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A COMMMENTARY ON THE VOLVELLES IN PETRUS APIANUS' *ASTRONOMICUM CÆSAREUM*

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Abstract: This commentary investigates the theoretical background of the volvelles in *Astronomicum Cæsareum* aided by a modern computer for calculations and simulations, and compares the result with the actual volvelles.

Keywords: Apianus, *Astronomicum Caesareum*, volvelle.

1 INTRODUCTION

Petrus Apianus (Figure 1)[1] was born Peter Bienewitz on 16 April 1495 in Leisnig in Saxony, Germany, son of a Martin Bienewitz, a shoemaker. He went to the Latin school in Rochlich and then from 1516 to 1519 studied at the University of Leipzig. During this time, he Latinized his name to Apianus, *biene* in German means bee which is *apis* in Latin. In 1519 he moved to Vienna to study at the university there. In 1521 he moved to Regensburg and then to Landshut. There he wrote *Cosmographicus Liber*, a work on astronomy and navigation, a famous book that was translated into many languages and appeared in numerous editions. In 1527 Petrus Apianus was called to the University of Ingolstadt as mathematician and printer. He started a print shop that was active between 1533 and 1540 that became well-known for its high-quality editions of works on geography and cartography. He became a favourite of the Holy Roman Emperor Charles V and was granted a printing monopoly in 1532 and 1534. In 1535 the Emperor granted him the right to display a coat of arms. In 1540, after having worked for almost a decade with its production, Petrus Apianus (1540a) published his *magnum opus*, *Astronomicum Cæsareum* (see Figure 2) dedicated to Charles V and his brother Ferdinand I of Spain. Many of the worked examples in the book deal with astronomical events at the dates of their births. Charles V paid for the printing of the book and promised Apianus the royal sum of 3,000 golden guilders, appointed him a court mathematician and knighted him. Petrus Apianus remained in Ingoldstadt until his death on 21 April 1552. He was married to Katharina Mosner, the daughter of a local councilman, and had 14 children with her; one of them, Philip, succeeded him as a mathematician.

Figure 1: Petrus Apianus.

Figure 2: The title page of *Astronomicum Cæsareum*.

The *Astronomicum Cæsareum* is a magnificent book with more than thirty volvelles or equatoria, a kind of paper computer, that, using the detailed instructions and examples in the text, allowed readers to calculate astronomical, chronological and also astrological phenomena. These computing devices were quite popular during the fifteenth and sixteenth centuries. Poulle (1980)


Astronomicum Cäsareum

Eine Grüntliche außlegung des
Büchs Astronomici Cäsarei/ Und seiner instru-
ment / darinne deß gantzen Hymmels
lauff on alle rechnung/ vnd kopf
brechen: zü ewigen zeytten: mit sampt
den finsternussen gefunden wirdt:
in Deutscher sprach auffs
kürtzest begriffen.

Figure 3: The title page of the German manual.

has written a comprehensive study of different kinds of equatoria. Doing by hand the calculations required, for instance, to find the true longitude of a planet, was a very difficult and time-consuming task. For a less specialised audience that did not need the high precision of a numerical calculation, volvelles provided a comfortable solution. Today there are about 120 copies extant of the *Astronomicum Cæsareum* (henceforth AC). Some of its volvelles have up to nine different parts with moving disks, each one printed with intricate and beautifully hand-coloured patterns. The second part of the book also contains a description of comets, noting that their tails pointed away from the Sun, and descriptions of different astronomical instruments. Apianus (1540b) also published a small manual of AC in German for the ordinary public (see Figure 3). A facsimile edition of AC was published in 1967 by Edition Leipzig with comments by Gingerich (1967).

The volvelles are based on the Ptolemaic geocentric model of the Universe in the version given in the Alfonsine Tables (e.g. see Alfonsine Tables; Poulle, 1984), although with some small amendments mainly to adjust the parameters to the meridian of Ingolstadt. Apianus implemented this astronomical model in a sequence of ingenuously conceived astronomical instruments. As will be shown below, the implementation is in general very accurate and you can easily imagine the immense amount of work used in computing the background data and then transferring it to the layout of the printed book. It is a bit ironic that just three years after the publishing of this magnificent work, Nicolaus Copernicus (1543) published *De Revolutionibus Orbium Cœlestium* on the heliocentric model that would replace the Ptolemaic model of the Solar System.

Each volvelle consists of a base that I will call the *mater,* drawn in a style to remind you of an astrolabe. On top of the mater additional disks can be attached, sometimes moving around different axes. In general, there is also one or more silk threads attached to the volvelle for setting or reading data.

My notation for angles is a,b;c,d, ... or b;c,d, ... where 'a' is the zodiacal sign (0–11), 'b' the angle in degrees within the sign, where 'c', 'd', ... represent arc minutes, seconds and subsequent fractions of 60. In some cases, I have used the zodiacal symbols for the signs in order to be compatible with AC. The correlation between zodiacal symbol and their numerical representation is given in Table 1.

A quantity like the mean longitude or the argument (angular distance from the apogee in the epicycle) of an astronomical object is basically calculated by the expression:

mean value = mean daily motion · days + radix,

where the radix is the value at the epoch; in AC this is taken as the birth of Christ or noon 1 January 1 CE and days are the elapsed days, possibly including fraction of days, from the epoch. The day is assumed to start at noon.

I have used several digital copies of AC on the internet, each with its own merits. The pictures in the copy from the New York Public Library (https://digitalcollections.nypl.org/) are complete, scanned with very high resolution and free of distortion. They are also in the public domain. I have used the page numbering from the copy in the Institut für Astronomie, Universität in Vienna (http://www.univie.ac.at/hwastro/). Other good copies can be found at

http://bvpb.mcu.es/es/catalogo_imagenes/grupo.cmd?posicion=1&path=11002499&forma=&presentacion=pagina
http://dx.doi.org/10.3931/e-rara-8724
http://www.rarebookroom.org/Control/appast/

I also made replicas of several of the volvelles. They were of great help in understanding the function and layout of the volvelles.

2 FUNDAMENTAL QUANTITIES

The Alfonsine model of the Universe assumes a constant precession of the sphere of fixed stars (*Stellarum fixarum*) relative to the equinox with a period of 49,000 years on which is superimposed a trepidation of the eighth sphere (*Octave Sphæræ*) that varies periodically with an amplitude of 9 and a period of 7,000 years. The

Table 1: Zodiacal signs.

Sign	0	1	2	3	4	5	6	7	8	9	10	11
Symbol	♈	♉	♊	♋	♌	♍	♎	♏	♐	♑	♒	♓
Degree	0	30	60	90	120	150	180	210	240	270	300	330

apogees (AUX) of the planets are supposed to partake of this movement.

- Daily motion of *Stellarum fixarum* v_E = 0; 0, 0, 4,20,41,17,12, one round in 49,000 years.
- Daily motion of *Octave sphæræ* v_Θ = 0; 0, 0,30,24,49, period 7,000 years.
- Radix for *Octave Sphæræ* Θ_0 = ♓ 29;12,34 = 359;12,34

The zero point of the fixed stars is calculated by Z = $v_E \cdot t + 9^\circ \cdot \sin(\Theta_0 + v_\Theta \cdot t)$, where t is the elapsed days from the epoch.

The epoch positions of the planetary apogees are:

- Sun and Venus: ♊ 11;25,23 = 71; 25, 23
- Mars ♋ 25;12,13,4 = 115;12,13,4
- Jupiter ♍ 3; 37,0,4 = 153;37,0,4
- Mercury ♎ 10;39,33,4 = 190;39,33,4
- Saturn ♏ 23;23,42,4 = 233;23,42,4

I measured the epicycle parameters of the planets, the Moon, and the Sun directly from pictures of the volvelles, taking the deferent radius as a unit. Within measuring errors, they agree very well with the Ptolemaic values (Pedersen, 1974: 423). For the mean longitudes and arguments, the radices differ somewhat from the Alfonsine Tables (Poulle, 1984: 124f). As AC only has a precision of one arc minute in its radices it is hard to make exact conclusions but using the longitude time difference between Toledo and Ingoldstadt, 1:29 hours, you find that for the longitude radices of the Sun, Saturn, and Mars and the argument radices for Saturn, Mars, Venus, and Mercury, the radices in the Alfonsine Tables, adjusted for this geographical longitude difference, agree to within one arc minute with those of AC. For Jupiter there is a 5′ extra correction in the mean longitude and argument radices and also the Moon's radices have extra corrections of the order of 5–6′.

- Mean daily motion in longitude of the Sun: v_S = 0;59, 8,19,37,19,13,56. Radix 9, 8;17.3
- Mean daily motion in longitude of Saturn: 0; 2, 0,35,17,40,21. Radix ♊ 14: 5
- Mean daily motion in longitude of Jupiter: 0; 4,59,15,27, 7,23,50. Radix ♎ 0:32
- Mean daily motion in longitude of Mars: 0;31,26,38,40, 5, 0. Radix ♉ 11:24

For the inner planets, Venus and Mercury, the mean daily motion in longitude is the same as for the Sun. For the outer planets the mean motion in argument is the difference between the mean solar and mean planetary longitude motions.

The radices for the arguments of the outer planets are:

- Saturn, radix: ♎ 24:12. Jupiter, radix: ♋ 7;45. Mars, radix: ♏ 26;53.
- Mean daily motion in argument of Venus: 0;36,59,27,23,59,31. Radix ♌ 9;20
- Mean daily motion in argument of Mercury: 3; 6,24, 7,42,40,52. Radix ♉ 15;12
- Mean daily motion in longitude of the Moon: v_M = 13;10,35, 1,15,11, 4,35. Radix ♌ 2: 8
- Mean daily motion in argument of the Moon v_A = 13; 3,53,57,30,21, 4,13. Radix ♎ 18; 2
- Mean daily motion of the Moon's node (retrograde): v_D = 0; 3,10,38, 7,14,49,10. Radix ♓ 28; 4
- Synodic month $m = 360 / (v_M - v_S)$ days = 29.530591 days
- Draconic month days $d = 360 / (v_M + v_D)$ days = 27.212223 days
- Tropical year 360 / v_S = 365.2425461 days

Apianus uses the Julian calendar with a mean year length of 365.25 days where normal years have 365 days and leap years 366 days. Years that are divisible by four are leap years.

3 THE VOLVELLES

3.1 The Sky Volvelle (page 20)

This is shown in Figure 4. The mater has a circular scale divided into the twelve zodiacal signs, each sign being subdivided into 30°. On top of this is a circular disk, centred on the mater, with five index tabs for reading off the apogees of the planets and the Sun, Venus and the Sun having the same tab. There is a thread attached to the centre of the top disk. There is also a main index (marked *AUX Communis*) on this disk for setting the zero point of the fixed stars. The constant precession is set with a scale on the mater, hidden below the top disk and graduated from 7,000 BCE to 7,000 CE with a change of 7.3469° per millennium, the zero of the scale being at ecliptic longitude 0°. On the top disk, this precession is then corrected for the trepidation (varying between ±9°) by an elliptical scale using another tab marked X. The positions of the apogee tabs, overlaid on Figure 4, agree well with the longitude values given in the Alfonsine Tables. I also calculated some values for the trepidation, and they also agree well with the points on the volvelle (see the red points in Figure 5).

In order to use the volvelle, align the central thread with the century value of the precession using the scale hidden under the top disk. Then move the main apogee index to the thread sett-

Figure 4: The sky volvelle.

ing. Next, align the thread to the century value of the elliptical trepidation scale and move tab X to this setting. Finally, you read off the positions of the apogee tabs. These positions are used in the volvelles for computing the true longitudes of the planets.

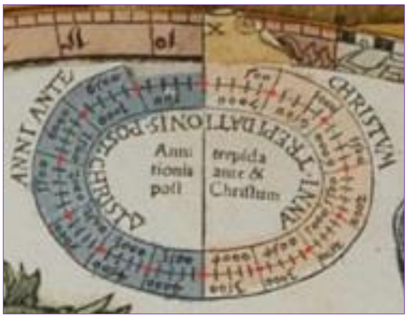


Figure 5: The trepidation ellipse.

3.2 The Equation of Time (page 24)

The mater (Figure 6) has a year scale with 365 days. There is a thread attached to the centre to read off the equation of time in minutes: seconds for a specific date. The equation of time differs from that in the Alfonsine Tables. The minimum is 0:0 on 1 February, first maximum 20:56 on 9 May, second minimum 12:52 on 16 July and second maximum 32:48 on 23 October. Table 2 below shows a comparison with a modern calculation. The location of the maxima and minima fits rather well with a date in the first half of the sixteenth century although it is hard to draw any safe conclusions on the date of origin from the values in AC. The equation of time has a built-in addition of about 15 minutes in order to avoid negative numbers.

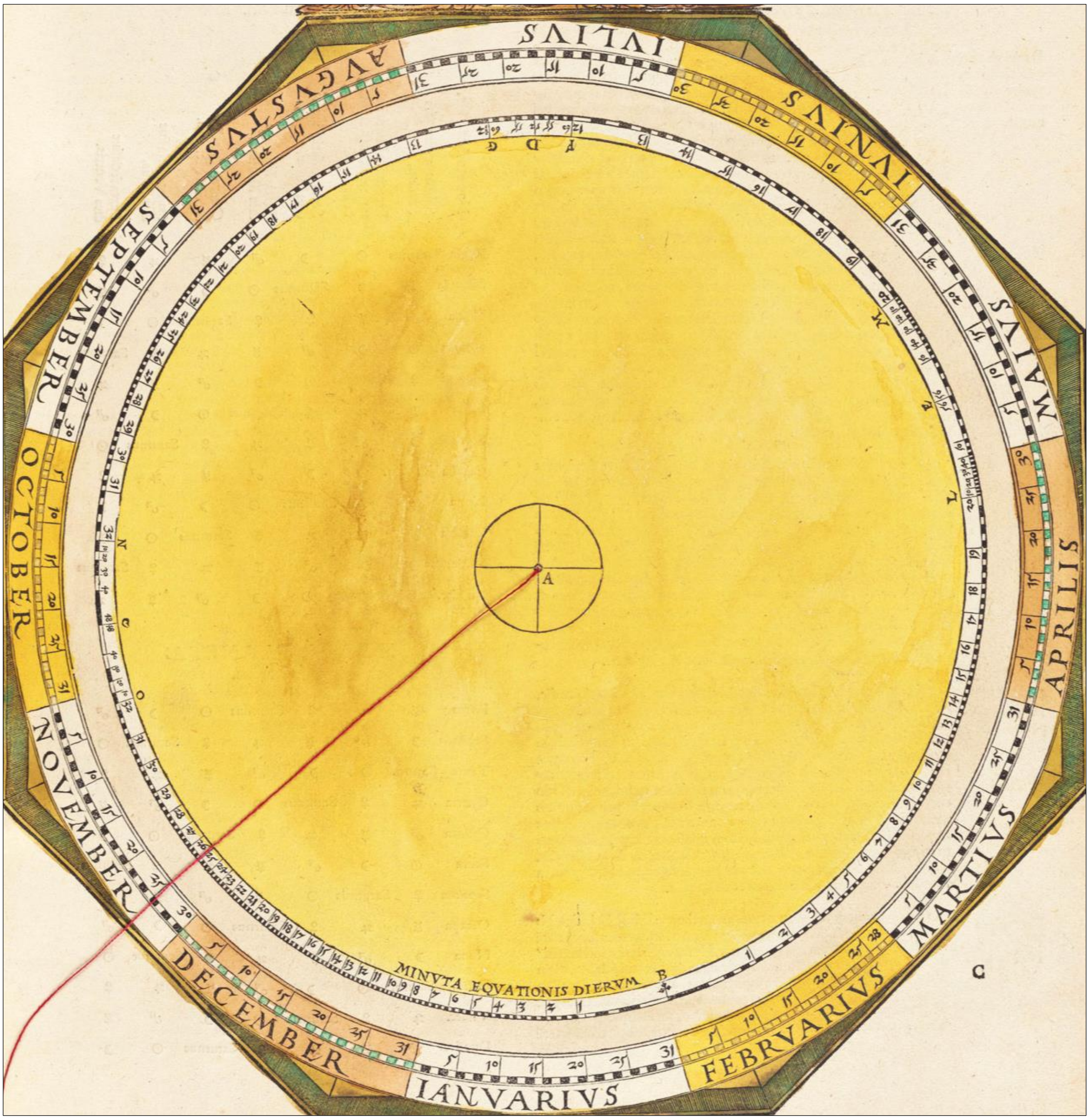


Figure 6: The equation of time volvelle.

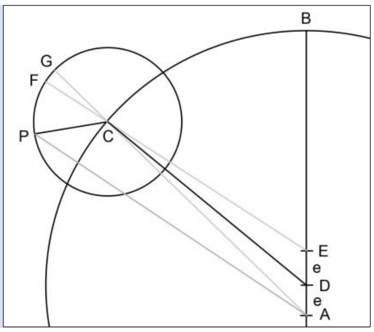
Table 2: Equation of time.

Year	Date	Min	Date	Max	Date	Min	Date	Max
1200	3 Feb	0	9 May	20:44	20 Jul	10:52	24 Oct	31:18
1300	2 Feb	0	8 May	20:23	19 Jul	10:30	24 Oct	31:15
1400	2 Feb	0	7 May	20:00	18 Jul	10:07	23 Oct	31:11
1500	1 Feb	0	6 May	19:40	17 Jul	9:44	23 Oct	31:07
1600	31 Jan	0	5 May	19:29	16 Jul	9:12	23 Oct	31:00
AC	1 Feb	0	5 May	20:56	16 Jul	12:52	23 Oct	32:48

3.3 The Outer Planets

In the Ptolemaic model an outer planet P (Figure 7) moves anti-clockwise on the epicycle circle with centre C. The centre of the epicycle moves anti-clockwise along the deferent circle BC with centre in D. B is the position of the apogee. The position of the mean centrum C is measured by the angle BEC, the mean distance or angle between the centre of the epicycle and the apogee. Point E is the equant introduced by

Figure 7: Ptolemy's model for an outer planet and Venus.



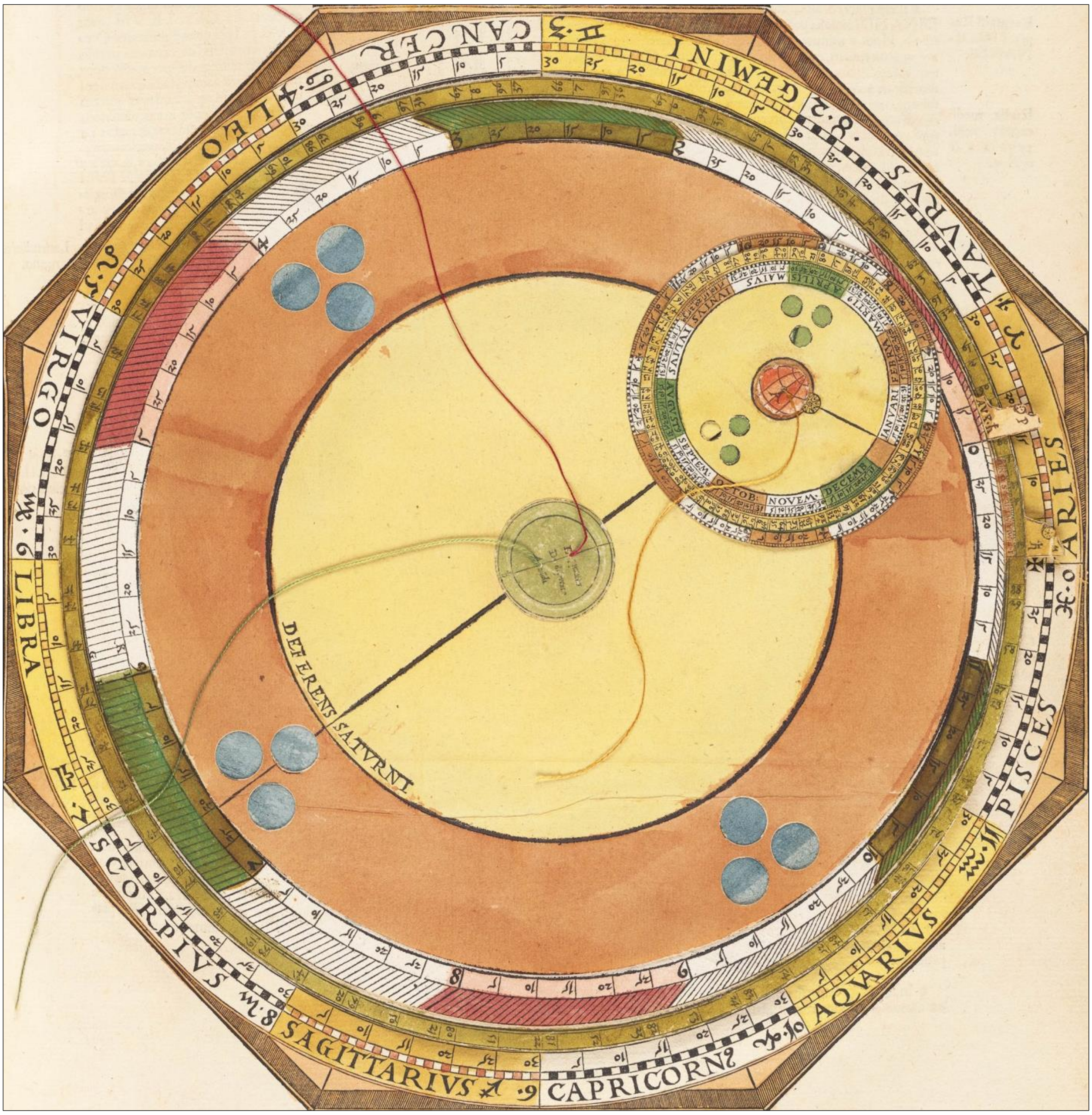


Figure 8: The Saturn volvelle.

Ptolemy in his *Almagest* (Neugebauer, 1969; Pedersen, 1974; Toomer, 1984). The (mean) argument is the angle FCP. The centre of the epicycle is seen from the point A, and the angle GCP is the true argument. Finally, the true position of the planet is the angle BAP. The points A and E are located on the opposite sides of centre D at distance e, the eccentricity.

3.3.1 The Longitude of Saturn (pages 27 and 28)

The volvelle (Figure 8) is preceded by tables for the mean longitudes and arguments for each elapsed century before and after Christ. The unit in the tables is sign: degree: minute. I have checked the tables by recalculation. Table 3 shows the calculated tables for the century mean longitudes and arguments, and larger typographical errors, typically a couple of arc minutes, are marked in red. There are two tabular columns for each quantity, the left one (CE, *Radices post Christum*) for centuries after Christ, the right one (BCE, *Radices ante Christum*) for centuries before Christ. The numbers in the tables are calculated using the 36,525 days in a Julian century and using the formula

$$\text{elapsed centuries} \cdot 36525 \cdot \text{daily motion} + \text{radix},$$

and restricting the result to be in the interval 0° to 360°. The year within the century, the month, and the day are set on the volvelle as will be shown below.

The layout of this volvelle is typical of those for the planets except for Mercury. The mater (Figure 9), representing the zodiac, is divided into signs and degrees. There are three axes,

Table 3: Calculated century mean longitudes and argument tables. The red figures are explained in the text.

	Longitude						Argument					
	CE			BCE			CE			BCE		
0	♊	14	5	♊	14	5	6	24	12	6	24	12
100	♏	7	33	♑	20	37	2	1	28	11	16	56
200	♈	1	1	♌	27	9	9	8	44	4	9	40
300	♌	24	29	♈	3	41	4	16	0	9	2	24
400	♑	17	57	♏	10	13	11	23	16	1	25	8
500	♊	11	25	♊	16	45	7	0	32	6	17	52
600	♏	4	54	♑	23	16	2	7	48	11	10	36
700	♓	28	22	♌	29	48	9	15	4	4	3	20
800	♌	21	50	♈	6	20	4	22	20	8	26	4
900	♑	15	18	♏	12	52	11	29	36	1	18	48
1000	♊	8	46	♊	19	24	7	6	52	6	11	32
1100	♏	2	14	♑	25	56	2	14	8	11	4	16
1200	♓	25	42	♍	2	28	9	21	24	3	27	0
1300	♌	19	10	♉	9	0	4	28	40	8	19	44
1400	♑	12	38	♏	15	32	0	5	56	1	12	28
1500	♊	6	6	♊	22	4	7	13	12	6	5	12
1600	♎	29	34	♑	28	36	2	20	28	10	27	56
1700	♓	23	3	♍	5	7	9	27	44	3	20	40
1800	♌	16	31	♈	11	39	5	5	0	8	13	24
1900	♑	9	59	♏	18	11	0	12	16	1	6	8
2000	♊	3	27	♊	24	43	7	19	32	5	28	52
2100	♎	26	55	♒	1	15	2	26	48	10	21	36
2200	♓	20	23	♍	7	47	10	4	4	3	14	20
2300	♌	13	51	♈	14	19	5	11	20	8	7	4
2400	♑	7	19	♏	20	51	0	18	36	0	29	48
2500	♊	0	47	♊	27	23	7	25	52	5	22	32
2600	♎	24	15	♒	3	55	3	3	8	10	15	16
2700	♓	17	43	♍	10	27	10	10	24	3	8	0
2800	♌	11	12	♈	16	58	5	17	40	8	0	44
2900	♑	4	40	♏	23	30	0	24	56	0	23	28
3000	♉	28	8	♋	0	2	8	2	12	5	16	12
3100	♎	21	36	♒	6	34	3	9	28	10	8	56
3200	♓	15	4	♍	13	6	10	16	44	3	1	40
3300	♌	8	32	♈	19	38	5	24	0	7	24	24
3400	♑	2	0	♏	26	10	1	1	16	0	17	8
3500	♉	25	28	♋	2	42	8	8	32	5	9	52
3600	♎	18	56	♒	9	14	3	15	48	10	2	36
3700	♓	12	24	♍	15	46	10	23	4	2	25	20
3800	♌	5	53	♈	22	17	6	0	20	7	18	4
3900	♐	29	21	♏	28	49	1	7	36	0	10	48
4000	♉	22	49	♋	5	21	8	14	52	5	3	32
4100	♎	16	17	♒	11	53	3	22	8	9	26	16
4200	♓	9	45	♍	18	25	10	29	24	2	19	0
4300	♌	3	13	♈	24	57	6	6	40	7	11	44
4400	♐	26	41	♐	1	29	1	13	56	0	4	28
4500	♉	20	9	♋	8	1	8	21	12	4	27	12
4600	♎	13	37	♒	14	33	3	28	27	9	19	57
4700	♓	7	5	♍	21	5	11	5	43	2	12	41
4800	♌	0	33	♈	27	37	6	12	59	7	5	25
4900	♐	24	2	♐	4	8	1	20	15	11	28	9
5000	♉	17	30	♋	10	40	8	27	31	4	20	53
5100	♎	10	58	♒	17	12	4	4	47	9	13	37
5200	♓	4	26	♍	23	44	11	12	3	2	6	21
5300	♋	27	54	♉	0	16	6	19	19	6	29	5
5400	♐	21	22	♐	6	48	1	26	35	11	21	49
5500	♉	14	50	♋	13	20	9	3	51	4	14	33
5600	♎	8	18	♒	19	52	4	11	7	9	7	17
5700	♓	1	46	♍	26	24	11	18	23	2	0	1
5800	♋	25	14	♉	2	56	6	25	39	6	22	45
5900	♐	18	42	♐	9	28	2	2	55	11	15	29
6000	♉	12	11	♋	15	59	9	10	11	4	8	13
6100	♎	5	39	♒	22	31	4	17	27	9	0	57
6200	♒	29	7	♍	29	3	11	24	43	1	23	41
6300	♋	22	35	♉	5	35	7	1	59	6	16	25

6400	♐	16	3	♐	12	7	2	9	15	11	9	9
6500	♉	9	31	♋	18	39	9	16	31	4	1	53
6600	♎	2	59	♒	25	11	4	23	47	8	24	37
6700	♒	26	27	♎	1	43	0	1	3	1	17	21
6800	♋	19	55	♉	8	15	7	8	19	6	10	5
6900	♐	13	23	♐	14	47	2	15	35	11	2	49
7000	♉	6	51	♋	21	19	9	22	51	3	25	33

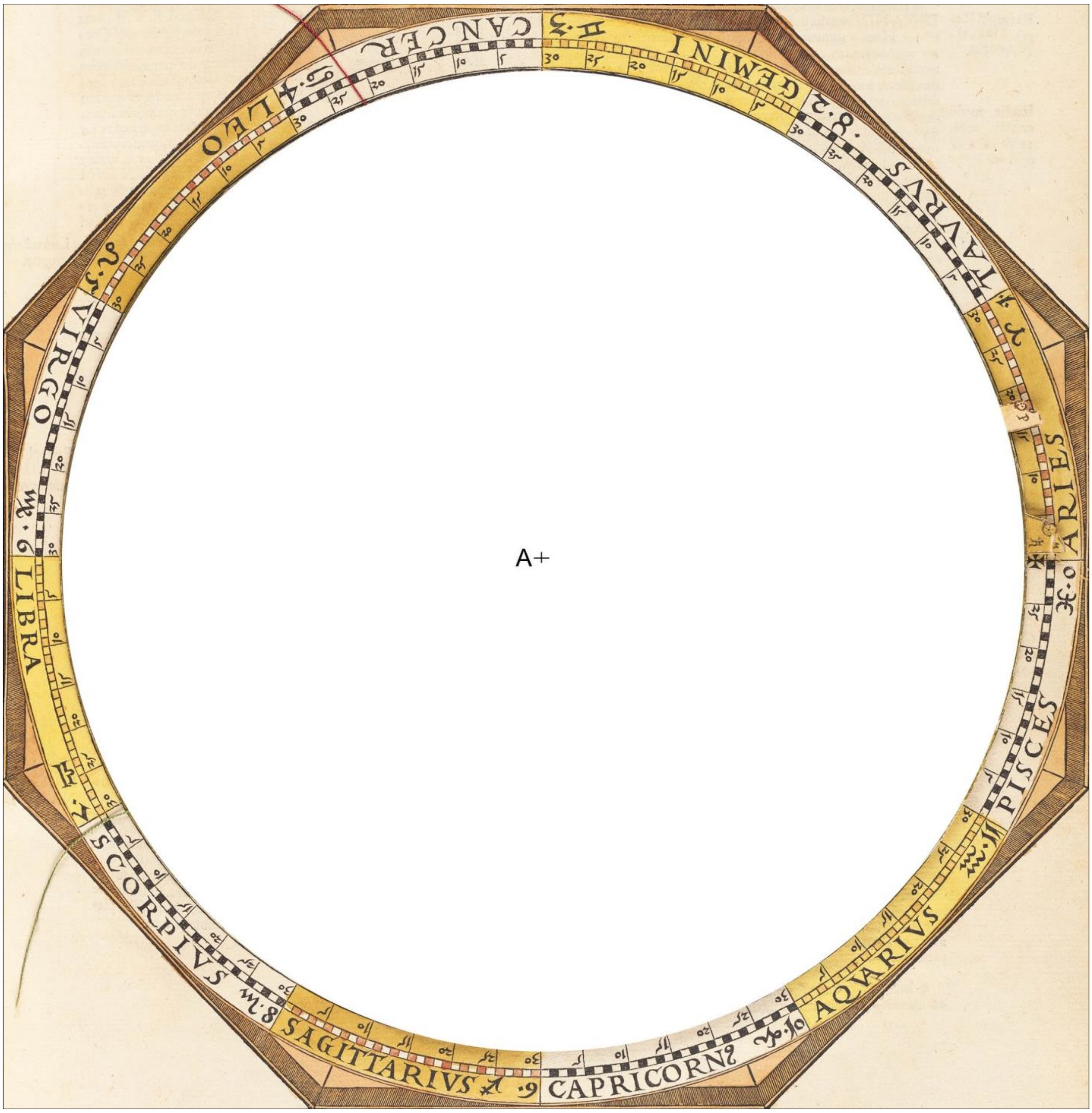


Figure 9: The mater.

A, D, and E. D is located exactly in the middle between A and E. On top of the mater and centred with it on the main axis A, is a disk M, with an index tab marked M and the symbol of the planet, year marks for setting the year and a smaller scale for setting the month and day. The angles for the year marks are calculated by the formula

(year· 365 + integer(year / 4)) · mean motion in longitude.

The angles are reduced to lie in the interval 0° to 360°. The second term inside the bracket corrects for the extra days in leap years. Tables 4a and 4b show the result of a computer calculation for years and months. Figure 10 shows the M disk with a comparison of the calculated year marks with the volvelle marks.

On top of disk M is a circular disk (Figure 11), centred on the main axis A, with an index tab marked AUX and P and with the planet sym-

Table 4a: Calculated longitude year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
59	1.82	64	63.02	69	124.19	75	197.58	23	281.37
30	7.02	35	68.19	40	129.39	46	202.78	82	283.23
89	8.88	94	70.04	99	131.21	17	207.98	53	288.43
1	12.23	6	73.39	11	134.56	76	209.84	24	293.63
60	14.08	65	75.25	70	136.41	47	215.01	83	295.45
31	19.25	36	80.45	41	141.61	18	220.21	54	300.66
90	21.11	95	82.27	100	143.47	77	222.06	25	305.86
2	24.45	7	85.62	12	146.82	48	227.27	84	307.71
61	26.31	66	87.47	71	148.64	19	232.43	55	312.88
32	31.51	37	92.68	42	153.84	78	234.29	26	318.09
91	33.33	96	94.53	13	159.04	49	239.49	85	319.94
3	36.68	8	97.88	72	160.90	20	244.69	56	325.14
62	38.53	67	99.70	43	166.07	79	246.52	27	330.31
33	43.74	38	104.90	14	171.27	50	251.72	86	332.17
92	45.59	97	106.76	73	173.12	21	256.92	57	337.37
4	48.94	9	110.10	44	178.33	80	258.78	28	342.57
63	50.76	68	111.96	15	183.50	51	263.94	87	344.39
34	55.96	39	117.13	74	185.35	22	269.15	58	349.60
93	57.82	98	118.98	45	190.55	81	271.00	29	354.80
5	61.17	10	122.33	16	195.76	52	276.20	88	356.65

Table 4b: Calculated longitude month marks.

Months	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.
Longitude marks	1.04	1.98	3.01	4.02	5.06	6.06	7.10	8.14	9.14	10.18	11.19	12.23

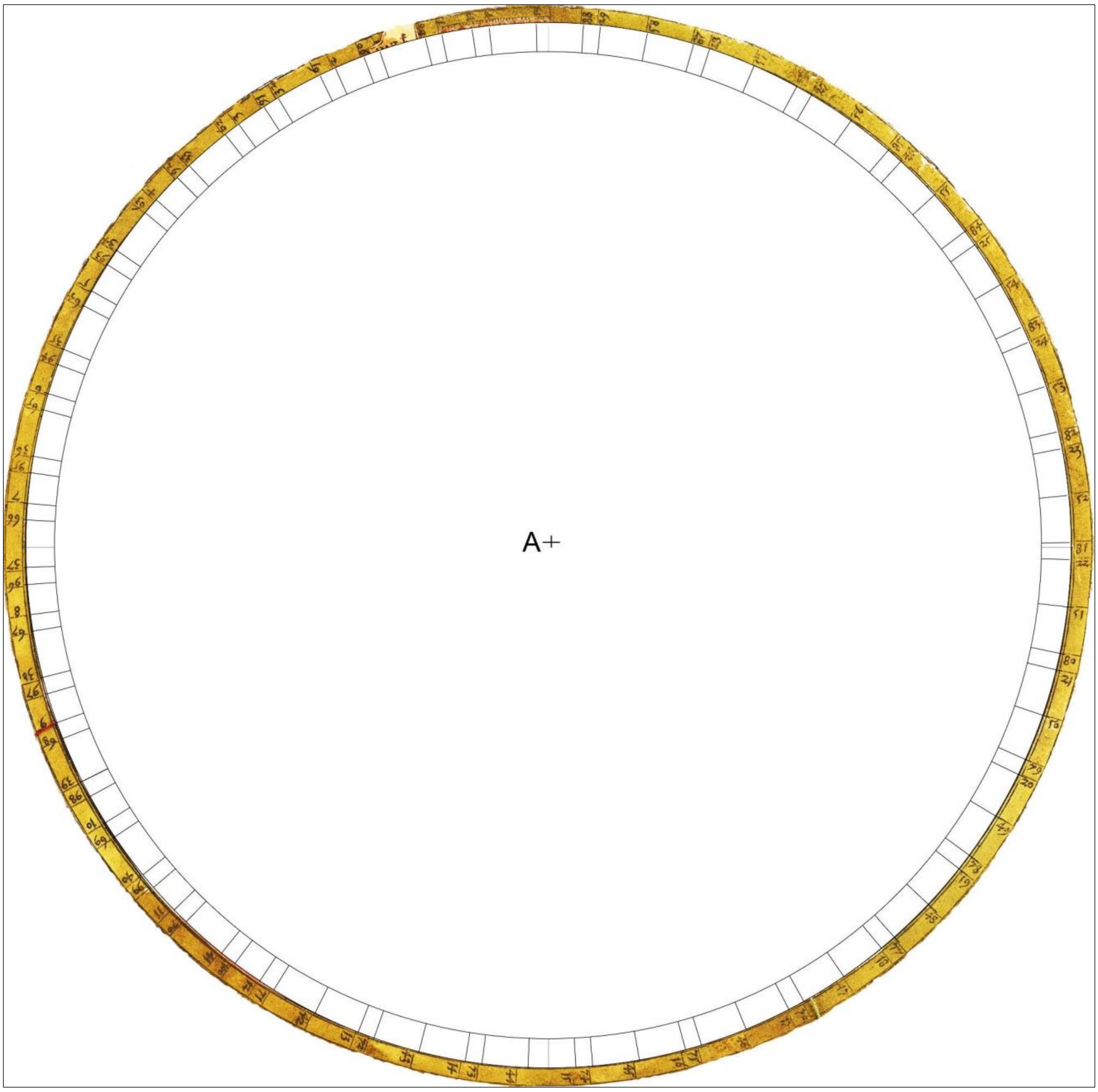


Figure 10: Disk M with comparison of calculated year marks on the disk.

