ C_Y(v) &= C_Y \sin v \\ C_N(v) &= C_N \sin 2v \end{aligned}$$

The paper by van Berlekom *et al.* (1975) gives the functions C_X, C_Y and C_N for typical tanker configurations based on wind tunnel tests. The wind velocity is modelled as the sum of a constant term and a stochastic term which characterizes the turbulence. Turbulence data is presented in Lumley and Panofsky (1964). The turbulence scale L is approximately proportional to altitude $L \approx 0{\cdot}9h$. For typical tankers the turbulence scale is thus smaller than the length of the ship. This means that it is not unreasonable to consider the random fluctuations as white noise. Introducing the expression (3.2) for the wind forces into the equation of motion gives the following linearized equations

$$\frac{d}{dt}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix}\delta + \begin{bmatrix} e_1 \\ e_2 \\ 0 \end{bmatrix} \tag{3.3}$$

Compare with (2.3). The effect of the wind in the linearized equations is thus that the coupling terms a_{13} and a_{23} and the disturbances e_1 and e_2 are added. The numerical values of a_{13} and a_{23} will depend on the angle v. The parameters

a_{13} and a_{23} may well change sign. The transfer function corresponding to (3.3) is

$$G(s) = \frac{b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3} \tag{3.4}$$

where

$$\begin{aligned} a_1 &= -a_{11} - a_{22} \\ a_2 &= a_{11}a_{22} - a_{12}a_{21} - a_{23} \\ a_3 &= a_{11}a_{23} - a_{13}a_{21} \\ b_2 &= b_{21} \\ b_3 &= a_{21}b_{11} - a_{11}b_{21} \end{aligned}$$

Compare with (2.4). Notice that in the presence of wind the transfer function relating heading to rudder angle will not necessarily contain an integrator. Experimental evidence of this is given in Åström and Källström (1976).

Wave-generated disturbances

There will be substantial forces and moments on a ship due to the motion of the sea. Several attempts have been made to model those forces. Zuidweg (1970) has made the simplifying assumption that the sea waves can be described as a plane sinusoidal wave. The forces and moments can then be approximately described as

$$\left.\begin{aligned} Y_{\text{waves}} &= \hat{Y}(\mu) \sin \omega t \\ N_{\text{waves}} &= \hat{N}(\mu) \cos \omega t \end{aligned}\right\} \tag{3.5}$$

where μ is the angle between the x-axis and the direction of wave propagation (see Fig. 4). Zuidweg also gives explicit expressions for the functions $\hat{Y}$ and $\hat{N}$. Introducing (3.5) into the equations of motion and linearizing, we find that the linearized equations have the form (3.3) where the elements a_{13} and a_{23} are sine- and cosine-functions of time. There will also be sinusoidal driving functions in the right-hand side of the linearized equation. Zuidweg's model will thus lead to a linear system with periodically varying parameters.

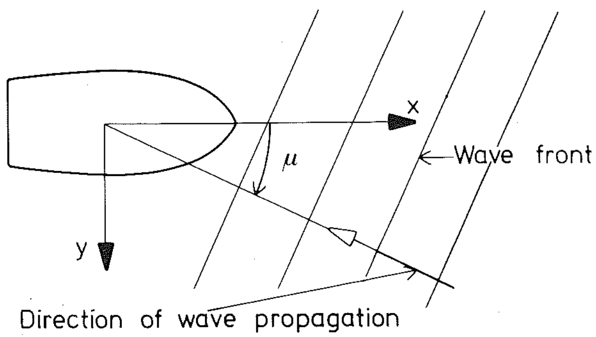

Figure 4.

It is an oversimplification to assume that the waves in the ocean can be described as a plane sinusoidal wave. A more fruitful approach is to assume that the level of the sea is a stochastic process. Such models have been determined both theoretically and empirically under many different conditions. The spectral density will vary significantly depending on wave height. An expository presentation is given by Price and Bishop (1974). The measured spectral densities indicate that the major contribution is from frequencies in the range 0·2 to 0·6 rad/s. Keeping in mind that the dominating time constants of a tanker are of the order of 50 s or more it is reasonable to consider the disturbances as white noise. Assuming that the sea-level is a random process the forces and moments can then be calculated in the same way as Zuidweg treated sinusoidal waves. This would lead to a linear system of the form (3.3) where a_{13}, a_{23}, e_1 and e_2 are random variables.

