Some distance properties of tailbiting codes

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Some Distance Properties of Tailbiting Codes

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I. INTRODUCTION

Tailbiting codes can be obtained by terminating rate R = b/c convolutional codes into block codes of length l c-tuples [1]. For simplicity, we consider only binary codes. The error correcting capability of a block code is estimated via its minimum distance \( d_{\text{free}} \). In order to describe the error correcting capability of a tailbiting code beyond the minimum distance \( d_{\text{min}} \), we define the active tailbiting segment distance \( d_{\text{act}} \). We also give an upper bound on the length of a tailbiting code so that \( d_{\text{min}} \) equals \( d_{\text{free}} \) of the corresponding convolutional code.

II. THE ACTIVE TAILBITING SEGMENT DISTANCE

Consider the convolutional code \( C \) encoded by a rate \( R = b/c \) encoder. The binary matrix \( \sigma_t \) denotes the encoder state at time \( t \). Let \( S_{[t_1, t_2]} = \{ \sigma_{t_1}, \ldots, \sigma_{t_2} \} \) be the set of state sequences that start in state \( \sigma_{t_1} \) and terminate in state \( \sigma_{t_2} \), and do not have any consecutive zero states with zero input in between. Let \( j_s \) be the smallest positive integer such that \( S_{[0, j_s]} \neq \emptyset \). The \( j \)th order active burst distance [2] is \( \alpha^j \triangleq \min_{j \geq j_s} \{ w_H(\sigma_j) \} \). For any code \( C \), \( \alpha^j \) is invariant over the set of its canonical encoders [2]. It is lower-bounded by a linearly increasing function \( \gamma_j \geq \alpha^j + \beta^j \), where \( \alpha \) is the asymptotic slope, and \( \beta^j \) is chosen as large as possible. Let \( j^* \) be the smallest number of steps that, starting in the allzero state, take us to any reachable state.

Definition 1 The \( j \)th order active tailbiting segment distance is \( \alpha_{\text{act}}^j \triangleq \min_{\sigma_t, \sigma_{t+1}, \ldots, \sigma_{t+j-1}} \{ w_H(\sigma_j) \} \), where \( \sigma \) denotes any possible encoder state.

Theorem 1 The active tailbiting segment distance is lower-bounded by \( \alpha_{\text{act}}^j \geq \alpha(j + 1) \), for all \( j \geq 0 \).

III. PROPERTIES OF TAILBITING CODES VIA THE ACTIVE DISTANCES

We define an incorrect path at the receiver to be any trellis path differing from the transmitted path. For any \( k_1, k_2 \) such that \( k_1 < k_2 \), let \( e_{[k_1, k_2]} \) be the Hamming weight of the error pattern \( e_{[k_1, k_2]} = e_{k_1} e_{k_1+1} \ldots e_{k_2} \). We also give an upper bound on the length of a tailbiting code so that \( d_{\text{min}} \) equals \( d_{\text{free}} \) of the corresponding convolutional code.

Example 1 A tailbiting code \( C_{\text{tb}} \) of length \( l = 18 \) 2-tuples encoded by a convolutional encoder. Then, for \( j_0 < l \), a maximum likelihood (ML) decoder corrects all error patterns \( e_{[0, j_0]} \) that satisfy \( e_{[0, j_0]} \leq \min \{ a_j^1, a_{j-1}^1 \} \) for \( 0 \leq j < l \), for \( 0 \leq l < l \). For \( j_0 \geq l \), all error patterns that satisfy \( e_{[0, l]} \leq \min \{ a_{l-1}^1 \} \) are corrected.

Consider a tailbiting code of length \( l \) c-tuples with minimum distance \( d_{\text{min}} \). The free tailbiting length \( l_{\text{free}} \) is the shortest length \( l \) for which \( d_{\text{min}} \) will remain equal to \( d_{\text{free}} \) of the corresponding convolutional code for all tailbiting lengths greater than or equal to \( l_{\text{free}} \). The free tailbiting length is upper-bounded by \( l_{\text{free}} \leq \lceil \alpha d_{\text{free}}/\alpha \rceil \).

IV. ENSEMBLE PROPERTIES OF THE ACTIVE TAILBITING SEGMENT DISTANCE

The concept of the active distances can be generalized to time-varying convolutional encoders.

Theorem 3 There exists a rate \( R = b/c \) convolutional code \( C \) encoded by a time-varying encoder of memory \( m \) such that \( d_{\text{free}} > \rho(f) + O(\log m) \), for \( f = O(\log m) \), \( m \rightarrow \infty \), where \( \rho = h^{-1}(1 - R) \) is the Gilbert-Varshamov parameter, \( h(\cdot) \) is the binary entropy function, and \( f_0 \) is the smallest integer satisfying \( 1 - R(f + 1)c/f_0 \geq 4 \log m \).

Using the Heller asymptotic bound we obtain

Theorem 4 There exists a tailbiting code \( C_{\text{tb}} \) encoded by a time-varying encoder of memory \( m \), such that \( l_{\text{free}} = \infty \).

REFERENCES