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SIMULTANEOUS CHANNEL AND SYMBOL MAXIMUM LIKELIHOOD ESTIMATION IN LAPLACIAN NOISE

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ABSTRACT

This paper treats channel estimation and signal detection in Laplacian noise. The received signal is assumed to be a transmitted signal which has been corrupted by an unknown channel, modeled as a FIR filter, the output being further disturbed by additive independent Laplacian noise. The transmitted signal is assumed to depend on an unknown parameter belonging to a known finite set.

The simultaneous maximum likelihood (ML) estimator of the unknown parameter, as well as of the FIR filter coefficients, is derived. The ML estimate of the channel can be obtained by using a linear programming approach and the decision about the parameter is based on the output from a set of generalized matched filters. Simulation results are included in order to illustrate the performance of the proposed receivers.

1. INTRODUCTION

A number of situations exist where the problem is to estimate a parameter in an observed signal [10]. Examples of such problems would be arrival time estimation, detection or classification of a signal, etc. It is normally assumed in these cases that the noise is Gaussian [9], [10]. Two common motives for this assumption are that the noise in many applications is approximately Gaussian according to the central limit theorem, and that the Gaussian assumption is analytically tractable.

Despite this, the problem of parameter estimation and signal detection in non-Gaussian noise has over the years been an active field of research [3], [7]. In part, this research is motivated by an interest in a better general understanding of estimation and detection, as well as by the fact that some noise environments actually are non-Gaussian. The Laplacian model is one example of a non-Gaussian noise model. This model has been used in studies of data quantization for signal detection schemes [1]. It has also been used in image processing [12] and in modeling noise encountered in communications at extremely low frequencies [2]. Noise described by this model is more impulsive in character than noise described by the commonly used Gaussian model. Estimation and detection in non-Gaussian environments is a notably more complex problem. The combined requirements for an analytically tractable model and for physical representation are often contradictory.

In many cases the model used for parameter estimation has the form $r = h * s_\theta + v$, where h is a known or unknown impulse response, v is independent additive noise and s_θ is a known signal depending on a parameter θ, which is a quantity to be estimated from the observed signal. This model is often used in conjunction with some common optimization criterion, such as the maximum likelihood (ML) criterion, the maximum a posteriori (MAP) criterion or the minimum mean square error (MMSE) criterion [10].

The estimation problem is generally more complex if the impulse response h in the model is both unknown and unquantified. The simultaneous ML estimate of h and θ can be calculated exactly, however, when certain restrictions are fullfilled. Such results have been reported when h is modeled as a finite impulse response (FIR) filter and the noise is either Gaussian or uniformly distributed [3], [5], or when the noise is Laplacian and h is a non-dispersive channel [3], [4].

In this paper we generalize the results presented in [3], [4] to the case when the channel is dispersive. We derive the simultaneous ML estimator of h and θ for the case when the noise is Laplacian and the channel modeled as a unknown FIR filter. The estimate of h is obtained using a linear programming approach and the decision about θ is based on the output from a set of generalized matched filters [3].

We examine the performance of receivers based on either the proposed ML estimators or the conventional least squares estimators in a case of binary communication. Examples of performance of the receivers are calculated by means of simulations and the results are compared when the noise is either Laplacian or Gaussian.

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2. SIGNAL MODEL

We consider the problem of estimating a parameter θ from an observed signal r, where r is modeled as

$$r(k) = (h*s_\theta)(k)+v(k) = \sum_{l=0}^{M-1} h(l)s_\theta(k-l)+v(k),\ k \in I. \tag{1}$$

In this model s_θ is a signal dependent on the unknown parameter θ, h is an unknown real impulse response of a channel with a known finite length M, v is independent zero-mean Laplacian noise, k is a discrete 'time' variable and I is the observation interval. The unknown parameter θ, whether random or non-random, is assumed to belong to a known and finite set Θ where θ is independent of the noise v. Further, it is assumed both that for all values of θ, $(h * s_\theta)(k) = 0$ for all k outside the observation interval and that some rough knowledge about the channel h is available (for example, that the parameters $h(l)$ are constrained by some lower and/or upper limits).