Summary

The influence of wind, waves and current are thus difficult to model. The disturbances will introduce couplings (a_{13} and a_{23}) from the heading to the sway and yaw equations as well as forcing terms (e_1 and e_2) in the linearized equations. The couplings are small, at least for moderate disturbance levels, and therefore often neglected. The forcing terms can be considered as the sum of constant and random components. Because of the long time constants of tankers it is reasonable to approximate the random components by white noise both for wind- and wave-generated forces. There is support for such assumptions in results from system identification applied to data from real tankers (see Åström and Källström (1976) and Åström *et al.* (1974)).

4. Criteria

The criteria to be used in the evaluation and design of autopilots will now be discussed. The criteria will depend on many factors like safety, propulsion economy and accuracy in path-keeping. When steering in confined waters with several other ships in the neighbourhood precision in path-keeping is the most important factor. When operating in open sea far from other ships propulsion economy is of major concern. For large tankers this factor is the most important one and the following discussion will therefore be limited to the influence of steering on propulsion efficiency. Deviations from the desired heading will give a loss due to a longer distance travelled. The rudder deflections introduced to counteract deviations in heading will, however, give retarding forces. There will also be retarding forces caused by deviations in sway velocity and yaw rate. The x-component of the equation of motion (2.1) can be written as

$$m[\dot{u}-rv-r^2x_G]=X(u, v, r, d, \delta, \dot{u})$$

where the forces can be approximated as

$$X \approx X_{\dot{u}}\dot{u}+X_{vr}vr+X_{\delta\delta}\delta^2+X_{uu}u^2+\ \dots$$

It was shown by Norrbin (1972) that the most important contribution to the increase of resistance due to course deviations comes from the term vr which represents the Coriolis acceleration due to the coupling of yaw and sway velocity. Assuming small perturbations around a straight line course Norrbin also showed that the average increase in resistance due to steering, given by the x-equation in (2.1), could be approximately described by

$$\Delta R/R = k[\bar{\psi}^2 + \lambda \bar{\delta}^2] \tag{4.1}$$

where R is the drag, ΔR the increase in drag due to steering, $\bar{\psi}^2$ and $\bar{\delta}^2$ are the averages of the squared heading and rudder deviations respectively. Norrbin (1972) gives the values $k = 0{\cdot}014$ and $\lambda = 0{\cdot}10$ for a typical tanker. The earlier analysis by Koyama (1967) which only included rudder losses and the loss due to the increased path gave $\lambda = 8$. When designing and evaluating autopilots for steering of tankers in open sea it therefore seems natural to use the following criterion

$$J = \frac{1}{T} \int_0^T [\psi^2(t) + \lambda \delta^2(t)]\, dt \tag{4.2}$$

We thus have one of the rare occasions when a quadratic performance criterion is physically well motivated and in particular when the weighting between state variable deviations and control actions is given *a priori*. The value of the loss function also corresponds directly to the relative increase in resistance. In a typical case one unit of the loss function (4.2) corresponds to 1·4% in the propulsion drag R.

Recalling that the deviations of the state variables that occur during normal steering are so small that the dynamics can be described by linearized models and that there are good reasons to characterize the disturbances due to wind and waves as random processes we find that the design of autopilots fits the framework of linear-quadratic–gaussian control theory. The design of a regulator for course keeping is therefore straightforward. For design of the turning regulator it is necessary to take the non-linearities into account. The optimal course keeping regulator based on the model (3.3) and the criterion (4.2) consists of a state feedback from heading, yaw rate and sway velocity while a regulator based on the simple Nomoto model is a state feedback from heading and yaw rate only, i.e. a PD-regulator. It is necessary to include a model of the disturbances, and the regulator will then also contain feedback from the disturbance states. A simple case is to model the disturbances as a constant but unknown moment. Together with the simple Nomoto model this leads to an ordinary PID-regulator which is the basis for most commercial autopilots. The regulators obtained using the more complete model (3.3) and a more detailed disturbance model will be more complex because it includes more state variables. It is then convenient to use a Kalman filter to provide reliable estimates of the state variables.

5. Variations in process dynamics and disturbances

It has been demonstrated in the previous sections that the problem of controlling a large tanker in normal course keeping is a control problem that fits nicely into the linear-quadratic formulation. The design of a regulator composed of a state feedback and a Kalman filter is therefore straightforward. Such regulators have been determined by Zuidweg (1970). The process dynamics will, however, vary depending on operating conditions such as speed trim, loading and ocean depth. The dynamics will vary significantly with speed. Since the speed dependence can be described analytically and since the speed also can be measured it is easy to eliminate the influence of the ship's speed by gain scheduling. The characteristics of the disturbances will also vary depending on changing winds and waves. The consequences of these changes will be investigated in this section. In particular, the consequences of using a regulator with fixed feedback gains will be explored. The purpose of the analysis is to develop an understanding for the different effects and to provide order-of-magnitude estimates. Simple models will therefore be used. The problem of selecting a fixed gain autopilot which gives a closed loop system with good properties for a ship with changing dynamics will first be investigated. This analysis is done purely based on deterministic arguments. The problem of selecting optimal autopilot parameters will next be investigated. This requires stochastic models.