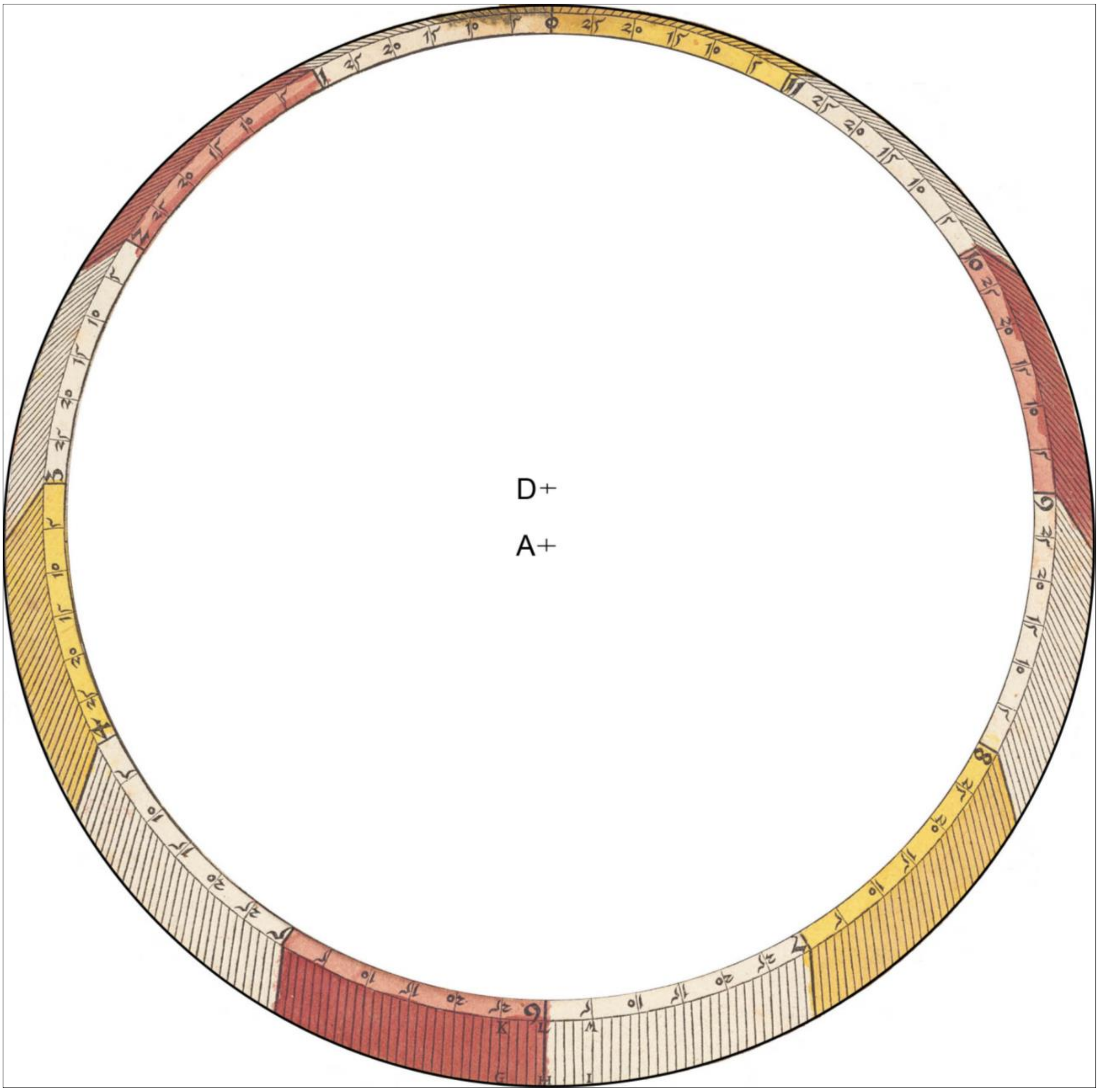


Figure 11: Disk P.

bol for setting the position of the apogee, read from the sky volvelle. It has a circular scale with the axis D as centre. The scale is graduated with the zodiacal signs, each one sign being sub-graduated from 0° to 30°, however, the centre of the graduation is the axis E. From the edge of this scale there are slanting lines for each degree that connect with the periphery of the disk itself divided into 360 equal parts. On top of this disk P is a circular disk (Figure 12) mounted inside the circular scale of disk P, its axis coinciding with the axis D on the P disk. This is the deferent disk, it is marked with a circle DEFERENS and the name of the planet. On the periphery of this circle, at point F, there is another disk attached (Figure 13), the epicycle base disk. It has a 360° graduation anticlockwise around its periphery with a cross marking the origin. On top of, and centred with the epicycle base disk, is the argument disk Y (Figure 14) with an index tab marked Y and a sequence of year, month and day marks for setting the argument for the year and date. There is a 'star' symbol showing the position of the planet on the epicycle; in the case of Mars, Venus and Mercury it is positioned on the periphery of the disk on the Y tab, and in the case of Jupiter and Saturn on a circle with a smaller radius. There is a thread attached to the centre of the Y disk.

The marks on disk Y are calculated using the same formula as for the longitude, but using instead the mean motion in argument and the argument radix. Table 5 shows the result of a calculation, and Figure 15 shows the actual Saturn Y disk with a comparison of the calculated year marks with the volvelle.

Finally, there is a cap (Figure 16) covering

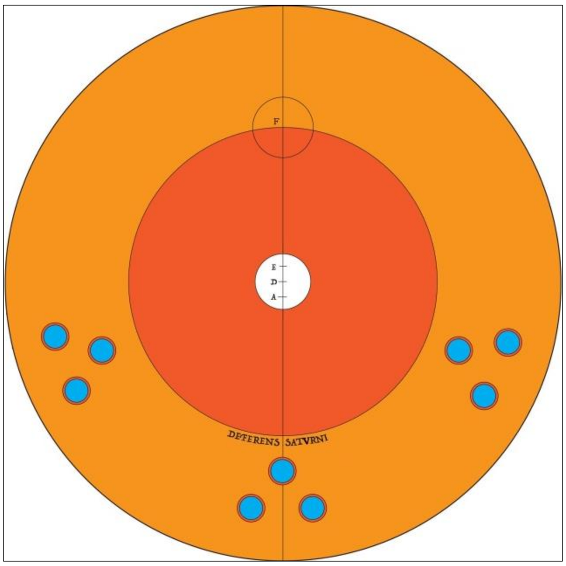


Figure 12: The deferent disk.

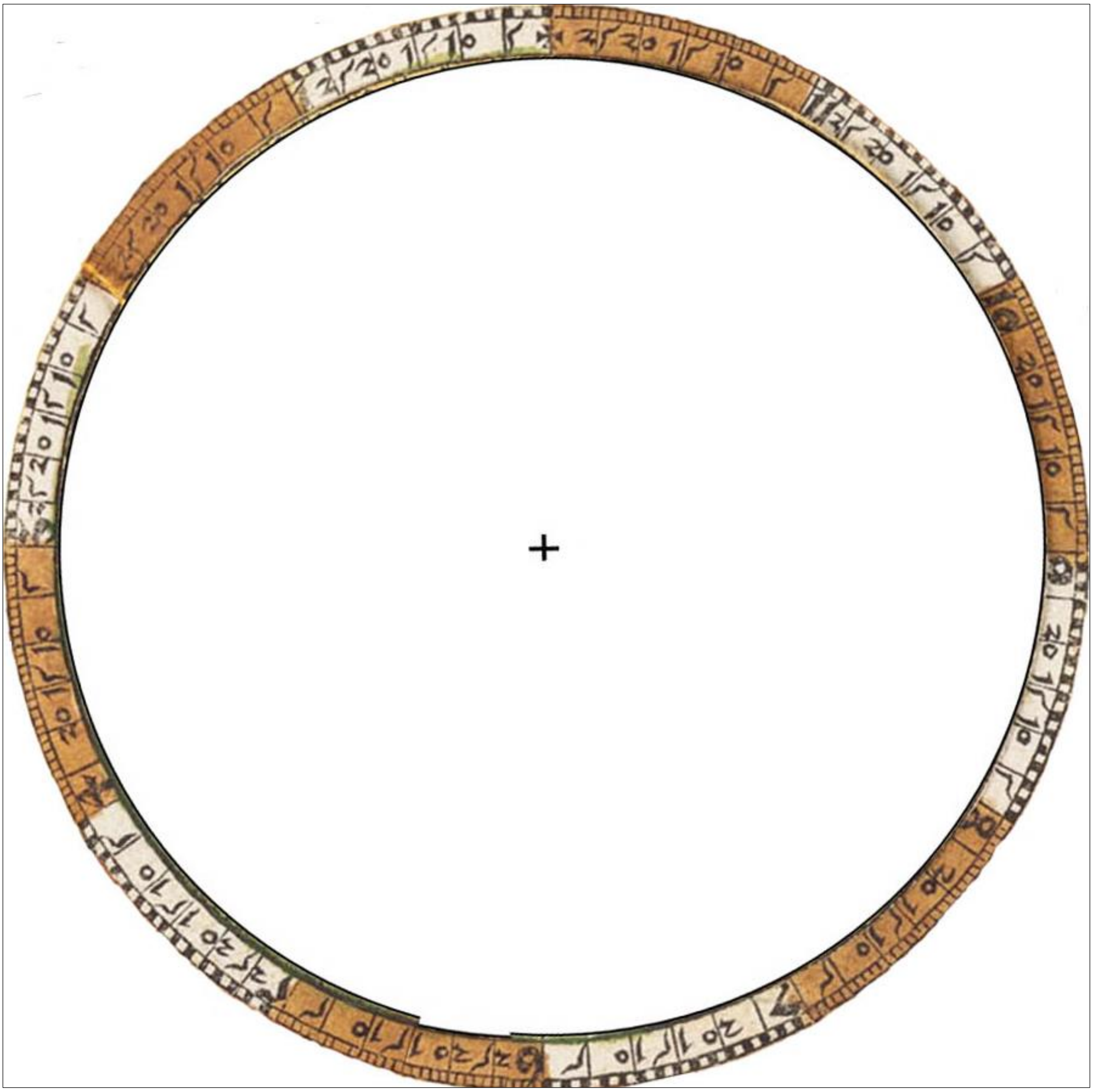


Figure 13: The epicycle base disk.

Table 5. Calculated argument year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
88	4.00	52	84.18	16	164.36	10	237.25	5	298.63
29	5.17	81	89.35	45	169.53	98	241.25	93	302.62
58	10.34	22	90.52	74	174.70	39	242.42	34	303.80
87	15.51	51	95.69	15	175.88	68	248.54	63	308.96
28	17.64	80	101.81	44	182.00	9	249.72	4	311.09
57	22.80	21	102.99	73	187.17	97	253.71	92	315.09
86	27.97	50	108.16	14	188.34	38	254.89	33	316.26
27	29.15	79	113.33	43	193.51	67	260.05	62	321.43
56	35.27	20	115.45	72	199.63	8	262.18	3	322.60
85	40.44	49	120.62	13	200.81	96	266.18	91	326.60
26	41.61	78	125.79	42	205.98	37	267.35	32	328.73
55	46.78	19	126.97	71	211.15	66	272.52	61	333.90
84	52.90	48	133.09	12	213.27	7	273.70	2	335.07
25	54.08	77	138.26	100	217.27	95	277.69	90	339.06
54	59.25	18	139.43	41	218.44	36	279.82	31	340.24
83	64.42	47	144.60	70	223.61	65	284.99	60	346.36
24	66.54	76	150.72	11	224.79	6	286.16	1	347.54
53	71.71	17	151.90	99	228.78	94	290.15	89	351.53
82	76.88	46	157.07	40	230.91	35	291.33	30	352.70
23	78.06	75	162.24	69	236.08	64	297.45	59	357.87

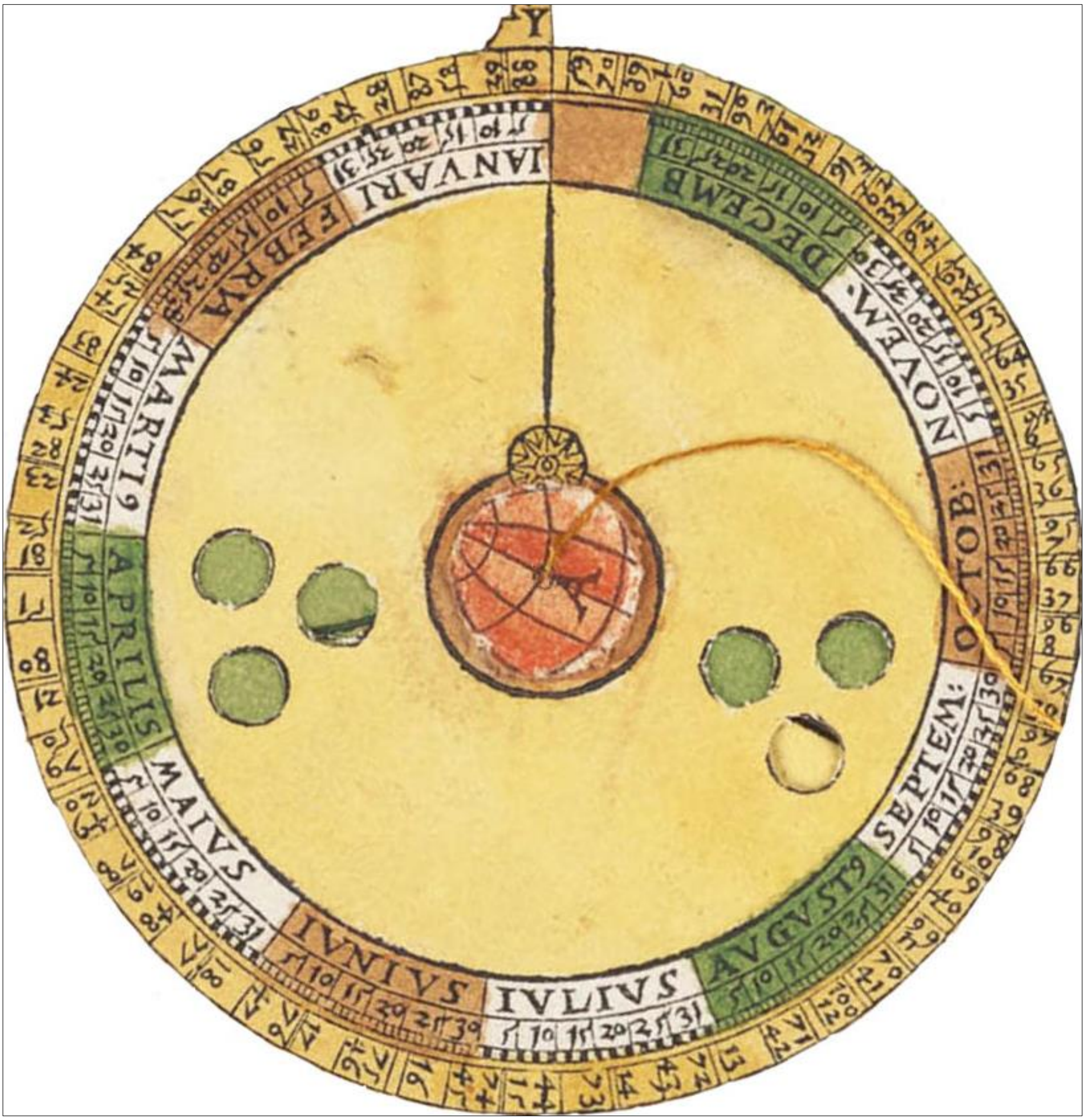


Figure 14: The Y disk.

the centre of the deferent disk. It is marked with letters E – Equans, D – Deferens, and A – Mundi. There are threads attached to the points E and A. The distance between points A and D and D and E is the eccentricity of the planet.

For the working of the volvelle I give a slightly redacted English translation from Apianus' German manual:

> If you want to find the positions of Saturn, Jupiter or Mars, where they are in the zodiac, then look carefully at this example. There is a well-known person [i.e. Charles V], born 1500 years after the birth of Christ on 23 February 15:44 hours after noon. In chapter 3 I have reduced this time to the meridian of Ingoldstadt to 16:23 hours. I have also corrected it for the unequal days (equation of time) in the next volvelle to 16:23:4 hours. This is the correct time on which to find the positions of the stars.
>
> As the birth was on 23 February, you understand that the year 1500 was not fully elapsed and that you therefore have to use the previous year that is elapsed and take 1499 and find the positions for this year. Search in the table of numbers, under the section Tabula Medii Motus Saturni (what I tell you about Saturn is the same for Jupiter and Mars) in the first column to the left, the year 1400, where the header says Radices Post Christum (the radix for the middle movement in longitude after the birth of Christ) there it is written as Capricorn 12:38. Then search by the year 1400 under the section Medii Argument Saturni and under the column Radices Post Christum, and you will find the century radix as 0,5;56.
>
> For this example you don't need more from the tables. Search the radix Capricorn 12;38 in the zodiac or mater scale of the volvelle and set the index tab M to it. There are 99 more elapsed years after 1400, and you must search for that year in the circular scale of disk M, you will find it close to Taurus 23°. Stretch thread A to that year and move tab M under the thread. You will also see a calendar on this disk close to tab M where each month is marked by a letter: January by J, February by F, March by M and so on. Each month is divided into six divisions, each one corresponding to five days but you have to note that the last division corresponds to six days in January, March, May, July, August, October, and December, and five days in April, June, September and November. In February the last division corresponds to three days for an ordinary year and four days for a leap year. Jupiter has the same graduation, but Mars has a finer graduation, each division corresponds to one day. When you have examined the calendar you need to find the day of this birth, 23 February (the 16:23 hours are not necessary because of the coarse graduation of the calendar). Stretch again the thread A to this date and again move tab M there. This disk is now set correctly.
>
> In chapter 4, you found that the longitude of the apogee of Saturn in 1500 was Sagittarius13:10. As this year is not elapsed we decrease the value by 1′. Set index tab P on the P disk to the value Sagittarius13:9 on the zodiac scale of the mater and this disk also is set correctly for the birth.
>
> Now you also have to set the deferent disk and the epicycle disks correctly for this hour of birth. Therefore search on the bottom epicycle disk, on which there is a cross marked, the radix for the mean argument, 0,5;56 and move index tab Y to that position. Search for the extra 99 years on the top disk (you will find it close to sign number 8 on the lower disk) and stretch the thread from the epicycle centre to it and move tab Y there, keep the disks locked and finally stretch the thread to 23 February 16 hours (you can skip the minutes) and move again the top disk with tab Y there and you have the epicycle disks correctly set.
>
> Further, you have to look for where the index tab M touches the peripheral P disk scale, that happens in 5,12;30. This location is called the mean centrum. Go from this point on the periphery of the P disk along the slanting line on the disk until it meets the scale of the disk and stretch thread E to it, that is from the centre of the equant to this point, and move the deferent (on which the epicycle is) with the epicycle centre under this thread, and align the cross of the bottom epicycle disk to be under the thread, directed outwards to the mater and not inwards to the

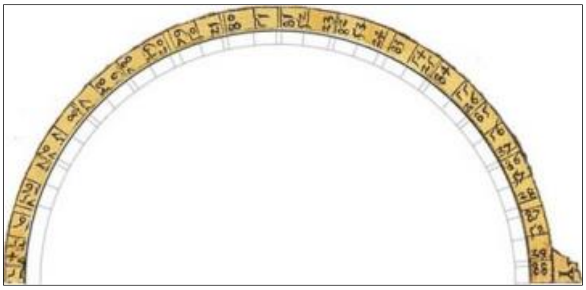


Figure 15: Epicycle disk Y with comparisons of the some of the year marks.

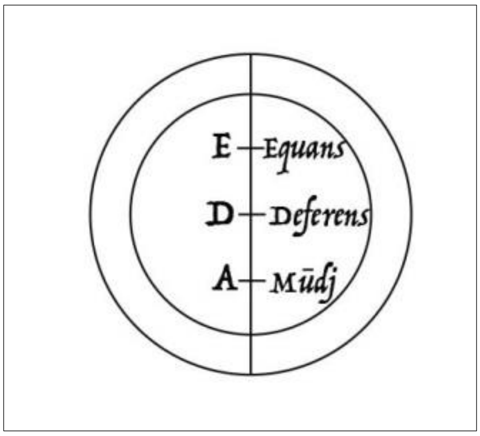


Figure 16: The cap.

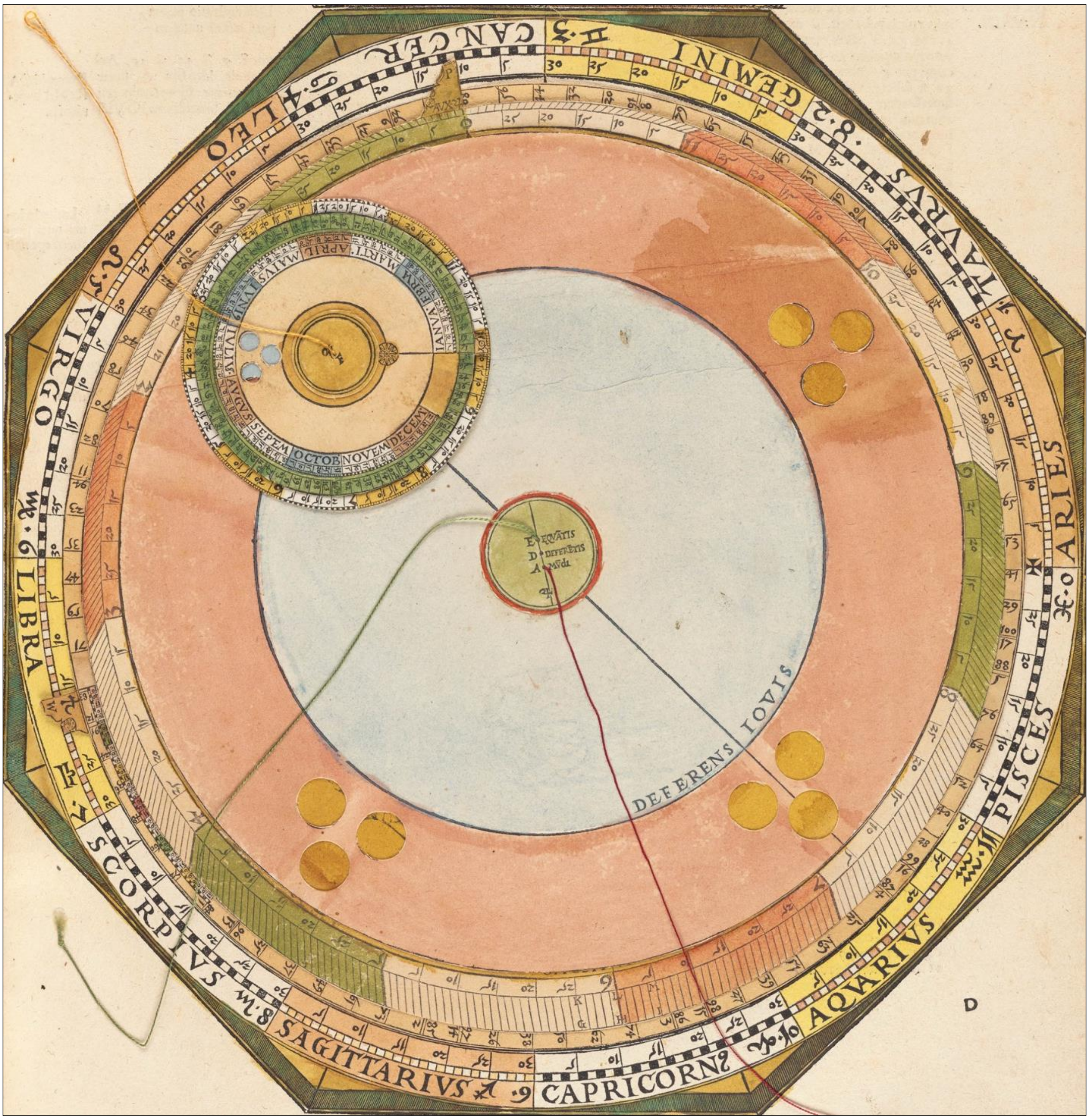


Figure 17: The Jupiter volvelle.

centre of the instrument, then you have set up the instrument correctly. Then, if you align the tread from A in the centre (where Mundi is written) with the centre point of the star, not far from the centre of the epicycle disk, then it shows Taurus 17:40 on the scale of the mater. That is the true position of Saturn that you have requested for this birth. If you then want to find the true centrum of Saturn, by which you can later find the latitude of Saturn in the ecliptic, then stretch thread A from the centre, through the centre of the epicycle disk and it hits the P disk scale in 5,10;20, that is called the true and equalled centrum. You can also find the true and equalled argument of Saturn. When you have stretched thread A through the centre of the epicycle disk, note where the thread crosses the scale of the bottom epicycle disk, the same point is the most distant point of the epicycle from the Earth, or the whole world, that is from centre A. It is called the true apogee by the astronomers. From the same point, the angle to tab Y is the true argument of Saturn, 9,17;52. That number is necessary to have when you later search for the latitude of Saturn in the latitude volvelle.

3.3.2 The Longitude of Jupiter (pages 31 and 32)

This volvelle (Figure 17) is constructed and works exactly as for Saturn. Table 6 shows the calculated century longitudes and arguments. Tables 7 and 8 show calculated year marks and Figures 18 and 19 comparisons with the volvelle marks.

Table 6: Calculated century mean longitudes and arguments for Jupiter.

	Longitude						Argument					
	CE			BCE			CE			BCE		
0	♎	0	32	♎	0	32	3	7	45	3	7	45
100	♓	6	45	♈	24	19	10	2	16	8	13	14
200	♌	12	58	♏	18	6	4	26	47	1	18	43
300	♑	19	11	♊	11	53	11	21	18	6	24	12
400	♊	25	24	♑	5	40	6	15	49	11	29	41
500	♐	1	37	♋	29	27	1	10	20	5	5	10
600	♉	7	50	♒	23	14	8	4	51	10	10	39
700	♎	14	3	♍	17	1	2	29	22	3	16	8
800	♓	20	16	♈	10	48	9	23	53	8	21	37
900	♌	26	29	♏	4	35	4	18	25	1	27	5
1000	♒	2	42	♉	28	22	11	12	56	7	2	34
1100	♋	8	55	♐	22	9	6	7	27	0	8	3
1200	♐	15	8	♋	15	56	1	1	58	5	13	32
1300	♉	21	21	♒	9	43	7	26	29	10	19	1
1400	♎	27	34	♍	3	30	2	21	0	3	24	30
1500	♈	3	47	♓	27	17	9	15	31	8	29	59
1600	♍	10	0	♎	21	4	4	10	2	2	5	28
1700	♒	16	13	♉	14	51	11	4	33	7	10	57
1800	♋	22	26	♐	8	38	5	29	4	0	16	26
1900	♐	28	39	♋	2	25	0	23	35	5	21	55
2000	♊	4	52	♑	26	12	7	18	6	10	27	24
2100	♏	11	6	♌	19	58	2	12	37	4	2	53
2200	♈	17	19	♓	13	45	9	7	8	9	8	22
2300	♍	23	32	♎	7	32	4	1	39	2	13	51
2400	♒	29	45	♉	1	19	10	26	10	7	19	20
2500	♌	5	58	♏	25	6	5	20	41	0	24	49
2600	♑	12	11	♊	18	53	0	15	12	6	0	18
2700	♊	18	24	♑	12	40	7	9	44	11	5	46
2800	♏	24	37	♌	6	27	2	4	15	4	11	15
2900	♉	0	50	♓	0	14	8	28	46	9	16	44
3000	♎	7	3	♍	24	1	3	23	17	2	22	13
3100	♓	13	16	♈	17	48	10	17	48	7	27	42
3200	♌	19	29	♏	11	35	5	12	19	1	3	11
3300	♑	25	42	♊	5	22	0	6	50	6	8	40
3400	♋	1	55	♐	29	9	7	1	21	11	14	9
3500	♐	8	8	♋	22	56	1	25	52	4	19	38
3600	♉	14	21	♒	16	43	8	20	23	9	25	7
3700	♎	20	34	♍	10	30	3	14	54	3	0	36
3800	♓	26	47	♈	4	17	10	9	25	8	6	5
3900	♍	3	0	♎	28	4	5	3	56	1	11	34
4000	♒	9	13	♉	21	51	11	28	27	6	17	3
4100	♋	15	26	♐	15	38	6	22	58	11	22	32
4200	♐	21	39	♋	9	25	1	17	29	4	28	1
4300	♉	27	52	♒	3	12	8	12	0	10	3	30
4400	♏	4	5	♌	26	59	3	6	32	3	8	58
4500	♈	10	18	♓	20	46	10	1	3	8	14	27
4600	♍	16	31	♎	14	33	4	25	34	1	19	56
4700	♒	22	44	♉	8	20	11	20	5	6	25	25
4800	♋	28	57	♐	2	7	6	14	36	0	0	54
4900	♑	5	10	♌	25	54	1	9	7	5	6	23
5000	♊	11	23	♑	19	41	8	3	38	10	11	52
5100	♏	17	36	♌	13	28	2	28	9	3	17	21
5200	♈	23	49	♓	7	15	9	22	40	8	22	50
5300	♎	0	2	♌	1	2	4	17	11	1	28	19
5400	♓	6	15	♈	24	49	11	11	42	7	3	48
5500	♌	12	28	♏	18	36	6	6	13	0	9	17
5600	♑	18	41	♊	12	23	1	0	44	5	14	46
5700	♊	24	54	♑	6	10	7	25	15	10	20	15
5800	♐	1	7	♋	29	57	2	19	46	3	25	44
5900	♉	7	20	♒	23	44	9	14	17	9	1	13
6000	♎	13	33	♍	17	31	4	8	48	2	6	42
6100	♓	19	46	♈	11	18	11	3	20	7	12	10
6200	♌	25	59	♏	5	5	5	27	51	0	17	39
6300	♒	2	13	♉	28	51	0	22	22	5	23	8

6400	♋	8	26	♐	22	38	7	16	53	10	28	37
6500	♐	14	39	♋	16	25	2	11	24	4	4	6
6600	♉	20	52	♒	10	12	9	5	55	9	9	35
6700	♎	27	5	♍	3	59	4	0	26	2	15	4
6800	♈	3	18	♓	27	46	10	24	57	7	20	33
6900	♍	9	31	♎	21	33	5	19	28	0	26	2
7000	♒	15	44	♉	15	20	0	13	59	6	1	31

Table 7: Calculated longitude year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
95	4.34	26	69.38	52	138.8	7	212.47	45	286.28
12	4.35	38	73.72	64	143.2	90	212.55	57	290.62
24	8.69	50	78.07	76	147.5	19	216.82	69	294.97
36	13.04	62	82.41	5	151.8	31	221.17	81	299.32
48	17.38	74	86.76	88	151.9	43	225.51	10	303.58
60	21.73	3	91.02	17	156.1	55	229.86	93	303.66
72	26.08	86	91.11	100	156.2	67	234.20	22	307.93
1	30.34	15	95.37	29	160.5	79	238.55	34	312.27
84	30.42	98	95.45	41	164.8	91	242.90	46	316.62
13	34.69	27	99.72	53	169.2	8	242.90	58	320.96
96	34.77	39	104.06	65	173.5	20	247.24	70	325.31
25	39.03	51	108.41	77	177.9	32	251.59	82	329.66
37	43.38	63	112.75	6	182.1	44	255.94	11	333.92
49	47.73	75	117.10	89	182.2	56	260.28	94	334.00
61	52.07	87	121.45	18	186.5	68	264.63	23	338.27
73	56.42	4	121.45	30	190.8	80	268.97	35	342.61
2	60.68	99	125.79	42	195.2	9	273.24	47	346.96
85	60.76	16	125.80	54	199.5	92	273.32	59	351.31
14	65.03	28	130.14	66	203.9	21	277.59	71	355.65
97	65.11	40	134.49	78	208.2	33	281.93	83	360.00

Table 8: Calculated argument year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
71	4.13	21	82.32	66	156.13	28	230.07	97	295.36
59	8.39	9	86.58	54	160.39	99	234.20	2	298.84
47	12.65	92	87.36	42	164.65	16	234.32	85	299.61
35	16.90	80	91.61	30	168.90	87	238.45	73	303.87
23	21.16	68	95.87	18	173.16	4	238.58	61	308.13
11	25.42	56	100.13	6	177.42	75	242.71	49	312.39
94	26.20	44	104.39	89	178.20	63	246.97	37	316.65
82	30.45	32	108.65	77	182.45	51	251.23	25	320.90
70	34.71	20	112.90	65	186.71	39	255.49	13	325.16
58	38.97	91	117.03	53	190.97	27	259.74	96	325.94
46	43.23	8	117.16	41	195.23	15	264.00	1	329.42
34	47.49	79	121.29	29	199.48	98	264.78	84	330.20
22	51.74	67	125.55	17	203.74	3	268.26	72	334.45
10	56.00	55	129.81	100	204.52	86	269.03	60	338.71
93	56.78	43	134.07	5	208.00	74	273.29	48	342.97
81	61.03	31	138.32	88	208.78	62	277.55	36	347.23
69	65.29	19	142.58	76	213.03	50	281.81	24	351.48
57	69.55	7	146.84	64	217.29	38	286.07	95	355.62
45	73.81	90	147.62	52	221.55	26	290.32	12	355.74
33	78.07	78	151.87	40	225.81	14	294.58	83	359.87

3.3.3 The Longitude of Mars (pages 35 and 36)

This volvelle (Figure 20) also is constructed and works exactly as for Saturn but uses the parameters of Mars (see Tables 9–11, and Figure 21).

On the epicycle disk of Mars, the argument mark for year 94 is grossly misplaced, it should be close to the year 15 mark in the lower right of Figure 22.

3.4 The Longitude of the Sun (pages 39 and 40)

The Ptolemaic model for the Sun is really quite simple: the Sun moves clockwise on an epicycle with a centre that moves anti-clockwise along the deferent circle with a fixed centre. The argument is the same as the longitude of the mean centrum.

The section starts with a table of the century mean longitudes of the Sun (Table 12a) calculated in the same way as for the planets. It is followed by a table with corrections (Table 12b) for individual years. The motion of the Sun in 'y' years is

daily motion · y · (365 + integer(y/4)) mod 360,

the last term in the bracket taking care of the leap days and the 'mod' function skipping multiples of 360°. The result will be a number that is

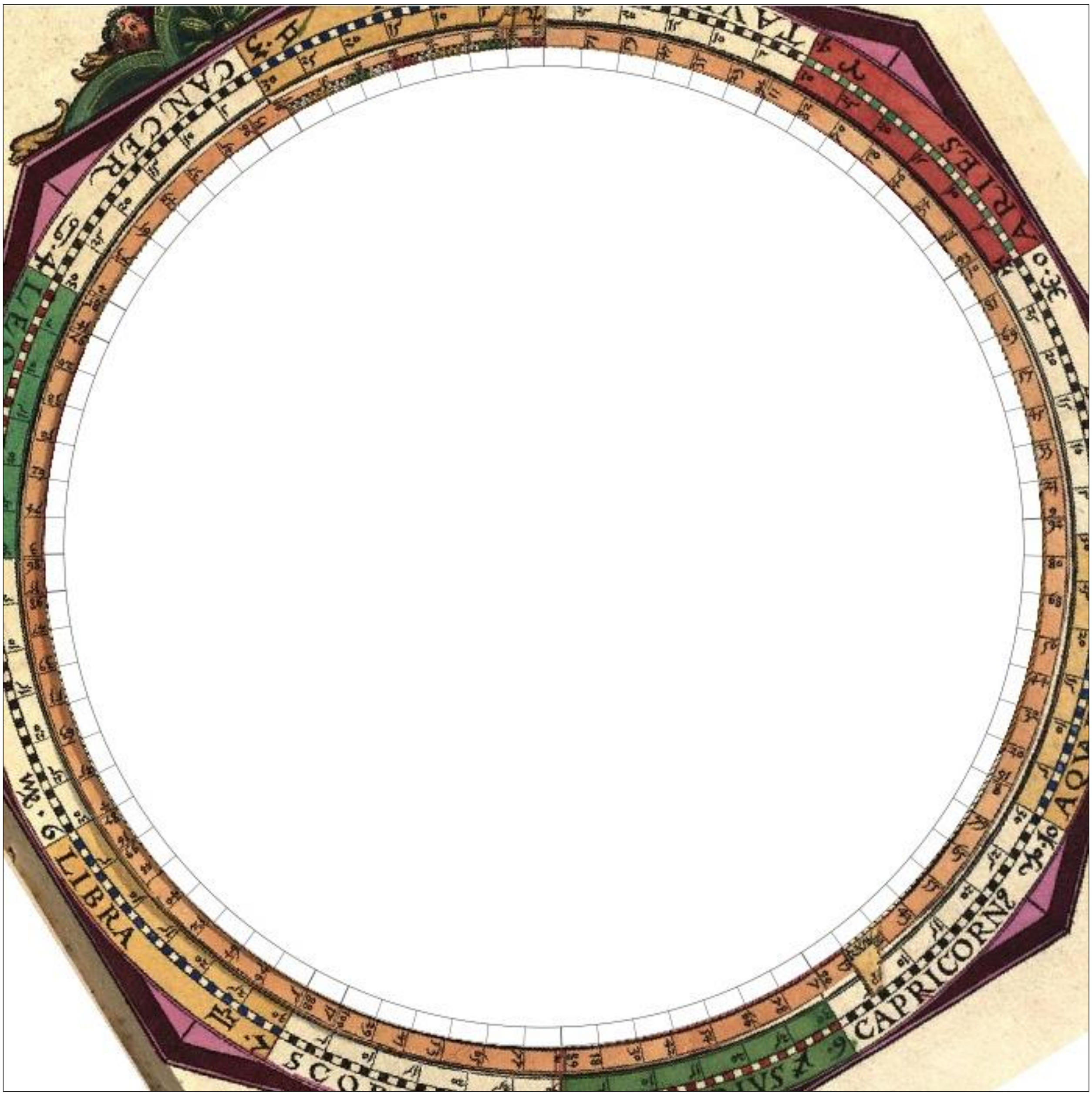


Figure 18: Comparison of the longitude year marks.

that is either close to 0° or to 360°. In the first case the correction is additive and marked with an A in the table. In the second case you subtract the number from 360° and get a new number close to 0° that is subtractive and is marked by an M in the table. Every fourth year in Apianus' table is marked by a small 'b' to indicate that it is a leap year (*bissextus*). The unit in this table is arc minute: arc second.

The solar volvelle (see Figure 23) has two disks on top of the mater, both centred on the mater. The mater has a scale with the zodiac. The next disk has a tab M for setting the century mean longitude, including the year correction, and a scale graduated with the months and days of the year for setting the date. The top disk has a tab for setting the apogee longitude and a circular angular scale slightly off-centre in order to account for the eccentricity of the Sun.


Figure 19: Comparison of some of the argument year marks.

