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ON LUNISOLAR CALENDARS AND INTERCALATION SCHEMES IN SOUTHEAST ASIA

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Abstract: This is a survey of different calendar intercalation schemes, mainly in Southeast Asia. The Thai and Burmese Calendars, superficially very similar, are shown to have quite different and interesting intercalation schemes. We also investigate similarities between the original Burmese Calendar and the *Romakasiddhânta* from India.

Keywords: Lunisolar calendar, intercalation, sidereal year, tropical year.

1 INTRODUCTION

In lunisolar calendars there are four sequences of time that need to be synchronised: the Sun, the solar calendar, the lunar calendar and the Moon. In order to achieve this synchronisation, different calendars have used different schemes. As a solar year is slightly more than 365 days and a solar calendar contains an integer number of days, there is a need for inserting leap days resulting in solar calendar years with 365 or 366 days. Such solar calendar years will be about 11 days longer than a lunar year with 12 synodic months with a mean value of about 29.5 days. In order to synchronise the lunar and solar calendars the usual procedure is to intercalate extra lunar months from time to time. Finally, in order to synchronise the lunar calendar with the Moon there may also be a need to intercalate extra days in the lunar calendar.

The Babylonians originally had a lunar calendar with months that started with the first visual appearance of the New Moon crescent. This automatically synchronised the lunar calendar with the Moon. The synchronisation with the Sun was done by requiring that the start of the lunar year would not deviate too far from the spring equinox, and if so an extra lunar month was inserted. From around the fourth century before the Christian Era the Babylonians switched to a more rigid system with 7 intercalary lunar months in 19 solar years, inserting at fixed years 3, 6, 8, 11, 14, 17 and 19 in each 19-year cycle, giving a total of 235 lunar months in each cycle. This so-called Metonic cycle is quite accurate: 19 tropical years of 365.2422 days are almost equal to 235 synodical months of 29.53059 days, $19 \cdot 365.2422 = 6939.6018$ days, $235 \cdot 29.53059 = 6939.6886$ days, and this gives a good synchronisation between the Sun and the lunar calendar. The same scheme was used in the early Jewish calendar and in some ancient Greek calendars.

2 INDIA

In India the backbone of the calendar is a sidereal solar calendar, i.e. a calendar where the solar year is determined by the return of the Sun to the same location relative to the fixed stars, in contrast to the tropical year that is based on the return of the Sun to the vernal point, the crossing between the ecliptic and the celestial equator. By the precession of the equinoxes, the vernal point is slowly receding relative to the stars by about 1° in 72 years. The sidereal year is slightly longer than the tropical year, its modern current value being 365.25626 days, while the tropical year has 365.2422 days. The early Hindu calendar that will be considered here, the original *Sûryasiddhânta*, uses a sidereal year of 365.25875 days (Billard, 1921). Each solar month starts when the Sun's mean longitude reaches a multiple of 30°, i.e. when the mean Sun enters a new zodiacal sign. This aligns the solar calendar with the Sun and has twelve solar months, each with the same length of 365.2587/12 = 30.43823 days. The *Sûryasiddhânta* lunar calendar is based on a mean synodic month of 29.530589 days. The fact that the synodic month is shorter than the solar month means that there will sometimes be two lunar months starting in the same solar month. The first of these two months is then an intercalary month. In this way there will be a lunar month intercalation when needed and the synchronisation between the solar and lunar calendars will be automatic. Later, true solar and lunar months were introduced, that made the intercalation system mathematically more complicated but it still used the same principle for intercalation.

3 SOUTHEAST ASIA

In Southeast Asia there are two calendars that are of special interest as regards intercalation: the Thai and the Burmese Calendars. They have great similarities but have fundamentally different intercalation schemes. Ôhashi (2006; 2009) and Komonjinda et al. (2017) give a good background on intercalation problems.

