



# LUND UNIVERSITY

## Some distance properties of tailbiting codes

Handlery, Marc; Höst, Stefan; Johannesson, Rolf; Zyablov, Viktor V.

*Published in:*  
[Host publication title missing]

*DOI:*  
[10.1109/ISIT.2001.936153](https://doi.org/10.1109/ISIT.2001.936153)

2001

[Link to publication](#)

*Citation for published version (APA):*  
Handlery, M., Höst, S., Johannesson, R., & Zyablov, V. V. (2001). Some distance properties of tailbiting codes. In [Host publication title missing] (pp. 290) <https://doi.org/10.1109/ISIT.2001.936153>

*Total number of authors:*  
4

### General rights

Unless other specific re-use rights are stated the following general rights apply:  
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# Some Distance Properties of Tailbiting Codes<sup>1</sup>

Marc Handlery, Stefan Höst, Rolf Johannesson  
Department of Information Technology  
Lund University, Box 118  
S-221 00 Lund, Sweden  
e-mail: {marc, stefanh, rolf}@it.lth.se

Victor V. Zyablov  
Inst. for Problems of Information Transm.  
Russian Academy of Science  
Moscow, Russia  
e-mail: zyablov@iitp.ru

**Abstract** — The active tailbiting segment distance for convolutional codes is introduced. Together with the earlier defined active burst distance, it describes the error correcting capability of a tailbiting code encoded by a convolutional encoder. Lower bounds on the new active distance as well as an upper bound on the ratio between tailbiting length and memory of encoder such that its minimum distance  $d_{\min}$  equals the free distance  $d_{\text{free}}$  of the corresponding convolutional code are presented.

## I. INTRODUCTION

Tailbiting codes can be obtained by terminating rate  $R = b/c$  convolutional codes into block codes of length  $l$   $c$ -tuples [1]. For simplicity, we consider only binary codes. The error correcting capability of a block code is estimated via  $d_{\min}$ . There is no description which error patterns with more than  $\lfloor \frac{d_{\min}-1}{2} \rfloor$  errors can be corrected. In order to describe the error correcting capability of a tailbiting code beyond the minimum distance argument, we define the active tailbiting segment distance  $a_j^{tbs}$ . We also give an upper bound on the length of a tailbiting code so that  $d_{\min}$  equals  $d_{\text{free}}$  of the corresponding convolutional code. This is useful when analyzing concatenated coding schemes containing tailbiting encoders.

## II. THE ACTIVE TAILBITING SEGMENT DISTANCE

Consider the convolutional code  $C$  encoded by a rate  $R = b/c$  encoder. The binary matrix  $\sigma_t$  denotes the encoder state at time  $t$ . Let  $S_{[t_1, t_2]}^{\sigma_s, \sigma_e}$ ,  $0 \leq t_1 < t_2$ , be the set of state sequences  $\sigma_{[t_1, t_2]} = \sigma_{t_1} \dots \sigma_{t_2}$  that start in state  $\sigma_s$  and terminate in state  $\sigma_e$ , and do not have two consecutive zero states with zero input in between. Let  $j_b$  be the smallest positive integer such that  $S_{[0, j_b+1]}^{0,0} \neq \emptyset$ . The  $j$ th order active burst distance [2] is  $a_j^b \triangleq \min_{S_{[0, j+1]}^{0,0}} \{w_H(v_{[0, j]})\}$ ,  $j \geq j_b$ . For any code  $C$ ,  $a_j^b$  is invariant over the set of its canonical encoders [2]. It is lower-bounded by a linearly increasing function  $a_j^b \geq \alpha j + \beta^b$ ,  $j \geq j_b$ , where  $\alpha$  is the asymptotic slope, and  $\beta^b$  is chosen as large as possible. Let  $j_r$  be the smallest number of steps that, starting in the allzero state, take us to any reachable state.

**Definition 1** The  $j$ th order active tailbiting segment distance is  $a_j^{tbs} \triangleq \min_{S_{[j_r, j_r+j+1]}^{\sigma, \sigma}} \{w_H(v_{[j_r, j_r+j]})\}$ , where  $\sigma$  denotes any possible encoder state.