Deterministic analysis

The linearized steering motion of a tanker travelling at constant speed can be approximately described by the transfer function (2.4).

$$G(s)=\frac{K(1+sT_3)}{s(1+sT_1)(1+sT_2)}\approx\frac{K}{s(1+sT_N)}=\frac{K_1}{s(s+a)} \tag{5.1}$$

which relates heading angle ψ to rudder angle δ. The parameters of the transfer function (5.1) will depend on the operating conditions. The variation with the ship's speed u can be approximately given as

$$\left.\begin{aligned} K&=K^0u/u^0\\ T_1&=T_1{}^0u/u^0\\ T_2&=T_2{}^0\,u^0/u\\ T_3&=T_3{}^0\,u^0/u\\ K_1&=K_1{}^0(u/u^0)^2\\ a&=a^0u/u^0 \end{aligned}\right\} \tag{5.2}$$

This follows from the empirical observation that the parameters of the state model (2.3) are approximately independent of ship speed provided that the length unit L is the ship length L and the time unit is L/u_0. Compare with Table 1. A simple dimensional analysis then gives (5.2).

Typical values of the parameters of the transfer function (5.1) for a tanker under different operating conditions are shown in Table 2.

Operating conditions	T_1	T_2	T_3	K	T_N	a	$K_1\times 10^6$
OC1	80	15	40	−0·013	50	0·020	−260
OC2	160	20	30	−0·040	150	0·007	−270
OC3	1000	25	60	−0·130	1000	0·001	−130
OC4	−300	30	65	0·040	−400	−0·003	−100

Table 2. Parameters of the transfer function (5.1) for a tanker under different operating conditions. The condition OC1 corresponds to ballast and OC3 and OC4 to full load. In OC4 the tanker has a forward trim. The speed is 8 m/s in all cases.

Apart from variations with speed the transfer function parameters will also change considerably with trim and loading. It follows from Table 2 that the ship is stable in operating conditions 1, 2 and 3 but unstable in 4. Also, notice that it is advantageous to use the parameters K_1 and a rather than K and T_N because K_1 varies less than K. The parameter K_1 changes only by a factor of 3 over the operating conditions shown in Table 2.

In the following it will be assumed that the ship dynamics can be characterized by the simple Nomoto model. Motivated by the numbers in Table 2 it will be assumed that the parameter $K_1{}^0$ is constant equal to $-2\times 10^{-4}\ \mathrm{s}^{-2}$ and that the parameter a^0 varies between $-0{\cdot}01\ \mathrm{s}^{-1}$ and $0{\cdot}01\ \mathrm{s}^{-1}$ depending on trim and loading. It is assumed that the variation of the parameters with the speed of the ship are given by (5.2).

To analyse the consequences of the parameter variations for controller design it will be assumed that the ship is controlled by a PD-regulator having the input–output relation

$$\delta(t)=k_1[\psi(t)-\psi_0]+k_2\,\frac{d\psi}{dt} \tag{5.3}$$

This regulator is equivalent to a state feedback controller for the Nomoto model. A more complete model will require a more complex controller. It may also be necessary to introduce an integral term to handle low frequency disturbances.

The rudder deflection is limited by about 40° in practice. It is also desirable to keep the rudder motions within reasonable limits. The limited control authority can be introduced by bounding the gains of the controller. Reasonable limits are

$$\left.\begin{aligned} |k_1| &\leqslant 10 \\ |k_2| &\leqslant 100\ \mathrm{s} \end{aligned}\right\} \tag{5.4}$$

This means that a rudder deflection of 20° is obtained either by a heading error of 2° or a yaw rate error of 0·2°/s. The consequences of increasing the bound on k_2 to 200 s will also be explored.