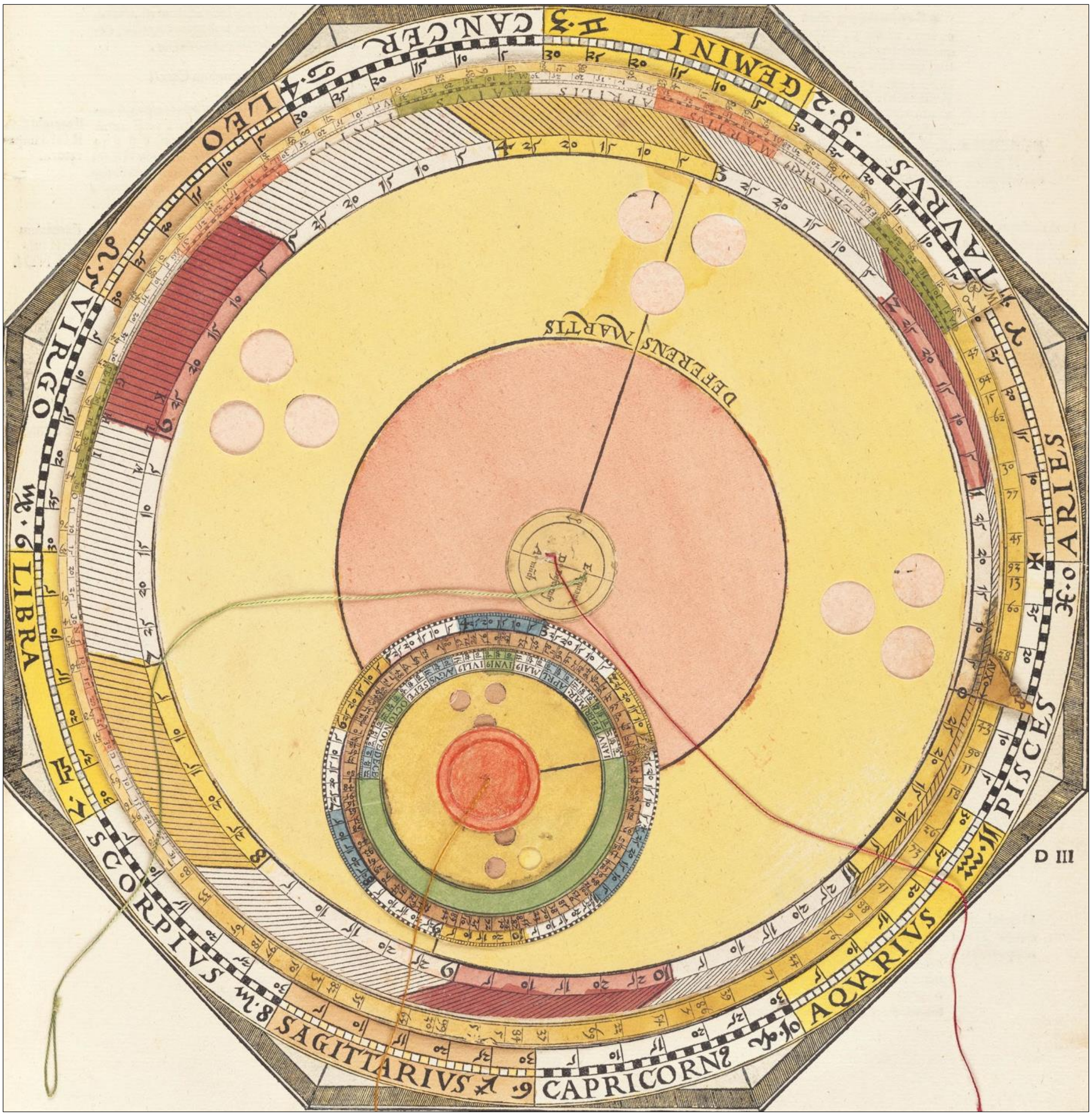


Figure 20: The Mars volvelle.

Table 9: Calculated century mean longitudes and arguments.

	Longitude						Argument					
	CE			BCE			CE			BCE		
0	1	11	24	1	11	24	7	26	53	7	26	53
100	3	12	59	11	9	49	5	26	2	9	27	44
200	5	14	34	9	8	14	3	25	12	11	28	34
300	7	16	8	7	6	40	1	24	21	1	29	25
400	9	17	43	5	5	5	11	23	30	4	0	16
500	11	19	18	3	3	30	9	22	39	6	1	7
600	1	20	53	1	1	55	7	21	49	8	1	57
700	3	22	28	11	0	20	5	20	58	10	2	48
800	5	24	3	8	28	45	3	20	7	0	3	39
900	7	25	37	6	27	11	1	19	16	2	4	30
1000	9	27	12	4	25	36	11	18	26	4	5	20
1100	11	28	47	2	24	1	9	17	35	6	6	11
1200	2	0	22	0	22	26	7	16	44	8	7	2
1300	4	1	57	10	20	51	5	15	53	10	7	53
1400	6	3	31	8	19	17	3	15	3	0	8	43
1500	8	5	6	6	17	42	1	14	12	2	9	34
1600	10	6	41	4	16	7	11	13	21	4	10	25
1700	0	8	16	2	14	32	9	12	30	6	11	16
1800	2	9	51	0	12	57	7	11	40	8	12	6

1900	4	11	26	10	11	22	5	10	49	10	12	57
2000	6	13	0	8	9	48	3	9	58	0	13	48
2100	8	14	35	6	8	13	1	9	8	2	14	38
2200	10	16	10	4	6	38	11	8	17	4	15	29
2300	0	17	45	2	5	3	9	7	26	6	16	20
2400	2	19	20	0	3	28	7	6	35	8	17	11
2500	4	20	54	10	1	54	5	5	45	10	18	1
2600	6	22	29	8	0	19	3	4	54	0	18	52
2700	8	24	4	5	28	44	1	4	3	2	19	43
2800	10	25	39	3	27	9	11	3	12	4	20	34
2900	0	27	14	1	25	34	9	2	22	6	21	24
3000	2	28	49	11	23	59	7	1	31	8	22	15
3100	5	0	23	9	22	25	5	0	40	10	23	6
3200	7	1	58	7	20	50	2	29	49	0	23	57
3300	9	3	33	5	19	15	0	28	59	2	24	47
3400	11	5	8	3	17	40	10	28	8	4	25	38
3500	1	6	43	1	16	5	8	27	17	6	26	29
3600	3	8	18	11	14	30	6	26	26	8	27	20
3700	5	9	52	9	12	56	4	25	36	10	28	10
3800	7	11	27	7	11	21	2	24	45	0	29	1
3900	9	13	2	5	9	46	0	23	54	2	29	52
4000	11	14	37	3	8	11	10	23	3	5	0	43
4100	1	16	12	1	6	36	8	22	13	7	1	33
4200	3	17	46	11	5	2	6	21	22	9	2	24
4300	5	19	21	9	3	27	4	20	31	11	3	15
4400	7	20	56	7	1	52	2	19	41	1	4	5
4500	9	22	31	5	0	17	0	18	50	3	4	56
4600	11	24	6	2	28	42	10	17	59	5	5	47
4700	1	25	41	0	27	7	8	17	8	7	6	38
4800	3	27	15	10	25	33	6	16	18	9	7	28
4900	5	28	50	8	23	58	4	15	27	11	8	19
5000	8	0	25	6	22	23	2	14	36	1	9	10
5100	10	2	0	4	20	48	0	13	45	3	10	1
5200	0	3	35	2	19	13	10	12	55	5	10	51
5300	2	5	9	0	17	39	8	12	4	7	11	42
5400	4	6	44	10	16	4	6	11	13	9	12	33
5500	6	8	19	8	14	29	4	10	22	11	13	24
5600	8	9	54	6	12	54	2	9	32	1	14	14
5700	10	11	29	4	11	19	0	8	41	3	15	5
5800	0	13	4	2	9	44	10	7	50	5	15	56
5900	2	14	38	0	8	10	8	6	59	7	16	47
6000	4	16	13	10	6	35	6	6	9	9	17	37
6100	6	17	48	8	5	0	4	5	18	11	18	28
6200	8	19	23	6	3	25	2	4	27	1	19	19
6300	10	20	58	4	1	50	0	3	37	3	20	9
6400	0	22	32	2	0	16	10	2	46	5	21	0
6500	2	24	7	11	28	41	8	1	55	7	21	51
6600	4	25	42	9	27	6	6	1	4	9	22	42
6700	6	27	17	7	25	31	4	0	14	11	23	32
6800	8	28	52	5	23	56	1	29	23	1	24	23
6900	11	0	27	3	22	21	11	28	32	3	25	14
7000	1	2	1	1	20	47	9	27	41	5	26	5

Table 10. Calculated longitude year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
79	1.46	6	68.23	44	142.30	3	213.85	88	284.59
32	5.31	85	70.21	76	147.60	82	215.83	41	287.92
64	10.61	38	73.54	29	150.93	35	219.16	73	293.22
17	13.94	70	78.84	61	156.23	67	224.47	26	296.55
96	15.92	23	82.17	14	159.56	20	228.32	58	301.86
49	19.24	55	87.48	93	161.54	99	229.77	11	305.18
2	22.57	8	91.33	46	164.87	52	233.62	90	307.16
81	24.55	87	92.78	78	170.17	5	236.95	43	310.49
34	27.88	40	96.63	31	173.50	84	238.93	75	315.79
66	33.18	72	101.94	63	178.80	37	242.25	28	319.64
19	36.51	25	105.26	16	182.65	69	247.56	60	324.95
98	38.49	57	110.57	95	184.11	22	250.89	13	328.27
51	41.81	10	113.90	48	187.96	54	256.19	92	330.25
4	45.66	89	115.88	1	191.29	7	259.52	45	333.58
83	47.12	42	119.20	80	193.26	86	261.50	77	338.89
36	50.97	74	124.51	33	196.59	39	264.82	30	342.21
68	56.28	27	127.83	65	201.90	71	270.13	62	347.52
21	59.60	59	133.14	18	205.22	24	273.98	15	350.84
100	61.58	12	136.99	97	207.20	56	279.29	94	352.82
53	64.91	91	138.45	50	210.53	9	282.61	47	356.15

Table 11: Calculated argument year markings.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
47	3.46	9	77.21	50	149.35	91	221.48	53	295.24
94	7.37	56	81.13	97	153.26	12	223.10	100	299.15
15	8.53	24	86.20	18	154.42	59	226.56	21	300.31
62	12.45	71	89.65	65	158.34	27	231.63	68	304.23
30	17.52	39	94.72	33	163.41	74	235.54	36	309.30
77	21.43	86	98.64	80	167.32	42	240.61	83	312.75
45	26.50	7	99.80	1	168.48	89	244.53	4	314.37
92	30.42	54	103.71	48	172.39	10	245.69	51	317.82
13	31.58	22	108.78	95	175.85	57	249.60	98	321.74
60	35.49	69	112.70	16	177.47	25	254.67	19	322.89
28	40.56	37	117.77	63	180.92	72	258.59	66	326.81
75	44.02	84	121.69	31	185.99	40	263.66	34	331.88
43	49.09	5	122.84	78	189.91	87	267.12	81	335.80
90	53.01	52	126.76	46	194.98	8	268.73	2	336.95
11	54.16	99	130.22	93	198.90	55	272.19	49	340.87
58	58.08	20	131.83	14	200.05	23	277.26	96	344.79
26	63.15	67	135.29	61	203.97	70	281.18	17	345.94
73	67.07	35	140.36	29	209.04	38	286.25	64	349.86
41	72.14	82	144.28	76	212.96	85	290.17	32	354.93
88	76.06	3	145.43	44	218.03	6	291.32	79	358.39

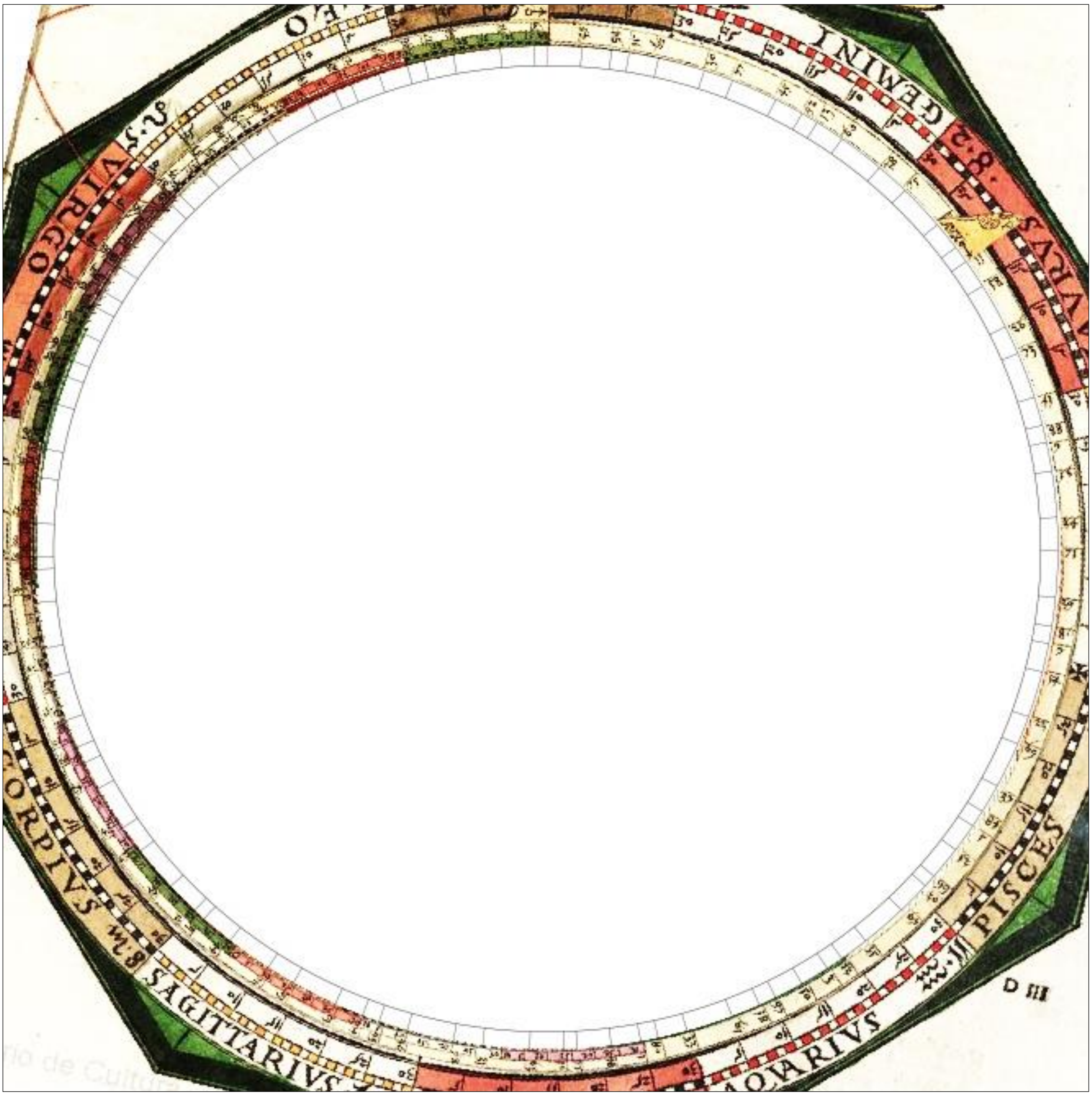


Figure 21: Comparison of the longitude year marks.

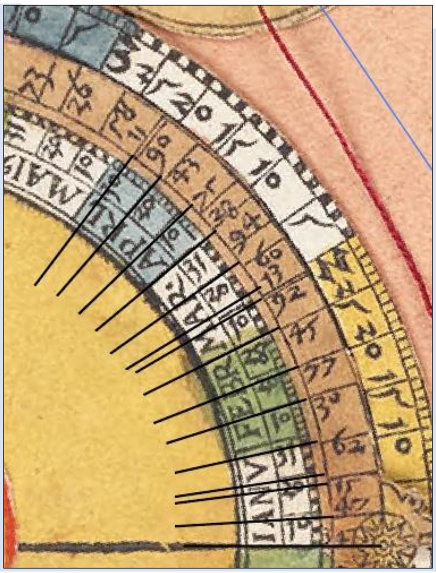


Figure 22: Comparison of the some of the argument year marks.

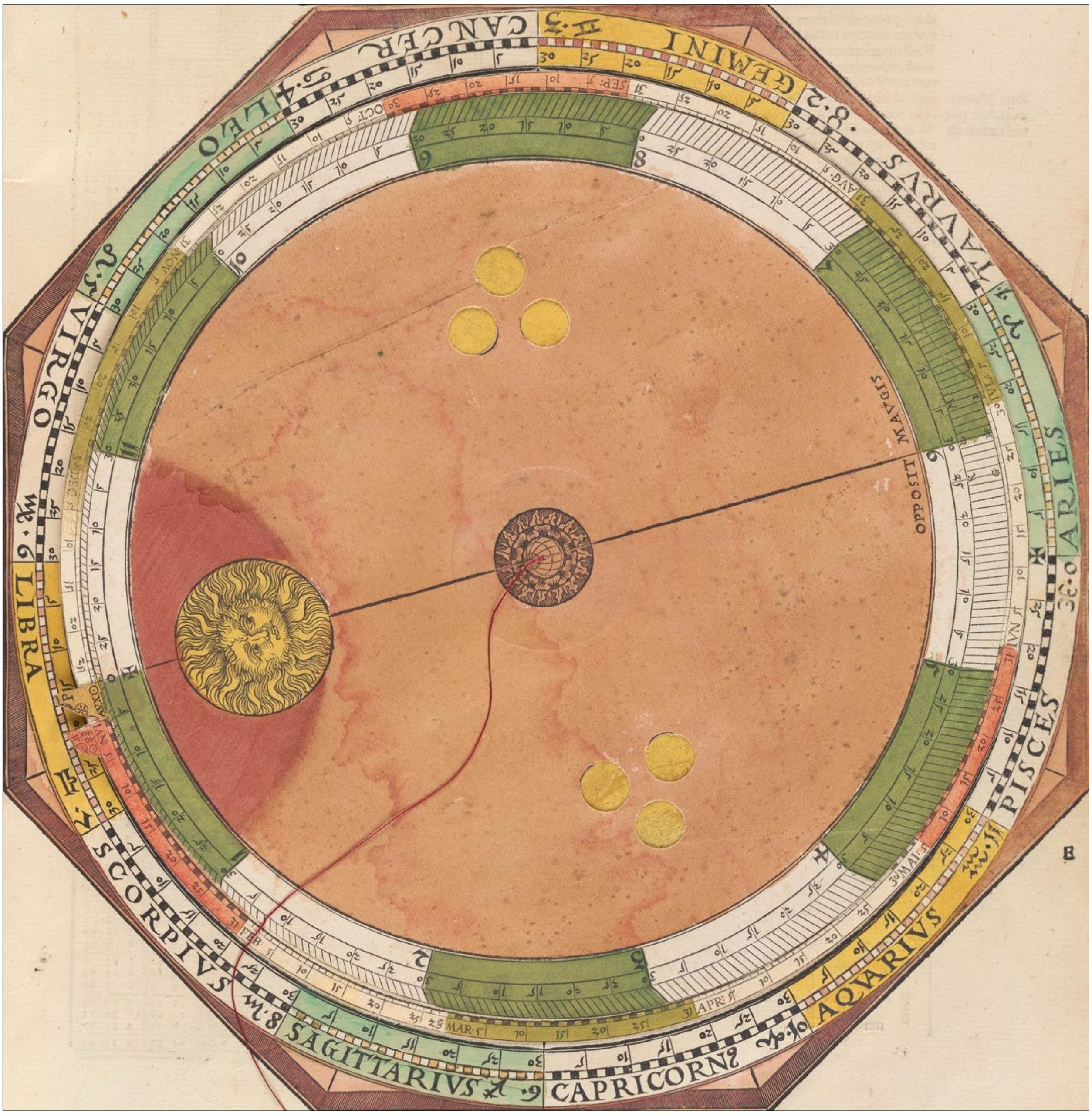


Figure 23: The solar volvelle.

Table 12a: Calculated mean century longitudes of the Sun.

	CE			BCE		
0	9	8	17	9	8	17
100	9	9	1	9	7	33
200	9	9	45	9	6	49
300	9	10	30	9	6	5
400	9	11	14	9	5	21
500	9	11	58	9	4	37
600	9	12	42	9	3	53
700	9	13	26	9	3	9
800	9	14	10	9	2	25
900	9	14	54	9	1	41
1000	9	15	38	9	0	56
1100	9	16	22	9	0	12
1200	9	17	6	8	29	28
1300	9	17	50	8	28	44
1400	9	18	34	8	28	0
1500	9	19	19	8	27	16
1600	9	20	3	8	26	32
1700	9	20	47	8	25	48
1800	9	21	31	8	25	4
1900	9	22	15	8	24	20
2000	9	22	59	8	23	36
2100	9	23	43	8	22	52
2200	9	24	27	8	22	8
2300	9	25	11	8	21	23
2400	9	25	55	8	20	39
2500	9	26	39	8	19	55
2600	9	27	23	8	19	11
2700	9	28	8	8	18	27
2800	9	28	52	8	17	43
2900	9	29	36	8	16	59
3000	10	0	20	8	16	15
3100	10	1	4	8	15	31
3200	10	1	48	8	14	47
3300	10	2	32	8	14	3
3400	10	3	16	8	13	19
3500	10	4	0	8	12	34
3600	10	4	44	8	11	50
3700	10	5	28	8	11	6
3800	10	6	12	8	10	22
3900	10	6	56	8	9	38
4000	10	7	41	8	8	54
4100	10	8	25	8	8	10
4200	10	9	9	8	7	26
4300	10	9	53	8	6	42
4400	10	10	37	8	5	58
4500	10	11	21	8	5	14
4600	10	12	49	8	3	45
4700	10	12	5	8	4	30
4800	10	13	33	8	3	1
4900	10	14	17	8	2	17
5000	10	15	1	8	1	33
5100	10	15	45	8	0	49
5200	10	16	30	8	0	5
5300	10	17	14	7	29	21
5400	10	17	58	7	28	37
5500	10	18	42	7	27	53
5600	10	19	26	7	27	9
5700	10	20	10	7	26	25
5800	10	20	54	7	25	41
5900	10	21	38	7	24	56
6000	10	22	22	7	24	12
6100	10	23	6	7	23	28
6200	10	23	50	7	22	44
6300	10	24	34	7	22	0
6400	10	25	19	7	21	16
6500	10	26	3	7	20	32
6600	10	26	47	7	19	48
6700	10	27	31	7	19	4
6800	10	28	15	7	18	20
6900	10	28	59	7	17	36
7000	10	29	43	7	16	52

Table 12b: Calculated annual correction to the mean longitudes of the Sun.

Year	M	Z		Year	M	Z	
1	14	20	M	51	21	51	M
2	28	40	M	52	22	55	A
3	43	1	M	53	8	35	A
4	1	46	A	54	5	45	M
5	12	34	M	55	20	6	M
6	26	54	M	56	24	41	A
7	41	15	M	57	10	21	A
8	3	32	A	58	3	59	M
9	10	48	M	59	18	20	M
10	25	9	M	60	26	27	A
11	39	29	M	61	12	6	A
12	5	17	A	62	2	13	M
13	9	2	M	63	16	34	M
14	23	23	M	64	28	13	A
15	37	44	M	65	13	52	A
16	7	3	A	66	0	28	M
17	7	16	M	67	14	48	M
18	21	37	M	68	29	59	A
19	35	58	M	69	15	38	A
20	8	49	A	70	1	17	A
21	5	31	M	71	13	2	M
22	19	51	M	72	31	44	A
23	34	12	M	73	17	24	A
24	10	35	A	74	3	3	A
25	3	45	M	75	11	17	M
26	18	5	M	76	33	30	A
27	32	26	M	77	19	9	A
28	12	21	A	78	4	49	A
29	1	59	M	79	9	31	M
30	16	20	M	80	35	16	A
31	30	40	M	81	20	55	A
32	14	6	A	82	6	35	A
33	0	13	M	83	7	45	M
34	14	34	M	84	37	2	A
35	28	55	M	85	22	41	A
36	15	52	A	86	8	20	A
37	1	32	A	87	5	59	M
38	12	48	M	88	38	48	A
39	27	9	M	89	24	27	A
40	17	38	A	90	10	6	A
41	3	17	A	91	4	13	M
42	11	2	M	92	40	33	A
43	25	23	M	93	26	13	A
44	19	24	A	94	11	52	A
45	5	3	A	95	2	28	M
46	9	17	M	96	42	19	A
47	23	37	M	97	27	58	A
48	21	10	A	98	13	38	A
49	6	49	A	99	0	42	M
50	7	31	M	100	44	5	A

For the Sun the working is that you take the century mean longitude from the table corrected for the individual year and set tab M to that value. Tab M is then corrected for the date and hour, using the months and day scales. Tab P on the top disk is set to the longitude of the apogee. The slanting lines are followed from tab M to the scale on the top disk and the thread from the centre is aligned with that point and followed back out to the zodiac scale of the mater where the true longitude is read.

3.5 The Inner Planets

3.5.1 The Longitude of Venus (pages 41 and 42)

The Ptolemaic model for Venus is identical with that for the outer planets (see Figure 24). The

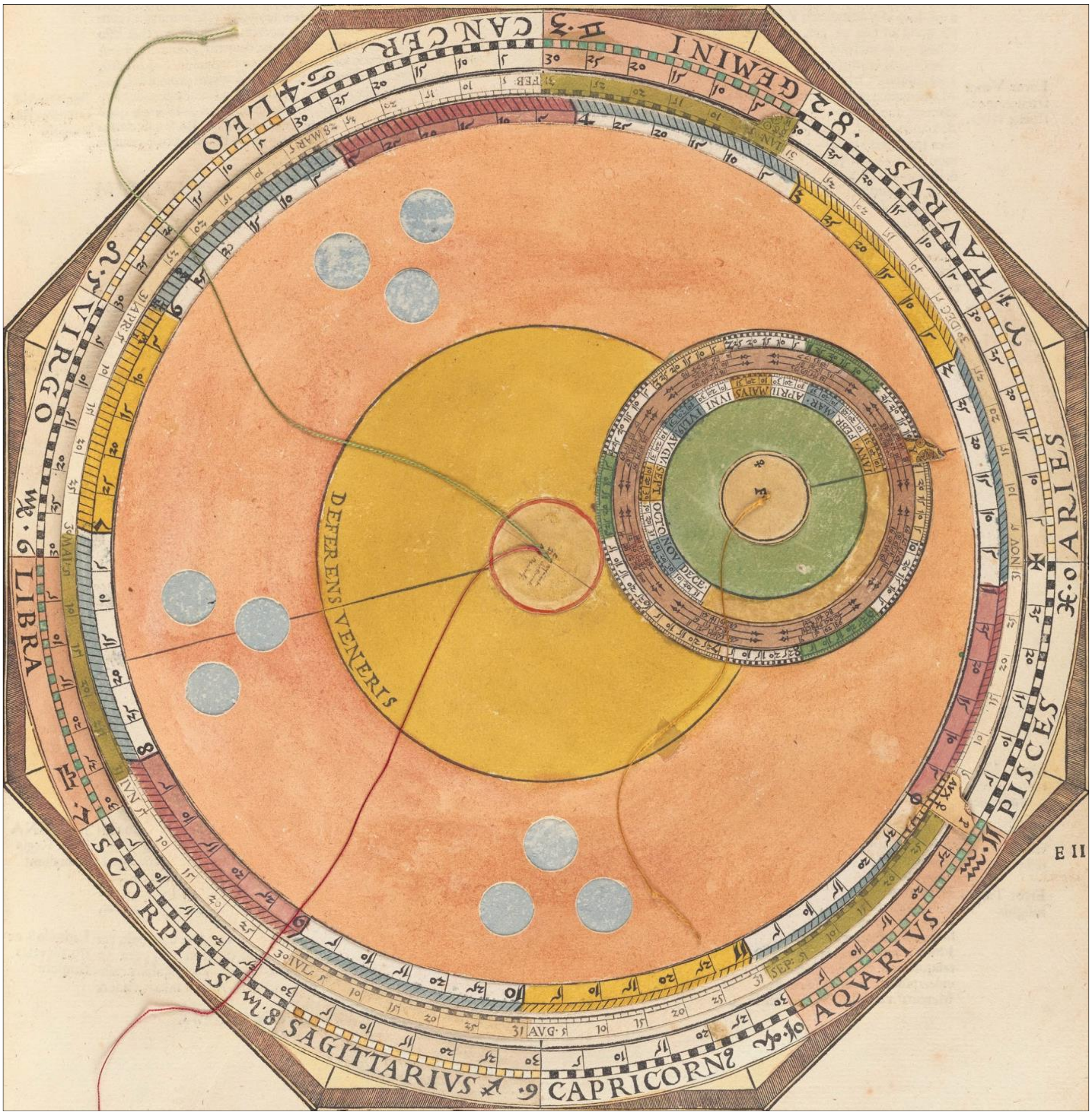


Figure 24: The Venus volvelle.

mean motion in longitude is the same as for the Sun and is marked and set in the same way using tab M marked with symbol for the Sun, Venus, and Mercury. On the page preceding the volvelle is a table (Table 13) for the century arguments.

Table 13: Calculated century arguments of Venus.

	CE			BCE		
0	4	9	20	4	9	20
100	10	27	34	9	21	6
200	5	15	48	3	2	52
300	0	4	3	8	14	37
400	6	22	17	1	26	23
500	1	10	31	7	8	9
600	7	28	45	0	19	55
700	2	17	0	6	1	40
800	9	5	14	11	13	26
900	3	23	28	4	25	12
1000	10	11	42	10	6	58
1100	4	29	57	3	18	43
1200	11	18	11	9	0	29
1300	6	6	25	2	12	15
1400	0	24	39	7	24	1
1500	7	12	54	1	5	46
1600	2	1	8	6	17	32
1700	8	19	22	11	29	18
1800	3	7	36	5	11	4
1900	9	25	51	10	22	49
2000	4	14	5	4	4	35
2100	11	2	19	9	16	21
2200	5	20	33	2	28	7
2300	0	8	48	8	9	52
2400	6	27	2	1	21	38
2500	1	15	16	7	3	24
2600	8	3	30	0	15	10
2700	2	21	45	5	26	55
2800	9	9	59	11	8	41
2900	3	28	13	4	20	27
3000	10	16	27	10	2	13
3100	5	4	42	3	13	58
3200	11	22	56	8	25	30

3300	6	11	10	2	7	16
3400	0	29	24	7	19	
3500	7	17	39	1	1	1
3600	2	5	53	6	12	47
3700	8	24	7	11	24	33
3800	3	12	21	5	6	19
3900	10	0	36	10	18	4
4000	4	18	50	3	29	50
4100	11	7	4	9	11	36
4200	5	25	18	2	23	22
4300	0	13	33	8	5	7
4400	7	1	47	1	16	53
4500	1	20	1	6	28	39
4600	8	8	15	0	10	25
4700	2	26	29	5	22	11
4800	9	14	44	11	3	56
4900	4	2	58	4	15	42
5000	10	21	12	9	27	28
5100	5	9	26	3	9	14
5200	11	27	41	8	20	59
5300	6	15	55	2	2	45
5400	1	4	9	7	14	31
5500	7	22	23	0	26	17
5600	2	10	38	6	8	2
5700	8	28	52	11	19	48
5800	3	17	6	5	1	34
5900	10	5	20	10	13	20
6000	4	23	35	3	25	5
6100	11	11	49	9	6	51
6200	6	0	3	2	18	37
6300	0	18	17	8	0	23
6400	7	6	32	1	12	8
6500	1	24	46	6	23	54
6600	8	13	0	0	5	40
6700	3	1	14	5	17	26
6800	9	19	29	10	29	11
6900	4	7	43	4	10	57
7000	10	25	57	9	22	43

Tables 14a and 14b show the calculated year and month marks of the argument on the epicycle disk. The marks are grouped into eight groups. Figure 25 shows a comparison with calculated year and month marks.

Table 14a: Calculated argument year marks.

Outer Ring		Middle Ring		Inner Ring	
Year	Angle	Year	Angle	Year	Angle
8	1.46	16	2.92	24	4.38
32	5.84	40	7.30	48	8.75
56	10.21	64	11.67	72	13.13
80	14.59	88	16.05	96	17.51
5	45.76	13	47.22	21	48.68
29	50.14	37	51.59	45	53.05
53	54.51	61	55.97	69	57.43
77	58.89	85	60.35	93	61.81
2	90.06	10	91.52	18	92.97
26	94.43	34	95.89	42	97.35
50	98.81	58	100.27	66	101.73
74	103.19	82	104.65	90	106.11
98	107.56				
7	135.81	15	137.27	23	138.73
31	140.19	39	141.65	47	143.11
55	144.57	63	146.03	71	147.49
79	148.95	87	150.40	95	151.86
4	180.73	12	182.19	20	183.65
28	185.11	36	186.57	44	188.02
52	189.48	60	190.94	68	192.40
76	193.86	84	195.32	92	196.78
100	198.24				
1	225.03	9	226.49	17	227.95
25	229.41	33	230.86	41	232.32
49	233.78	57	235.24	65	236.70
73	238.16	81	239.62	89	241.08
97	242.54				
6	270.79	14	272.25	22	273.70
30	275.16	38	276.62	54	279.54
		46	278.08		
62	281.00	70	282.46	78	283.92
86	285.38	94	286.84		
3	315.09	11	316.54	19	318.00
27	319.46	35	320.92	43	322.38
51	323.84	59	325.30	67	326.76
75	328.22	83	329.68	91	331.13
99	332.59				

3.5.2 The Longitude of Mercury (pages 41 and 42)

The Mercury volvelle is shown in Figure 26. The Ptolemaic model for Mercury is quite involved (Figure 27). In addition to the ordinary epicycle the centre C of the epicycle and the deferent is dragged clockwise by a crank system around a small circle with the radius of the eccentricity and centred on D. The position of the mean centrum is the angle BEC, and the crank angle BHK is equal to this angle. The argument is the angle FCP as before and the true argument is the angle GCP. Finally, the location of the true centrum is the angle BAP. The mean motion in longitude is the same as for the Sun and is marked and set in the same way.

The Mercury volvelle has a more complicated structure than the other planetary volvelles. On top of the mater and centred on it is disk with an index M and symbols of the Sun, Venus, and Mercury for setting the mean longitude. It has a year scale to set the date with index M once the solar radix of the year has been set. On top of this disk is the P disk for setting the apogee. It has a set of slating lines, one for each degree that connects the periphery of the disk with an inner zodiacal scale, offset from the main axis in the same ways as for the other planets and graduated clockwise and starting from index P. Along the periphery, the slanting lines are graduated anti-clockwise also with a zodiacal scale and starting from P. This scale is used to measure the elongation of the mean longitude from the apogee, i.e. the mean centrum. On top of disk P is another disk with an index tab Q and on top of this disk is the deferent disk. The axis of deferent disk is made to rotate around a centre H in a small circle (Parui Cir:) that is attached to the disk below and moves with it, and there are now four points marked on the cover cap: D (Deferentis), H, E (Equantis), A (Mundi). Two threads are attached to the points A and E. Finally the epicycle disks for setting the argu-

Table 14b: Calculated argument month marks.

Months	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.
Angle	19.11	36.37	55.49	73.98	93.09	111.59	130.70	149.81	168.31	187.42	205.92	225.03

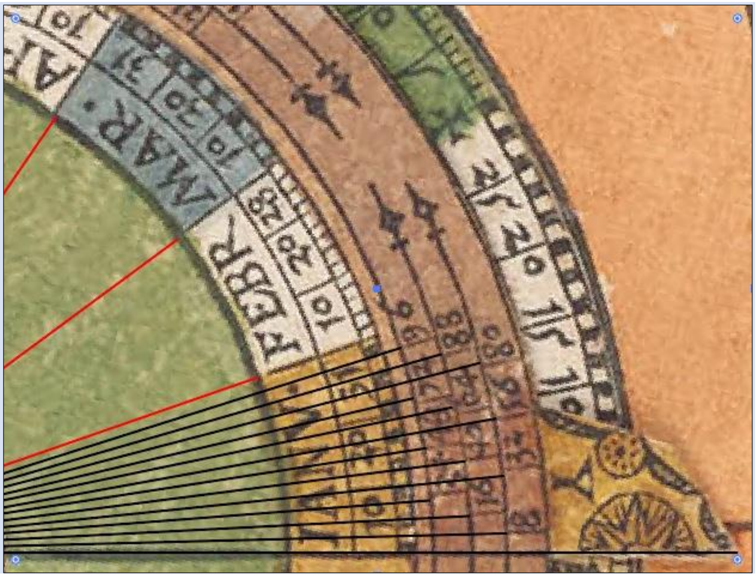


Figure 25: Comparison of some argument year (black) and month (red) marks.

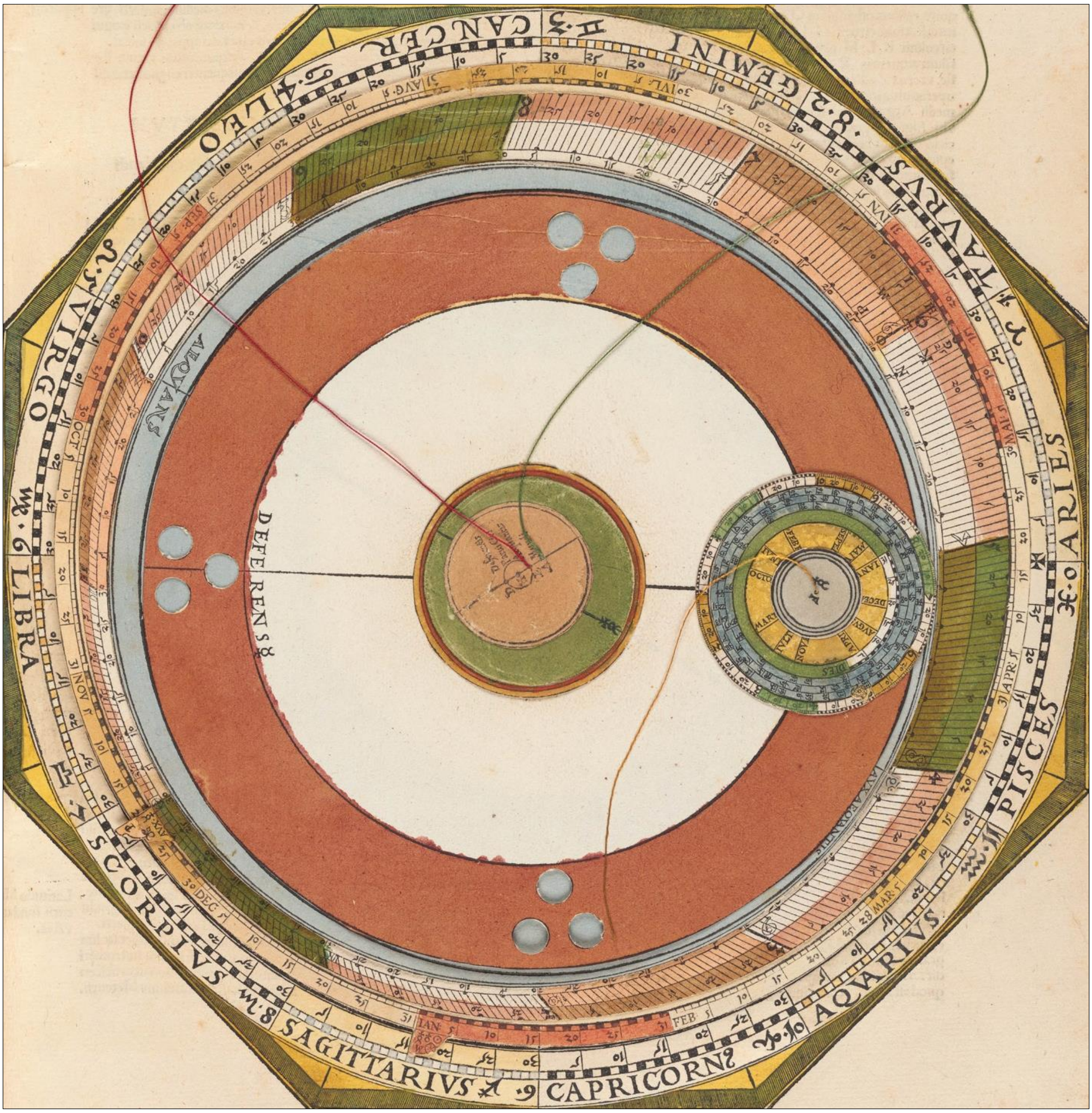


Figure 26: The Mercury volvelle.

