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# A PRIORI BOUNDS ON THE ONSET FREQUENCY OF WIDEBAND ANTENNAS

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## ABSTRACT

This paper presents new bounds on the onset frequency and partial realized gain of wideband antennas. The result is a sum rule quantifying the antenna performance in terms of its low-frequency properties via certain static boundary-value problems. The theoretical findings are compared with numerical simulations using the method of moments.

## 1. INTRODUCTION

This conference paper is based on a recent approach on physical bounds on antennas set forth in Refs. 1 and 2. For this purpose, consider an antenna of arbitrary shape modeled by linear and time-translational invariant constitutive relations in terms of the electric and magnetic susceptibilities  $\chi_e = \chi_e(\mathbf{x})$  and  $\chi_m = \chi_m(\mathbf{x})$ , respectively.<sup>1</sup> Based on the holomorphic properties of the forward scattering dyadic, a sum rule for the partial realized gain  $g$  (with respect to the spatial  $\hat{\mathbf{k}}$ -direction and electric  $\hat{\mathbf{e}}$ -polarization) is derived in Refs. 1 and 2, *viz.*,

$$\int_0^\infty \frac{g(k; \hat{\mathbf{k}}, \hat{\mathbf{e}})}{k^4} dk = \frac{\eta}{2} \left( \hat{\mathbf{e}}^* \cdot \boldsymbol{\gamma}(\chi_e) \cdot \hat{\mathbf{e}} + (\hat{\mathbf{k}} \times \hat{\mathbf{e}}^*) \cdot \boldsymbol{\gamma}(\chi_m) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{e}}) \right), \quad (1)$$

where  $\eta = \eta(-\hat{\mathbf{k}}, \hat{\mathbf{e}}^*)$  is a real-valued number in the unit interval  $[0, 1]$ . Here, the static polarizability dyadic  $\boldsymbol{\gamma}$  is defined by ( $\ell$  takes any of the values e and m depending on whether the problem is of electric or magnetic nature)

$$\boldsymbol{\gamma}(\chi_\ell) = \sum_{i,j=1}^3 (\hat{\mathbf{a}}_i \cdot \boldsymbol{\gamma}_{ij}) \hat{\mathbf{a}}_i \hat{\mathbf{a}}_j,$$

where  $\hat{\mathbf{a}}_1$ ,  $\hat{\mathbf{a}}_2$  and  $\hat{\mathbf{a}}_3$  form an arbitrary set of linearly independent unit vectors, and

$$\boldsymbol{\gamma}_{ij} = \int_{\mathbb{R}^3} \chi_\ell(\mathbf{x}) (\hat{\mathbf{a}}_j - \nabla \psi_j(\mathbf{x})) dV_{\mathbf{x}}.$$

<sup>1</sup>The results in this paper are formulated for isotropic susceptibilities, but they can easily be extended to include anisotropic or bi-anisotropic material models.

The scalar potential  $\psi_j$  is the unique solution of the static boundary-value problem

$$\begin{cases} \nabla \cdot ((\chi_\ell(\mathbf{x}) + 1) \nabla \psi_j(\mathbf{x})) = \hat{\mathbf{a}}_j \cdot \nabla \chi_\ell(\mathbf{x}) & \mathbf{x} \in \mathbb{R}^3, \\ \psi_j(\mathbf{x}) = \mathcal{O}(x^{-2}) \text{ as } x \rightarrow \infty \end{cases}$$

where  $x = |\mathbf{x}|$ . It is surprising to see that the integral on the left-hand side of (1) is related to the static or low-frequency behavior of the antenna.

As an example of how (1) can be used in modern antenna design, consider a planar antenna  $\Lambda$  enclosed by a circular disk  $\Lambda_+ = \{\mathbf{x} \in \mathbb{R}^3 : x \leq a\}$  of radius  $a$ .<sup>2</sup> Let  $\hat{\mathbf{n}}$  denote the outward-directed unit normal vector of the disk, and choose  $\hat{\mathbf{k}} = \hat{\mathbf{n}}$  and  $\hat{\mathbf{e}} = \hat{\boldsymbol{\rho}}$ , corresponding to a direction of observation and an electric polarization which are perpendicular and parallel to the disk, respectively. Introduce the frequency band  $f \in [3.1, 10.6]$  GHz, or equivalently  $k \in [0.65, 2.22]$  cm<sup>-1</sup>, as the appropriate frequency band for ultra-wideband (UWB) communication in North America. Assume that  $\Lambda$  is specified to have a partial realized gain

$$g(k; \hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) \geq \begin{cases} g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) k^4 / k_1^4 & k \in [0, k_1] \\ g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) & k \in [k_1, k_2], \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where  $k_1 = 0.65$  cm<sup>-1</sup> and  $k_2 = 2.22$  cm<sup>-1</sup>. Then, for a given threshold  $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})$ , it is desirable to determine the smallest radius  $a$  such that it is feasible for  $\Lambda$  to have a partial realized gain which satisfies (2).

Based on (2), a straightforward calculation of (1) yields

$$\int_0^\infty \frac{g(k; \hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{k^4} dk \geq g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) \left( \frac{1}{k_1^3} + \int_{k_1}^{k_2} \frac{dk}{k^4} \right) = \frac{g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{3} \frac{4k_2^3 - k_1^3}{k_1^3 k_2^3}.$$

From the analysis in Ref. 1, it follows that the polarizability dyadics for the perfectly electric conducting circular disk are  $\boldsymbol{\gamma}(\chi_e) = 16a^3 \mathbf{I}_\perp / 3$  and  $\boldsymbol{\gamma}(\chi_m) = \mathbf{0}$ ,

<sup>2</sup>Here, the support  $\Lambda$  is defined by  $\Lambda = \Lambda_e \cup \Lambda_m$ , where  $\Lambda_\ell = \{\mathbf{x} \in \mathbb{R}^3 : \chi_\ell(\mathbf{x}) \neq 0\}$  and  $\ell$  takes any of the values e and m.

respectively, where  $\mathbf{I}_\perp = \mathbf{I} - \hat{\mathbf{n}}\hat{\mathbf{n}}$  denotes the projection dyadic in  $\mathbb{R}^3$ . Hence, by inserting (??) into (1), one obtains

$$\frac{g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{a^3} \leq 0.55\eta(-\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}),$$

where  $a$  now measures the radius of the disk in units of centimeters. For example, by invoking the upper bound  $\eta(-\hat{\mathbf{k}}, \hat{\mathbf{e}}^*) < 1$ , it is concluded that the minimum radius of the disk is 1.8 cm for  $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 3$ , and 1.9 cm for  $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 4$ . For many antennas,  $\eta$  is close to 1/2 and a more realistic bound is therefore 2.2 cm and 2.4 cm for  $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 3$  and  $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 4$ , respectively.

The conference presentation will focus on the use of this sum in antenna design, and how static considerations can offer fundamental insights into the behavior of wideband antennas, *e.g.*, by establishing estimates on the onset antenna frequency. The theoretical findings will be compared with several numerical simulations using the method of moments.

## 2. REFERENCES

- [1] M. Gustafsson, C. Sohl, and G. Kristensson. Physical limitations on antennas of arbitrary shape. *Proc. R. Soc. A*, 463:2589–2607, 2007.
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