The case when $M = 1$ has been discussed in [4]. In this case the parameter $h(0)$ can be interpreted as an unknown gain A of a channel with, for instance, a known non-negative lower bound.

3. THE MAXIMUM LIKELIHOOD RECEIVER

The probability density function of zero-mean Laplacian noise v is

$$f_v(x) = \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|x|/\sigma},\quad x \in \mathbf{R}, \tag{2}$$

where σ^2 is the variance of the noise.

Let the channel h be represented by a vector in an M-dimensional Euclidean space $\mathbf{R}^M$. Assume that h is restricted to belong to some subset $\mathcal{H}$ of $\mathbf{R}^M$, where $\mathcal{H}$ is chosen to match some given a priori knowledge about the channel. The likelihood function for parameters (h, θ) is

$$\begin{aligned} L_1(h,\theta) &= \prod_{k\in I} f_v(r(k) - (h * s_\theta)(k)) \\ &= \prod_{k\in I} \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|r(k)-(h*s_\theta)(k)|/\sigma}. \end{aligned} \tag{3}$$

The maximum likelihood estimate of h for a given θ, is the member of $\mathcal{H}$ which maximizes $L_1(h,\theta)$. By observing that

$$L_1(h,\theta) \propto \exp\left(-\sum_{k\in I} |r(k) - (h * s_\theta)(k)|\right) \tag{4}$$

it is seen that

$$\hat{h}(\theta) = \arg\left\{\min_{h\in\mathcal{H}} \sum_{k\in I_\theta} |r(k) - (h * s_\theta)(k)|\right\}, \tag{5}$$

where $I_\theta \subseteq I$ is defined by $\{k : (h * s_\theta)(k) \neq 0$ for any $h \in \mathcal{H}\}$. Thus, I_θ is the union of the support for $h * s_\theta$ over all $h \in \mathcal{H}$. In the case when h is a pure gain factor, the estimate $\hat{h}(\theta)$ is given by the weighted median [3], [4].

The simultaneous maximum likelihood estimate of (h, θ) is $(\hat{h}(\hat{\theta}), \hat{\theta})$, where

$$\hat{\theta} = \arg\left\{\max_{\theta\in\Theta} L_1(\hat{h}(\theta), \theta)\right\}, \tag{6}$$

c.f. the generalized likelihood ratio test [10]. A block structure for calculating an estimate $\hat{\theta}$ of θ when the channel impulse response h is unknown is shown in Figure 1.

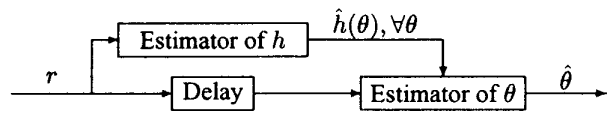

Figure 1: *A block structure for calculating an estimate $\hat{\theta}$ of θ when the channel's impulse response is unknown. The calculations of $\hat{h}(\theta)$ must be repeated for all possible values of θ.*

The channel estimate

Suppose that the set $\mathcal{H}$ is defined using linear constraints, e.g., by constraining the parameters $h(l)$ by some lower limit $h_0(l)$ so that $h(l) \geq h_0(l)$. In this case the estimate $\hat{h}(\theta)$ in (5) can be obtained using a linear programming formulation as elaborated below. This approach is based on the l_1-solution of an overdetermined system of linear equations by linear programming [11].

Suppose first that $\mathcal{H}$ is given by $h(k) \geq 0, \forall k$. The estimate $\hat{h}(\theta)$ in (5) is then given by the following formulation

$$\begin{cases} \min \sum |e(k)| \\ S_\theta h + e = r \\ h \geq 0, \end{cases} \tag{7}$$

where S_θ is a convolution matrix with elements $s_\theta(k-l)$, r and e are vectors containing the observed signal $r(k)$ and the error signal $e(k) = r(k) - (h * s_\theta)(k)$, respectively, $k \in I_\theta$ and $l = 0, 1, \ldots, M-1$.