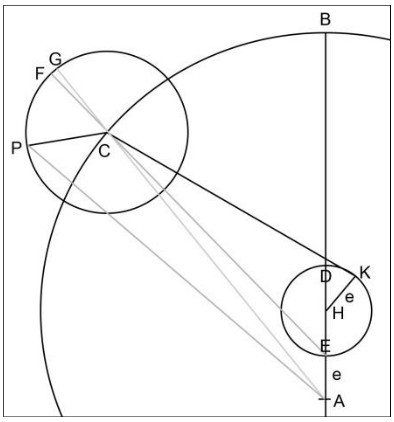


Figure 27: Ptolemy's model for Mercury.

ment are attached to the deferent circle. The computed century arguments of Mercury are shown in Table 15.

Table 15: Computed table for the century arguments of Mercury.

	CE			BCE		
0	1	15	12	1	15	12
100	3	27	30	11	2	54
200	6	9	48	8	20	36
300	8	22	7	6	8	17
400	11	4	25	3	25	59
500	1	16	43	1	13	41
600	3	29	1	11	1	23
700	6	11	20	8	19	4
800	8	23	38	6	6	46
900	11	5	56	3	24	28
1000	1	18	14	1	12	10
1100	4	0	33	10	29	51
1200	6	12	51	8	17	33
1300	8	25	9	6	5	15
1400	11	7	27	3	22	57
1500	1	19	46	1	10	38
1600	4	2	4	10	28	20
1700	6	14	22	8	16	2
1800	8	26	40	6	3	44
1900	11	8	59	3	21	25
2000	1	21	17	1	9	7
2100	4	3	35	10	26	49
2200	6	15	53	8	14	31
2300	8	28	11	6	2	13
2400	11	10	30	3	19	54
2500	1	22	48	1	7	36
2600	4	5	6	10	25	18
2700	6	17	24	8	13	0
2800	8	29	43	6	0	41
2900	11	12	1	3	18	23
3000	1	24	19	1	6	5
3100	4	6	37	10	23	47
3200	6	18	56	8	11	28
3300	9	1	14	5	29	10
3400	11	13	32	3	16	52
3500	1	25	50	1	4	34
3600	4	8	9	10	22	15
3700	6	20	27	8	9	57
3800	9	2	45	5	27	39
3900	11	15	3	3	15	21
4000	1	27	22	1	3	2
4100	4	9	40	10	20	44
4200	6	21	58	8	8	26
4300	9	4	16	5	26	8
4400	11	16	34	3	13	50
4500	1	28	53	1	1	31
4600	4	11	11	10	19	13
4700	6	23	29	8	6	55
4800	9	5	47	5	24	37
4900	11	18	6	3	12	18
5000	2	0	24	1	0	0
5100	4	12	42	10	17	42
5200	6	25	0	8	5	24
5300	9	7	19	5	23	5
5400	11	19	37	3	10	47
5500	2	1	55	0	28	29
5600	4	14	13	10	16	11
5700	6	26	32	8	3	52
5800	9	8	50	5	21	34
5900	11	21	8	3	9	16
6000	2	3	26	0	26	58
6100	4	15	45	10	14	39
6200	6	28	3	8	2	21
6300	9	10	21	5	20	3
6400	11	22	39	3	7	45
6500	2	4	57	0	25	27
6600	4	17	16	10	13	8
6700	6	29	34	8	0	50
6800	9	11	52	5	18	32
6900	11	24	10	3	6	14
7000	2	6	29	0	23	55

Tables 16a and 16b show the calculated year and month marks on the epicycle disk. The inner ring has years 80–100, the middle ring 41–79, the outer ring 1–40. Figure 28 shows a comparison of some of the argument year (black) and month (red) marks.

Table 16a: Calculated argument year marks.

Outer Ring		Middle Ring		Inner Ring	
Year	Angle	Year	Angle	Year	Angle
33	5.08	79	0.79	99	15.25
20	14.46	66	10.17	86	24.63
7	20.73	53	19.54	93	48.47
40	28.92	73	34.01	80	57.84
27	35.19	60	43.38	100	72.30
14	44.57	47	49.65	87	78.57
1	53.95	67	64.11	94	102.41
34	59.03	54	73.49	81	111.79
21	68.41	41	82.87	88	135.63
8	77.78	74	87.95	95	156.36
28	92.25	61	97.33	82	165.74
15	98.52	48	106.71	89	189.57
2	107.89	68	121.17	96	213.41
35	112.98	55	127.44	83	219.68
22	122.35	42	136.81	90	243.52
9	131.73	75	141.90	97	267.36
29	146.19	62	151.28	84	276.74
16	155.57	49	160.65	91	297.47
3	161.84	69	175.11	98	321.31
36	170.03	56	184.49	85	330.68
23	176.30	43	190.76	92	354.52
10	185.68	76	198.95		
30	200.14	63	205.22		
17	209.52	50	214.60		
4	218.89	70	229.06		
37	223.98	57	238.44		
24	233.35	77	252.90		
11	239.62	64	262.28		
44	247.81	51	268.55		
31	254.08	71	283.01		
18	263.46	58	292.38		
5	272.84	45	301.76		
38	277.92	78	306.84		
25	287.30	65	316.22		
12	296.68	52	325.60		
32	311.14	72	340.06		
19	317.41	59	346.33		
6	326.79	46	355.71		
39	331.87				
26	341.25				
13	350.62				

Table 16b: Calculated argument month marks.

Months	April	August	Dec.	Jan.	May	Sept.	Feb.	June	Oct.	March	July	Nov.
Angle	12.80	34.93	53.95	96.31	109.11	128.13	183.30	202.31	224.44	279.60	298.62	317.64

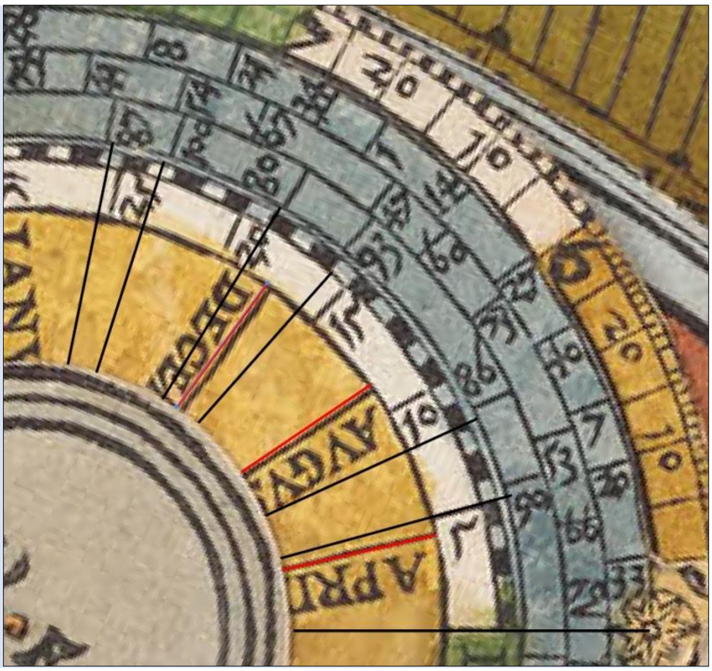


Figure 28: Comparison of some of the argument year (black) and month (red) marks.

To work the volvelle you first set the mean longitude with index M in the same way as for the Sun. Then index P is set to the longitude of the apogee. By noting the value on the peripheral, anti-clockwise zodiacal scale of disk P for the position of index M you determine the mean centrum. You then set index Q to the same value on the inner end of the slanting line by the clockwise scale. This will be approximately on the opposite side of index M relative to index P. The epicycle top disk is set to the argument. The epicycle disks (with the upper set disk locked) are moved with their centre under the thread from E stretched to the inner end of the slanting line from index M, and the epicycle disks are rotated such that the cross of the lower disk is aligned with the E thread. Finally the thread from A is stretched through the star symbol on the top epicycle disk and the true longitude read off from the main zodiac scale of the mater.

3.6 The Latitudes of the Planets (pages 30, 34, 38, 44, and 48)

The latitudes of the planets are calculated according to the Ptolemaic/Alfonsine scheme (Pedersen, 1974). The needed input parameters to the volvelles are the true centrum and the true argument found in the volvelle of the respective planet. The values of these parameters are set using the thread from the centre of the volvelle with its sliding bead, the value for the centrum by using the azimuthal scale of the volvelle and the argument by using the wedge scale at the top. The value of the latitude can then be read from the line below the bead. Figure 29 shows a comparison of calculated points on Apianus' latitude volvelles from the analysis in Gislén (2017). The red points are generated by a Java application that calculates the latitudes using the data and methods of the Alfonsine Tables. The volvelles for the outer planets in general agree well with the simulation. For the inner planets the agreement is sometimes less perfect, which is maybe not astonishing considering the very complicated calculations needed for these latitudes.

3.7 The Moon

The Ptolemaic model for the Moon (Figure 30) is slightly more involved than the model for the outer planets and has some similarities with the model for Mercury. The centre of the deferent circle is attached to D that rotates clockwise around a smaller circle with centre A. The epicycle moves anti-clockwise on the deferent circle. The Moon moves clockwise on the epicycle. B is the location of the Sun. The angle BAC is the elongation from the Sun, equal to the angle BAD. FCP is the mean argument and GCP the true argument. The true place of the Moon relative to the Sun is then the direction of AP.

The basis of all the Moon volvelles are three tables with the mean longitudes, arguments, and the positions of the node (Caput Draco) (Table 17).

3.7.1 The Longitude of the Moon (page 50)

This volvelle (Figure 31) has six disks stacked on the mater. The first disk, M, is centred on the

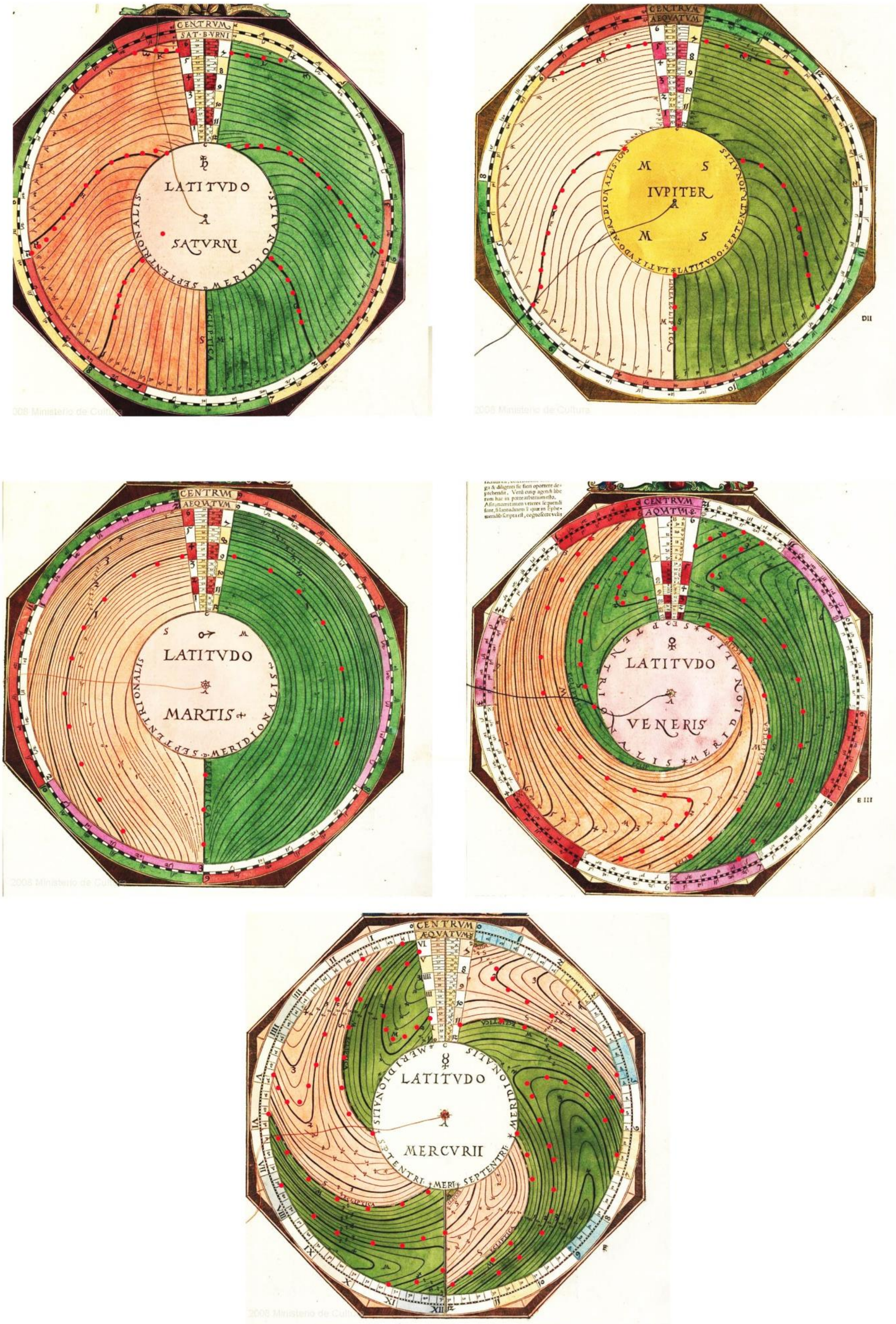


Figure 29: Simulated points on the latitude volvelles.

mater and has an index tab marked M and with a Sun symbol. It is used for setting the longitude of the Sun. The periphery is divided into 100 equally spaced parts anti-clockwise. Inside this there is a scale with a year of 365 days and a scale graduated symmetrically clockwise and anti-clockwise from 0 to 250, starting from M. This is used to read the elongation and then set the double elongation, as will be described below.

On top of disk M there is a disk P, also centred on the main axis. It has an index tab marked P and the text INDEX ME·MO· ☽ and is used for setting the mean longitude of the Moon.

Figure 30: Ptolemy's model for the Moon.

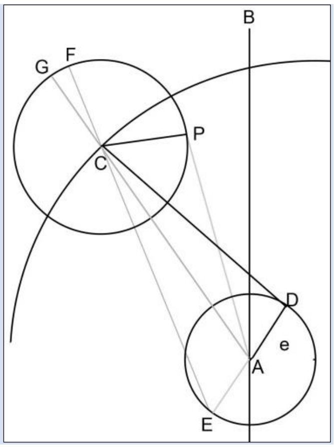


Table 17: Calculated Moon tables for centuries.

	Longitude						Argument						Node					
	CE			BCE			CE			BCE			CE			BCE		
0	4	2	8	4	2	8	6	18	21	6	18	21	8	28	4	8	28	4
100	2	9	57	5	24	19	1	7	3	11	29	39	4	13	55	1	12	13
200	0	17	46	7	16	30	7	25	45	5	10	57	11	29	45	5	26	23
300	10	25	35	9	8	41	2	14	28	10	22	14	7	15	36	10	10	32
400	9	3	24	11	0	52	9	3	10	4	3	32	3	1	27	2	24	41
500	7	11	13	0	23	3	3	21	52	9	14	50	10	17	18	7	8	50
600	5	19	2	2	15	14	10	10	34	2	26	8	6	3	8	11	23	0
700	3	26	51	4	7	25	4	29	16	8	7	26	1	18	59	4	7	9
800	2	4	40	5	29	36	11	17	59	1	18	43	9	4	50	8	21	18
900	0	12	29	7	21	47	6	6	41	7	0	1	4	20	41	1	5	27
1000	10	20	18	9	13	58	0	25	23	0	11	19	0	6	31	5	19	37
1100	8	28	7	11	6	9	7	14	5	5	22	37	7	22	22	10	3	46
1200	7	5	56	0	28	20	2	2	47	11	3	55	3	8	13	2	17	55
1300	5	13	45	2	20	31	8	21	30	4	15	12	10	24	4	7	2	4
1400	3	21	33	4	12	43	3	10	12	9	26	30	6	9	54	11	16	14
1500	1	29	22	6	4	54	9	28	54	3	7	48	1	25	45	4	0	23
1600	0	7	11	7	27	5	4	17	36	8	19	6	9	11	36	8	14	32
1700	10	15	0	9	19	16	11	6	18	2	0	24	4	27	26	0	28	42
1800	8	22	49	11	11	27	5	25	1	7	11	41	0	13	17	5	12	51
1900	7	0	38	1	3	38	0	13	43	0	22	59	7	29	8	9	27	0
2000	5	8	27	2	25	49	7	2	25	6	4	17	3	14	59	2	11	9
2100	3	16	16	4	18	0	1	21	7	11	15	35	11	0	49	6	25	19
2200	1	24	5	6	10	11	8	9	49	4	26	53	6	16	40	11	9	28
2300	0	1	54	8	2	22	2	28	31	10	8	11	2	2	31	3	23	37
2400	10	9	43	9	24	33	9	17	14	3	19	28	9	18	22	8	7	46
2500	8	17	32	11	16	44	4	5	56	9	0	46	5	4	12	0	21	56
2600	6	25	21	1	8	55	10	24	38	2	12	4	0	20	3	5	6	5
2700	5	3	10	3	1	6	5	13	20	7	23	22	8	5	54	9	20	14
2800	3	10	59	4	23	17	0	2	2	1	4	40	3	21	45	2	4	23
2900	1	18	48	6	15	28	6	20	45	6	15	57	11	7	35	6	18	33
3000	11	26	37	8	7	39	1	9	27	11	27	15	6	23	26	11	2	42
3100	10	4	26	9	29	50	7	28	9	5	8	33	2	9	17	3	16	51
3200	8	12	15	11	22	1	2	16	51	10	19	51	9	25	7	8	1	1
3300	6	20	4	1	14	12	9	5	33	4	1	9	5	10	58	0	15	10
3400	4	27	53	3	6	23	3	24	16	9	12	26	0	26	49	4	29	19
3500	3	5	42	4	28	34	10	12	58	2	23	44	8	12	40	9	13	28
3600	1	13	31	6	20	45	5	1	40	8	5	2	3	28	30	1	27	38
3700	11	21	20	8	12	56	11	20	22	1	16	20	11	14	21	6	11	47
3800	9	29	9	10	5	7	6	9	4	6	27	38	7	0	12	10	25	56
3900	8	6	58	11	27	18	0	27	47	0	8	55	2	16	3	3	10	5
4000	6	14	47	1	19	29	7	16	29	5	20	13	10	1	53	7	24	15
4100	4	22	36	3	11	40	2	5	11	11	1	31	5	17	44	0	8	24
4200	3	0	24	5	3	52	8	23	53	4	12	49	1	3	35	4	22	33
4300	1	8	13	6	26	3	3	12	35	9	24	7	8	19	26	9	6	42
4400	11	16	2	8	18	14	10	1	18	3	5	24	4	5	16	1	20	52
4500	9	23	51	10	10	25	4	20	0	8	16	42	11	21	7	6	5	1
4600	8	1	40	0	2	36	11	8	42	1	28	0	7	6	58	10	19	10
4700	6	9	29	1	24	47	5	27	24	7	9	18	2	22	48	3	3	20
4800	4	17	18	3	16	58	0	16	6	0	20	36	10	8	39	7	17	29
4900	2	25	7	5	9	9	7	4	49	6	1	53	5	24	30	0	1	38
5000	1	2	56	7	1	20	1	23	31	11	13	11	1	10	21	4	15	47
5100	11	10	45	8	23	31	8	12	13	4	24	29	8	26	11	8	29	57

5200	9	18	34	10	15	42	3	0	55	10	5	47	4	12	2	1	14	6
5300	7	26	23	0	7	53	9	19	37	3	17	5	11	27	53	5	28	15
5400	6	4	12	2	0	4	4	8	20	8	28	22	7	13	44	10	12	24
5500	4	12	1	3	22	15	10	27	2	2	9	40	2	29	34	2	26	34
5600	2	19	50	5	14	26	5	15	44	7	20	58	10	15	25	7	10	43
5700	0	27	39	7	6	37	0	4	26	1	2	16	6	1	16	11	24	52
5800	11	5	28	8	28	48	6	23	8	6	13	34	1	17	7	4	9	1
5900	9	13	17	10	20	59	1	11	50	11	24	52	9	2	57	8	23	11
6000	7	21	6	0	13	10	8	0	33	5	6	9	4	18	48	1	7	20
6100	5	28	55	2	5	21	2	19	15	10	17	27	0	4	39	5	21	29
6200	4	6	44	3	27	32	9	7	57	3	28	45	7	20	29	10	5	39
6300	2	14	33	5	19	43	3	26	39	9	10	3	3	6	20	2	19	48
6400	0	22	22	7	11	54	10	15	21	2	21	21	10	22	11	7	3	57
6500	11	0	11	9	4	5	5	4	4	8	2	38	6	8	2	11	18	6
6600	9	8	0	10	26	16	11	22	46	1	13	56	1	23	52	4	2	16
6700	7	15	49	0	18	27	6	11	28	6	25	14	9	9	43	8	16	25
6800	5	23	38	2	10	38	1	0	10	0	6	32	4	25	34	1	0	34
6900	4	1	26	4	2	50	7	18	52	5	17	50	0	11	25	5	14	43
7000	2	9	15	5	25	1	2	7	35	10	29	7	7	27	15	9	28	53

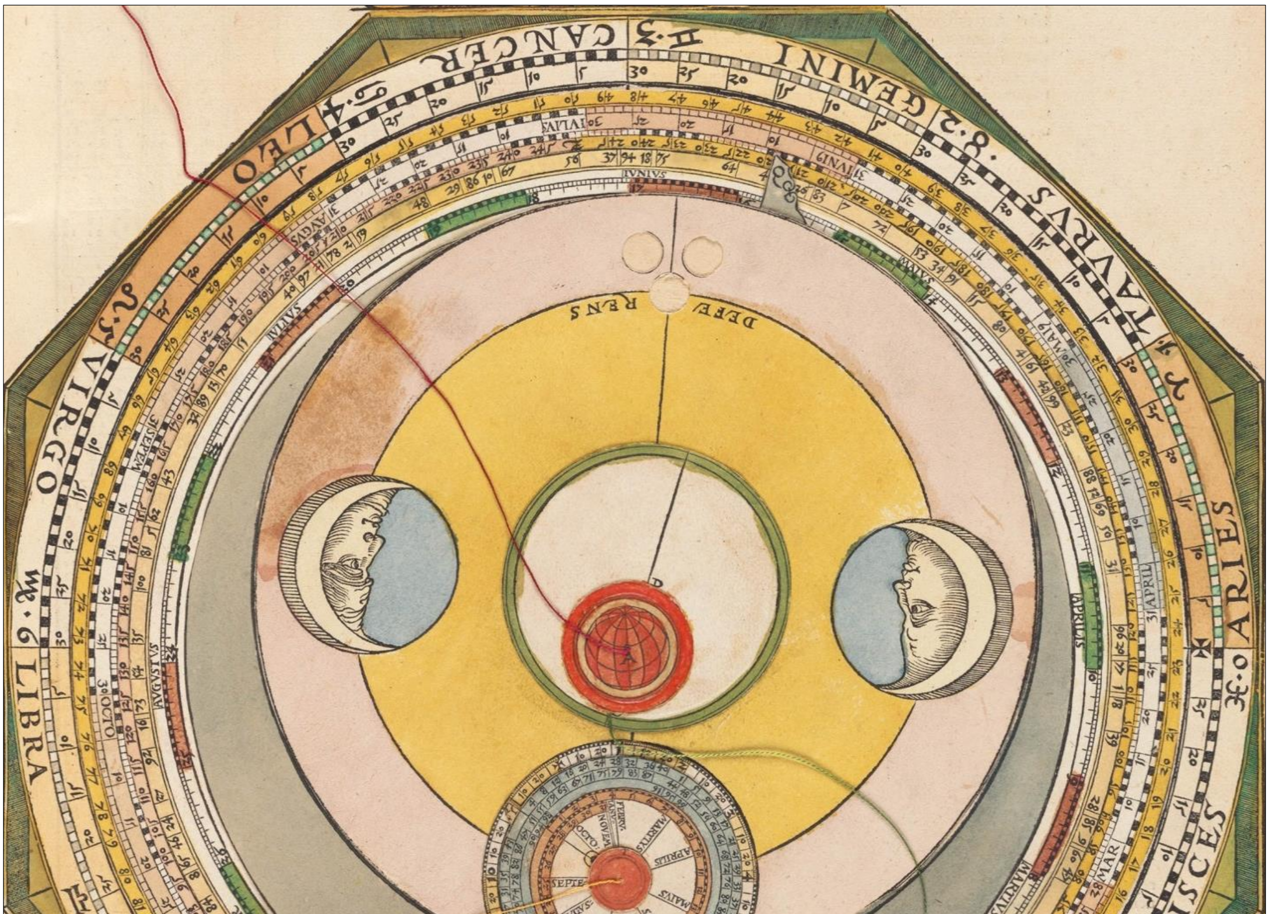


Figure 31: The lunar longitude volvelle.

There are year marks (Table 18a) located anti-clockwise, with angles given by

(year·365 + integer(year/4)) · lunar mean motion in longitude

and also month and days marks (Table 18b). Because the Moon moves more than 360° in a month, the layout of the days is a spiral and the months are a little cramped. Figure 32a shows

Table 18a: Lunar mean longitude year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
76	3.54	25	73.66	69	151.50	18	221.62	43	295.28
11	9.58	44	77.84	12	152.14	94	225.16	62	299.46
87	13.12	55	87.42	88	155.68	37	225.80	5	300.10
30	13.76	74	91.60	23	161.72	56	229.98	81	303.64
49	17.94	17	92.24	99	165.26	67	239.55	24	304.28
68	22.12	93	95.78	42	165.90	10	240.19	100	307.82
3	28.15	36	96.41	61	170.07	86	243.73	35	313.85
79	31.69	47	105.99	4	170.71	29	244.37	54	318.03
22	32.33	66	110.17	80	174.25	48	248.55	73	322.21
98	35.87	9	110.81	15	180.29	59	258.13	16	322.85
41	36.51	85	114.35	91	183.83	2	258.77	92	326.39
60	40.69	28	114.99	34	184.47	78	262.31	27	332.43
71	50.27	39	124.57	53	188.65	21	262.95	46	336.61
14	50.91	58	128.75	72	192.83	97	266.49	65	340.79
90	54.45	1	129.38	7	198.87	40	267.13	8	341.43
33	55.09	77	132.92	83	202.41	51	276.70	84	344.97
52	59.26	20	133.56	26	203.04	70	280.88	19	351.00
63	68.84	96	137.10	45	207.22	13	281.52	95	354.54
6	69.48	31	143.14	64	211.40	89	285.06	38	355.18
82	73.02	50	147.32	75	220.98	32	285.70	57	359.36

Table 18b: Lunar mean longitude month marks.

Months	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.
Angle	48.47	57.41	105.88	141.17	189.64	224.93	273.40	321.86	357.16	45.62	80.92	129.38

Figure 32a: Comparison of the year marks.

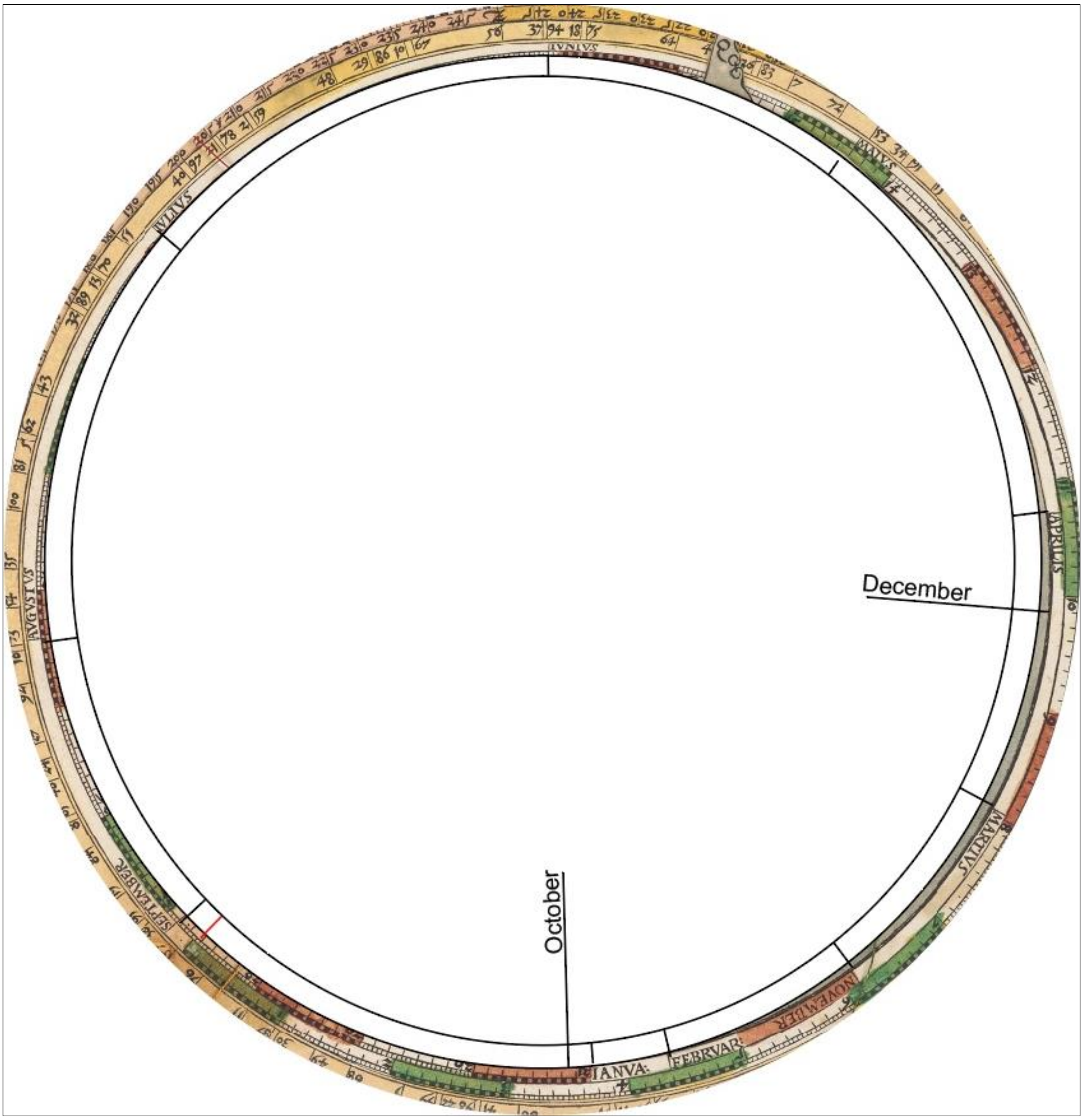


Figure 32b: Comparison of the month marks.

a comparison between calculated year marks and the volvelle disk, while Figure 32b shows a comparison with the calculated month marks. December and October were not visibly marked on the volvelle that I studied.

On top of disk P is the elongation disk Q, centred on the main axis, for setting the elongation with an index tab Q. On top of disk Q is the deferent disk, which rotates around an off-centre axis relative to the main axis on disk Q and moving with it. On top of the deferent disk, on the deferent circle sits the epicycle base disk with a 360° graduation clockwise along its periphery. Finally, centred on this disk there is a top disk with a tab Y and with year, month, and day marks for setting the argument. The marks on this disk are calculated by the same formula as the mean longitude marks but using the argument daily motion. The marks are arranged in three rings (see Table 19a).

Inside these marks is a spiral of 31 days (Table 19b), each day being subdivided into four six-hour parts. Finally, more centrally are the month marks (Table 19c). A partially reconstructed disk is shown in Figure 33.

There is a cap on the deferent disk with one thread A from the main axis and a second thread E off-centre and fixed to and following the motion of disk Q.

The working of the volvelle is as follows: The solar longitude is set with disk M in the same was as before by the thread from A, using the century mean solar longitude table, the year

Table 19a: Calculated argument year marks.

Outer Ring				Middle Ring				Inner Ring	
Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
4	7.95	6	185.39	51	1.54	49	184.10	94	0.25
8	15.90	10	193.34	55	9.49	53	192.05	98	8.20
12	23.84	14	201.29	59	17.44	57	200.00	91	81.02
16	31.79	18	209.23	63	25.38	61	207.94	95	88.97
20	39.74	22	217.18	67	33.33	65	215.89	99	96.92
24	47.69	26	225.13	71	41.28	69	223.84	88	174.86
28	55.64	30	233.08	75	49.23	73	231.79	92	182.81
32	63.59	34	241.03	79	57.18	77	239.74	96	190.76
36	71.53	38	248.98	83	65.13	81	247.68	100	198.70
40	79.48	42	256.92	87	73.07	85	255.63	89	263.58
1	88.72	3	266.16	44	87.43	46	264.87	93	271.53
5	96.67	7	274.11	48	95.38	50	272.82	97	279.48
9	104.62	11	282.06	52	103.33	54	280.77	90	352.30
13	112.57	15	290.01	56	111.27	58	288.72		
17	120.51	19	297.96	60	119.22	62	296.66		
21	128.46	23	305.90	64	127.17	66	304.61		
25	136.41	27	313.85	68	135.12	70	312.56		
29	144.36	31	321.80	72	143.07	74	320.51		
33	152.31	35	329.75	76	151.01	78	328.46		
37	160.25	39	337.70	80	158.96	82	336.40		
41	168.20	43	345.64	84	166.91	86	344.35		
2	177.44			45	176.15	47	353.59		

Table 19b: Computed day marks.

Day	Angle	Day	Angle	Day	Angle	Day	Angle
1	13.07	9	117.59	17	222.11	25	326.63
2	26.13	10	130.65	18	235.17	26	339.69
3	39.20	11	143.72	19	248.24	27	352.76
4	52.26	12	156.78	20	261.30	28	5.82
5	65.33	13	169.85	21	274.37	29	18.89
6	78.39	14	182.91	22	287.43	30	31.95
7	91.46	15	195.98	23	300.50	31	45.02
8	104.52	16	209.04	24	313.56		

Table 19c: Calculated month marks.

Months	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.
Angle	45.02	50.83	95.85	127.80	172.81	204.76	249.78	294.79	326.74	11.76	43.71

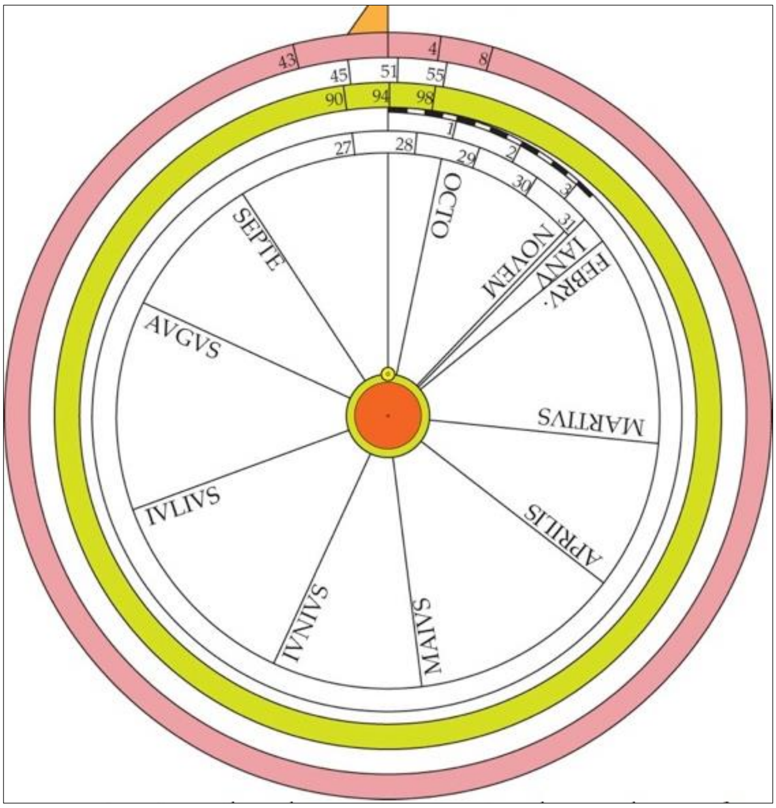


Figure 33: The top disk partially reconstructed.

Figure 34: The lunar latitude volvelle.

correction and the date scale. Then the lunar mean longitude is set in the same way on disk P. Then the elongation between the Moon and the Sun is read off from the elongation scale 0–250. The tab of the elongation disk Q is set on the same number, but on the opposite side of tab M. The thread from A is stretched to the mean longitude of the Moon and the deferent disk rotated such that the centre of the epicycle disk falls under the thread. Thread E is stretched to the centre of the epicycle disk and the cross mark on its lower disk is aligned with the thread and with the cross pointing away from E. The Y tab of the epicycle disk is set to the argument using the central thread and the year, month and day marks. Finally, the thread from A is stretched over the small yellow Moon patch near the centre and continued out to the zodiacal scale on the mater where the true longitude of the Moon is read off.

3.7.2 The Latitude of the Moon (page 52)

A circular disk (Figure 34) is mounted centrally on the mater that has a zodiac scale. The disk has two tabs, one for the ascending node (dragon's head, CAPVT) and one for the descending node (dragon's tail, CAVDA). There is a sequence of year marks and a smaller scale for setting the month and day. The year marks are calculated in the usual way but using the daily motion of the node (Caput Draco). In the centre of the disk is a picture of a dragon. Once the position of the node has been set on the scale of the mater, the latitude can either be read off by using the thread to mark the longitude of Moon on the zodiac of the mater or by marking the argument of latitude, the difference between the Moon's longitude and the longitude of the node, on the scale on the periphery of the top disk. The latitude is read off from one of the arched scales surrounding the dragon. Effectively the volvelle computes the latitude of the Moon as 5°· sin(argument of latitude).

Table 20 shows the calculated year angles and Figure 35 a comparison of the computed year marks with the marks on the volvelle.