3.1 Thailand

Like the *Sûryasiddhânta*, the Thai Calendar uses a sidereal solar year of 365.25875 days, expressed as 292207 days in 800 years (Faraut, 1910; Eade, 1995; 2000). The starting date or epoch is 21 March 638 CE (Christian Era), the Chulasakarat Era. On this date, the (mean) vernal equinox, there was also an annular solar eclipse visible from India. A normal solar year has 365 days. In order to synchronise the solar calendar with the sidereal Sun, a leap day is inserted to give a leap year of 366 days, the insertion being determined by the value of a quantity called the *kammacubala*. The *kammacubala* is computed by integer division:

(year · 292207 + 373)/800 = integer part: remainder

The number 373 is an epochal constant. The integer part, increased by one, is the *ahargana* or the number of elapsed solar days since the epoch. The *kammacubala* is 800 minus the remainder. It expresses the time in 1/800 of a solar day that remains of the new year day from the time when the mean solar longitude is zero. When the *kammacubala* is less than or equal to 207, there will be a leap year. The significance of the number 207 is apparent when we see that 207/800 = 0.25875, the excess of a sidereal solar year over 365 days.

Example: Compute the *ahargana* and the new year *kammacubala* of the Chulasakarat year 1238.

(1238 · 292207 + 373)/800 = 452190: 639

The *ahargana* is 452190 + 1 = 452191 and the *kammacubala* = 800 – 639 = 161. The year is a solar leap year.

Both the Thai and the Burmese Calendars use the Indian and Babylonian concept of *tithi*, the length of a lunar day or one thirtieth of a synodic lunar month. The value used is that a *tithi* equates to 692/703 solar days. This gives the length of the synodic month as 692/703 · 30 = 29.53058321 days, a value very close to the modern value of 29.5305880 days. A continuous fraction development of the ratio 29.5305880/30 gives the successive approximations 62/63, 63/64, 692/703, 8367/8500 ... The chosen ratio 692/703 is the one that gives a satisfactory accuracy and that is reasonably simple to handle, actually the error using this approximation is only one day in about 17000 years.

The normal lunar year consists of 12 months with alternating lengths of 29 and 30 days, in total 354 days. An extra month of 30 days is intercalated in order to synchronise the lunar and solar calendar giving a lunar year of 384 days. The insertion of the intercalary month is governed by the requirement that the solar year should start in the month of Caitra (the first month of the lunar calendar), or possibly slightly later. If the next lunar year threatens to start later than 6 Vaisakha (the second lunar month) there will be an intercalary month. This will push back the start of the solar year and automatically aligns the solar and lunar calendars. We can easily compute the average frequency of this intercalation. Nineteen sidereal solar years have 19 · 365.25875 = 6939.91625 days. This is 6939.91625/29.53058321 = 235.0077613 synodic months. 19 years each with 12 synodic months contain 19·12 = 228 months. Thus, we need 235.0077613 – 228 = 7.0077613 intercalary lunar months in a 19-year period in order to keep the solar and lunar calendars aligned. This is very close to a Metonic intercalation, though the intercalation pattern will not be fixed but will recede slowly within the 19-year cycle.

In order to synchronise the lunar calendar with the (mean) Moon there will also be intercalary days. To manage this intercalation, the Thai Calendar uses a quantity called the *avoman*. The *avoman* measures the excess of *tithis* relative to elapsed solar days. As a *tithi* equals 692/703 solar days, a solar day is equal to 703/692 *tithis* or 1 + 11/692 *tithis*, i.e. the excess *tithi* is 11 in units of 1/692 of a solar day. The cumulative excess is computed by

(*ahargana* · 11 +650)/692 = periods of 692 : *avoman*

Another way of writing this is (*ahargana*·11 + 650) MOD 692 = *avoman*. Note that if the result is 0 it should be replaced by 692. Again the number 650 is an epochal constant.

Example: Compute the new year *avoman* for Chulasakarat 1238. (452191 · 11 + 650)/692 = 7188 : 655. The *avoman* is 655.