Influence of ship velocity

The influence of the ship's velocity on the performance of the closed loop system will first be explored. The closed loop system is of second order. It follows from the eqns. (5.1), (5.2) and (5.3) that the characteristic equation of the closed loop system is given by

$$s^2+s[a^0(u/u^0)-k_2K_1{}^0(u/u^0)^2]-k_1K_1{}^0(u/u^0)^2=0 \tag{5.5}$$

The system has thus

$$\left.\begin{aligned} \omega &= (u/u^0)\sqrt{(-k_1K_1{}^0)} \\ \zeta &= \frac{a^0-k_2K_1{}^0(u/u^0)}{\sqrt{(2-k_1K_1{}^0)}}=\zeta_0\,\frac{a^0-k_2K_1{}^0(u/u^0)}{a^0-k_2K_1{}^0} \end{aligned}\right\} \tag{5.6}$$

The characteristic frequency is thus proportional to the ship's speed. The relative damping ζ is in general increasing with increasing ship speed because the $K_1{}^0$ is negative and k_2 is positive. If the controller gains are set so that the closed loop system has desired properties at the nominal speed u^0 it is thus found that the characteristic frequency ω will decrease with decreasing speed. The relative damping will also decrease. A numerical example illustrates the orders of magnitude involved.

Example 1

Consider a tanker with $a^0 = 0{\cdot}01$, $K_1{}^0 = -2 \times 10^{-4}$ and $k_2 = 100$. Equation (5.6) gives

$$\zeta = \zeta_0 \frac{2(u/u^0)+1}{3}$$

The relative damping will thus decrease by $\frac{1}{3}$ if the speed is decreased by a factor of 2.

Similarly for an unstable ship with $a = -0{\cdot}01$ eqn. (5.6) gives

$$\zeta = \zeta_0[2(u/u_0) - 1]$$

In this case a reduction in speed by a factor of two will thus result in an unstable closed loop system. □

It is thus found that if the controller settings are constant changes in speed will result in changes in the closed loop characteristic frequency and damping. For unstable ships the changes may result in an unstable closed loop system. Since the ship's speed is easily measured it is, however, straightforward to eliminate the influence of changes in the ship's speed, by letting the parameters of the controller be functions of the ship speed. This is called *gain scheduling* in the terminology of aircraft flight control systems. Gain scheduling can be accomplished in several different ways.

In some cases it is acceptable and even desirable that the closed loop characteristic frequency is a linear function of the ship's speed. It follows from (5.6) that the proportional gain k_1 should then be a constant. By choosing the derivative gain k_2 as

$$k_2 = k_2{}^0\, u_0/u \tag{5.7}$$

it also follows from (5.6) that the relative damping of the closed loop system will be invariant with the ship's speed. In this case the response of the path of the ship as drawn on a chart will be invariant with the ship's speed. This type of gain-scheduling is therefore called *path scheduling*.

It is desired to have the time response of the controlled ship invariant with the ship's speed it follows from (5.6) that the controller gains should be the following functions of the ship's speed

$$\left.\begin{aligned} k_1 &= (u^0/u)^2 k_1{}^0 \\ k_2 &= (u^0/u)^2[k_2{}^0 - (a^0/K_1{}^0)(1 - u/u^0)] \end{aligned}\right\} \tag{5.8}$$

This type of gain scheduling is called *time scheduling* because it makes the time responses of the closed loop system invariant with the ship's speed. Notice that time scheduling is much more difficult to implement than path scheduling because it requires that the parameter a is known. This parameter will change significantly with trim and loading.

Influence of trim and loading

It has been found that the influence of the forward speed of the ship can be eliminated by gain scheduling. When analysing the effects of changes in trim and loading it will therefore be assumed that the ship's velocity u is constant. As before it is assumed that the ship dynamics is given by the Nomoto model (5.1) with $K_1 = -2 \times 10^{-4}$. The parameter a may take any value in the range $(-0{\cdot}01, 0{\cdot}01)$ depending on loading and trim. It is easy to show that the closed loop poles that are obtained with the regulator (5.3) whose parameters are constrained by (5.4) are those shown in Fig. 5.

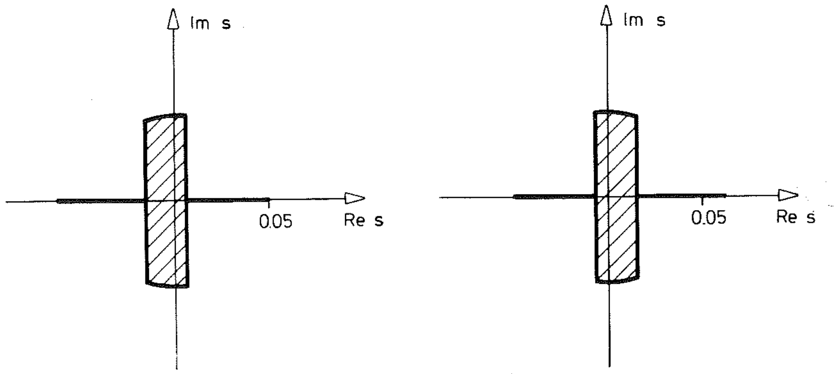

Figure 5. Possible positions of the closed loop poles obtained by a regulator (5.2) whose parameters are constrained by (5.4).