**Theorem 1** The active tailbiting segment distance is lower-bounded by  $a_j^{tbs} \geq \alpha(j+1)$ , for all  $j \geq 0$ .

<sup>1</sup>This research was supported in part by the Swedish Academy of Science in cooperation with the Russian Academy of Sciences and in part by the Swedish Research Council for Engineering Sciences under Grant 97-723 and 98-501.

## III. PROPERTIES OF TAILBITING CODES VIA THE ACTIVE DISTANCES

We define an *incorrect path* at the receiver to be any trellis path differing from the *transmitted path*. For any  $k_1, k_2 < l$ , let  $e_{[k_1, k_2]}$  denote the Hamming weight of the error pattern  $e_{[k_1, k_2]} = e_{k_1} e_{k_1+1} \dots e_{k_2}$ , where  $e_i$ ,  $0 \leq i < l$ , are  $c$ -tuples and all indices are evaluated modulo  $l$ . Then, we have

**Theorem 2** Consider a tailbiting code  $C^{tb}$  of length  $l$   $c$ -tuples encoded by a convolutional encoder. Then, for  $j_b < l$ , a maximum likelihood (ML) decoder corrects all error patterns  $e_{[0, l-1]}$  that satisfy  $e_{[t, t+j \bmod l]} < \min \{a_j^b/2, a_{l-1}^{tbs}/2\}$  for  $0 \leq t < l$ ,  $j_b \leq j < l$ . For  $j_b \geq l$ , all error patterns that satisfy  $e_{[0, l-1]} < a_{l-1}^{tbs}/2$  are corrected.

**Example 1** A tailbiting code of length  $l = 18$  2-tuples encoded by  $G(D) = (1 + D + D^2 \quad 1 + D^2)$  with  $d_{\min} = 5$  is used on a binary symmetric channel. From Theorem 2 follows that

$$e_{[0, 17]} = 10\ 00\ 00\ 01\ 00\ 00\ 00\ 00\ 01\ 00\ 00\ 00\ 10\ 00\ 00\ 00\ 00\ 00$$

is corrected by an ML decoder although the error pattern contains four channel errors which exceeds  $\lfloor \frac{d_{\min}-1}{2} \rfloor$ .

Consider a tailbiting code of length  $l$   $c$ -tuples with minimum distance  $d_{\min}$ . The free tailbiting length  $l_{\text{free}}$  is the shortest length  $l$  for which  $d_{\min}$  will remain equal to  $d_{\text{free}}$  of the corresponding convolutional code for all tailbiting lengths greater than or equal to  $l_{\text{free}}$ . The free tailbiting length is upper-bounded by  $l_{\text{free}} \leq \lfloor d_{\text{free}}/\alpha \rfloor$ .

## IV. ENSEMBLE PROPERTIES OF THE ACTIVE TAILBITING SEGMENT DISTANCE

The concept of the active distances can be generalized to time-varying convolutional encoders.

**Theorem 3** There exists a rate  $R = b/c$  convolutional code  $C$  encoded by a time-varying encoder of memory  $m$  such that  $a_j^{bs} > \rho(j+1)c + O(\log m)$ , for  $j = O(m) \geq j_0$   $m \rightarrow \infty$

where  $\rho = h^{-1}(1-R)$  is the Gilbert-Varshamov parameter,  $h(\cdot)$  is the binary entropy function, and  $j_0$  is the smallest integer satisfying  $(1-R)(j+1)c \geq 4 \log m$ .

Using the Heller asymptotic bound we obtain

**Theorem 4** There exists a tailbiting code  $C^{tb}$  encoded by a time-varying encoder of memory  $m$ , such that  $\lim_{m \rightarrow \infty} l_{\text{free}}/m \leq \frac{1}{2\rho}$ .

## REFERENCES

- [1] G. Solomon and H. C. A. van Tilborg, "A connection between block and convolutional codes," *SIAM J. Appl. Math.*, vol. 37, 1979.
- [2] S. Höst, R. Johannesson, and V. V. Zyablov, "Active distances for convolutional codes", *IEEE Trans. Inform. Theory*, vol. 44, Mar. 1999.