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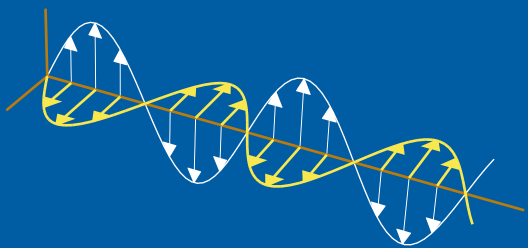

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**Abstract**

A previous investigation has shown that at normal angle of incidence, the integral of the reflectance over wavelength is bounded for a flat metal backed absorber. The bound is applicable to any absorber made of linear, time-invariant, causal and passive materials. We generalize the physical bound to arbitrary angle of incidence and polarization. Different design examples and numerical calculations are provided to investigate the inequalities. It is shown that the theoretical limit for TE polarization results in fair approximations of the integral of the reflectance over wavelength but the TM polarization overestimates the integral value. A simple relation for estimating the optimal thickness of a nonmagnetic absorber is suggested for arbitrary angle of incidence.

# 1 Introduction

The main goal of an absorber design is to achieve the desired frequency response with the minimum possible total thickness. For normal angle of incidence, Rozanov has shown that the integral of the reflectance over wavelength is bounded by a theoretical limit [10]. The inequality applies to any flat metal backed absorber consist of linear, time-invariant, causal and passive materials. Therefore, all the metamaterial, electromagnetic band-gap and frequency selective surface based absorber are included [3–9]. The physical bound can be used efficiently to estimate the minimum possible thickness of a nonmagnetic absorber for normal incident illumination. Although conventional methods of absorbers are usually formulated for normal angle of incidence, there are cases that the performance of the absorber at oblique angles becomes important [2, 9]. For such applications it is necessary to generalize the physical bound. This paper considers the generalization of the physical bound to arbitrary angle of incidence. It is shown that theoretical limit becomes polarization dependent at oblique angles of incidence. To proceed the time dependence $e^{-i\omega t}$ is assumed throughout the analysis. The reflection coefficient is treated as a function of the free space wavelength $\lambda$ and is defined as the quotient of the amplitude of the incident and reflected electric fields.

# 2 The physical bound for normal incidence

The reflection coefficient has no poles in the lower half-plane but may have nulls there. If the nulls are located at $\lambda_1, \ldots \lambda_n, \ldots$ then the function

$$\tilde{R}(\lambda) = R(\lambda)\frac{(\lambda-\lambda_1^*)\ldots(\lambda-\lambda_n^*)\ldots}{(\lambda-\lambda_1)\ldots(\lambda-\lambda_n)\ldots}$$

where $*$ denotes complex conjugation, has neither poles, nor zeros, in the lower half-plane. Hence the logarithm of $\tilde{R}(\lambda)$ is an analytic function in the lower half-plane and the Cauchy theorem can be applied. Integrate $\tilde{R}(\lambda)$ along the real axis and close the contour with semi-circle $\mathcal{C}_\infty$ in the lower half-plane. Notice that $|\tilde{R}(\lambda)| = |R(\lambda)|$

at real wavelengths and that the real part of $\ln(R(\lambda))$ is an even function of $\lambda$. The real part of the Cauchy integral over the contour transforms to

$$\begin{aligned} \mathrm{Re} \int_C \ln(\tilde{R}(\lambda))\, d\lambda &= 2 \int_0^\infty \ln|R(\lambda))|\, d\lambda \\ &+ \mathrm{Re} \int_{C_\infty} \ln(R(\lambda))\, d\lambda \\ &+ \mathrm{Re} \int_{C_\infty} \ln\left( \frac{(\lambda-\lambda_1^*)\ldots(\lambda-\lambda_n^*)\ldots}{(\lambda-\lambda_1)\ldots(\lambda-\lambda_n)\ldots} \right) d\lambda = 0 \end{aligned} \tag{2.1}$$

Consider a slab occupying the region $0 < z < d$ where for $z < 0$ there is vacuum and at $z = d$ there is a perfectly conducting plate. An incident plane wave is reflected from the slab. The reflection coefficient for the electric field is denoted $R$. The low frequency behavior of the reflection coefficient is now used. For a single slab the zeroth and first order terms are $R(\lambda) = -1 - 4\pi i(\frac{\mu_s}{\mu_0})d/\lambda$ where $d$ is the thickness and $\mu_s$ is the static value of the (total) permeability of the slab [1]. The value of $\mu_s$ is assumed to be real. Thus along $C_\infty$ one can use

$$\ln(R(\lambda)) = \ln(-1) + \ln(1 + 4\pi i \frac{\mu_s d}{\lambda \mu_0}) = i\pi + 4\pi i \frac{\mu_s d}{\lambda \mu_0} + O(\lambda^{-2})$$

Then

$$\int_0^\infty \ln|R(\lambda)|\, d\lambda = -2\pi^2 \mu_s d/\mu_0 - \pi \sum_n \mathrm{Im}\lambda_n$$