Table 20: Calculated year marks.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
56	3.13	78	68.61	100	134.15	48	208.39	52	285.76
19	7.45	41	72.99	7	135.35	11	212.72	15	290.08
75	10.58	97	76.12	63	138.48	67	215.84	71	293.21
38	14.95	4	77.37	26	142.85	30	220.22	34	297.59
94	18.08	60	80.49	82	145.98	86	223.35	90	300.71
1	19.33	23	84.82	45	150.36	49	227.72	53	305.09
57	22.46	79	87.94	8	154.73	12	232.10	16	309.47
20	26.83	42	92.32	64	157.86	68	235.23	72	312.59
76	29.96	98	95.45	27	162.18	31	239.55	35	316.91
39	34.28	5	96.69	83	165.31	87	242.68	91	320.04
95	37.41	61	99.82	46	169.69	50	247.05	54	324.42
2	38.66	24	104.20	9	174.06	13	251.43	17	328.79
58	41.78	80	107.32	65	177.19	69	254.55	73	331.92
21	46.16	43	111.65	28	181.56	32	258.93	36	336.30
77	49.29	99	114.77	84	184.69	88	262.06	92	339.42
40	53.66	6	116.02	47	189.01	51	266.38	55	343.75
96	56.79	62	119.15	10	193.39	14	270.76	18	348.12
3	57.99	25	123.53	66	196.52	70	273.88	74	351.25
59	61.11	81	126.65	29	200.89	33	278.26	37	355.62
22	65.49	44	131.03	85	204.02	89	281.38	93	358.75

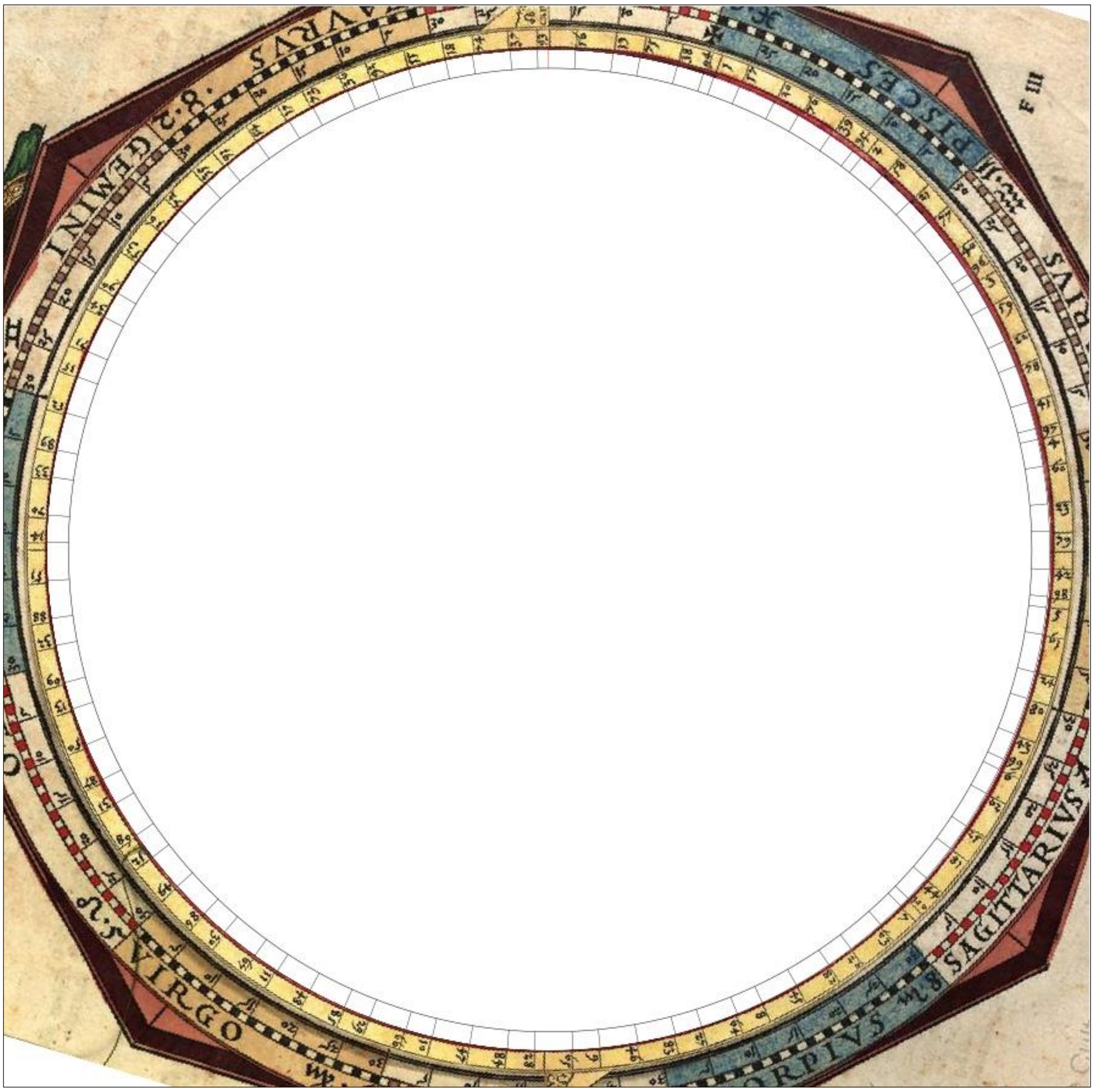


Figure 35: Comparison with the volvelle marks.

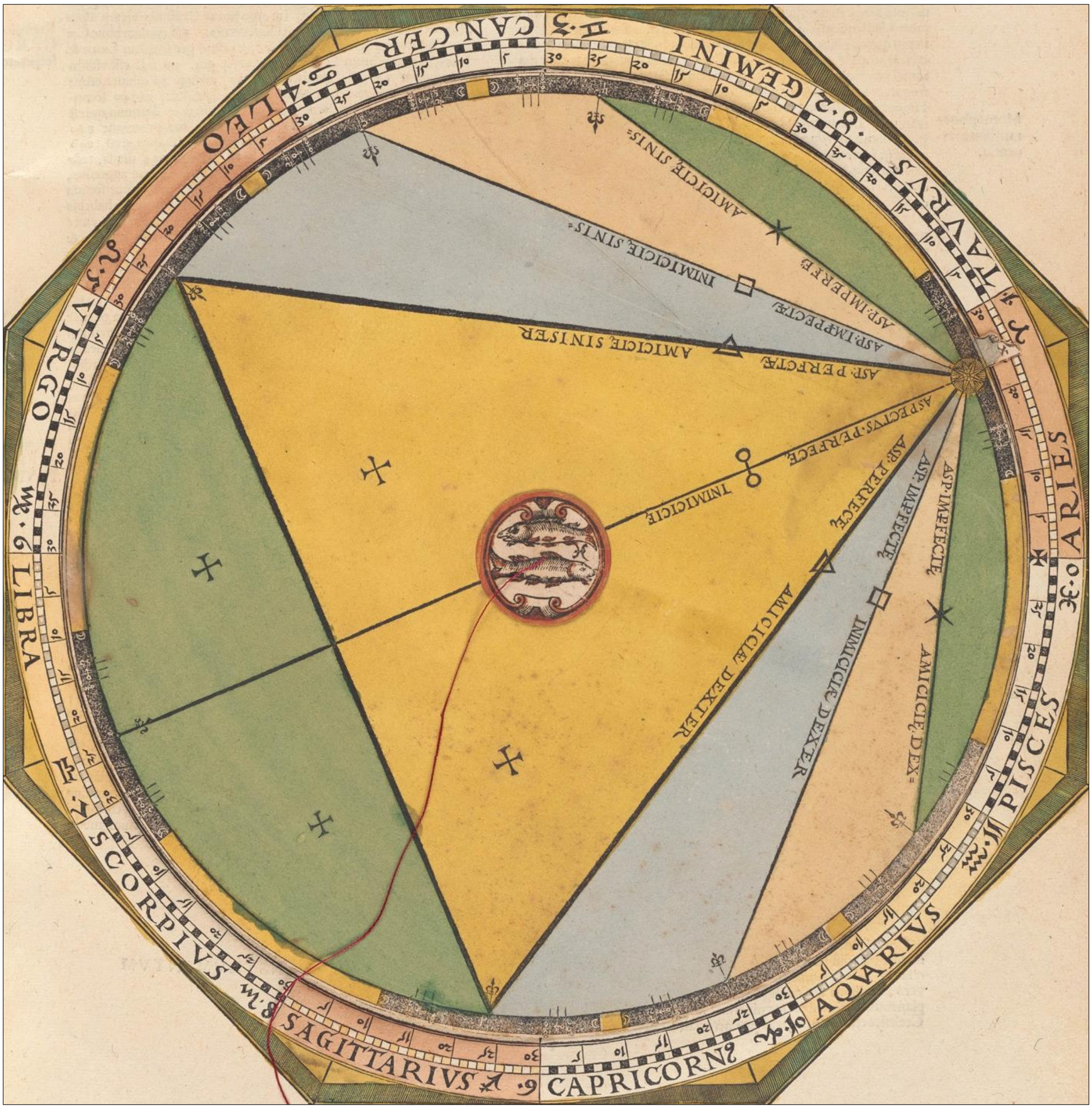


Figure 36: The planetary aspects volvelle.

3.8 The Planetary Aspects (pages 54, 56, and 58)

The volvelle (Figure 36) on page 54 computes the aspects, or difference in longitude, between two astronomical bodies. The five aspects are: conjunction (0°), sextile (60°), quartus (90°), trine (120°) and opposition (180°). The different planets can be within different angular sectors around the exact aspect: Mercury and Venus ±7°, Mars ±8°, Saturn and Jupiter ±9° and the Moon ±12°, while the Sun ±15° as is symmetrically marked for each of the aspects marked on the volvelle. There is a moveable disk on top of the mater with an index X that can be set for the longitude of one planet and the thread from the centre that can be set for the longitude of another planet or the Moon or the Sun.

The volvelles in pages 56 and 58 of AC (see Figures 37 and 38) are used to compute the time to reach an aspect between the Moon and a planet or between two planets, simply computed as the ratio between the difference in longitude and the difference in angular speed. A thread with a small sliding bead is attached centrally. The difference, irrespective of sign, in daily motion is set radially with the bead and the difference in longitude, irrespective of sign, is set azimuthally with the thread, and the time is read off from the position of the bead.

3.9 Lunar Phases and Nodes (pages 59 and 61)

This volvelle (Figure 39) is intended to be used in sequence with the following five volvelles to calculate the time and circumstances of eclipses.

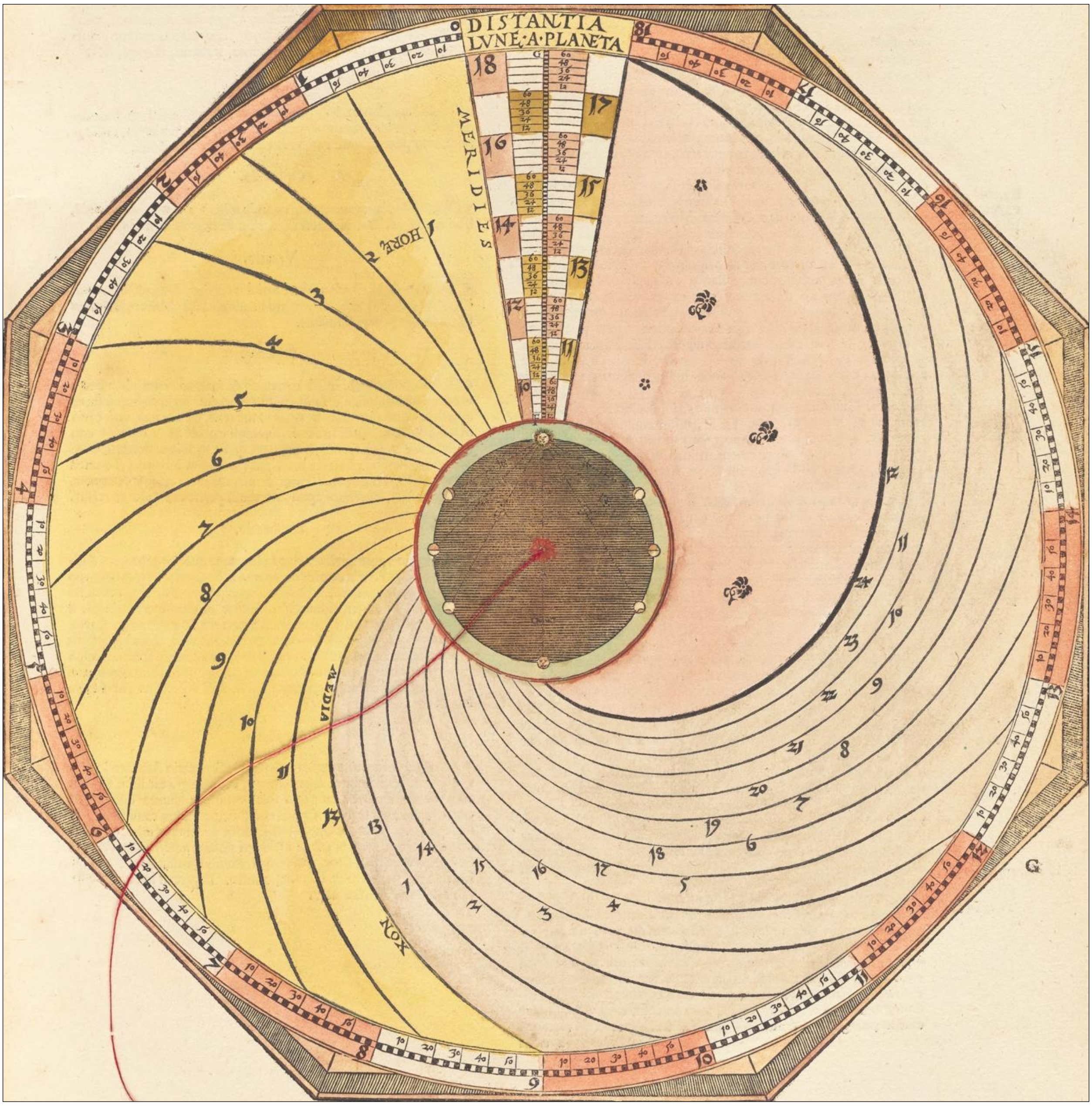


Figure 37: Aspect time for the Moon and a planet.

The table on page 59 of AC gives the day and time for each century in January for conjunctions of the Moon and the Sun and the Moon and its node. The unit in the table is days: hours: minutes in January, the first occurrence of a conjunction in that month. The numbers in the table are calculated in the following way. First the number of months in a century is calculated by dividing the number of Julian days in the centuries (century number · 36525) with the lunar month length, synodic for the conjunctions with the Sun and draconic for the conjunctions with the node. This will result in an integer number of lunar months and a fraction. Multiplying one minus the fraction with the number of days in the lunar month and adding the radix (24:10:24 days for the synodic month and 23:6:33 days for a draconic month) will give the days remaining until the first conjunction in the century, i.e. the date of the conjunction in January. Whenever the date is larger than 31, a synodic or draconic month is subtracted in order to have the conjunction occur in January. This will generate Table 21. For the draconic dates there is a systematic error of up to +5 minutes at the end of Apianus' tables for centuries before Christ.

The volvelle on page 61 computes phases of the Moon and also approximate eclipse dates. The mater has a circular scale with a 365-day year. The bottom moveable disk is mounted centrally on the mater and has a tab marked X and a set of year marks. The excess in days of '*y'* years over 12 synodic months is

$a_S = y \cdot (365 - 12 \cdot m) + \text{integer}((y - 1)/4) + 1$, m = days in a synodic month.

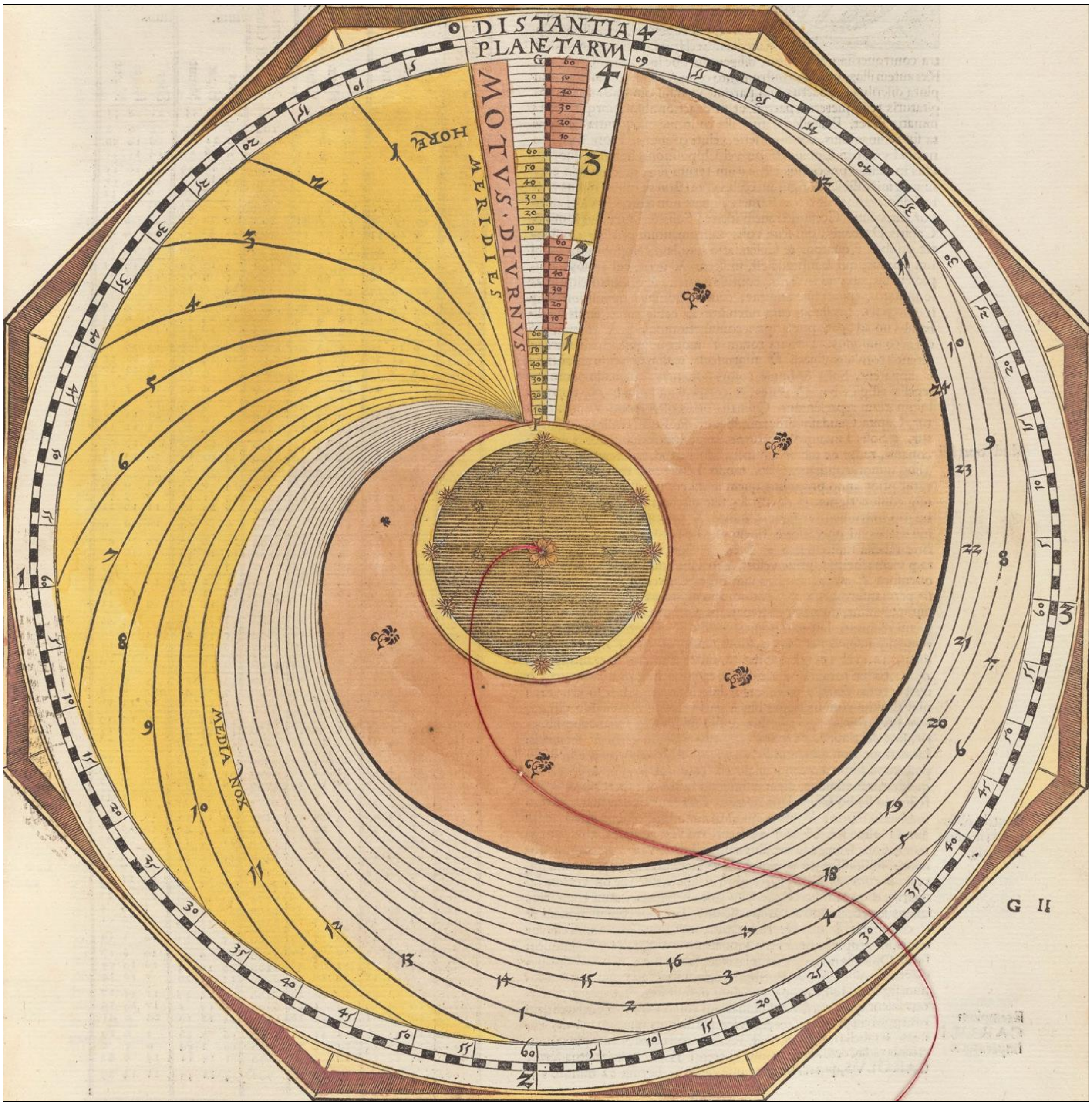


Figure 38: Aspect time for two planets.

This excess is normalised to be within one synodic month by setting

$E_S = a_S - m \cdot \text{integer}(a_S / m)$

Finally, in order to group the angles in ten-year portions

$a = (E_S + m \cdot \text{integer}(y / 10)) \cdot 360/365$,

where the last factors convert the days to degrees. This will move the group exactly an integer number of lunations backwards in the year and avoids having the year marks on top of each other.

The result is shown in Table 22. It is compared with the actual marks on the volvelle disk in Figure 40. There is a gap g between the index tab X and the start of the year numbers that is $g = 365 - 11 \cdot m = 40.163$ days or 39.61°.

A lunar year of 12 lunations is 12·29.530591 = 354.367 days that is 365 – 354.367 = 10.633 days shorter than a normal year of 365 days. Thus, if we start with lunation 1 on 1 January, lunation 12 will fall 10.633 days before the start of the next year. The previous lunation will then be 10.633 + 29.530 = 40.163 days before the end of that year. The intention of the gap is to keep Roman numbered new and full moons on disk Y (see below) within the year.

On top of the previous disk is a second centrally aligned disk. It has an index tab Z and is used to set the excess days in years over 13 draconic months, $13 \cdot d = 353.76$ days. The calculation is done almost as for the synodic excess but using the days in a draconic month and using 13 draconic months.

Figure 39: Lunar phases volvelle.

Table 21: Calculated conjunction times for centuries.

	Solar conjunction						Nodal conjunction					
	CE			BCE			CE			BCE		
0	24	10	24	24	10	24	23	6	33	23	6	33
100	28	18	35	20	2	13	17	1	51	2	6	10
200	3	14	2	15	18	2	10	21	8	8	10	52
300	7	22	13	11	9	51	4	16	26	14	15	35
400	12	6	23	7	1	41	25	16	49	20	20	17
500	16	14	34	2	17	30	19	12	7	27	0	59
600	20	22	45	27	22	3	13	7	24	6	0	36
700	25	6	56	23	13	52	7	2	42	12	5	19
800	29	15	7	19	5	41	0	21	59	18	10	1
900	4	10	34	14	21	30	21	22	23	24	14	43
1000	8	18	45	10	13	19	15	17	40	3	14	20
1100	13	2	56	6	5	8	9	12	58	9	19	3
1200	17	11	6	1	20	58	3	8	15	15	23	45
1300	21	19	17	27	1	31	24	8	39	22	4	27
1400	26	3	28	22	17	20	18	3	56	1	4	4
1500	0	22	55	18	9	9	11	23	14	7	8	47
1600	5	7	6	14	0	58	5	18	31	13	13	29
1700	9	15	17	9	16	47	26	18	55	19	18	11
1800	13	23	28	5	8	36	20	14	12	25	22	54

1900	18	7	38	1	0	25	14	9	30	4	22	31
2000	22	15	49	26	4	59	8	4	47	11	3	13
2100	27	0	0	21	20	48	2	0	5	17	7	55
2200	1	19	27	17	12	37	23	0	28	23	12	38
2300	6	3	38	13	4	26	16	19	46	2	12	15
2400	10	11	49	8	20	15	10	15	3	8	16	57
2500	14	20	0	4	12	4	4	10	21	14	21	39
2600	19	4	11	0	3	53	25	10	44	21	2	22
2700	23	12	21	25	8	27	19	6	2	0	1	59
2800	27	20	32	21	0	16	13	1	19	6	6	41
2900	2	15	59	16	16	5	6	20	37	12	11	23
3000	7	0	10	12	7	54	0	15	55	18	16	6
3100	11	8	21	7	23	43	21	16	18	24	20	48
3200	15	16	32	3	15	32	15	11	35	3	20	25
3300	20	0	43	28	20	5	9	6	53	10	1	7
3400	24	8	53	24	11	55	3	2	11	16	5	50
3500	28	17	4	20	3	44	24	2	34	22	10	32
3600	3	12	31	15	19	33	17	21	51	1	10	9
3700	7	20	42	11	11	22	11	17	9	7	14	51
3800	12	4	53	7	3	11	5	12	27	13	19	34
3900	16	13	4	2	19	0	26	12	50	20	0	16
4000	20	21	15	27	23	33	20	8	7	26	4	59
4100	25	5	26	23	15	22	14	3	25	5	4	35
4200	29	13	36	19	7	12	7	22	43	11	9	18
4300	4	9	3	14	23	1	1	18	0	17	14	0
4400	8	17	14	10	14	50	22	18	23	23	18	43
4500	13	1	25	6	6	39	16	13	41	2	18	19
4600	17	9	36	1	22	28	10	8	59	8	23	2
4700	21	17	47	27	3	1	4	4	16	15	3	44
4800	26	1	58	22	18	50	25	4	39	21	8	27
4900	30	10	8	18	10	40	18	23	57	0	8	3
5000	5	5	35	14	2	29	12	19	15	6	12	46
5100	9	13	46	9	18	18	6	14	32	12	17	28
5200	13	21	57	5	10	7	0	9	50	18	22	11
5300	18	6	8	1	1	56	21	10	13	25	2	53
5400	22	14	19	26	6	29	15	5	31	4	2	30
5500	26	22	30	21	22	18	9	0	48	10	7	12
5600	1	17	57	17	14	7	2	20	6	16	11	55
5700	6	2	7	13	5	57	23	20	29	22	16	37
5800	10	10	18	8	21	46	17	15	47	1	16	14
5900	14	18	29	4	13	35	11	11	4	7	20	56
6000	19	2	40	0	5	24	5	6	22	14	1	39
6100	23	10	51	25	9	57	26	6	45	20	6	21
6200	27	19	2	21	1	46	20	2	3	26	11	3
6300	2	14	29	16	17	35	13	21	20	5	10	40
6400	6	22	39	12	9	25	7	16	38	11	15	23
6500	11	6	50	8	1	14	28	17	1	17	20	5
6600	15	15	1	3	17	3	22	12	19	24	0	47
6700	19	23	12	28	21	36	16	7	36	3	0	24
6800	24	7	23	24	13	25	10	2	54	9	5	7
6900	28	15	34	20	5	14	3	22	11	15	9	49
7000	3	11	1	15	21	3	24	22	35	21	14	31

Table 22: Calculated year mark positions.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
3	3.32	25	65.20	41	120.44	60	178.01	82	239.89
6	6.64	28	67.54	44	122.78	63	181.34	85	243.22
9	9.97	20	69.05	47	126.10	66	184.66	88	245.55
1	11.47	23	72.37	42	130.93	69	187.98	80	247.06
4	13.81	26	75.69	45	134.25	61	189.49	83	250.38
7	17.13	29	79.01	48	136.59	64	191.82	86	253.70
2	21.96	21	80.52	40	138.09	67	195.14	89	257.03
5	25.28	24	82.86	43	141.42	62	199.97	81	258.53
8	27.62	27	86.18	46	144.74	65	203.30	84	260.87
19	29.43	38	87.99	57	146.55	76	204.12	95	262.68
11	30.94	30	89.50	52	150.39	79	207.45	98	266.01
14	34.26	33	92.82	55	153.72	71	208.95	90	267.51
17	37.59	36	95.16	58	157.04	74	212.28	93	270.83
12	41.43	39	98.48	50	158.55	77	215.60	96	273.17
15	44.75	31	99.99	53	161.87	72	219.44	99	276.49
18	48.07	34	103.31	56	164.20	75	222.76	91	278.00
10	49.58	37	106.63	59	167.53	78	226.08	94	281.32
13	52.90	32	110.47	51	169.03	70	227.59	97	284.64
16	55.24	35	113.80	54	172.36	73	230.91	100	286.98
22	61.88	49	118.93	68	176.51	87	235.06	92	288.49

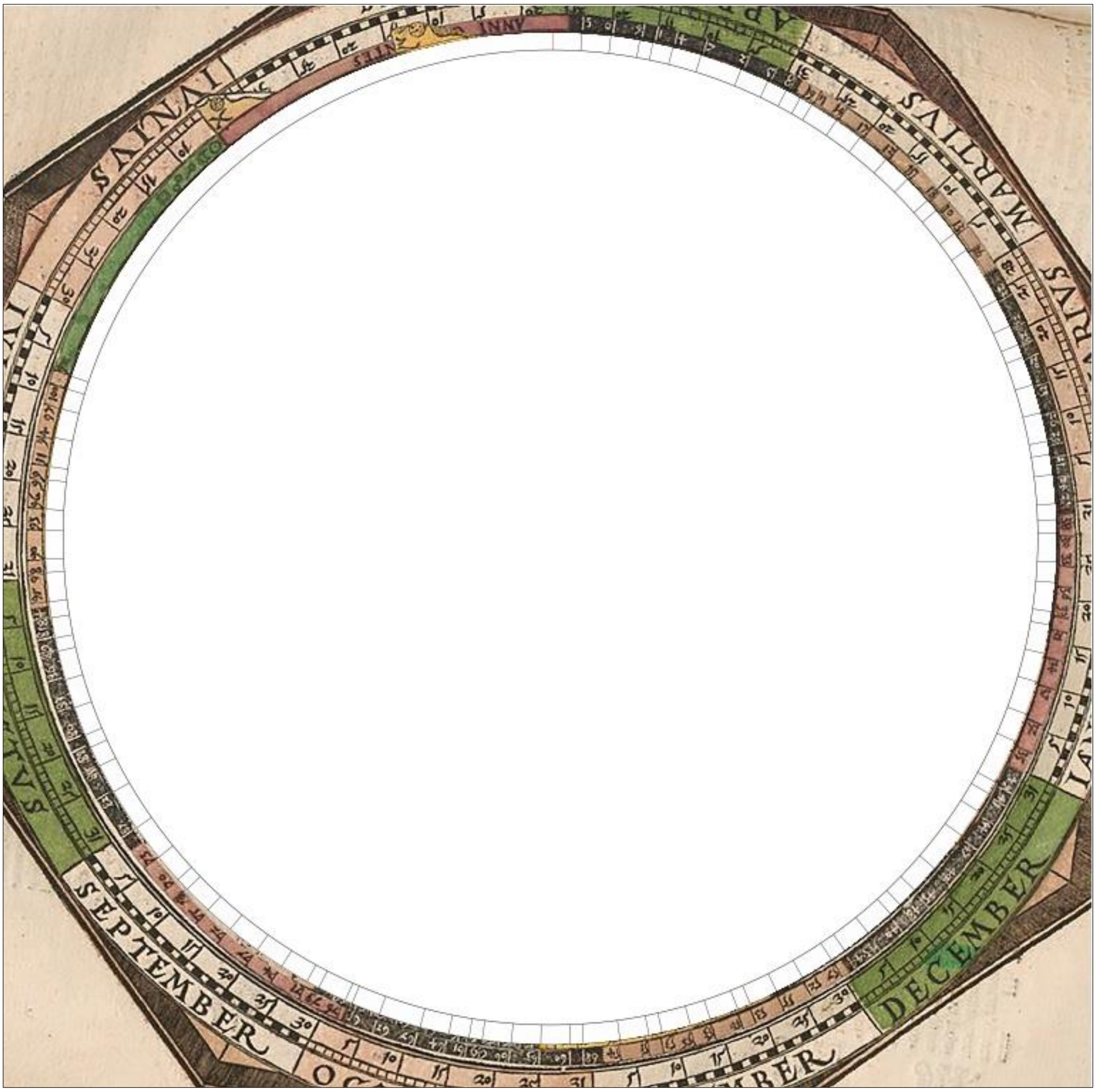


Figure 40: Comparison of year marks.

$a_S = y \cdot (365 - 13 \cdot d) + \text{integer}((y - 1)/4) + 1$, d = days in a draconic month.

The angles are grouped in the same way as for the synodic angles. The result is shown in Table 23. It is compared with the actual marks on the volvelle disk in Figure 41. There is a gap g between the index tab Z and the start of the year numbers that is $g = 365 - 12 \cdot d = 38.4533$ days or 37.927°.

On top of the Z disk is the disk for Moon phases. It has an index tab Y and an outer band with Moon symbols, black Moons for New Moons, white Moons for Full Moons and crescent Moons in between for quarter moons. Inside this band is another band with Roman numbers I, II, III,…,XII and in between these numbers symbols for conjunction and opposition. Between the numbers I and XII is a cross for a Full Moon and a double cross for a New Moon. The New Moons have an angular spacing of

$a_n = n \cdot m \cdot 360/365$, $n = 0, 1, 2, \ldots, 11$ corresponding to moons I, II, III,…, XII.

There is a 'forward' New Moon for n = 12, marked with a double cross, which may fall on the subsequent year. As before, m is the number of days in a synodic month. The Full Moons are located at angles for half-integer n:s, and there is a 'backward' Full Moon for $n = -0.5$ that is marked by one cross that may fall in the previous year. The quarter Moons are generated by taking n = 0.25, 0.75 and so on.

Figure 42a shows the New Moons generated by the formula, compared with the volvelle marks. Figure 42b shows a comparison with the corresponding Full Moons. The extra Moons are marked by green lines.

Table 23: Calculated year mark positions.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
5	3.73	29	61.02	43	111.84	69	165.46	88	219.02
3	7.41	22	61.96	48	114.59	62	166.39	81	220.94
8	10.15	27	64.70	41	116.51	67	169.14	86	223.69
1	12.07	20	65.64	46	119.25	60	170.07	84	227.36
6	14.82	25	69.37	44	122.93	65	173.80	89	231.09
4	18.50	23	73.05	49	126.66	63	177.48	82	232.03
9	22.23	28	75.79	42	127.60	68	180.22	87	234.77
2	23.16	21	77.71	47	130.34	61	182.15	80	235.71
7	25.90	26	80.45	40	131.27	66	184.89	85	239.44
19	27.71	38	82.26	57	136.81	76	190.37	90	242.18
12	28.65	31	83.20	50	137.75	74	195.04	95	244.92
17	32.38	36	85.94	55	140.49	79	197.78	100	247.67
10	33.31	34	90.60	53	145.15	72	198.72	93	249.59
15	36.06	39	93.35	58	147.90	77	202.45	98	252.33
13	40.72	32	94.28	51	148.83	70	203.38	91	253.27
18	43.46	37	98.01	56	151.58	75	206.13	96	256.01
11	44.40	30	98.95	54	156.24	73	210.79	94	260.68
16	47.14	35	101.69	59	158.98	78	213.53	99	263.42
14	51.81	33	106.36	52	159.92	71	214.47	92	264.36
24	57.29	45	108.16	64	161.73	83	216.28	97	268.08

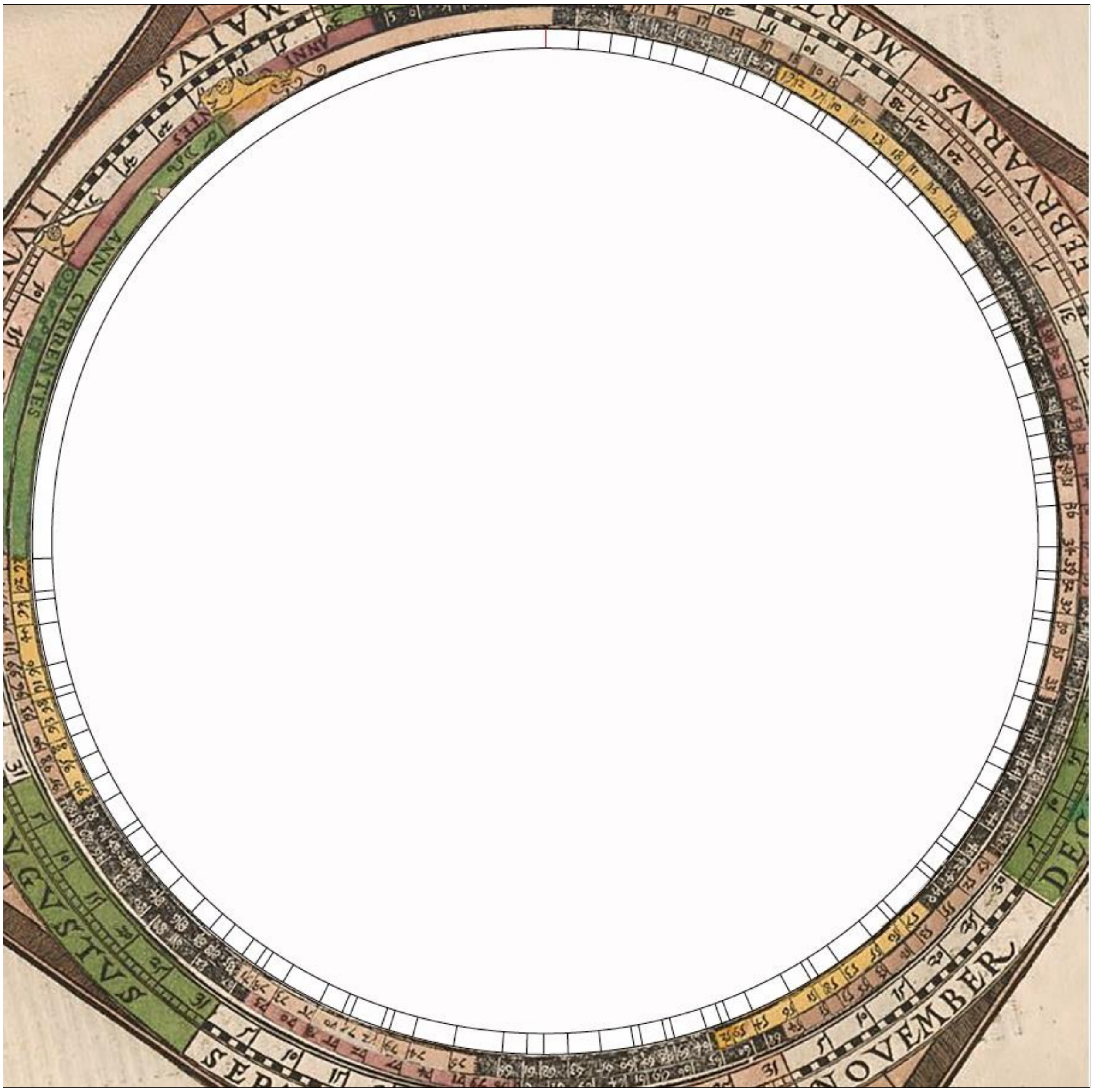


Figure 41: Comparison of year marks.

