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Bocharova, Irina; Kudryashov, Boris; Johannesson, Rolf; Ståhl, Per

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Searching for tailbiting codes with large minimum distances

Irina E. Bocharova and
Boris D. Kudryashov
Dept. of Inform. Systems
University on Airspace
Instrumentation
St.-Petersburg, 190000, Russia,
e-mail: {Irina,Boris}@kb.spb.su

Rolf Johannesson and
Per Ståhl
Dept. of Inform. Technology
Lund University
P.O. Box 118
SE-221 00 LUND, Sweden,
e-mail: {Rolf,Per}@it.lth.se

Abstract — **Tailbiting trellis representations of linear block codes with an arbitrary sectionalization of the time axis are studied. A new lower bound on the maximal state complexity of an arbitrary tailbiting code is derived. The asymptotic behavior of the derived bound is investigated. Some new tailbiting representations for linear block codes of rates $R = 1/c$, $c = 2,3,4$ are presented.**

I. Introduction

Tailbiting is a technique to terminate a convolutional code into a block code [1]. We focus on constructions and bounds for sectionalized tailbiting trellises since they may have less complexity than non-sectionalized ones.

We consider an $(N, K, d_{\min})$ binary linear block code $\mathcal{C}$ with a generator matrix $G = \{\mathbf{r}_i\}, i = 1, ..., K$. We say that G is given in *tailbiting span form* if it consists of rows such that (circular) $\text{start}(\mathbf{r}_i) \neq \text{start}(\mathbf{r}_j)$ and $\text{end}(\mathbf{r}_i) \neq \text{end}(\mathbf{r}_j)$, $i \neq j$, where $\text{start}(\mathbf{x})$ and $\text{end}(\mathbf{x})$ denote the (circular) number of the first and the last nonzero section in the vector $\mathbf{x}$, respectively. The ith section of $\mathbf{r}_j$ is *active* if $i \in [\text{start}(\mathbf{r}_j), \text{end}(\mathbf{r}_j))$. The maximal state complexity or μ-state complexity of the trellis is defined [2] as $\mu = \max_i\{\log_2 |A_i|\}$, where $|A_i|$ denotes the number of rows where the ith section is active.

II. Lower bound on the state complexity for tailbiting codes

Theorem 1 *The state complexity μ of a linear $(N, K, d_{\min})$ tailbiting code is lower-bounded by*

$$\mu \geq \mu_0 = \left\lceil \max_{j=1,...,K} \{RN_{\min}(j, d_{\min}) - j\} \right\rceil .$$

Moreover, if μ_0 is odd $\mu \geq \max\{\mu_0, d_{\min}(K + 1)/N - 1\}$.

Denote by $\zeta = \mu/N$ the relative trellis complexity. Then we have the following asymptotic behavior of ζ as $N \to \infty$,

$$\zeta \geq \max_{\theta\in[2\delta,1]} \{\theta [R - R_{\max}(\delta/\theta)]\},$$

where $\delta = d_{\min}/n$, and $R_{\max}(\cdot)$ is the McEliece-Rodemich-Rumsey-Welch upper bound.

III. Search techniques and results

We have used the bound in Theorem 1 to find an efficient (in sense of state complexity) tailbiting representation for an (N, K) linear block code using time-invariant convolutional codes of rate $R = 1/c$, $c = 2,3,4$, and state complexity (constraint length) μ. We exploit two kinds of methods to reject weak codes. The first one includes rules for rejecting weak encoders of convolutional codes. The second one rejects those encoders among the accepted ones which generate poor tailbiting codes. Some search results are presented in the following table.

$N, K, d_{\min}(\hat{d}_{\min})$	$\mu(\hat{\mu})$	Generators
56,28,12(12-14)	9(8)	477,1505
58,29,12(12-14)	9(8)	433,1275
60,30,12(12-14)	9(8)	217,1665
62,31,12(12-15)	8(8)	435,657
64,32,12(12-16)	8(8)	235,557
66,33,12(12-16)	8(8)	235,557
68,34,13(13-16)	11(9)	4315,5651
72,36,14(15-18)	13(10)	4473,32611
74,37,14(14-18)	11(10)	1353,7461
76,38,14(14-18)	11(10)	1145,7173
78,39,14(15-18)	10(10)	1473,2275
82,41,14(14-20)	10(10)	1157,3455
84,42,14(15-20)	10(10)	1157,3455
92,46,16(15-22)	13(11)	5447,21675
94,47,16(16-22)	12(11)	5135,14477
96,48,16(16-22)	12(11)	5135,14477
110,55,18(18-25)	15(14)	23077,173255
84,28,22(22-27)	11(10)	2215,5467,7647
96,32,24(24-30)	12(11)	2153,11625,17557
99,33,24(24-32)	11(11)	4467,5725,6373
102,34,24(24-32)	11(11)	4465,5357,6373
105,35,25(24-33)	13(13)	20447,25315,37317
108,36,26(24-34)	13(13)	20465,31327,34773
111,37,26(25-34)	13(13)	20445,31527,35757
114,38,26(26-36)	13(13)	20445,31653,37673
120,40,28(28-37)	14(14)	41127,63663,72575
112,28,32(32-40)	11(11)	4447,5277,6335,7533
116,29,32(32-42)	11(11)	4445,6353,6537,7673

Almost all codes meet the Brower-Verhoeff (BV) lower bound $\hat{d}_{\min}$ on the minimum distance for linear codes and achieve the lower bound $\hat{\mu}$ on the state complexity. All presented codes are new best known quasi-cyclic codes. The code (111,37,26) is better than any previously known linear code with the same length and dimension, and the codes (92,46,16), (105,35,25) and (108,36,26) are better than any previously known codes with the same length and dimension.

References

[1] G. Solomon and H. C. A. van Tilborg, "A connection between block and convolutional codes," *SIAM J. Appl. Math.*, vol. 37, pp. 358-369, 1979.

[2] A. R. Calderbank, G. D. Forney, Jr. and A. Vardy, "Minimal tail-biting trellises: the Golay code and more," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1435-1455, 1999.

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