Energy conservation ensures that $|R(\lambda)| < 1$ and hence $\ln|R(\lambda)| \leq 0$, for real $\lambda$. The last term on the right hand side is always positive since the imaginary part of $\lambda_n$ is negative. Thus the two terms on the right hand side have different signs and the following inequality holds:

$$|\int_0^\infty \ln|R(\lambda)|\, d\lambda| \leq 2\pi^2 \mu_s d/\mu_0$$

where $\mu_s$ is the static permeability and $d$ the thickness of the slab. For an absorber with $N$ layers these results are generalized to

$$|\int_0^\infty \ln|R(\lambda)|\, d\lambda| \leq 2\pi^2 \sum_{n=1}^{N} \mu_{s,n} d_n/\mu_0 \tag{2.2}$$

Eq. 2.2 can be used to estimate the optimal thickness of a nonmagnetic absorber for a desired frequency response. By rearranging the above equation it can be shown that:

$$d \geq \frac{|\int_0^\infty \ln|R(\lambda)|\, d\lambda|}{2\pi^2} \tag{2.3}$$

where $d$ is the total thickness of the nonmagnetic absorber.

# 3 Generalization to Oblique Angle of Incidence

Assume a planar structure in the region $0 < z < \ell$ for which the complex permittivity $\epsilon(z)$ and complex permeability $\mu(z)$ are $z$-dependent. We examine the low frequency behavior of the reflection coefficient for such a structure.

## 3.1 TE Polarization

The incident field is a TE-wave where

$$\begin{aligned}\boldsymbol{E}(y,z) &= \hat{\boldsymbol{x}}E(z)e^{\mathrm{i}k_y y}\\ \boldsymbol{H}(y,z) &= (\hat{\boldsymbol{y}}H_y(z)+\hat{\boldsymbol{z}}H_z(z))e^{\mathrm{i}k_y y}\end{aligned}$$

where the transverse component $k_y = k_0 \sin\alpha_0$ of the wave vector is constant. Here $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the wave number of vacuum and $\alpha_0$ is the angle between the wave vector of the incident wave and the $z$axis. From Maxwell equations it follows that

$$\begin{aligned}\frac{\partial}{\partial z}\begin{pmatrix}E(z)\\ H_y(z)\end{pmatrix} &= \begin{pmatrix}0 & \mathrm{i}\omega\mu(z)\\ \mathrm{i}\left(\omega\epsilon(z)-\dfrac{k_y^2}{\omega\mu(z)}\right) & 0\end{pmatrix}\begin{pmatrix}E(z)\\ H_y(z)\end{pmatrix}\\ &= D(z)\begin{pmatrix}E(z)\\ H_y(z)\end{pmatrix}\end{aligned} \tag{3.1}$$

We diagonalize the matrix $D(z)$ in order to decompose the electromagnetic field into right- and left moving waves. The eigenvalues are $\lambda_{1,2} = \pm\mathrm{i}k_z(z) = \pm\mathrm{i}(\omega^2\epsilon(z)\mu(z) - k_y^2)^{1/2}$ and the eigenvectors are

$$\begin{pmatrix}a_1\\ a_2\end{pmatrix} = \begin{pmatrix}1\\ \dfrac{k_z}{\omega\mu}\end{pmatrix} \text{ and } \begin{pmatrix}1\\ -\dfrac{k_z}{\omega\mu}\end{pmatrix}$$

The matrix

$$\Lambda(z) = \begin{pmatrix}1 & 1\\ \dfrac{k_z(z)}{\omega\mu(z)} & -\dfrac{k_z(z)}{\omega\mu(z)}\end{pmatrix}$$

diagonalizes the matrix $D$. The decomposed fields are

$$\begin{aligned}\begin{pmatrix}E^+(z)\\ E^-(z)\end{pmatrix} &= \Lambda^{-1}(z)\begin{pmatrix}E(z)\\ H_y(z)\end{pmatrix}\\ &= \frac{1}{2}\begin{pmatrix}1 & \dfrac{\omega\mu(z)}{k_z(z)}\\ 1 & -\dfrac{\omega\mu(z)}{k_z(z)}\end{pmatrix}\begin{pmatrix}E(z)\\ H_y(z)\end{pmatrix}\end{aligned} \tag{3.2}$$

Divide the region $[0,\ell]$ into $N$ intervals of length $\Delta z$ and assume that $D(z)$ is constant in each interval. Then

$$\begin{pmatrix}E(z_n)\\ H_y(z_n)\end{pmatrix} = \exp(-D(z_{n+1})\Delta z)\begin{pmatrix}E(z_{n+1})\\ H_y(z_{n+1})\end{pmatrix}$$

Since $D(z) = \mathcal{O}(\lambda^{-1})$ the low frequency approximation to first order in $\Delta z/\lambda$ is

$$\begin{pmatrix} E(z_n) \\ H_y(z_n) \end{pmatrix} = (I - D(z_{n+1})\Delta z) \begin{pmatrix} E(z_{n+1}) \\ H_y(z_{n+1}) \end{pmatrix}$$