The closed loop poles that can be obtained with a controller having higher authority namely

$$\left.\begin{aligned} |k_1| &\leqslant 10 \\ |k_2| &\leqslant 200 \text{ s} \end{aligned}\right\} \tag{5.9}$$

are shown in Fig. 6.

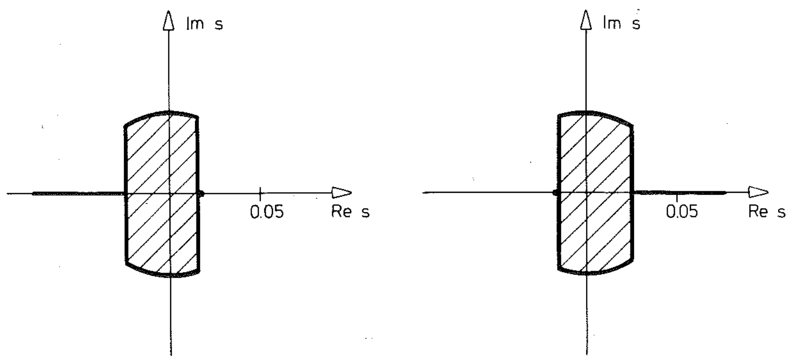

Figure 6. Possible positions of the closed loop poles obtained by a regulator (5.9).

It is clear from Fig. 5 and Fig. 6 that the main difficulty in controlling a ship that may be both stable and unstable is to provide sufficient damping. The analysis also shows that the essential limitation is due to the limited control authority (5.4) or (5.9). To further illustrate the trade-offs in selecting the parameters in an autopilot with fixed gains Fig. 7 shows the root loci of the closed loop poles with respect to the parameter a for different fixed gain regulators.

Figure 7 illustrates the difficulty in finding constant autopilot gains that will give good performance for all loading conditions. The controller in Fig. 7B will be acceptable if the parameter a only varies over the interval $0{\cdot}01 \leqslant a \leqslant 0$.