By introducing the substitution $e = u - v, u \geq 0, v \geq 0$, the problem (7) can be reformulated as the following linear program.

$$\begin{cases} \min \sum u(k) + v(k) \\ S_\theta h + u - v = r \\ h \geq 0, u \geq 0, v \geq 0 \end{cases} \tag{8}$$

The formulation (8) is now in standard form for solution by the two–phase simplex algorithm [8]. Since this algorithm will yield an optimum *basic* solution and since the

algorithm guarantees non–singularity of the basis, the variables $u(k)$ and $v(k)$ cannot be simultaneously basic [11]. Hence, if one of the variables is greater than zero, then the other must be equal to zero, and we have $u(k) + v(k) = \pm(u(k) - v(k)) = \pm e(k) = |e(k)|$.

The formulation (7) can be generalized to include any linear constraints on h. Suppose that $\mathcal{H}$ is given by $\mathcal{H} = \{h : Bh \leq b, Ch = c\}$, where B and C are matrices and b and c are vectors. Introduce the substitution $h = x - y$ and the slack variables z, where $x \geq 0, y \geq 0$ and $z \geq 0$.

The generalized linear programming formulation is

$$\begin{cases} \min \sum u(k) + v(k) \\ S_\theta x - S_\theta y + u - v = r \\ Bx - By + z = b \\ Cx - Cy = c \\ x \geq 0, y \geq 0, z \geq 0, u \geq 0, v \geq 0. \end{cases} \tag{9}$$

The linear program (9) is also in standard form. Since the additional linear constraints on h do not influence the column–structure corresponding to the variables u and v, it is concluded that $u(k) + v(k) = |e(k)|$ by the same argument as above.

The maximum likelihood estimate (6) of the symbol θ can be based on the output from a set of generalized matched filters [3], [6]. This set contains filters matched to s_θ for all possible values of θ. The single difference to the ordinary matched filter is that the multipliers are replaced by non-linear functions in the generalized matched filter. For Laplacian noise the characteristic of the non-linearities is given by the soft-limiter and the output from the generalized matched filter, for a given channel estimate $\hat{h}(\theta)$, is given by [3], [4]

$$L(\hat{h}(\theta), \theta) = \sum_{k \in I_\theta} |r(k) - (\hat{h}(\theta) * s_\theta)(k)|. \tag{10}$$

The output $L(\hat{h}(\theta), \theta)$ is related to the value of $L_1(\hat{h}(\theta), \theta)$, given by (3), and can be used to estimate θ according to (6).

4. EXAMPLES OF PERFORMANCE

Consider the binary hypothesis test problem with hypotheses H_0 and H_1 corresponding to the parameter values $\theta = \theta_0$ and $\theta = \theta_1$, respectively. In the case discussed in this section the hypotheses are

$$\begin{aligned} H_0 &: s_{\theta_0}(k) = -s(k), \forall k \\ H_1 &: s_{\theta_1}(k) = s(k), \forall k, \end{aligned} \tag{11}$$

where the signal is assumed to be $s = [1\ 2\ 3\ 2\ 1]$. This might be a model of a communication system using antipodal signaling.

The probability of error, p_e, for four different receivers has been calculated by means of simulations when the noise is either independent, zero-mean, unit-variance Laplacian or Gaussian. Hypotheses H_0 and H_1 are equally likely. The signal to noise ratio (SNR) is defined by

$$\text{SNR} = 10 \log \left\{ \frac{\sum_{k \in I_\theta} [(h * s_\theta)(k)]^2}{\sigma^2} \right\}, \tag{12}$$

where h is the true impulse response of the channel which in the calculations is assumed to be $h = A[1\ 2\ 3\ 2\ 1]$ (this channel is discussed in [9]). The gain parameter A is varied in order to obtain different SNR's.

The receivers used are based on the Neyman-Pearson detector and are constructed according to Fig. 1. The estimators of h and θ used in the receivers are optimal according to the ML-criterion when the noise is either Laplacian or Gaussian. These receivers are denoted D_{Lap} and D_{Gauss}, respectively. Further, the performance of the corresponding optimal receivers when the channel is known is calculated. Those receivers are denoted $D_{\text{Lap}}^{h \text{ known}}$ and $D_{\text{Gauss}}^{h \text{ known}}$, respectively.