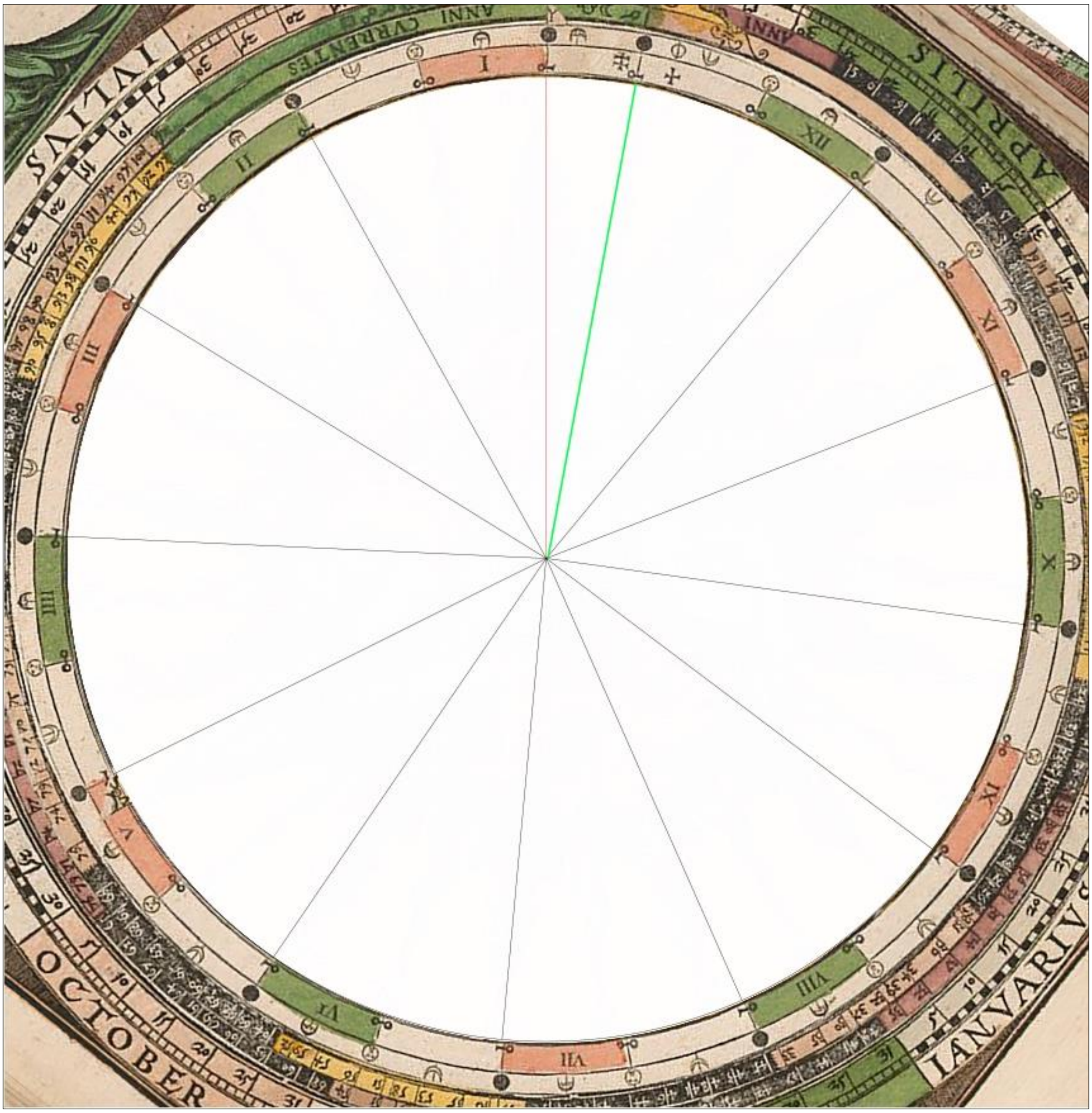


Figure 42a: New Moon marks.

Finally, there is a top disk with dragon heads and tails and with a tab V. The positions of the heads and tails are generated by the same formula as above but instead using the days in a draconic month:

$a_n = n \cdot d \cdot 360/365$, n = 0, 1,2,…,13 for heads 1, 2, 3,…,14 and with half integer n:s for the tails.

Each head and tail is marked by its respective number. There is a dragon's tail marked with a cross, that is the tail coming before head 1, for $n = -0.5$. Figures 43a and 43b show a comparison with computer-generated radial lines compared with the heads and tails of the dragons.

To use the volvelle, the X index is set to the radix days of the century in January taken from the table for the conjunctions with the Sun. Then the Z index is set to the century days in January of the conjunction with the node. This volvelle does not use elapsed centuries but current centuries. Disks X and Y are then locked in their set positions relative to one another. The Y disk tab is set close to the middle of January. The thread is lined up with the year of the century on the X disk and the Y disk is rotated such that the nearest New Moon on the Y disk is aligned with the thread. The dates of the different lunar phases can now be read from the symbols on the Y disk. In the same way, the V disk tab is set close to the middle of January. Note the dragon's head that is closest to the year mark on the Z disk and use the thread to rotate the V disk such that this head is aligned with the year mark of the Z disk. The days of the passages of the nodes can then be read off from the top disk. Where the number of the head or tail of a dragon

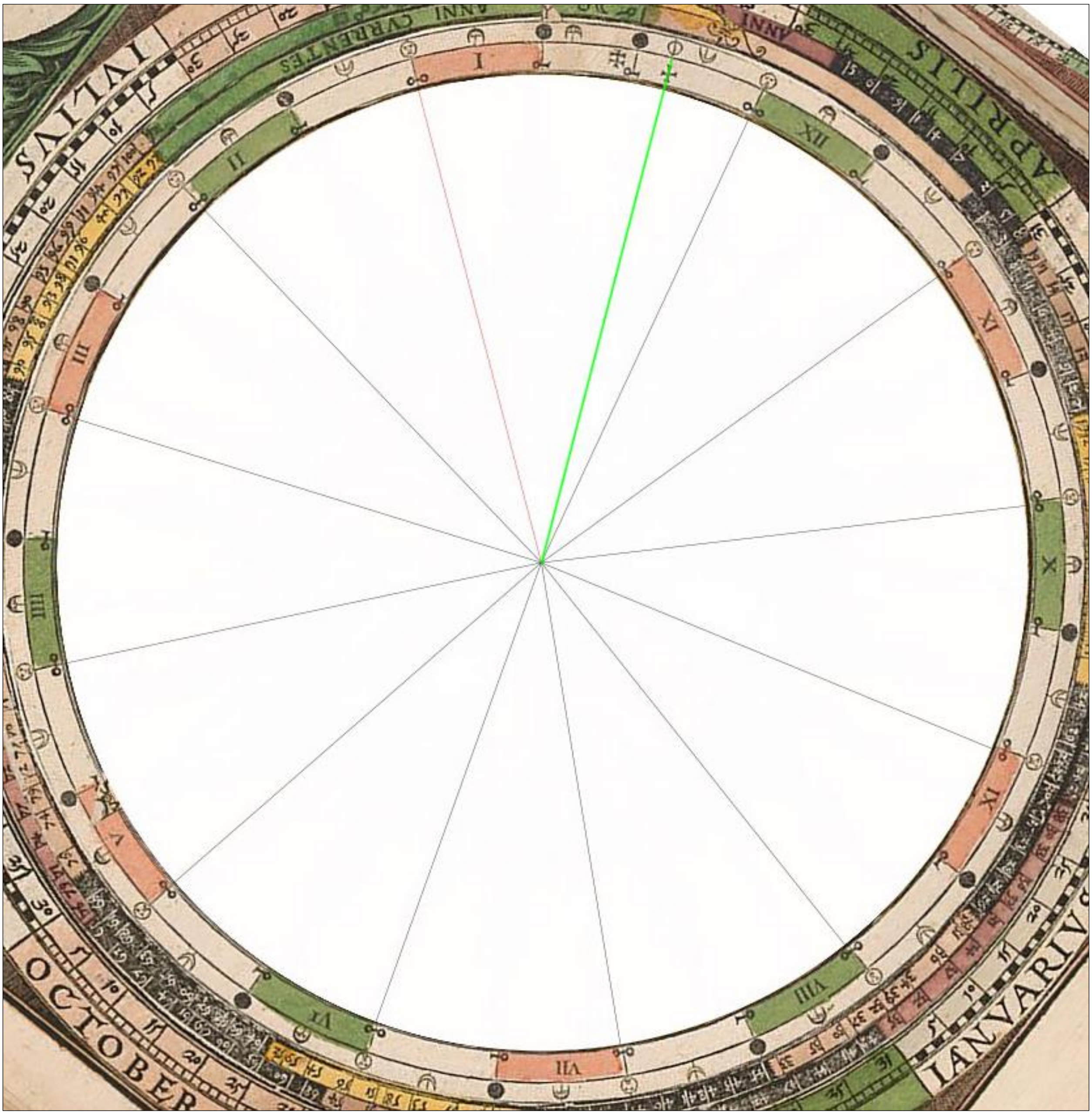


Figure 42b: Full Moon marks.

is close to the number of a New or Full Moon, you expect an eclipse of the Sun or the Moon and you will get an approximate date for the event. The numbers of the chosen conjunctions can be remembered and then used with the following volvelles to compute the mean time of a New or Full Moon or an eclipse. The lunar phases marked with crosses may fall in the previous or subsequent year.

3.10 More Precise Eclipse Times (page 62)

This volvelle (Figure 44) has a rather complex layout. It allows you to compute more precise eclipse times. It is a kind of zoomed-up version of the previous volvelle. It uses data from the previous volvelle where the date of a possible eclipse has been found. If it is an opposition (Full Moon) and occurs in the night, it may be a lunar eclipse, but if it is a conjunction (New Moon) and occurs in the day, it may be a solar eclipse. The mater has a circular graduation of 31 days for a full turn, each day being subdivided into hours. The days spiral inwards and anti-clockwise to 31 December. As July and August both have 31 days, the same set of 31 days can be used.

On top of the mater is a disk centred on the mater with an index tab marked XY. It has a set of year marks. Let y be the year. Then determine an integer n such that

$(n - 1) \cdot m < y \cdot 365 + \text{integer}((y - 1) / 4) + 1 < n \cdot m$

and calculate the angle $a = ((n \cdot m - y \cdot 365 + \text{integer}((y - 1) / 4) + 1) \cdot 360 / 31$. The last factors convert from the 31-day graduation to degrees.

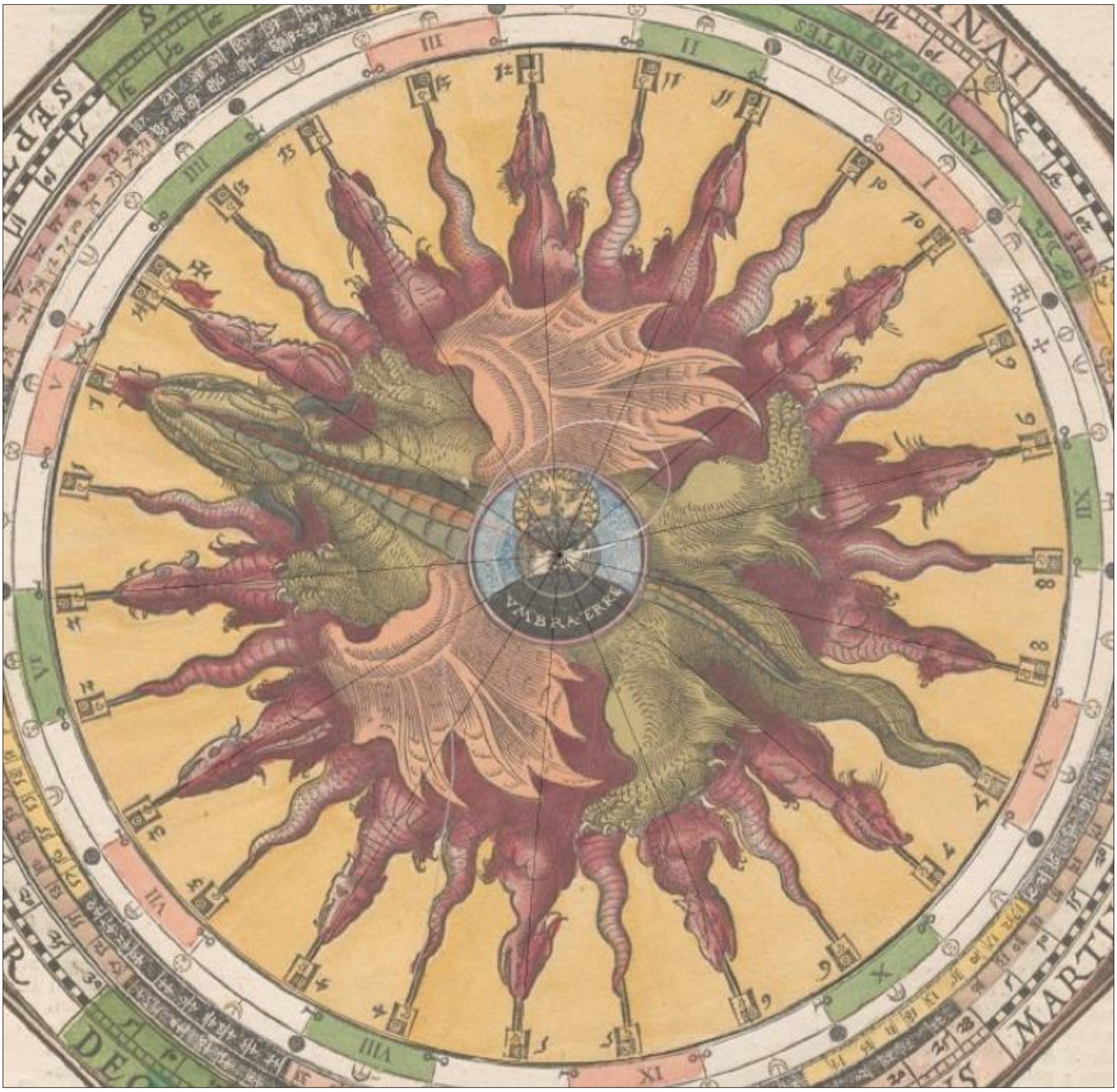


Figure 43a: The dragons' heads.

This procedure will generate a table of year marks (Table 24), compared with the marks of the volvelle disk in Figure 45.

In a band inside the band with year marks, the XY disk has a series of Moon symbols, Full Moons and New Moons.

A synodic month is 31 – 29.530591 = 1.469409 days shorter than a full turn on the 31-day scale of the mater. Converted to degrees this is 1.460409 ·360 / 31 = 17.064° (clockwise). Full Moon I of the Moon band starts half a lunation later than New Moon I. A half synodic month contains 14.765295 days or 171.468° anti-clockwise = 188.532° clockwise. Thus, we have New Moons at angles 17.064° · *n* and full moons at 188.532° + 17.064° · *n*, n = 0,1,2,…. We get Table 25 and a comparison with the volvelle in Figure 46.

On top of this disk is a disk with an index tab ZV. It has a set of year marks, computed with the same procedure as for the XY disk but instead using the number of days in a draconic month. The table generated is shown in Table 26 and the disk marks are compared in Figure 47. Year 59 is missing on the volvelle disk and year 35 is slightly misplaced.

Inside the band with year marks are five bands with pictures of dragons' heads and tails, each having a number from 1 to 14. Behind each head or tail is a sector with a Sun and a Moon icon.

A draconic month is 31 – 27.212223 = 3.787777 days shorter than a full turn of 31 days. Converted to degrees this is 3.787777·360/31 = 43.987° (clockwise). A dragon's tail is displaced half a draconic month anti-clockwise or 0.5·27.212223 = 13.60611 days or 158.006° anti-clockwise or 201.994°

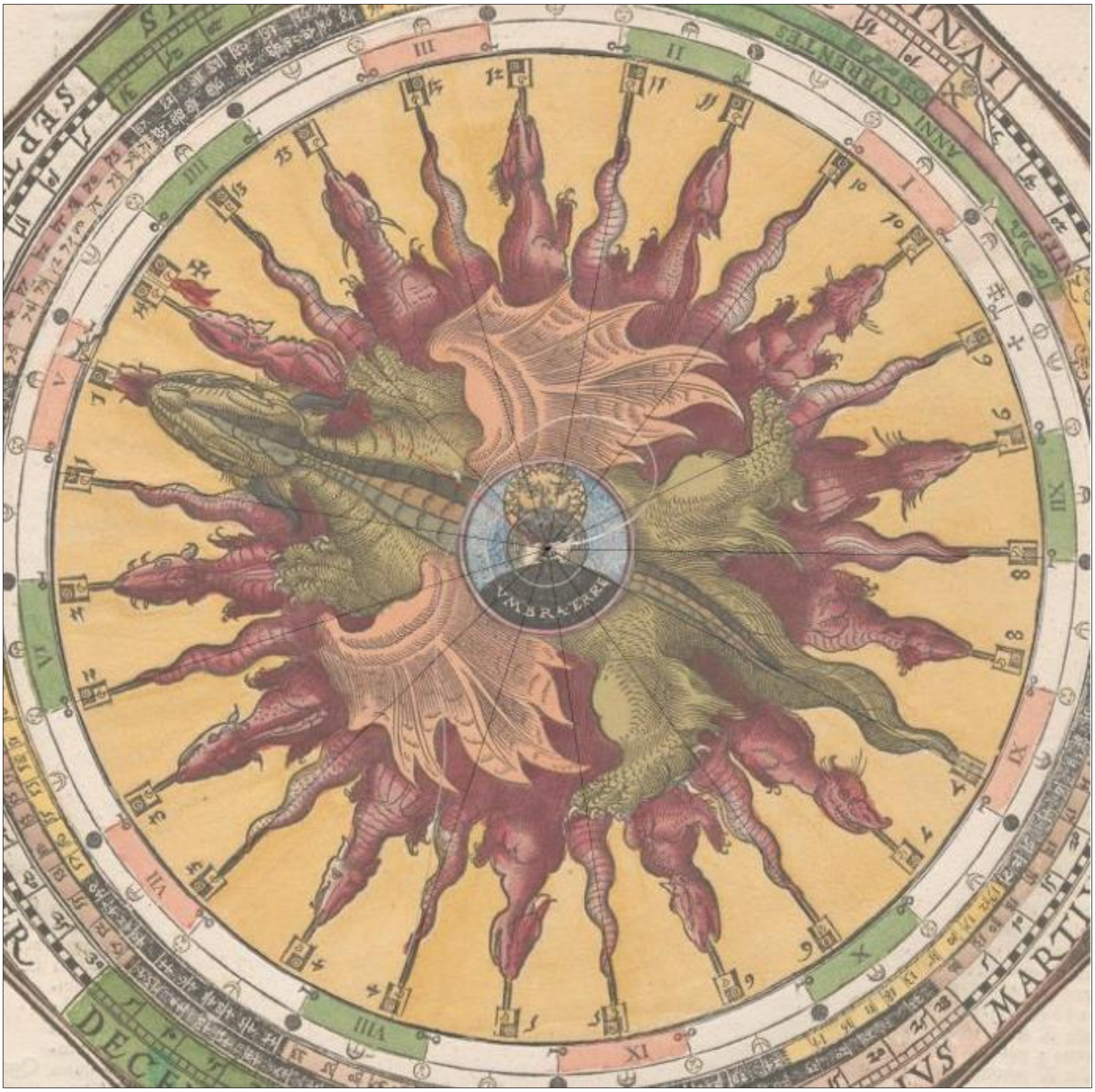


Figure 43b: The dragons' tails.

clockwise. Thus, the heads have positions 43.987°·*n* and the tails 201.994° + 43.987˚· *n*. This generates Table 27, which is compared with the volvelle in Figure 48. Tail number 14 should have an angle of 53.8246 by the formula. However, it falls on the following year and has to be augmented by 185.4707 to 239.2753. This is also why it is located in the separate fifth, innermost band. The excess of 14 draconic months over a year of 365 days is 15.971 days or 15.971·360 / 31 = 185.4707°. Heads are in band 1 and 2 and tails in 3, 4, and 5.

Finally, there is a topmost disk (DIES ET HORÆ) with a tab T and graduated counter-clockwise from 0 to 28 and just repeats the angles of 28 days and hours on the spiral scale.

The XY disk is set to the day, hour and minute as indicated by the century table for the conjunction of the Moon with the Sun. The setting is corrected to the year on the XY disk by aligning the tread with the year and rotating the XY tab to coincide with the thread. The ZV disk is set in the same manner using the century node conjunction table and the year mark on the ZV disk. The volvelle is now set to show the positions of the New and Full Moons and the position of the nodes. The date and time of a solar or lunar eclipse can now be found at positions where a New or Full Moon aligns with a node and the Latin number of the syzygy agrees with the number of the head or tail of a dragon.

In some of the volvelles there are two special marks at the end of January (Figure 44). The first one is a star with node symbol (✶☊) located at 27 January 5.1 hours, and the second one

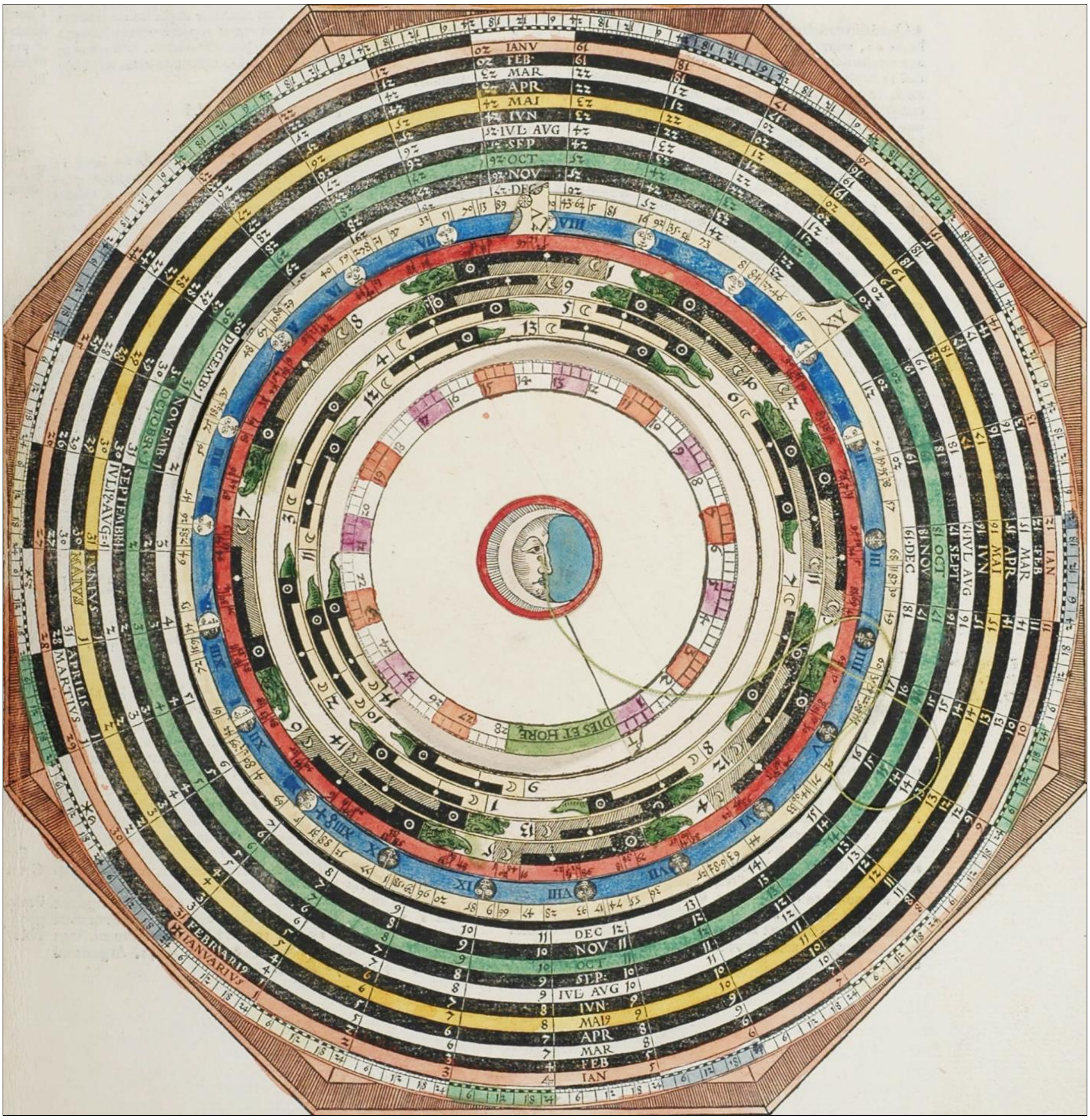


Figure 44: The volvelle for eclipse times.

Table 24. Calculated year marks on the XY disk.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
65	6.91	32	71.00	7	141.23	58	208.62	90	279.62
46	10.52	97	77.91	64	142.00	39	212.23	14	282.46
27	14.14	21	80.75	53	151.75	96	213.00	71	283.23
84	14.91	78	81.52	34	155.37	20	215.84	52	286.84
8	17.75	2	84.37	91	156.14	85	222.75	41	296.60
73	24.66	59	85.14	15	158.98	9	225.59	98	297.37
54	28.27	40	88.75	72	159.75	66	226.37	22	300.21
35	31.89	29	98.50	61	169.50	47	229.98	79	300.98
92	32.66	86	99.27	42	173.12	28	233.59	3	303.82
16	35.50	10	102.12	99	173.89	93	240.50	60	304.59
81	42.41	67	102.89	23	176.73	17	243.34	49	314.35
5	45.25	48	106.50	80	177.50	74	244.12	30	317.96
62	46.02	37	116.25	4	180.34	55	247.73	87	318.73
43	49.64	94	117.03	69	187.25	36	251.34	11	321.57
100	50.41	18	119.87	50	190.87	25	261.09	68	322.35
24	53.25	75	120.64	31	194.48	82	261.87	57	332.10
89	60.16	56	124.25	88	195.25	6	264.71	38	335.71
13	63.00	45	134.00	12	198.09	63	265.48	95	336.48
70	63.77	26	137.62	77	205.00	44	269.09	19	339.32
51	67.39	83	138.39	1	207.84	33	278.85	76	340.10

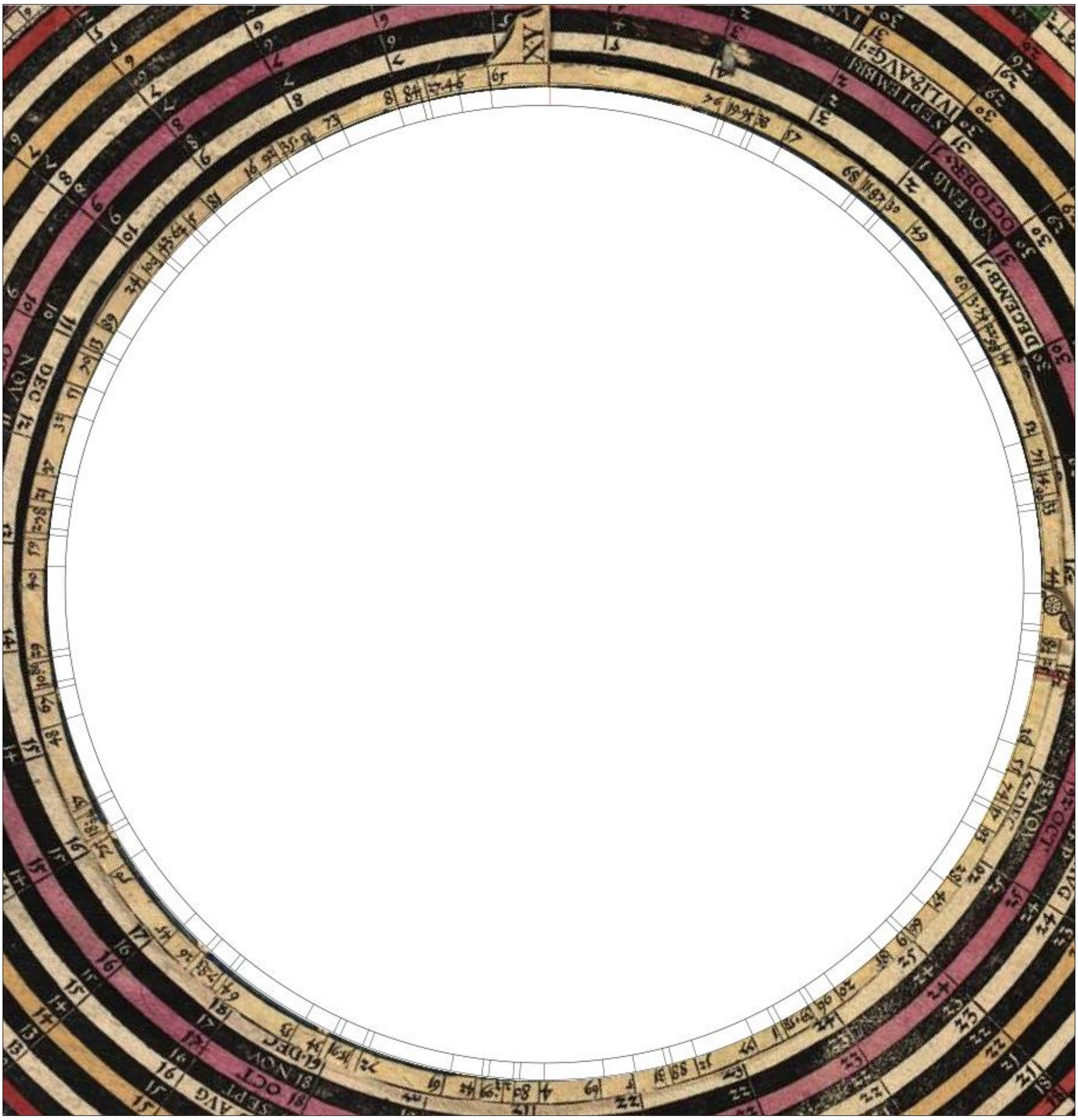


Figure 45: Comparison with the disk marks.

is a star with a Moon symbol (✶ ☽) at 29 January 12.7 hours. These are the lengths of a draconic and synodic month respectively, counted from the beginning of the year and marked by a

Table 25: Calculated positions of the New and Full Moons.

Number	New Moon	Full Moon
I	0.00	188.53
II	17.06	205.60
III	34.13	222.66
IIII	51.19	239.72
V	68.26	256.79
VI	85.32	273.85
VII	102.38	290.92
VIII	119.45	307.98
IX	136.51	325.04
X	153.58	342.11
XI	170.64	359.17
XII	187.70	16.24
XIII	204.77	

cross. If the dragon's head or the lunation of the year occurs after this mark it has to be set back by one or more lunar months. This is done by using the central T disk with which you can measure the excess over the respective mark and instead set that excess counted from the start of January.

3.11 The Mean Time of a Syzygy (page 64)

This (Figure 49) is a further zoom-up of the previous volvelle. The volvelle has a mater divided into 12 hours, subdivided into minutes. It has two disks, one on top of the other. The bottom disk has an index tab Q and a set of year marks. The marks are calculated by first determining an integer for each year such that

$(n-1)\ \ m < y\ \ 365 + \text{integer}\,((y-1)/4) < n \cdot m,$

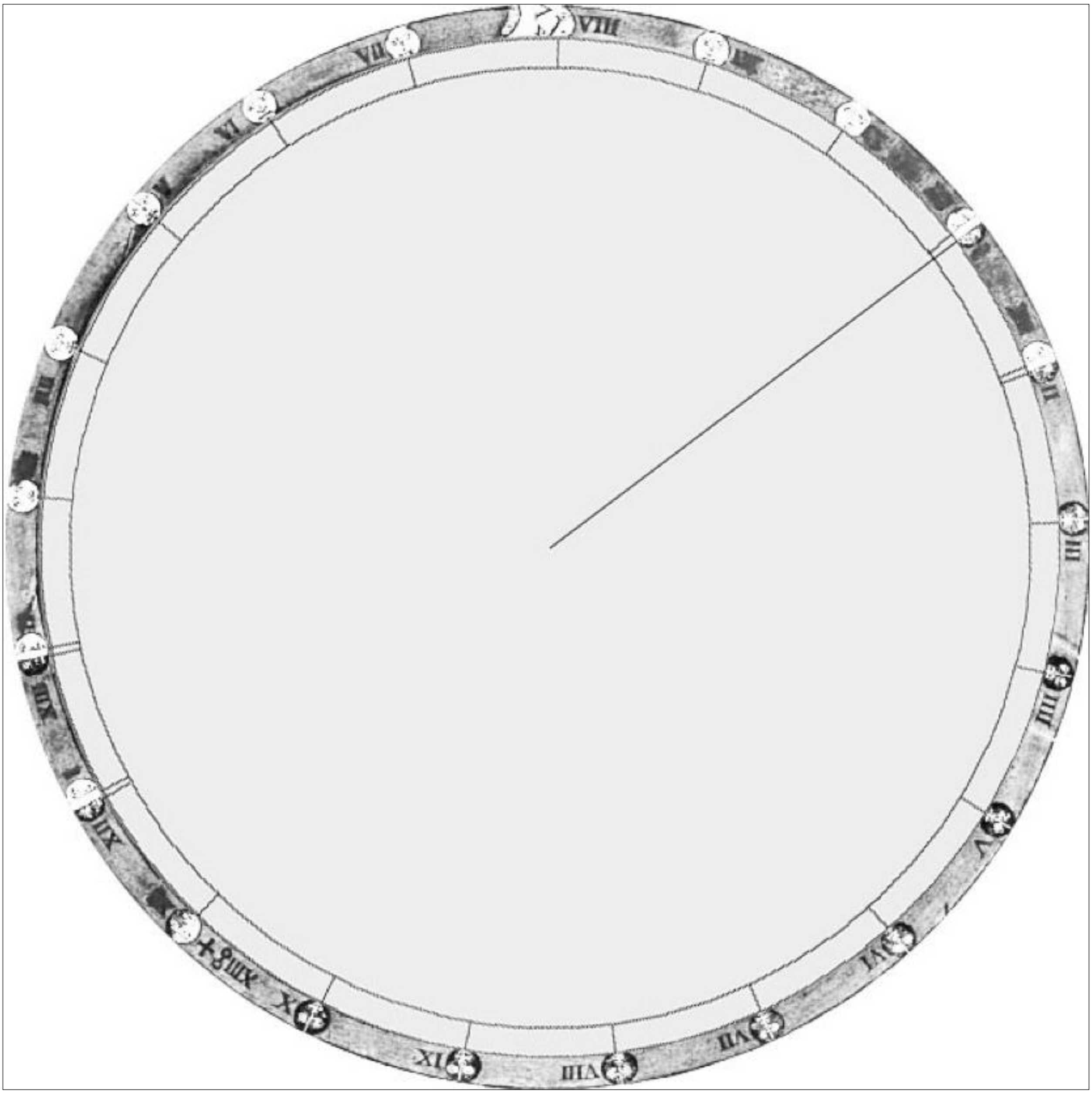


Figure 46: Comparison with the volvelle.

Table 26: Calculated year marks on the ZV disk.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
26	0.77	99	58.59	89	123.18	72	188.36	17	250.83
71	2.89	35	66.72	25	131.31	98	189.13	36	252.19
97	3.66	61	67.48	44	132.67	8	196.49	62	252.95
7	11.02	80	68.84	70	133.43	34	197.26	43	263.21
33	11.79	16	76.97	6	141.56	79	199.38	69	263.97
52	13.15	42	77.74	51	143.69	15	207.51	88	265.33
78	13.91	87	79.86	77	144.45	41	208.28	5	272.1
14	22.04	23	87.99	96	145.81	60	209.64	24	273.46
59	24.17	49	88.76	13	152.58	86	210.4	50	274.23
85	24.93	68	90.12	32	153.94	22	218.53	95	276.35
21	33.06	94	90.88	58	154.71	67	220.66	31	284.48
40	34.42	4	98.25	39	164.96	93	221.42	57	285.25
66	35.19	30	99.01	65	165.73	3	228.79	76	286.61
2	43.32	75	101.14	84	167.09	29	229.55	12	294.74
47	45.44	11	109.27	1	173.86	48	230.91	38	295.5
73	46.21	37	110.03	20	175.22	74	231.68	83	297.63
92	47.57	56	111.39	46	175.98	10	239.81	19	305.76
9	54.34	82	112.16	91	178.11	55	241.93	45	306.52
28	55.7	18	120.29	27	186.24	81	242.7	64	307.88
54	56.46	63	122.41	53	187	100	244.06	90	308.65

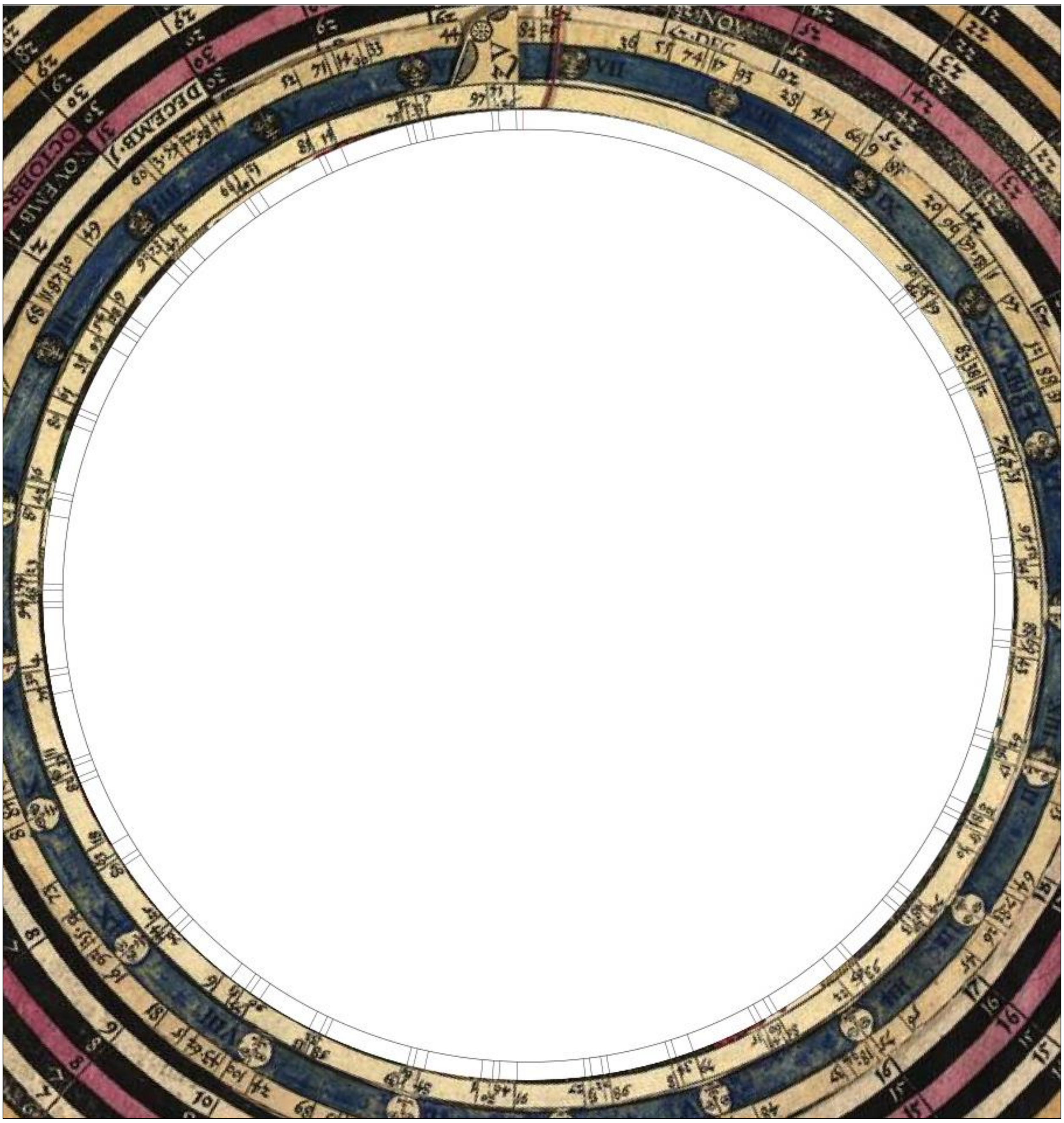


Figure 47: Comparison with the disk marks.

Table 27. Calculated head and tail positions.

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Head	0.00	43.99	87.97	131.96	175.95	219.94	263.92	307.91	351.90	395.88	79.87	123.86	167.84	211.83
Tail	201.99	245.98	289.97	333.96	17.94	61.93	105.92	149.90	193.89	237.88	281.86	325.85	9.84	239.28

then set the angle $a = 360 \cdot \text{frac}\,(n \cdot m - y \cdot 365 + \text{integer}((y-1)/4))$. frac(x) is the fractional part of a number, for instance frac(10.578) = 0.578. This procedure generated Table 28 and Figure 50.