The *avoman* increases by 11 each day. During a normal solar year, the *avoman* increases by (365 · 11) MOD 692 = 555, during a solar leap year by 555 + 11 = 566. The distance of these numbers from 692 is 137 and 126 respectively. An increase of 555 is, in modular language, the same as a decrease of 137. If the new year *avoman* of a normal year is equal to 137 or less it means that the lunar calendar needs an intercalary day; for a leap year there will be an intercalary day if the new year *avoman* is equal to 126 or less. In the Thai Calendar scheme, the intercalary day is not allowed to be inserted in a year with an intercalary month. If this would happen by the rule above, the intercalary day has to be relocated to an adjacent year. A lunar year with an intercalary day will then have 355 days. The canonical rules for this relocation are quite complicated and there is evidence that the day intercalation sometimes was made locally according to other rules but in the long run with the correct number of intercalary days. It is easy to calculate the frequency of the day intercalation, irrespective of the precise insertion rules. The probability of having a leap year is 207/800, the probability of a normal year (800–207)/800 = 593/800. The probability of an intercalary day is 126/692 and 137/692 respectively. Thus, the joint probability is

126/692 · 207/800 + 137/692 · 593/800 = 0.1938638

or on average 19·0.1938638 = 3.6834122 days in 19 years. We have the relation

19 sidereal solar years = 19·365.25875 = 6939.91625 days

19 normal lunar years with 354 days + 7.077613 intercalary months with 30 days give

19 · 354 + 7.077613 · 30 = 6936.232839 days

Add to this the 3.6834122 days from the day intercalation and we get 6936.232839 + 3.6834122 = 6939.91625, exactly matching the solar year days. Thus, the Thai intercalation scheme achieves a perfect synchronisation of the solar and lunar calendars and the Moon.

Since the system was implemented locally over a wide area for centuries, as is evidenced by monastic inscriptions, it may be concluded that the numbers that caused the system to function from year to year were easily memorised and reliably passed from generation to generation.

3.2 Burma (Myanmar)

The Burmese Calendar (see Htoon-Chan, 1918; Irwin, 1909) uses the same epoch, 21 March 638 CE, as the Thai Calendar. For reasons that will be evident, I will deal with the original *Makaranta* version of the Burmese Calendar, before the changes that were made to it in the nineteenth century and later, when the intercalation pattern and some of the calculation schemes were modified by the *Thandeikta* scheme.

The Burmese Calendar also uses the approximation 692/703 for a *tithi*. However, it uses the Metonic 19-year cycle with a fixed pattern of seven years with an intercalary month, the years 2, 5, 7, 10, 13, 15, and 18 in the cycle. The intercalary month is placed after the fourth lunar month Waso or, in the Arakanese Calendar, after the first lunar month Tagu. We have 235 synodic months of 692/703 · 30 days = 6939.687055 days corresponding to a solar year of 6939.687055 / 19 = 365.2466871 days. This is a *tropical* solar year, identical with Hipparchus' tropical year, 365 + 1/4 – 1/300 days.

Nineteen years with seven intercalary lunar months contain 19 · 354 + 7 · 30 = 6936 calendar days. We see that there is a need for 6939.687055 – 6936 = 3.687055 intercalary days in order to synchronise the lunar calendar with the Moon.

The Burmese Calendar uses the *avoman* in order to determine when to insert these intercalary days, but in a different way to the Thai Calendar. An intercalary day can only be inserted in years that also have an intercalary month, contrary to the Thai scheme. There will now be three kinds of years: normal years with 354 days; years with an intercalary month having 384 days; and years with both an intercalary month and an intercalary day with 385 days. The *avoman* is calculated with no reference at all to the solar calendar or the Sun. The *avoman* used as an intercalation indicator is that of the intercalated second Waso Full Moon date (2WFM). The scheme to calculate the *avoman* is:

1) Take the year from the epoch, multiply it by 12 to get the months, add 4 to the product in order to arrive at the end of the first Waso, the number of months will be $m_0 = year \cdot 12 + 4$.
2) Now add the number of intercalary months. There are seven of them in each 19-year period. Then the number of intercalary months is $m_1 = m_0 \cdot 7/(19 \cdot 12) = 7 \cdot m_0 / 228$.
3) The total number of elapsed lunar months is $m = m_0 + m_1$.
4) Convert this to *tithis* by multiplying by 30: $t_0 = m \cdot 30$.
5) Add the number of elapsed *tithis*, 14, until the second Waso Full Moon and get $t = t_0 + 14$.
6) The *tithi* excess or the *avoman* then is $(t \cdot 11 + 650)$ MOD 703. As before, 650 is an epochal constant.