The results of the simulations for Laplacian and Gaussian noise are shown in Figs. 2 and 3, respectively. In the figures p_e is shown as a function of SNR. The results indicate, as expected, that the best performance is achieved when the channel is known and the optimal receiver for the actual noise is used. When the channel is unknown, each ML receiver has a comparatively high performance for low SNR's, whereas for high SNR's this is not always the case. For Laplacian noise and high SNR's the receiver designed for Gaussian noise has a better performance than the one designed for Laplacian noise. This is because the error decisions occur mainly when there are large errors in the channel estimate (this is the case for both types of noise at high SNR's). Those large errors are more frequently encountered when the l_1-norm is used to estimate the channel than when the l_2-norm is used. This last result can be compared to the corresponding observation in uniformly distributed noise, where the channel estimate has a higher fraction of large errors when the l_∞-norm is used than when the l_2-norm is used [3], [5].

5. SUMMARY

Parameter estimation and detection in Laplacian noise have been discussed. The received signal is assumed to be a transmitted signal which has passed through an unknown channel, which is modeled as a FIR filter, the output being further corrupted by additive independent Laplacian noise. The transmitted signal is assumed to be dependent on an unknown parameter in a known finite set.

The simultaneous maximum likelihood (ML) estimator of the unknown parameter and of the FIR filter coefficients

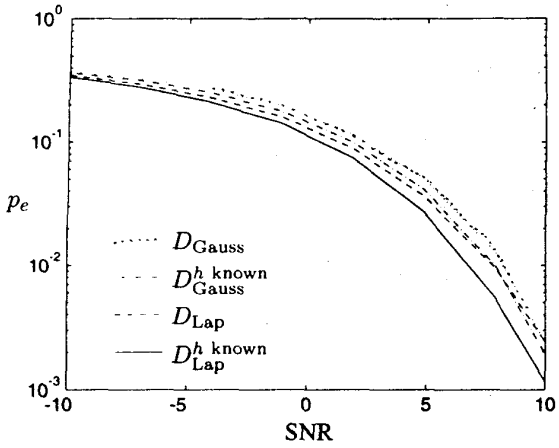

Figure 2: *The probability of error, p_e versus the SNR when the noise is Laplacian for four different detectors.*

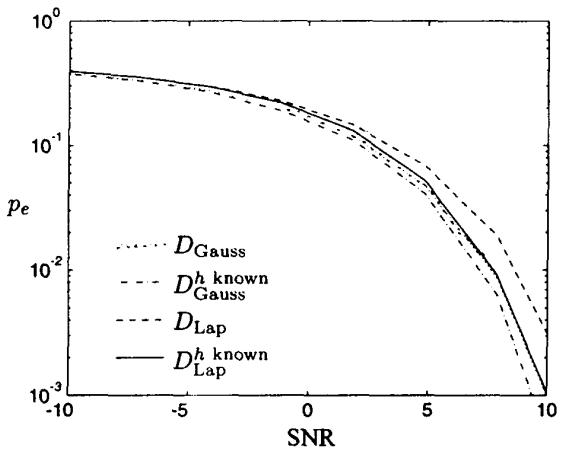

Figure 3: *The probability of error, p_e versus the SNR when the noise is Gaussian for four different detectors.*

has been derived. The estimators derived comprise two parts. Assuming a given transmitted signal, the first part estimates the channel using a linear programming approach and the second part uses this estimate of the channel to calculate a measure, which is related to the probability of the assumed transmitted signal being the true transmitted signal, e.g., the likelihood function.

Simulation results are included for a case of antipodal communication in order to compare the performance of the proposed ML receiver to the performance of the ordinary least square receiver (i.e., the ML receiver in Gaussian noise). The results indicate that each ML receiver has the best performance at low SNR's in in their respective noise environments. At high SNR's in a Laplacian noise environment the Gaussian ML receiver sometimes performs better than the Laplacian ML receiver. The Laplacian ML receiver's poorer result on these occassions is due to larger errors in the channel estimate, which may cause wrong decisions about the signal parameter.

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