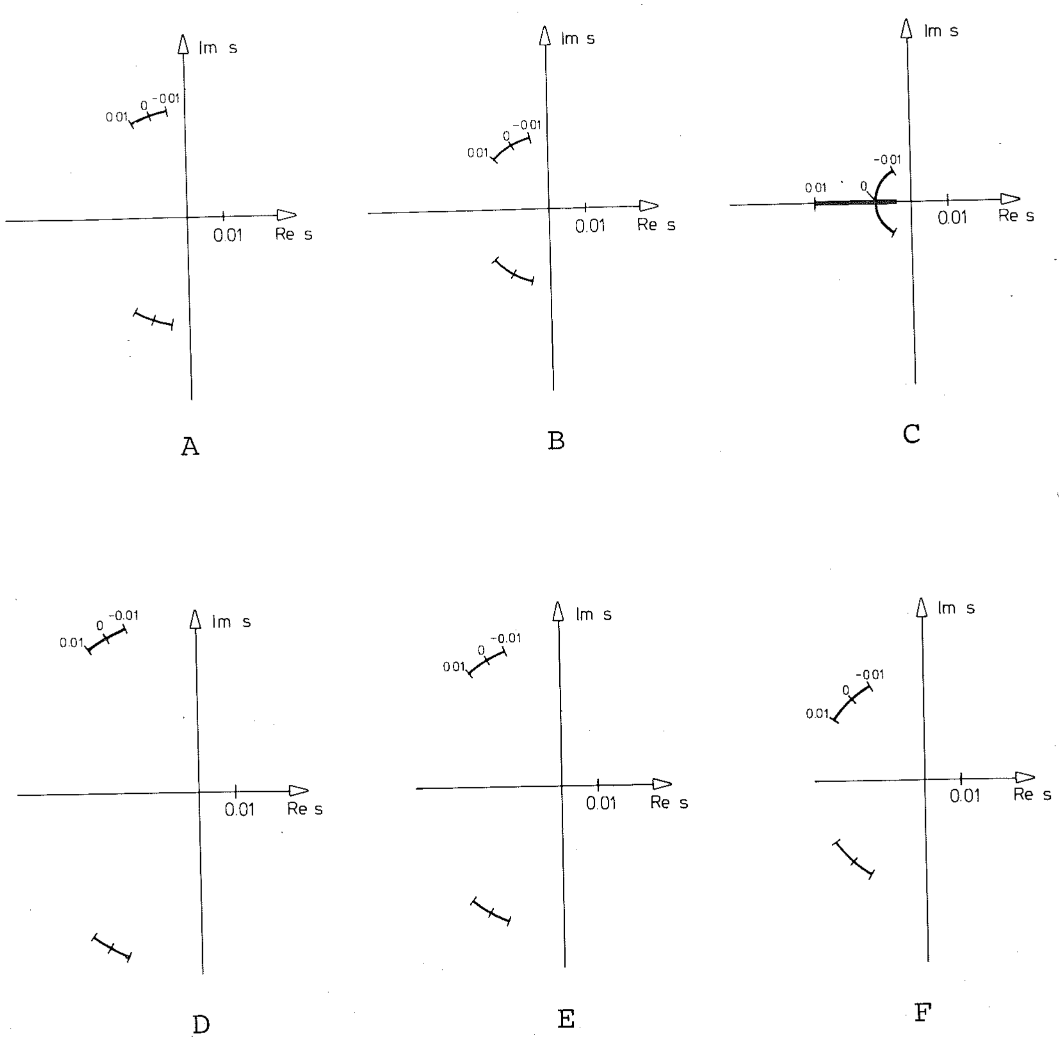

Figure 7. Root loci for the closed loop characteristic equation with respect to the parameter a for different autopilot gains.

A : $k_1 = 4{\cdot}5$, $k_2 = 100$ s B : $k_1 = 2$, $k_2 = 100$ s
C : $k_1 = 0{\cdot}5$, $k_2 = 100$ s D : $k_1 = 12{\cdot}5$, $k_2 = 200$ s
E : $k_1 = 8$, $k_2 = 200$ s F : $k_1 = 4{\cdot}5$, $k_2 = 200$ s

It will, however, give a very low damping when a is negative, i.e. for loading conditions which give an unstable dynamics. The controller in Fig. 7C which has lower proportional gain will give acceptable damping for unstable loading conditions. It will, however, give a response which is too much damped for stable loading conditions. The controllers with higher derivative gain $k_2 = 200$ s will give better performance. In particular the controller in Fig. 7F ($k_1 = 4{\cdot}5$ and $k_2 = 200$ s) will give acceptable performance for all loading conditions. A comparison between A, B, C and D, E, F shows clearly the drastic differences in sensitivity to variations in process parameters when the gain k_2 is increased from 100 s to 200 s.

The deterministic analysis shows that it may be possible to find a fixed gain controller which gives acceptable performance for all loading conditions. It is favourable to choose the derivative gain k_2 as high as possible. With the low derivative gain ($|k_2| \leqslant 100$) the closed loop system will be poorly damped for $a = -0{\cdot}01$ and over-damped for $a = 0{\cdot}01$. However, with the constraint (5.9) it is possible to find controller gains so that the relative damping is in the range $0{\cdot}50 \leqslant \xi \leqslant 0{\cdot}83$ which is acceptable.

The results obtained will hold qualitatively for the more complicated models. In practice it is also desirable to introduce an integral term in the controller. To obtain a good response it is then desirable to increase the damping a little by decreasing the proportional gain.

Stochastic analysis

The heading signal is normally taken from the gyro compass. There will be disturbances in this signal due to limited resolution and measurement noise. The rate feedback is obtained by taking the derivative of the heading or from a rate gyro. In both cases there will be disturbances in the measured signal. The disturbances acting on the ship will also vary considerably due to changing wind, waves and currents. A simplified case will again be analysed to provide the qualitative aspects of the trade-off required. Using the simple Nomoto model, the closed loop system can be represented by the block diagram in Fig. 8.

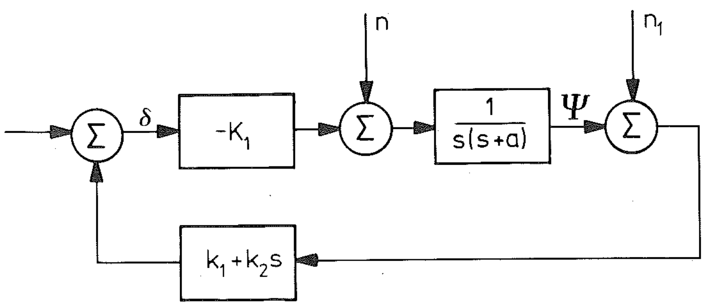

Figure 8. Block diagram of ship with autopilot.