Example: Compute the 2WFM *avoman* for the year 1242 that was a year with an intercalary month, number 7 in the 19-year cycle:

(1242 · 12 + 4) = 14908
14908 · 7/228 = 457
Total elapsed months 14908 + 457 = 15365
Elapsed *tithis* 15365 · 30 + 14 = 460964
(460964 · 11 + 650) MOD 703 = 515, the 2WFM *avoman*.

Once the 2WFM *avoman* has been calculated for one of the intercalary years in the Metonic sequence it is very easy to compute the 2WFM *avoman* for any subsequent year in the sequence by adding one of two numbers. The interval between two years in the intercalary sequence can either be two or three years. In a two-year interval there are two normal years with twelve months plus one intercalary month. Thus, 2 · 12 + 1 = 25 months = 25·30 *tithis* = 750 *tithis*. The excess (750 · 11) MOD 703 = 517 is the *avoman* change. As we consider a difference, the epochal constant will cancel.

In a three-year interval we have 3 · 12 + 1 = 37 months = 37 · 30 *tithis* = 1110 *tithis*, and the excess (1110 · 11) MOD 703 = 259 is the *avoman* change.

The rule for inserting an intercalary day is this: If the 2WFM *avoman* of a year is larger than the previous 2WFM *avoman,* there will be an intercalary day. If you take the case of a two-year interval it is easy to see that this statement is equivalent to saying that the previous 2WFM *avoman* lies in the interval [1, 186]. If you add 517 to any number in this interval the result will be less than 703 and thus larger, if it is outside of this interval, the sum will be larger than 703 and by the modulus condition be reduced by 703 and thus be smaller than the original number. In the same way for a three-year interval, if the previous 2WFM *avoman* lies in the interval [1, 444], addition of 259 will result in a larger *avoman*.

We can now calculate the mean number of intercalary days in a 19-year cycle. There are always two two-year intervals in a Metonic intercalation cycle and five three-year intervals. For a two-year interval the probability of an intercalary day will be 186/703. The corresponding probability for a three-year interval is 444/703. The total mean number of intercalary days in the cycle is then $2 \cdot 186/703 + 5 \cdot 444/703 = 2592/703$ solar days.

235 synodic months correspond to $235 \cdot 30$ *tithis* or $235 \cdot 30 \cdot 692/703$ solar days. 235 calendar months have $19 \cdot 354 + 7 \cdot 30 = 6936$ days. The difference is $235 \cdot 30 \cdot 692/703 - 6936 = 2592/703$ days.

Thus, the Burmese day intercalation scheme precisely compensates for this difference and synchronises the lunar calendar with the Moon and also the lunar calendar with the tropical year. To this extent there is no need for a solar calendar.

An interesting question is how the Burmese chose the intercalation pattern in the Metonic cycle. A possible answer could look like this:

The number of intercalary months is given by $m_1 = 7 \cdot m_0 / 228$ where we use integer division and m_0 is the number of elapsed normal months. Each time this expression increases by one unit, we will have a new intercalary month. Now, suppose that we place ourselves at the end of Waso, month four. We can now calculate for what years in a 19-year sequence there has been a new intercalary month at this moment. Using $m_0 = \textit{cycle year} \cdot 12 + 4$ in the expression for m_0, we find the years 3, 6, 8, 11, 14, 16, 19, which means that the previous years, 2, 5, 7, 10, 13, 15, 18 must have had an intercalary month. This is exactly the intercalation pattern used in the Burmese Calendar. The Arakanese Calendar inserts the intercalary month after Tagu, the first month. Using $m_0 = \textit{cycle year} \cdot 12 + 1$, we generate the intercalation pattern 2, 5, 8, 10, 13, 16, 18, which is actually a variant found (Chatterjee, 1996; Eade, 1995).