The top disk has an index tab P. It has two bands with Moon icons, the outer band with New Moons and the inner one with Full Moons. The New Moons have Roman numbers I – XIII, the Full Moons I – XI, and one Full Moon is marked by a cross. Angles for the respective Moons are generated by:

New Moons $360 \cdot \text{frac}((n-1) \cdot m)$ $n = 1,2,\ldots,13$
Full Moons $360 \cdot \text{frac}((n-0.5) \cdot m)$, $n = 1, 2,..,11$
Crossed Full Moon $360 \cdot \text{frac}(-1.5)$ $m = -106.52° = 253.48°$

We get Table 29 and Figure 51. New Moon IIII is slightly misplaced and Full Moon VI much so.

First tab P is set to the value of the hours for the century from the century table concerning conjunctions with the Sun. Then tab Q is set on the year within the century. You can now read off the times of the day for the sequence of New and Full Moons of the year.

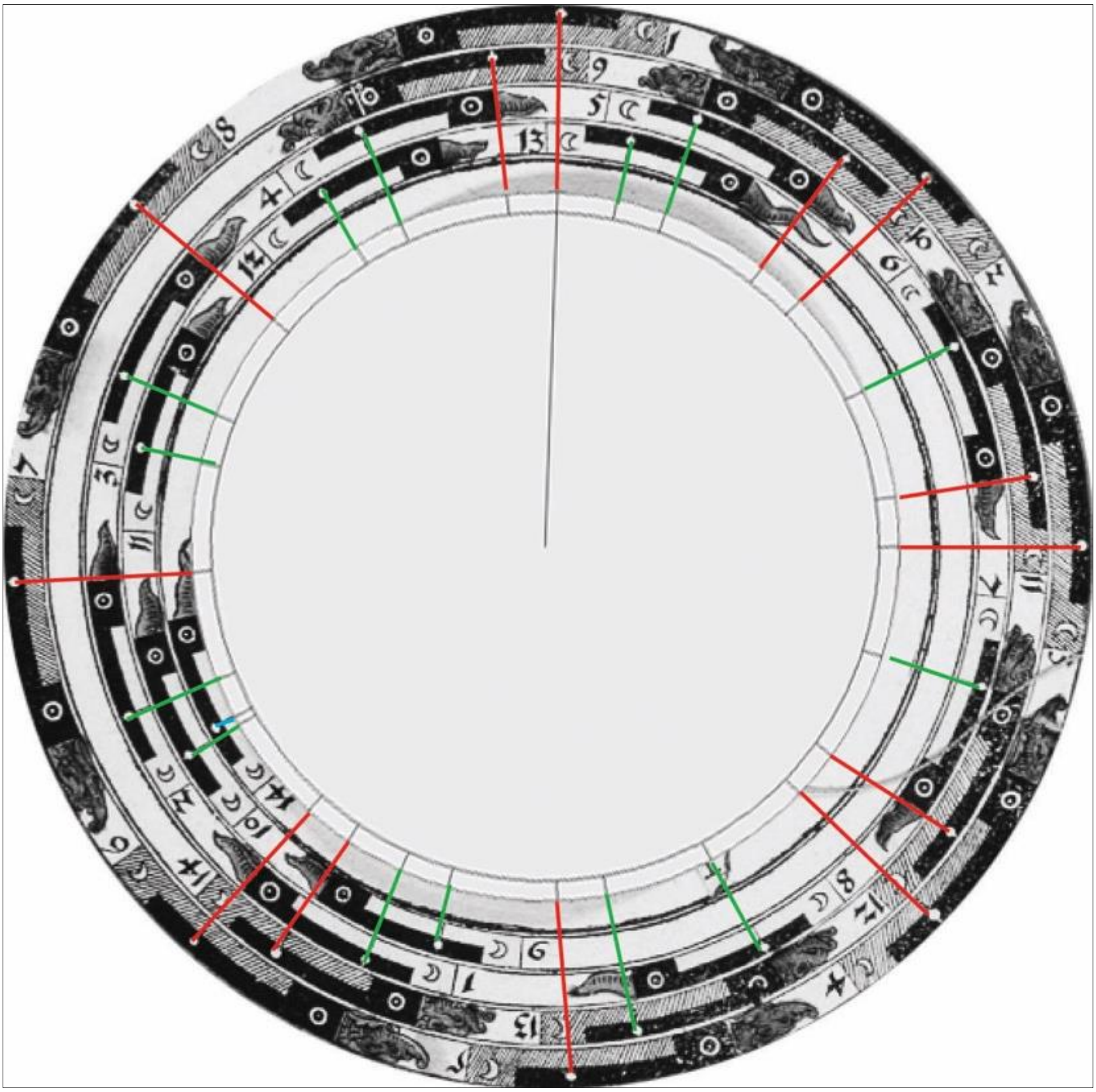


Figure 48: Comparison with the volvelle.

3.12 True Syzygy Time (page 66)

The syzygy volvelle (Figure 52) needs two input quantities, the argument of the Sun and the argument of the Moon. The former is set in the wedge at the top, the latter on the azimuthal scale. There is a thread attached to the centre with a sliding bead to set these quantities. The bead is placed on the thread using the wedge scale and the thread then rotated to the correct position on the azimuthal scale. On the main area of the volvelle are a set of lines, each representing a correction time to the mean syzygy and this correction can be read off by the position of the bead. An interesting feature of this volvelle is that the upper wedge for setting the argument of the Sun is non-linear, in order to get a more regular spacing of the equal-time lines.

At a syzygy, the elongation between the Sun and the Moon is 0°or 180°. However, the earlier volvelles have computed the time of the mean syzygy based on the mean longitudes of the Moon and the Sun. Thus, we need to correct the longitudes of the Moon and the Sun to true longitudes. In the Ptolemaic model the correction to the mean longitude of the Sun is given by

δ_S = –arctan(e_S sin (argument) / (1 + e_S cos (argument))),

where e_S , the eccentricity of the Sun, is 2;30/60 = 0.04167.

The Ptolemaic model for the Moon is more complicated than that of the Sun and the calculation of the correction δ_M to the lunar longitude is correspondingly more complicated. The correction to the elongation is given by $\Delta\lambda = \delta_S - \delta_M$. On order to reach the true syzygy the Sun and the Moon have to move during a time ΔT =

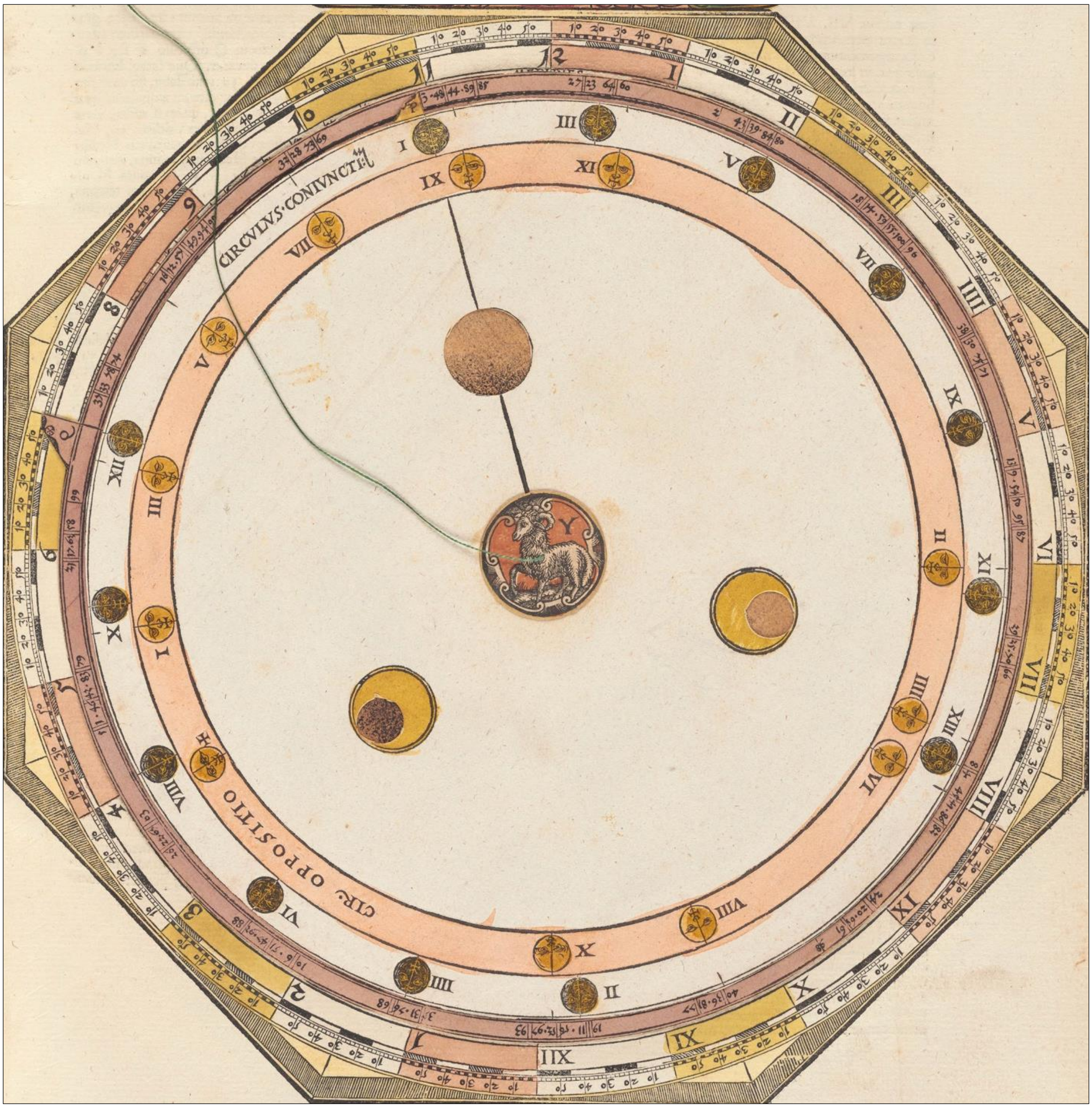


Figure 49: The volvelle for mean syzygy time.

Table 28: Calculated year marks on the Q disk.

Year	Angle	Year	Angle	Year	Angle	Year	Angle	Year	Angle
37	3.82	85	65.37	71	140.17	65	214.22	6	285.95
33	4.20	27	78.24	13	153.05	61	214.59	51	289.02
78	7.26	23	78.62	9	153.42	98	218.41	47	289.40
74	7.64	64	82.06	54	156.49	40	231.29	92	292.46
16	20.52	60	82.44	50	156.87	36	231.67	88	292.84
12	20.89	2	95.32	95	159.93	81	234.73	26	306.09
57	23.96	43	98.76	91	160.31	77	235.11	22	306.47
53	24.34	39	99.14	87	160.69	19	247.99	67	309.53
49	24.71	84	102.20	29	173.56	15	248.36	63	309.91
94	27.78	80	102.58	25	173.94	11	248.74	5	322.79
90	28.16	18	115.83	70	177.00	56	251.81	1	323.17
32	41.03	14	116.21	66	177.38	52	252.18	46	326.23
28	41.41	59	119.28	8	190.26	97	255.25	42	326.61
73	44.47	55	119.65	4	190.64	93	255.63	83	330.05
69	44.85	100	122.72	45	194.08	35	268.50	79	330.43
7	58.11	96	123.10	41	194.46	31	268.88	21	343.30
3	58.48	38	135.97	86	197.52	76	271.94	17	343.68
48	61.55	34	136.35	82	197.90	72	272.32	62	346.75
44	61.93	30	136.73	24	210.77	68	272.70	58	347.12
89	64.99	75	139.79	20	211.15	10	285.58	99	350.57

Figure 50: Comparison with the volvelle.

$(\delta_S - \delta_M) / (v_M - v_S)$, where v_M and v_S are the angular velocities of the Moon and the Sun respectively that depend on their respective arguments. The time correction, ΔT, is a function of the arguments of the Sun and the Moon.

A detailed analysis of this volvelle with simulations of the points on the volvelle can be found in Gislén (2017b). You also find some interesting facts about syzygy volvelles in Kremer (2011).

3.13 The Semi-diameters of the Moon and the Lunar Eclipse Shadow (page 68)

The mater of the volvelle (Figure 53) is graduated along the rim with a scale with the zodiacal signs 0 – 11, subdivided into degrees. It is used to set the Moon's argument, which essentially measures the distance to the Moon, or to set the argument of the Sun. Inside this scale is a scale offset from the centre and graduated symmetrically from the top clockwise and anti-clockwise from 0 to 56. Finally, there is a large black circu-

Table 29: Calculated positions of the New and Full Moons.

New Moon	Angle	Full Moon	Angle
I	0	I	275.51
II	191.01	II	106.52
II	22.03	III	297.53
IIII	213.04	IIII	128.54
V	44.05	V	319.56
VI	235.06	VI	150.57
VII	66.08	VII	341.58
VIII	257.09	VIII	172.6
IX	88.1	IX	3.61
X	279.11	X	194.62
XI	110.13	XI	25.63
XII	301.14	†	253.48
XIII	132.15		

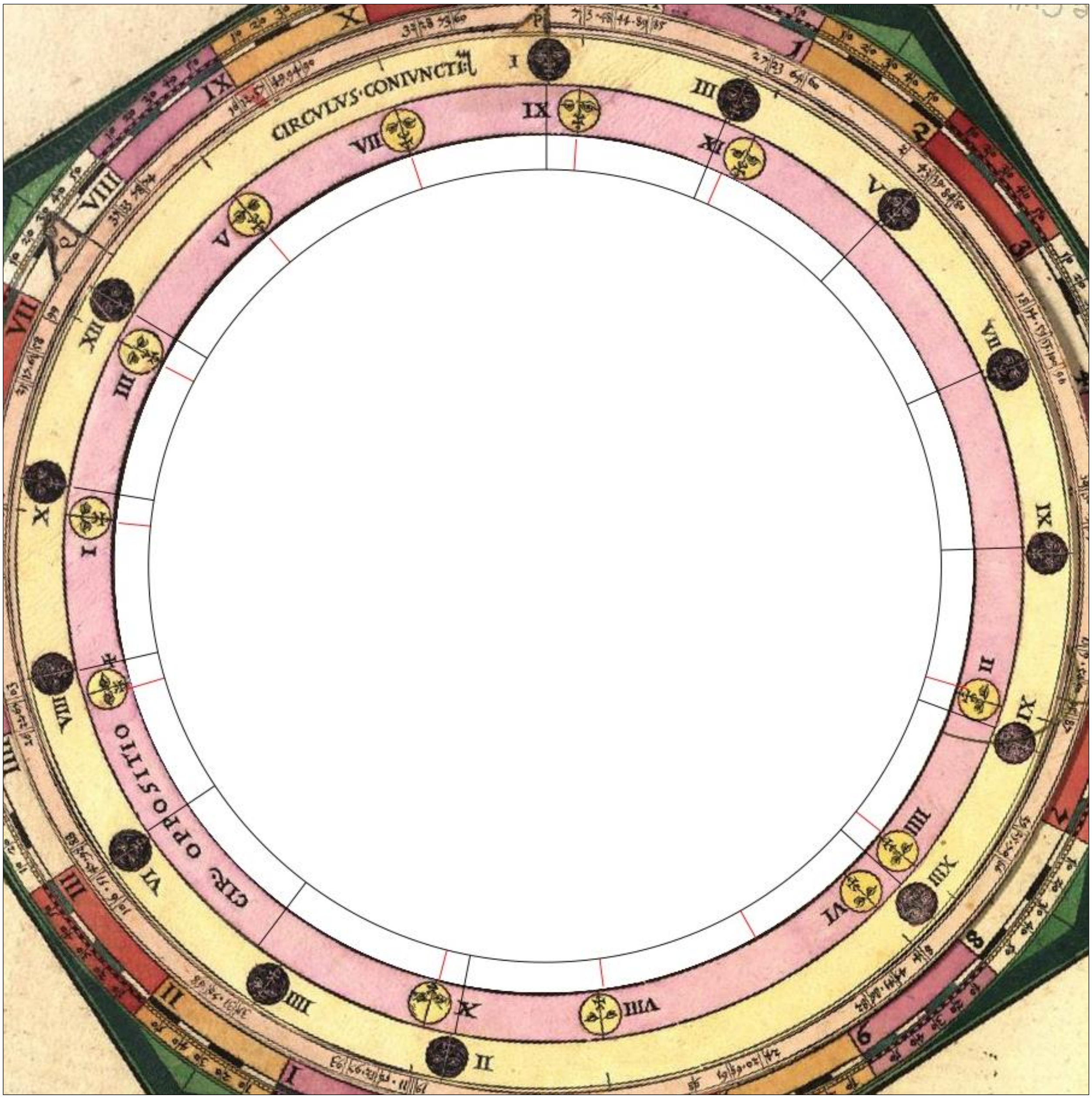


Figure 51: Comparison with the volvelle disk.

lar area representing the Earth's shadow and a yellow central circular area representing the Moon and finally a thread attached to the centre of the volvelle. Both these areas are displaced from the main centre of the volvelle. The ratio of the radii of the two areas is 2.6, a Ptolemaic value for lunar eclipse. In the Alfonsine Tables, the angular size of the of radius of the shadow is a function of the lunar argument and varies between 46′ 47″ and 37′ 41″. The ratio of these angles, 1.24, is also very precisely the ratio between the largest and smallest distances from the centre to the periphery of the shadow area and of the largest and smallest distances from the centre to the periphery of the Moon area. The Ptolemaic theory also assumes that because of the varying distance to the Sun, the radius of the shadow has a correction, depending on the argument of the Sun, that is proportional to the square of the sine of the solar argument, and with a maximum value of 56″. This is the purpose of the offset scale inside the outer scale on the mater.

The working of the volvelle is double. Using the central thread to set the argument of the Moon, the radii of the shadow and the Moon can be read off. Then, using the thread to set the solar argument, the small correction to the shadow radius can be read off from the scale graduated from 0 to 56. These quantities can then be used for the graphical construction of lunar eclipses that is illustrated by the volvelles on pages 70, 72 and 74.

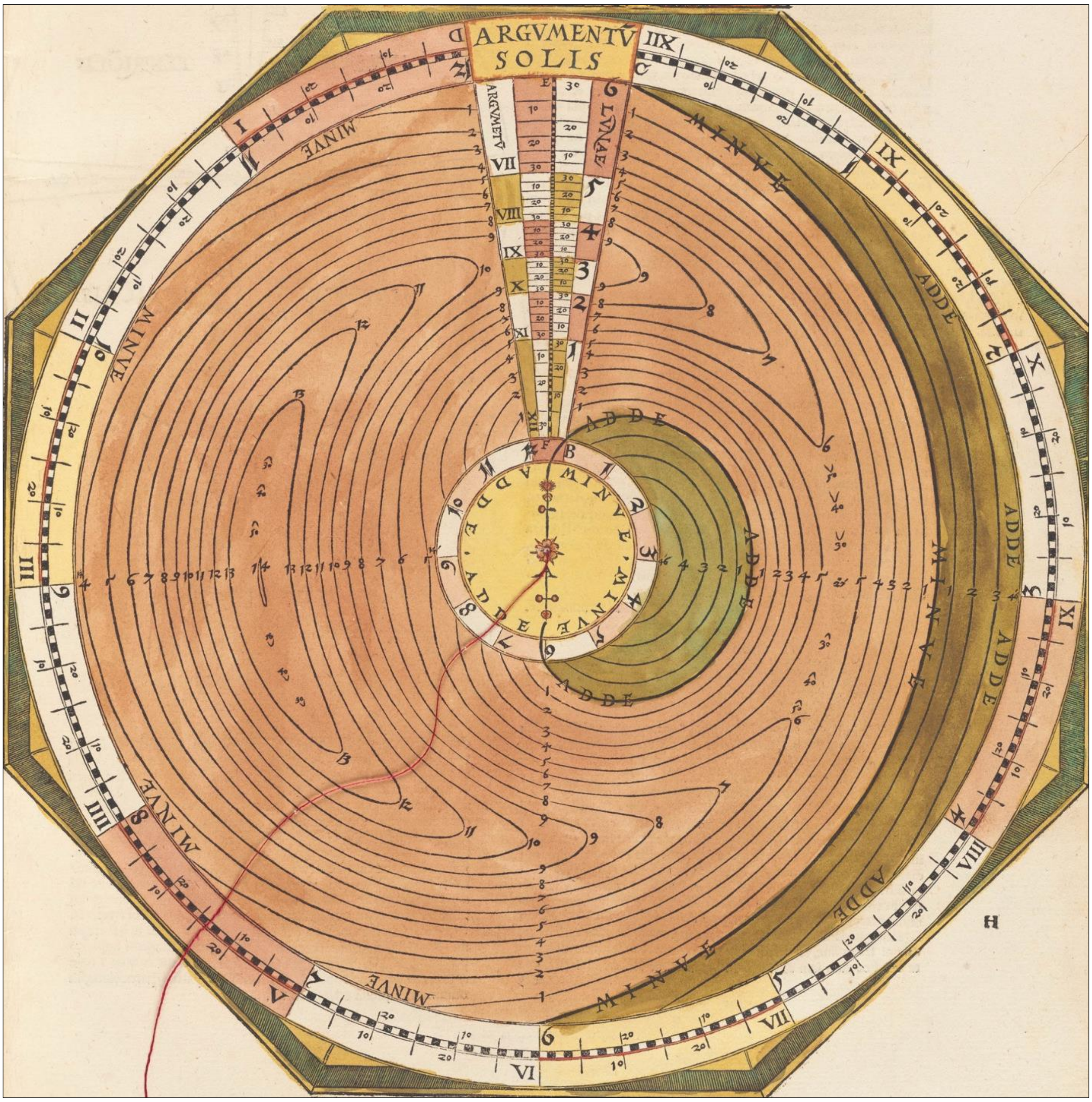


Figure 52: The syzygy volvelle.

3.14 The Size, Duration and Angular Movement in a Lunar Eclipse (page 76)

This volvelle (Figure 54) is used to compute several aspects of a lunar eclipse: the size of the eclipse (PVNCTA ECLIPTICA), the time of the total phase (MORA MEDIA), the time of the partial phase (TEMPVS CASVS), and the movement in longitude of the Moon during the eclipse (MINVTA GRA MOTVS LUNE). The theory of this volvelle is treated in detail by Gislén (2016). The input to this volvelle are the arguments of the Moon and the Sun that are set on the top panel, using the two centrally attached threads, one of which has a sliding bead. Once the bead has been set, the thread can be laid over a panel and stretched to the latitude of the Moon (MINVTA LATITVDINES LUNAE). The line below the bead will then show the desired quantity. Using the theory, it is possible to simulate the different curves on the panels of the volvelle (see the coloured points in Figure 54).

3.15 Planetary Conjunctions (page 86)

This volvelle (Figure 55) has three moveable disks attached on top of each other to the main axis of the volvelle. The disks have tabs for Saturn, Jupiter and Mars respectively, and have year marks to set the year and date, as for the corresponding longitude volvelles. On the page preceding the volvelle, the tables for the century values of their mean longitudes are repeated. For more precise calculations, the true longitudes computed by the earlier volvelles can be used.

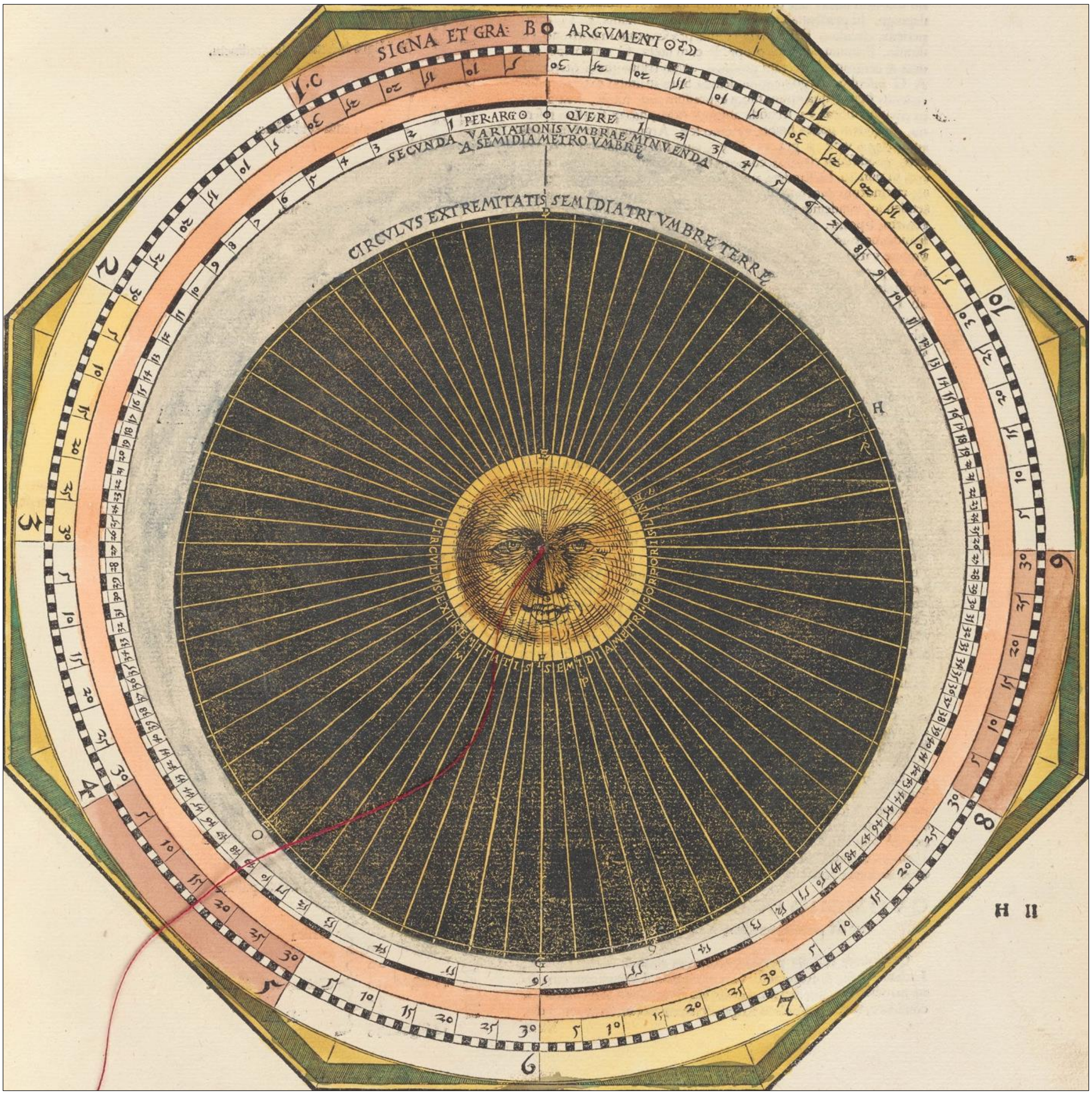


Figure 53: Lunar shadow volvelle.

3.16 Finding the Ascendant at the Birth (page 88)

This is one of the odder volvelles (see Figure 56). The ascendant or rising sign at the hour of birth was used for the construction of horoscopes, which at the time was one of the important tasks of some astronomers. The principle of the volvelle is the belief that the ascendant at the conception is the same as the longitude of the Moon at birth, and that the longitude of the Moon at the conception is the same as ascendant at birth. Even if you only know the ascendant approximately at birth, this volvelle is said to compute the true time of conception, and the longitude of the Moon can then be computed, leading to an improved value for the ascendant at birth.

The mater has a scale with the zodiac. On top of this, and attached to the main axis, is a disk with the circumference divided into 30 parts labelled 258 to 288, each part subdivided into 24 hours. Inside this is a scale with the days of the year. The disk has a tab Q and is also marked by ASCENDENS. On top of this is a circular disk, also moving around the main axis with a tab R and a small sector scale DIES MORI with 30 divisions labelled 258 to 288. The width of the sector precisely corresponds to 30 days on the year scale of disk Q. The thread from the centre is set to the longitude of the Moon at birth on the scale of the mater. The tab Q of the ascendant disk at birth is set to the longitude of the ascendant at birth on the same scale. By the thread you can now read off how many days and hours the child has been in the womb of the mother. Further, set the thread to the date and hour of birth and then move the tab R of the top

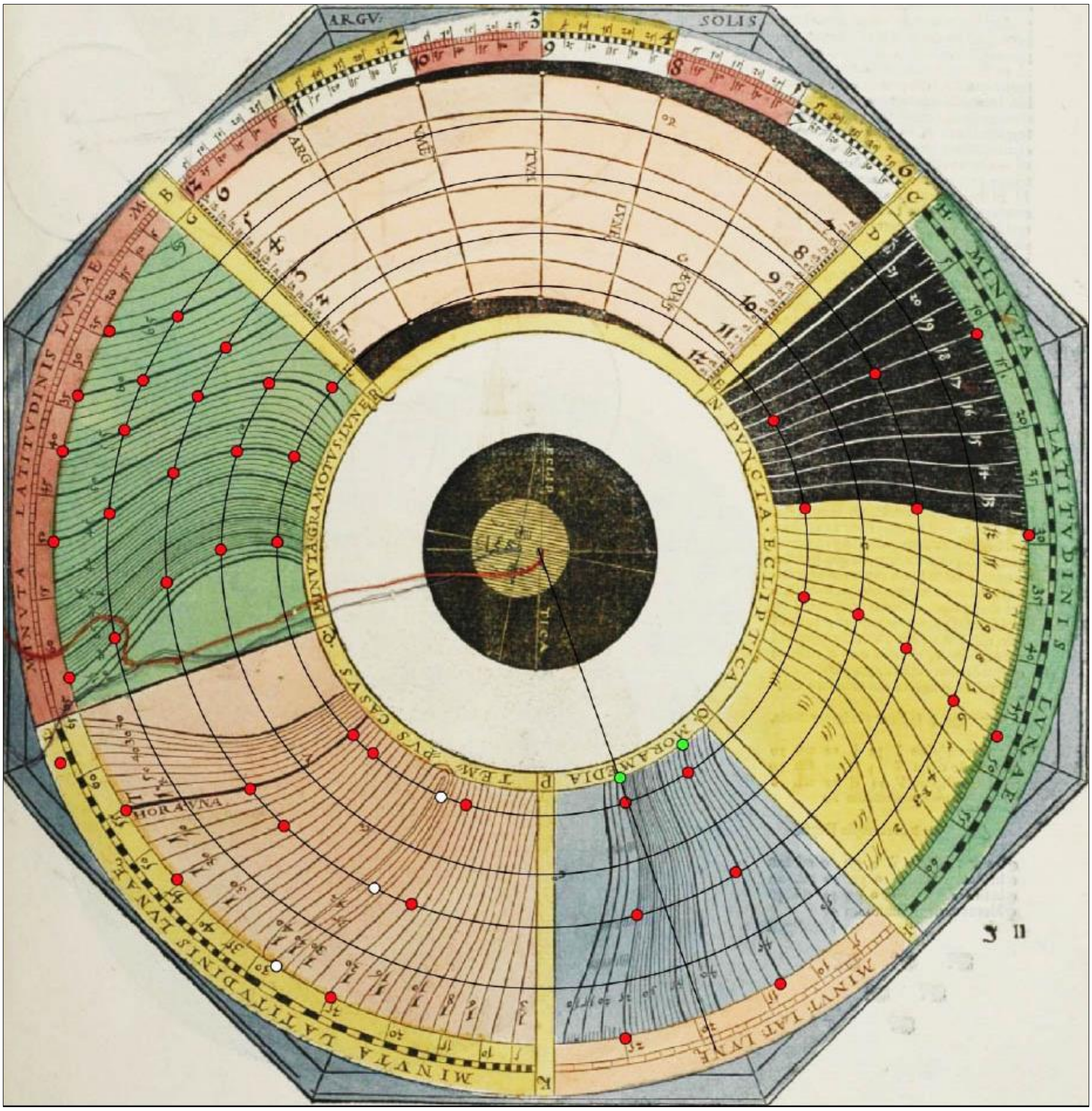


Figure 54: The lunar eclipse volvelle.

disk such that the number of days and hours in the DIES MORE scale is against thread, and you can read off the date and hour of the conception at tab R. The top disk merely performs a subtraction of the days and hours found above from the time of birth. The time of conception can then be used to compute the longitude of the Moon at that time and by that the ascendant at birth.

3.17 *PARS FORTUNÆ* (page 90)

This is also an astrological volvelle (Figure 57). *Pars Fortunæ* is computed by two formulas: for day horoscopes it is the longitudes of the ascendant + the Moon – the Sun, while by night the roles of the Moon and the Sun are switched and the volvelle performs these mathematical operations. *Pars Fortunæ* is said to represent worldly success and to be associated with health as well in some rather unclear way.

There is a mater with a zodiac scale. There are two disks on top of one other and centred on the main axis. Three input numbers are required: the ascendant, and the longitudes of the Moon and of the Sun. If the birth is by night, the bottom disk is set to the longitude of the Moon and the top disk to the longitude of the Sun. Both disks are then moved together until the tab of the bottom disk points to the ascendant longitude on the mater. Tab T will then point to the *Pars Fortunæ* or longitude of luck. If the birth is by day, the bottom disk is instead set to the solar longitude and the top disk to the lunar one.

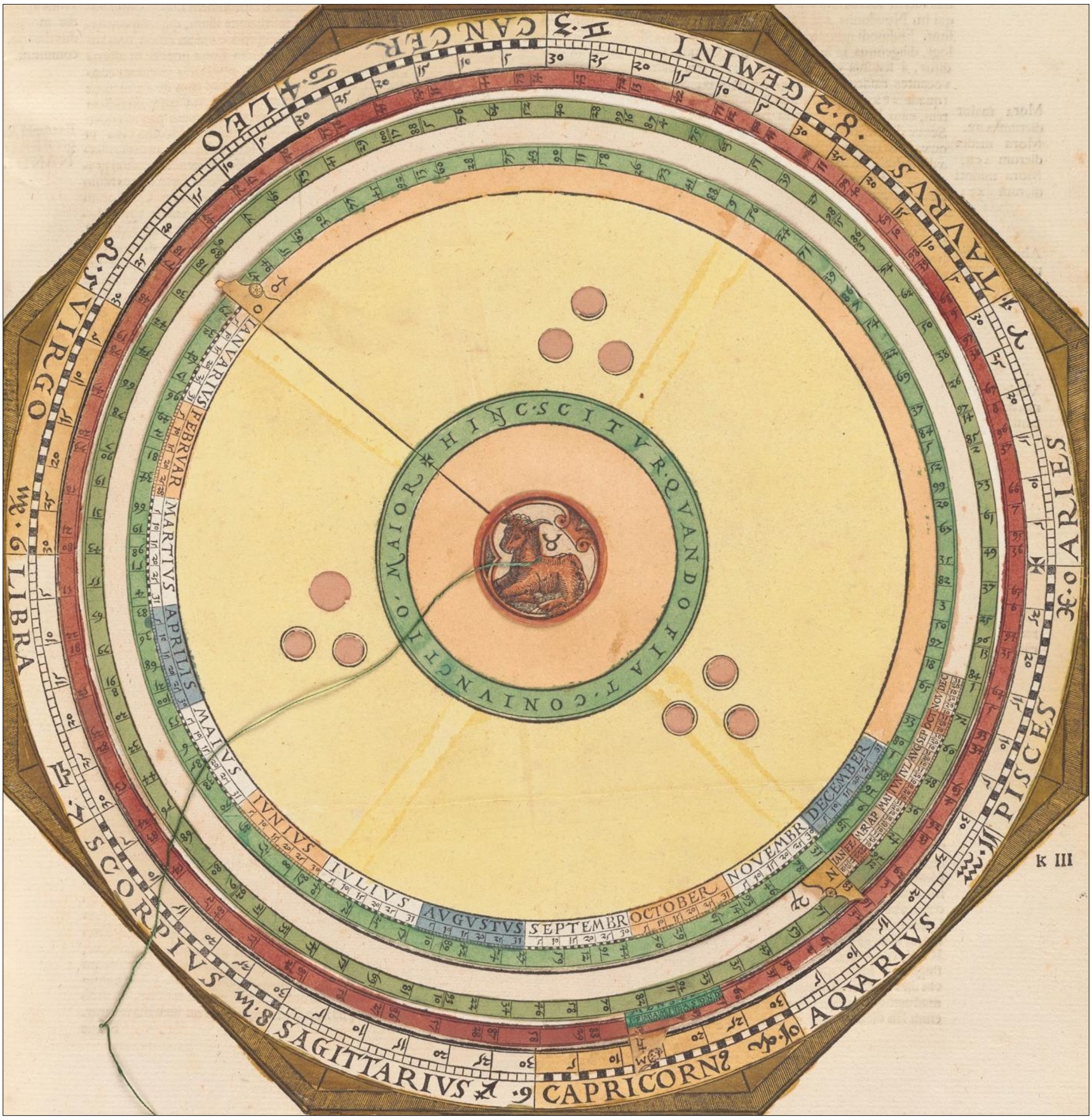


Figure 55: The volvelle for planetary conjunctions.

3.18 The Golden Number and the Key for the Moveable Seasons (page 92)

This volvelle (Figure 58) does the computation of the golden number (AUREUS NUMER) or the year in a 19-year cycle using a century letter value taken from the table on the previous page, set on by a tab X on the top disk of the volvelle and reading off the golden number using the central thread and the year within the century. The volvelle then also shows the key for the moving ecclesial feats, CLAVIS FESTOR MOBILIUM. The implemented formula is that the golden number G = (year / 19) mod 19 + 1. For instance, 1,500 CE gives a remainder of 18 and a golden number of 19. This volvelle uses current years, not elapsed years.