Assuming small perturbations the relation between the heading and the disturbances can be described by the equation

$$\Psi(s)=\frac{N(s)+K_1(k_1+k_2s)N_1(s)}{s^2+s(a-K_1k_2)-K_1k_1} \tag{5.10}$$

where Ψ, N and N_1 are the Laplace transforms of heading angle, disturbance torque acting on the ship and noise in the heading measurement. The important factor is the relative magnitude of the torque disturbance and the measurement noise. In a simplified analysis it is not unreasonable to model n and n_1 as independent white noises with spectral densities $2\pi\phi$ and $2\pi\phi_1$ respectively. It is easy to show that the mean square heading error is

$$E\psi^2=-\frac{\phi+(-K_1^3\,k_1k_2^2+K_1^2\,k_1^2)\phi_1}{2K_1k_1(a-K_1k_2)} \tag{5.11}$$

Minimization of the heading error gives the following optimal controller settings

$$\left.\begin{aligned} K_1k_1&=-\sqrt{(\phi/\phi_1)}\\ K_1k_2&=+a-\sqrt{[a^2+2\sqrt{(\phi/\phi_1)}]}\end{aligned}\right\} \tag{5.12}$$

Notice that the optimal gains only depend on the ratio ϕ/ϕ_1. The minimum value of the loss function is

$$E\psi^2=\phi_1[a^2+2\sqrt{(\phi/\phi_1)}-a] \tag{5.13}$$

The optimal controller settings for different signal-to-noise ratios are given in Table 3.

ϕ/ϕ_1		10^{-8}	10^{-7}	10^{-6}
k_1		0·50	1·6	5
k_2	$a=$ 0·01	37	85	180
	$a=$ 0	71	126	223
	$a=-0{\cdot}01$	137	185	279

Table 3. Autopilot gains which minimize the mean square heading error.

The ratio ϕ/ϕ_1 expresses the relation between the disturbances from wind and waves and those due to measurement errors. The ratio will depend on sea conditions in a fairly complicated way. The general tendency is, however, that ϕ/ϕ_1 increases with increasing wind and waves. The ranges given in Table 3 correspond approximately to a factor of 10 in wind velocity. It thus follows from Table 3 that low values of controller gains are optimal under nice weather conditions and that the gains should be increased in bad weather. The table also indicates that the proportional gain k_1 is changed most and that the variations in k_2 are smaller. It also follows from Table 2 that the value of the proportional gain does not depend on a. This gain is thus the same for a stable ($a = 0{\cdot}01$) and an unstable ($a = -0{\cdot}01$) ship. The rate gain will, however, depend on a. The variation in k_2 with a is larger for small ϕ/ϕ_1, i.e. nice sea conditions.

The minimum values of the variance of the heading error are shown in Table 4. It is seen from the table that the heading error will vary significantly with the operating conditions. Notice in particular that under fair operating conditions ($\phi/\phi_1 = 10^{-8}$) there is a significant difference between the minimal loss function for a stable and an unstable ship. This difference is smaller for operating conditions which correspond to heavier seas ($\phi/\phi_1 = 10^{-6}$).

a	ϕ/ϕ_1		
	10^{-8}	10^{-7}	10^{-6}
0·01	1·0	2·3	4·9
0	1·9	3·4	6·1
−0·01	3·7	5·1	7·6

Table 4. Relative variances of heading errors for optimal regulators under different operating conditions. The case $a = 0{\cdot}01$ and $\phi/\phi_1 = 10^{-8}$ is arbitrarily chosen as the reference.

Table 4 also shows that the minimal loss will vary significantly with the operating condition.

The deterministic analysis indicated that it was possible to choose constant autopilot gains which gave a well damped system for all operating conditions. Table 5 shows the performances of different fixed gain controllers.

The table shows the ratios between the loss of different fixed gain controllers and that of an optimally tuned controller. The table shows for example that a controller which is optimally tuned for a marginally stable ship ($a = 0$) and nice sea conditions ($\phi/\phi_1 = 10^{-8}$) will perform very poorly for bad sea conditions. Table 5 shows that if this controller is used for unstable loading conditions ($a = -0{\cdot}01$) and bad sea conditions ($\phi/\phi_1 = 10^{-6}$) then the loss function is 22·3 times larger than the loss of an optimally tuned controller. This agrees well with the empirical observation that an autopilot which is well tuned for nice sea conditions frequently is switched off in favour of manual control when the sea conditions get worse. The explanation is that the gains are too low to give good performance in bad sea conditions.