However, the problem is that the Burmese Calendar uses the Hindu sidereal solar year for which its lunar calendar in not well suited. The Burmese lunar Calendar will not, like the Thai Calendar, be locked to the solar calendar, with the result that the sidereal solar new year will slowly drift forward in the lunar calendar. This strongly indicates that the Burmese lunar Calendar was introduced separately from the Hindu sidereal solar calendar and probably at an earlier time. Around the year 1100 in the Burmese era (BE = CE 1738) this drift had become an acute problem and the Burmese calendarists started to change the month intercalation pattern and also tried other criteria for intercalating days, completely destroying the original consistency of the lunar calendar. Today, there is no canonical method of setting up future Burmese Calendar dates and the intercalation is determined from time to time by a Committee of Calendarists.

4 THE *ROMAKASIDDHÂNTA*

I will finish by looking at the scheme of the *Romakasiddhânta*, one of the calendar schemes described in the *Pañcasiddântikâ* by *Varahâmihira* and which has many features that are identical to the Burmese Calendar (Neugebauer and Pingree, 1970; Sastry, 1993; van der Waerden, 1988). The epoch of the *Romakasiddhânta* is 21 March 505 CE or 427 in the Šaka Era, a date when there was a conjunction of the Sun and the Moon on the vernal equinox. The *Romakasiddhânta* uses the Metonic cycle for the month intercalation and Hipparchus' tropical solar year exactly as in the original Burmese Calendar. It also uses the approximation 692/703 for the *tithi*. The procedure for determining the *ahargana* uses methods that are identical to the Burmese methods for computing the *avoman*.

In order to compute the *ahargana* you need the year, month and day. Take the year, subtract 427 and multiply the result by 12 to get the elapsed normal lunar months. Add the elapsed months M of the year to get the total number of elapsed normal months: $m_0 = (\textit{year} - 427)\ 12 + M.$ Compute the number of intercalary months: $m_1 = 7 \cdot m_0/228$. The total number of elapsed lunar months then is $m = m_0 + m_1$. Convert to *tithis* and add the elapsed number of *tithis* t_0 of the month. The total number of elapsed *tithis* then is $t = 30 \cdot m + t_0$. To get the number of elapsed solar days we have to subtract the excess *tithis* $T = (t \cdot 11 + 514)/703$, where 514 is an epochal constant. Then the elapsed calendar days, the *ahargana*, a, is $a = t - T$.

Note that, as in the Burmese Calendar, there is no reference at all to any solar calendar in the calculations. The *ahargana* is used for calculating the longitudes of the Sun, the Moon and the planets.

5 CONCLUDING REMARKS

Superficially, the Thai and Burmese Calendars are similar. However, when going into the details of their intercalation schemes, it is found that the methods are quite different. The Thai Calendar has a precise synchronisation between the sidereal Sun, the solar calendar, the lunar calendar and the (mean) Moon. The Burmese Calendar accurately synchronises the tropical solar year, the lunar calendar and the Moon, but fails to accommodate the sidereal year. The Burmese Calendar has several features in common with the *Romakasiddhânta* scheme, which makes it quite probable that the original Burmese Calendar was influenced by the *Romakasiddhânta* and later imported the Hindu sidereal solar year.

Note that the treatment of the Thai and Burmese Calendars by Ôhashi (2006; 2009) does not take into account the fundamental differences between these two calendars, and it also lacks several of the important details of the intercalations. Consequently, he draws conclusions that differ from those presented here.

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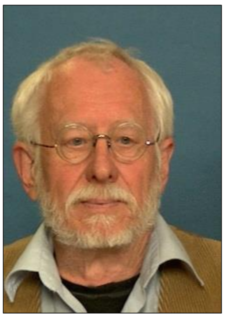

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