The key for the moveable seasons is used in the volvelle on page 96 to determine the Easter Sunday that then in turn determines the moveable ecclesial feasts. In the Julian calendar, Easter Sunday is the first Sunday after the first ecclesial Full Moon, the Easter Full Moon on or after the ecclesial equinox, 21 March. The Easter Full Moon is calculated by cyclical reckoning, and is in general different from the astronomical Full Moon. In fact, in the Julian Calendar the Easter Full Moon drifts away from the true Moon by more than three days per millennium. The Easter Full Moon is determined by first computing the epact E = (11 · (G – 1)) mod 30. In the Julian Calendar the epact is the age of the Moon on 22 March. The Easter Full Moon occurs when the epact is 14 and Easter Sunday can occur the day after, at the earliest. If the

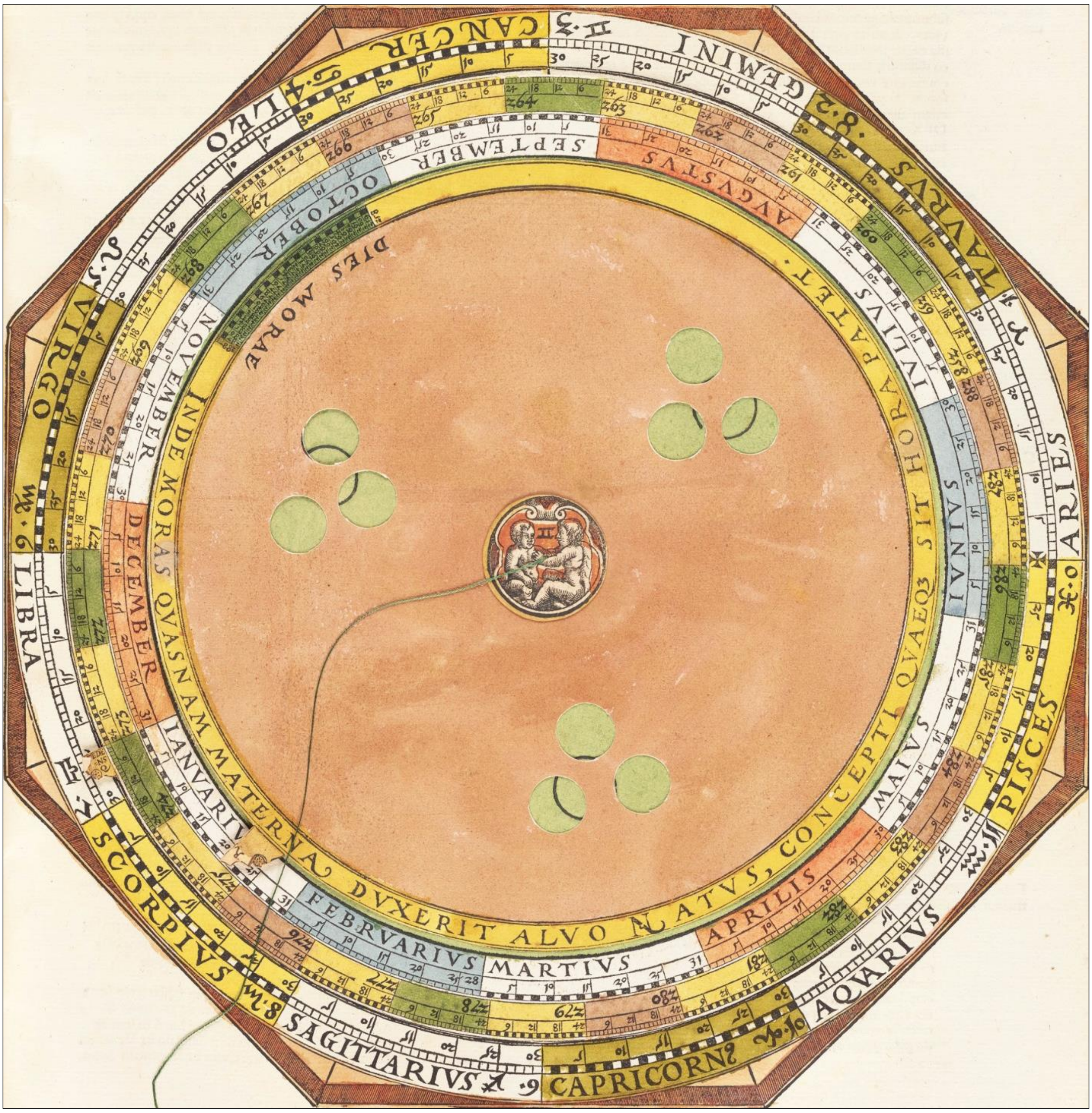


Figure 56: The volvelle for finding the ascendant.

epact is equal or less than 15 the Easter Full Moon can then occur 15 – E days after the equinox; if it is more than 15, it occurs 30 days later, i.e. after 45– E days. The key counts these days from 11 March, starting the count on that date, thus if the epact is equal or less than 15, the key will be 26 – E, otherwise 56 – E. This will give Table 30 with the relation between the golden number, G, the epact and the key number, which are the numbers (except for the epact) that you find on the volvelle. It also shows the date of the Easter Full Moon (EFM) that is uniquely determined by the golden number.

3.19 The Dominical Letter and the Solar Cycle (page 94)

This volvelle is shown in Figure 59. The dominical letter determines the Sundays of the calendar. The Julian Calendar with a leap year every four years, combined with the seven-day week, generates a cycle of 4·7 = 28 years after which the weekdays of the year will fall on the same dates. Each day of the calendar year is labelled sequentially with the seven letters A to G, starting with A on 1 January. There exist several mnemonic phrases to remember the sequence of dominical letters for the first day of each month, ADDGBEGCFADF.[2] A given year will have a dominical letter, a leap year will have two, the first one to be used before 29 February, the second one on or after. The days of the year with the dominical letter of the year will all be Sundays. A procedure for calculating the dominical letter of a year is de Morgan's rule.[3] Calculate S = 7 – (year +integer(year/4)–3) mod 7. The dominical letter is then A if S = 1, B if S =

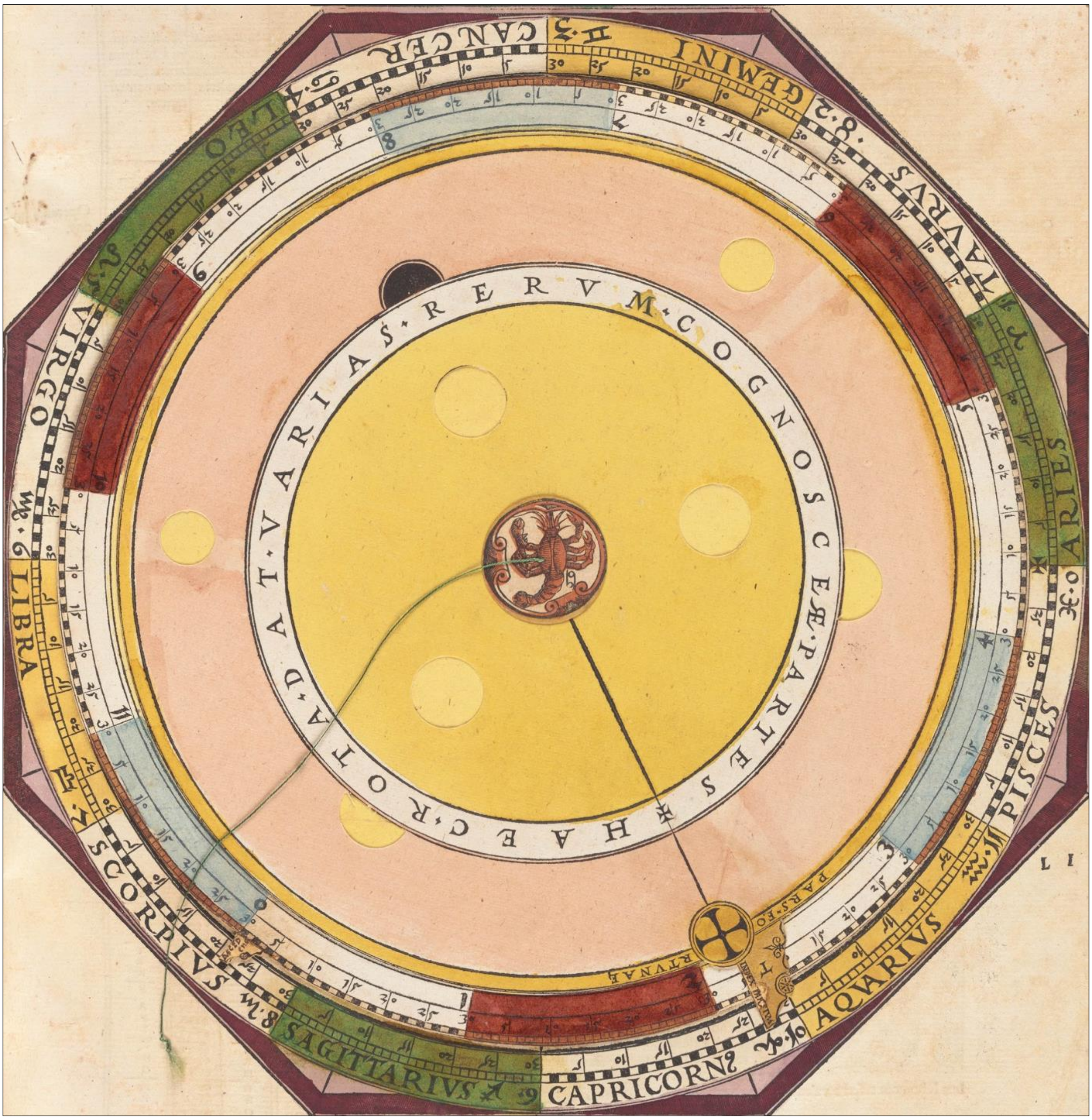


Figure 57: The volvelle for *Pars Fortunæ*.

2 and so on. If the year is a leap year, you will get the dominical letter to use before 29 February, from that date you use the preceding letter.

Here is an example: year = 1502, S = 7 – (1502 + integer(1502/4)–3) mod 7 = 7 – (1502 + 375 – 3) mod 7 = 7 – 1874 mod 7 = 7 – 5 = 2. The dominical letter will be B.

For the volvelle you first get a radix number for the century from a table preceding the volvelle. Using that radix number, you set the tab X of the volvelle to the radix and read off the dominical letter from the mater, using the thread from the centre and the century year in the spiral scale. The volvelle also shows the number in the 28-year solar circle. This number is computed as the year mod 28. The dominical letter is used for determining Easter Sunday in the next volvelle.

3.20 The Moveable Ecclesian Feasts (page 96)

This volvelle (Figure 60) has a mater with the months and days of the year and the sequence of dominical letters and a moveable upper disk with a tab for Easter Sunday, PASCHA OSTER TAG. There is a key icon labelled CLAVIS PASCAE pointing to 11 March. The ecclesial equinox was on 21 March. However, because the Julian mean year of 365.25 days is longer than the true tropical year of 365.2422 days, the true equinox would slowly recede in the calendar and at Apianus' time the true equinox occured approximately on 11 March, which is probably the reason why he put the Easter key at that

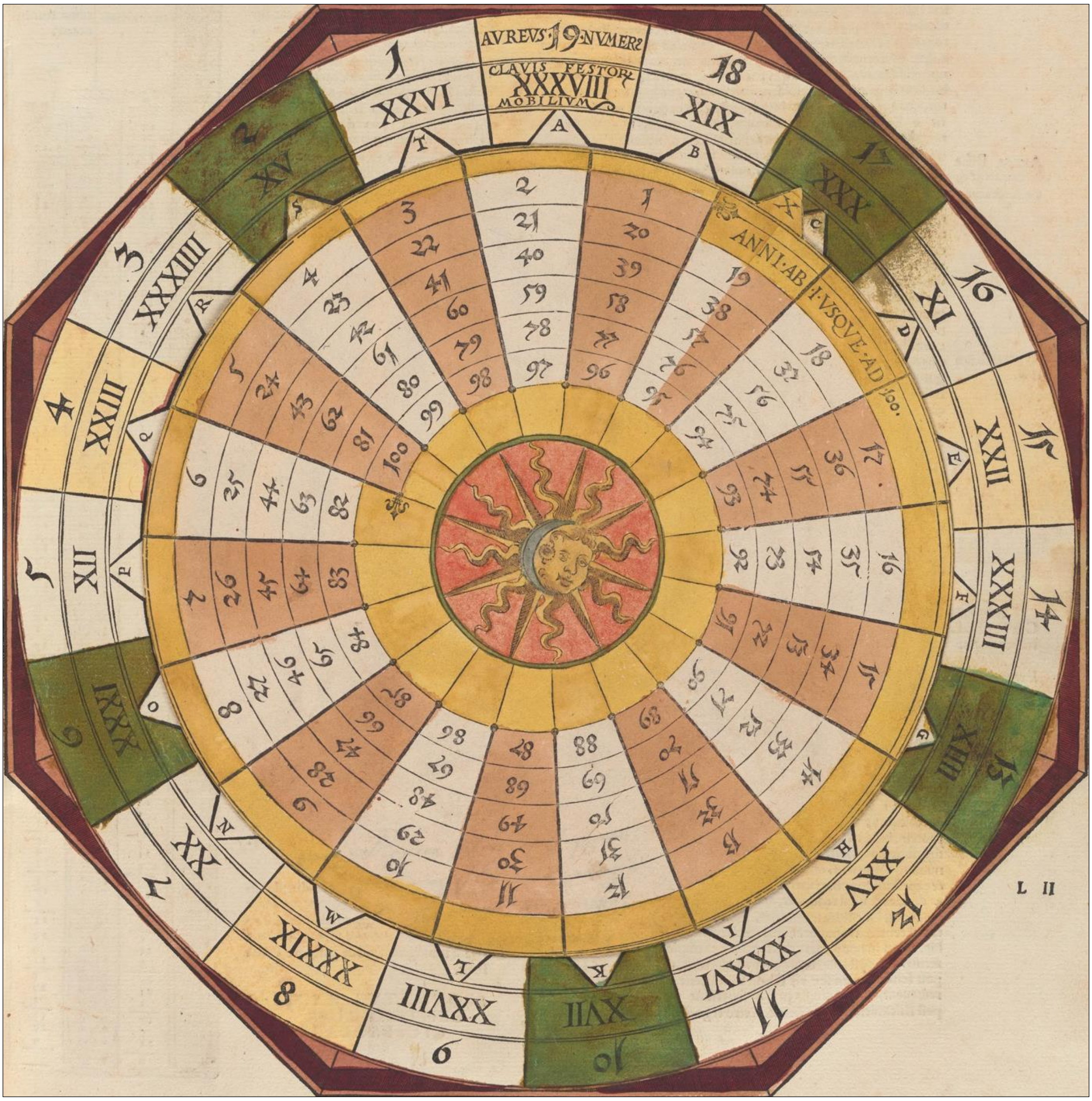


Figure 58: The golden number volvelle.

date. The recession of the equinox would eventually lead to the Gregorian Calendar reform in CE 1582.

Using the key value from the volvelle on page 92 and the dominical letter from the volvelle on page 94, you count off the key, starting with one on 11 March to reach the date of the Easter Full Moon. You then search the first date with the given dominical letter after this date and this will be Easter Sunday of the year.

Here is an example: 1540 CE has the key 15 and as it is a leap year it has two dominical letters D and C. As we deal with a date after 29 February, the dominical letter to use is C. Counting off the key starting from 11 March you reach 25 March. The first dominical C after that date is found on 28 March, which will be the date of Easter Sunday. Setting the tab on this

Table 30: Golden numbers, epacts, Easter keys, and Easter Full Moons.

G	Epact	Key	EFM Date
1	0	26	5 April
2	11	15	25 March
3	22	34	13 April
4	3	23	2 April
5	14	12	22 March
6	25	31	10 April
7	6	20	30 March
8	17	39	18 April
10	9	17	27 March
11	20	36	15 April
12	1	25	4 April
13	12	14	24 March
14	23	33	12 April
15	4	22	1 April
16	15	11	21 March
17	26	30	9 April
18	7	19	29 March
19	18	38	17 April

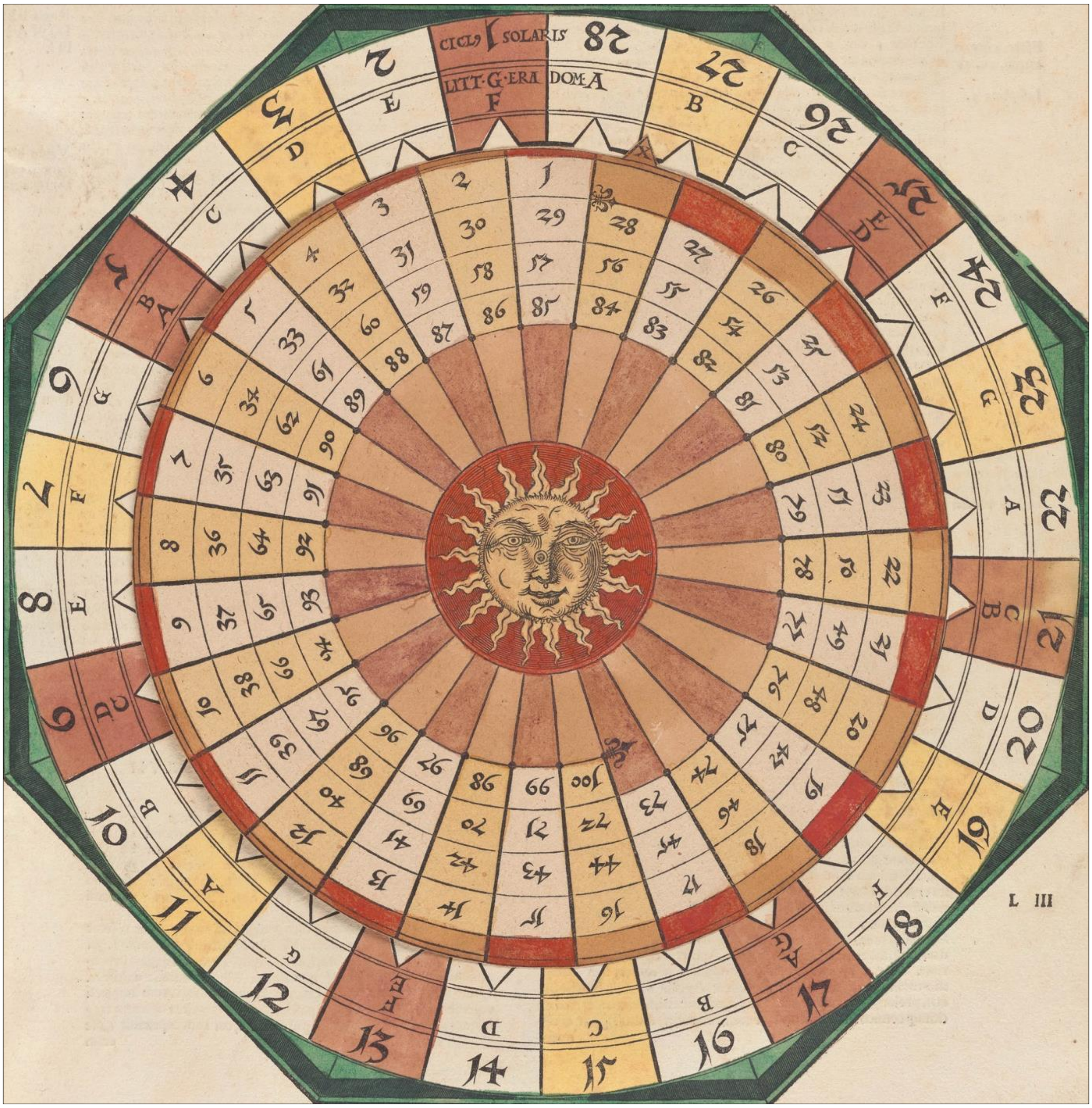


Figure 59: The dominical letter volvelle.

date you can now read off the dates of the other ecclesial feasts. In the Julian Calendar there are precisely 19 dates on which the ecclesial Easter Full Moon can occur. Combining this number with the solar cycle of 28 years after which the weekdays fall on the same date, we get the Easter cycle of 28 · 19 = 532 years after which Easter Sunday falls on the same date in the calendar.

3.21 The Equinox Volvelle (page 98)

The equinox volvelle (Figure 61) consists of the mater and a moveable top disk that can be rotated around the central axis. The mater is graduated by the fourteen days on which the equinox can fall in the selected time interval 1,300 CE–3,000 CE. Thus, a day corresponds to an angle 360 / 14 = 25.714°. Each day is sub-graduated by 24 hours from 12 hours midnight to noon to 12 hours midnight. Night and day portions are indicated by an outer circular band with white and black partitions. In the middle of the night part are the words MEDIA NOX or just NOX, in the middle of the day part the words DIES or MERI(DIES) or just M. In the innermost circular band of the volvelle are eighteen fixed indexes marked 1300 to 3000 for centuries after Christ. The index for century 1500 is marked with M. When the specified year is a leap year the February dates on the top of the mater should be 28 and 29 instead of 27 and 28.

The moveable top disk has five equally spaced tabs marked B, C, D, E, and F. Below each of the tabs is a group of twenty years of the century, grouped in five concentric circular bands, each having a sequence of four years.

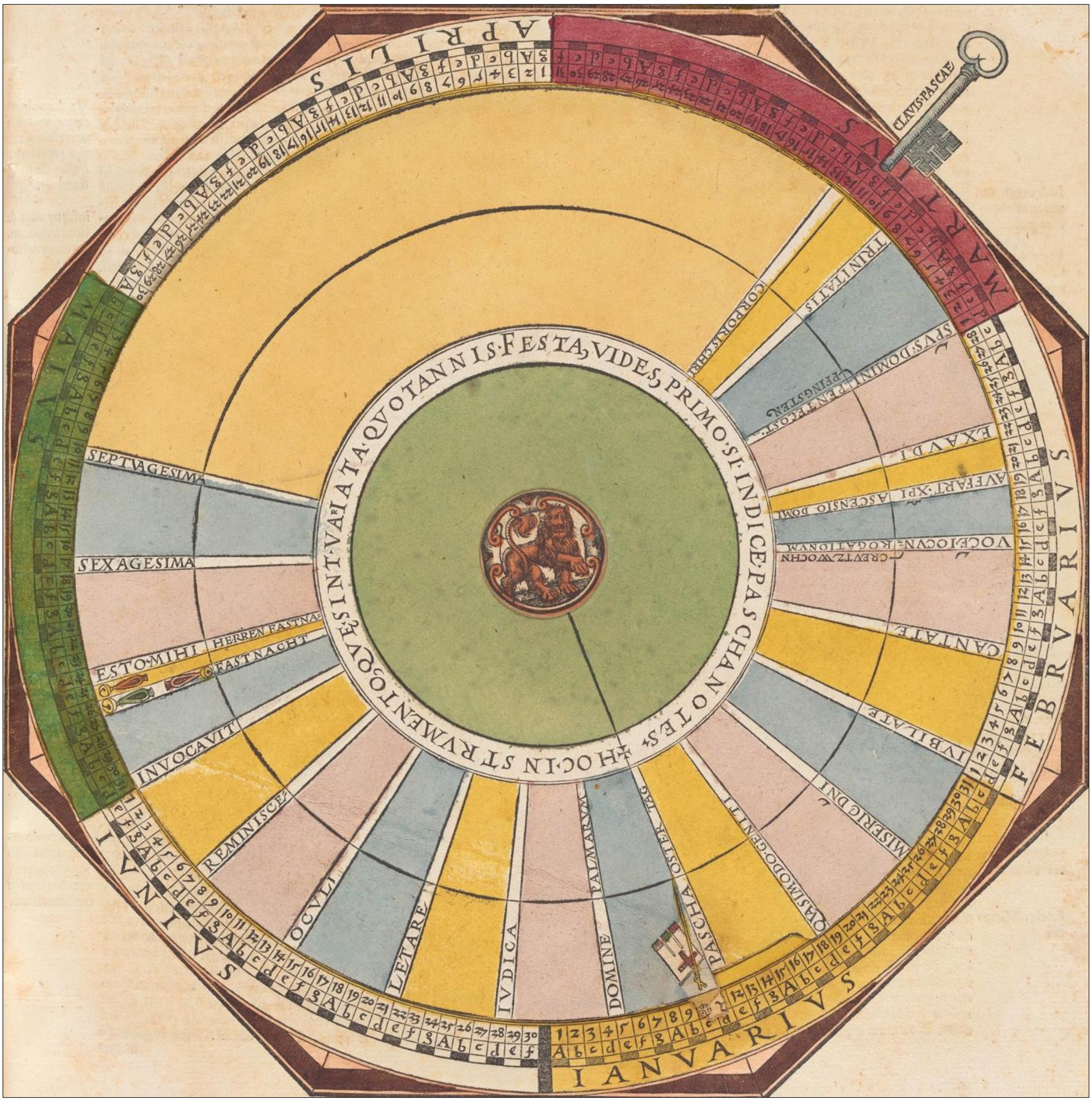


Figure 60: The ecclesial feasts volvelle.

In the circular band immediately below the tab there is a scale from 12 hours midnight over noon to 12 hours midnight. To the right of this scale are the letters OC (occident, west) and to the right OR (orient, east). This scale is used for geographical time difference corrections for locations west or east of Ingoldstadt, the meridian used for AC. Finally, there is a thread from the centre of the volvelle to be used for reading off the time of the equinox.

The instrument uses the Julian Calendar with a mean year length of 365.25 days. The tropical year used in AC is 365.2425461 days. A Julian century is 36,535 – 36,524.25461 = 0.74539 days longer than the tropical year. In the volvelle this corresponds to an angular interval of 0.74539 · 360/14 = 19.167°. Figure 62 shows a series of century radial lines with this angular separation, starting from the index 1300. The agreement with the volvelle is excellent. This angular condition only fixes the relative positions of the century indices. Apianus needed to calculate the absolute position of one of them. The one marked with M for 1,500 CE is the natural choice. In fact, if you calculate the true equinox for 1,500 CE, using the Alfonsine Tables, you get 10 March, 5:13 hours after noon. Adding to this Apianus' tabulated longitude time difference with Toledo, 1:29 hours, you arrive at 10 March, 6:42 hours after noon, which is very nearly what you find on the volvelle (Figure 62, red line from index M).

In a 20-year period there are 7,305 Julian Calendar days and 7,304.850922 tropical days. The difference is 0.149078 days or 3:35 hours. This difference is used in the top disk. Each

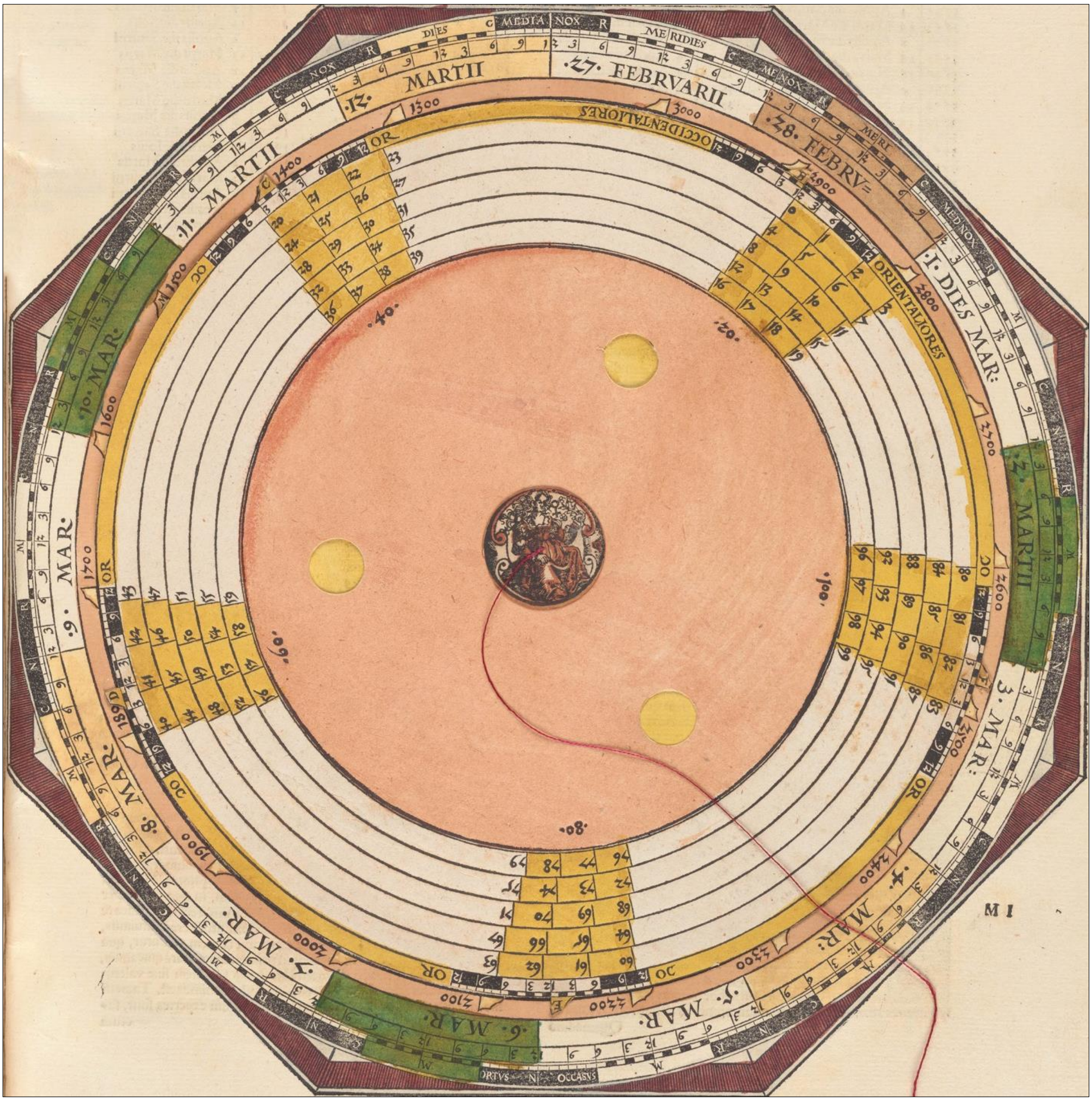


Figure 61: The equinox volvelle.

group of 20 years is successively displaced precisely this number of hours anti-clockwise relative to index of the group as you go from index to index. In a 4-year period there are 1,461 calendar days and 1,460.970184 tropical solar days. The difference is 0.029816 days or 0:43 hours. The four-year groups within each 20-year group are displaced anti-clockwise relative to each other by this amount. The difference between a tropical year and a normal Julian year and is 365.2425462 – 365 = 0.2425462 days or 5:49 hours. In each four-year group, the years are successively displaced clockwise by this amount. All this accounts perfectly for the layout of the volvelle.

The working of the volvelle is that the tab of the top disk with the 20-year group containing a specified year is placed on top of the index of the chosen century. The thread is aligned with the line representing the specified year in the 20-year group, and the time and day of the spring equinox are read out from the mater. If necessary, the time is corrected for east-west longitude time difference, using the time scale below the top disk tab. To find the corresponding autumn equinox you add six months, three days, and 42 minutes. Due to the difficulty in reading the time scale the result has an effective precision of about 10 minutes. In the examples given by Apianus in AC, he states the time with a precision of minutes, but he has clearly obtained his result by some other means of computation.

There is a more detailed explanation of the theory behind this volvelle in Gislén (2019).

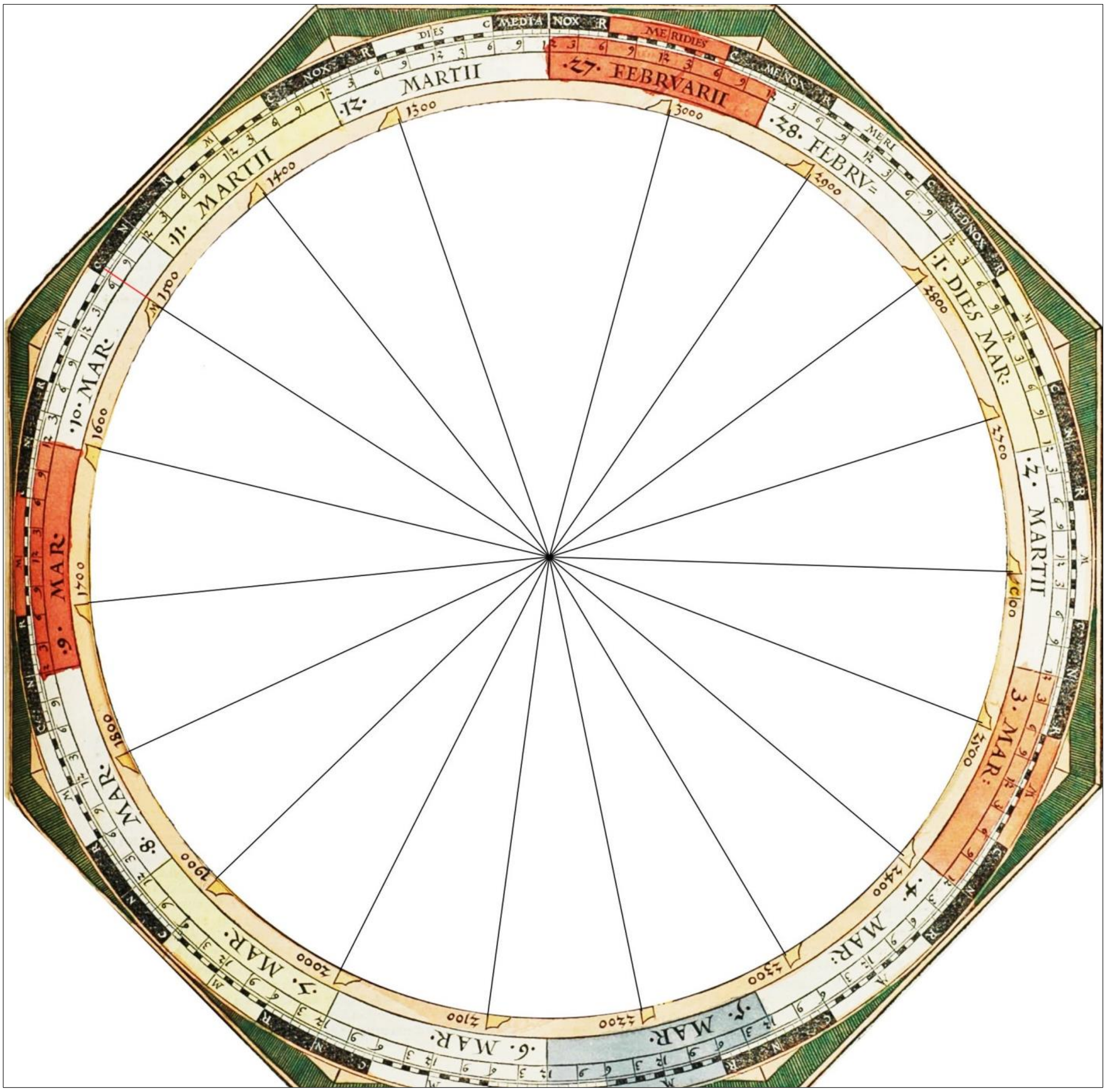


Figure 62: The location of the century indices.

3.22 The Medical Volvelle (page 102)

The last volvelle (Figure 63) is purely astrological, deals with the connection between astrology and medicine, and uses the longitude of the Moon as input. I have not investigated the details of this volvelle.

4 CONCLUDING REMARKS

In preparing the tables, comparisons and simulations I have been aided by a computer, Java programming and the Adobe applications Photoshop and Illustrator. In spite of these powerful tools it has been a major undertaking, and it is easy to appreciate the difficulties of inventing, preparing, computing and printing such a project as the *Astronomicum Cæsareum*, when you consider that all the calculations had to be done by hand and the volvelles then had to be carved by the wood-cutter. This is undoubtedly one of the most extraordinary books in the world.

5 ACKNOWLEDGEMENT

I am very grateful to Dr Christopher Eade for critically checking the manuscript.

6 NOTES

1. The foregoing biographical overview is based largely on data in Draxler and Lippitsch, (2012), Galle (2007) and Gingerich (1971).
2. Guillaume Durand (1230–1296) used the Latin mnemonic Alta Domat Dominus, Gratis Beat Equa Gerentes Contemnit Fictos, Augebit Dona Fideli.
3. Augustus de Morgan, British mathematician and logician (1806–1871).

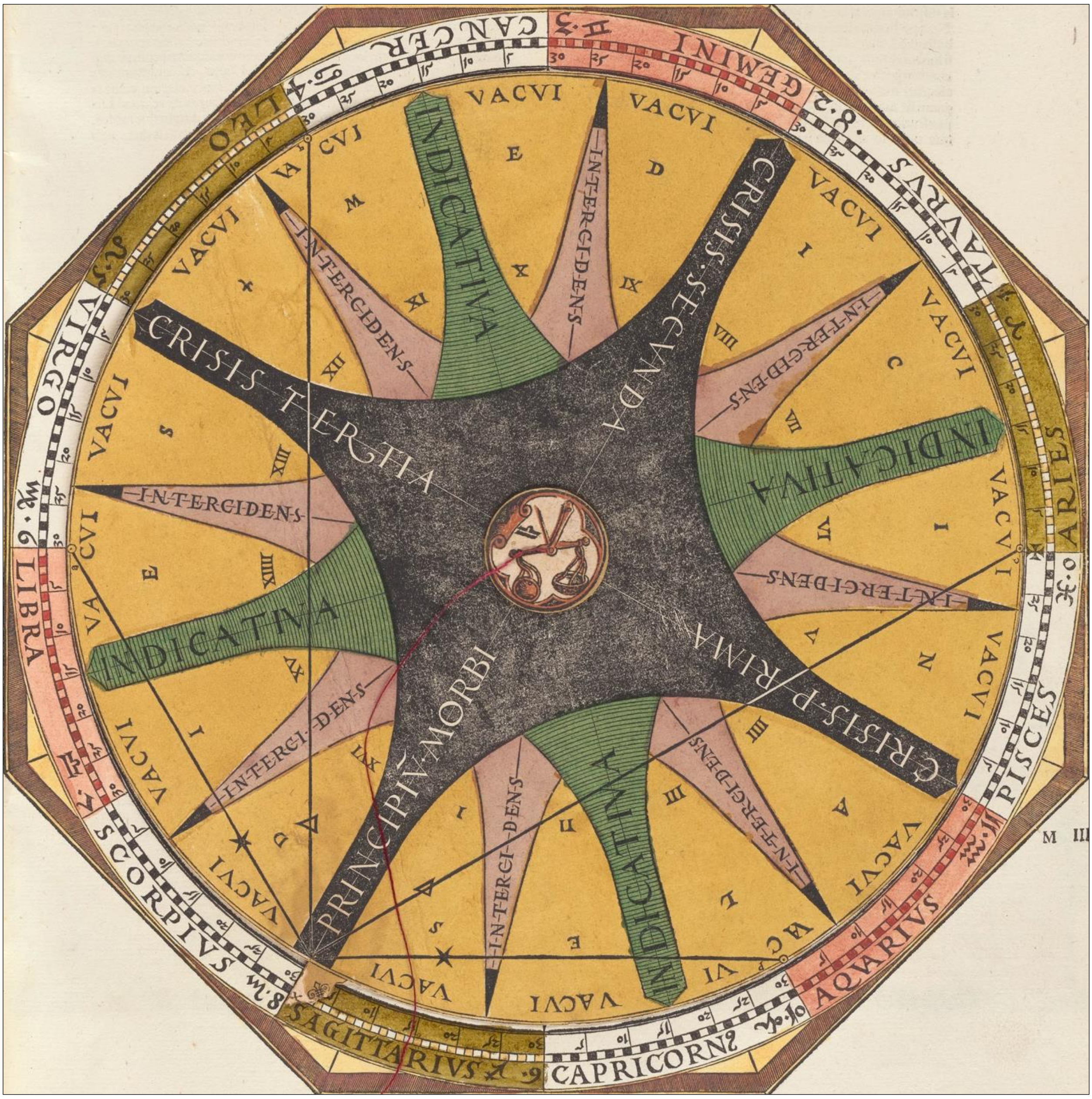


Figure 63: The astrology and medicine volvelle.

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