In Table 5 are also shown the performance of a fixed gain controller which is optimal for $\phi/\phi_1 = 10^{-6}$ and $a = -0{\cdot}01$, i.e. bad sea conditions and unstable loading. This regulator has high gains and its performance changes only a little with different operating conditions. Notice, however, that the regulator gives a very poor performance for the stable ship under nice sea conditions. The loss is 4·3 times larger than the loss of the optimal regulator. The heading errors are thus more than two times larger than necessary. The reason for this is that the errors in the heading measurement are fed back through the high gain thereby creating unnecessarily large rudder motions. The benefits of adaptive control are clearly seen from the table.

$k_1=0{\cdot}5,\ k_2=37$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	1·0	2·0	8·2
0	1·2	3·1	15·5
−0·01	∞	∞	∞

$k_1=1{\cdot}6,\ k_2=85$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	1·6	1	2·0
0	1·3	1·1	2·5
−0·01	1·7	1·8	4·8

$k_1=5,\ k_2=179$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	3·4	1·5	1
0	2·3	1·3	1·0
−0·01	1·6	1·2	1·1

$k_1=0{\cdot}5,\ k_2=71$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	1·1	1·6	6·0
0	1	1·8	8·1
−0·01	1·8	4·2	22·3

$k_1=1{\cdot}6,\ k_2=126$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	1·9	1·1	1·6
0	1·4	1	1·8
−0·01	1·2	1·1	2·4

$k_1=5,\ k_2=224$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	3·8	1·7	1·0
0	2·4	1·4	1
−0·01	1·6	1·2	1·0

$k_1=0{\cdot}5,\ k_2=137$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	1·7	1·4	4·1
0	1·2	1·3	4·4
−0·01	1	1·4	5·6

$k_1=1{\cdot}6,\ k_2=185$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	2·5	1·2	1·4
0	1·6	1·1	1·5
−0·01	1·2	1·0	1·6

$k_1=5,\ k_2=279$

a	ϕ/ϕ_1 10^{-8}	10^{-7}	10^{-6}
0·01	4·3	1·9	1·1
0	2·6	1·5	1·0
−0·01	1·6	1·2	1

Table 5. Ratio of loss functions of constant gain controllers to optimally tuned controllers.

It follows from (5.13) that the loss function is proportional to the spectral density of the measurement noise. This shows that it is beneficial to provide careful filtering of the measured signals. The analysis is admittedly based on a simplified model. This has the advantage that the influence of changing ship dynamics and changes in the environment can easily be investigated analytically. The simple model gives the main properties of the problem. To provide reliable quantitative information it is necessary to use more complicated models of the ship and its environment. Such studies based on simulation of more complete models and experiments on full-scale ships shows that the simple model given here gives the correct qualitative features.

6. Conclusions

Models for the steering dynamics of tankers, for disturbances due to current, wind and waves and for performance have been given. It has been concluded that course-keeping can be formulated as a linear quadratic control problem.

For turning even at moderate rates it is, however, necessary to use non-linear models of the tanker steering dynamics. The influences of parameter variations have been investigated. It has been shown, based on an analysis of a simple model, that a tanker which may be both stable and unstable depending on the operating conditions can be controlled by a PID-controller. It is possible to use a regulator with fixed gains to provide a reasonable damping over a wide operating range. Such a fixed gain regulator will, however, have a poor performance in many of the operating conditions. The performance criterion can be improved significantly by tuning the controller parameters. There is a trade-off between elimination of disturbances due to wind and waves and elimination of effects of measurement errors. The general characteristics of the optimal controllers is that both proportional gain and rate gain are increased with increasing force disturbances. The changes in proportional gain are larger than the changes in the rate gain.

The minimum value of the performance index changes considerably with the operating conditions. The mean square heading error is substantially larger under bad weather conditions. There are also large differences in heading error in fair weather when the loading conditions are changed so that the ship changes from being stable to being unstable.

There is a good incentive to decrease the measurement noise by careful filtering. An adaptive autopilot which automatically tunes its parameters for optimum performance as defined by minimum propulsion resistance can be designed using the concept of self-tuning regulators discussed in Åström and Wittenmark (1972, 1973) and Åström *et al.* (1975). Such a regulator has been designed by Källström *et al.* (1979). The regulator has been tested on several different tankers and it has been in continuous operation on one tanker for more than a year.

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