From this we get in the limit $\Delta z \to 0$ and $|\lambda| \to \infty$

$$\begin{pmatrix} E(0) \\ H_y(0) \end{pmatrix} = \left( I - \int_0^d D(z)\, dz \right) \begin{pmatrix} E(\ell) \\ H_y(\ell) \end{pmatrix}$$

We use Eq. (3.2)

$$\begin{pmatrix} E^+(0) \\ E^-(0) \end{pmatrix} = \Lambda^{-1}(0) \begin{pmatrix} 0 \\ H_y(0) \end{pmatrix}$$

Let $E^+(0) = 1$ and $E^-(0) = R_{\mathrm{TE}}$ =reflection coefficient and solve for $R_{\mathrm{TE}}$. The result is

$$R_{\mathrm{TE}} = -\left( 1 + 2\mathrm{i}\frac{k_z(0)}{\mu_0} \int_0^\ell \mu(z)\, dz \right) + \mathcal{O}(\lambda^{-2})$$

For stratified slab with piecewise constant $\epsilon$ and $\mu$ the above expression simplifies to:

$$\begin{aligned} R_{\mathrm{TE}} &= -\left( 1 + 2i\frac{k_{0z}}{\mu_0} \sum_{n=1}^{N} \mu_n d_n \right) + \mathcal{O}(\lambda^{-2}) \\ &= -\left( 1 + 2i\frac{2\pi \cos\alpha_0}{\lambda\mu_0} \sum_{n=1}^{N} \mu_n d_n \right) + \mathcal{O}(\lambda^{-2}) \end{aligned} \tag{3.3}$$

where $N$ is the number of layers in the medium.

## 3.2 TM Polarization

In the TM-case we assume the electric and magnetic fields $\boldsymbol{E}(y,z) = (\hat{\boldsymbol{y}}E_y(z) + \hat{\boldsymbol{z}}E_z(z))\, e^{\mathrm{i}k_y y}$ and $\boldsymbol{H}(y,z) = \hat{\boldsymbol{x}}H(z)e^{\mathrm{i}k_y y}$. From Maxwell's equations we get

$$\frac{\partial}{\partial z}\begin{pmatrix} E_y(z) \\ H(z) \end{pmatrix} = \begin{pmatrix} 0 & -\mathrm{i}\dfrac{k_z^2(z)}{\omega\epsilon(z)} \\ -\mathrm{i}\omega\epsilon(z) & 0 \end{pmatrix} \begin{pmatrix} E_y(z) \\ H(z) \end{pmatrix}$$

A comparison with the TE-case gives that $\dfrac{k_z(0)}{\omega\mu_0}$ is exchanged for $\dfrac{\omega\epsilon_0}{k_z(0)}$ and $\mu(z)$ is exchanged for $\dfrac{k_z(z)^2}{\omega\epsilon(z)}$. Thus in the low frequency limit the reflection coefficient reads

$$R_{\mathrm{TM}} = -\left( 1 + 2\mathrm{i}\frac{\epsilon_0}{k_z(0)} \int_0^\ell \frac{k_z(z)^2}{\epsilon(z)}\, dz \right) + \mathcal{O}(\lambda^{-2})$$

For a stratified slab with piecewise constant $\epsilon$ and $\mu$ the above expression simplifies to:

$$R_{\mathrm{TM}} = -\left(1 + 2i\frac{\epsilon_0}{\lambda\cos\alpha_0}\sum_{n=1}^{N}\frac{2\pi(\mu_n\epsilon_n/\mu_0\epsilon_0 - \sin^2\alpha_0)}{\epsilon_n}d_n\right) \tag{3.4}$$

### 3.3 The physical limit for oblique incidence

The low frequency expressions for the reflection coefficients (Eqs. 3.3, 3.4) are utilized in Eq. (2.1). Calculations show that the integral of the reflectance over wavelength ($|\int_0^\infty \ln|R(\lambda)|d\lambda|$) has the following upper limits for a multilayered absorber:

$$\begin{cases} 2\pi^2\cos\alpha_0 \mathrm{Re}\left\{\sum\limits_{n=1}^{N}(\mu_{s,n}/\mu_0)d_n\right\} & \mathrm{TE} \\ \dfrac{2\pi^2}{\cos\alpha_0}\mathrm{Re}\left\{\sum\limits_{n=1}^{N}\dfrac{(\mu_{s,n}\epsilon_{s,n} - \mu_0\epsilon_0\sin^2\alpha_0)}{\mu_0\epsilon_{s,n}}d_n\right\} & \mathrm{TM} \end{cases} \tag{3.5}$$

From the above equation it is seen that with the increase of angle the theoretical limit for TE-case decreases but for TM-case it increases unless all layers have $\epsilon = \epsilon_0$ and $\mu = \mu_0$. Unless $\epsilon = \epsilon_0$ and $\mu = \mu_0$ in all layers it is seen that as $\alpha_0 \to \pi/2$

$$|\int_0^\infty \ln|R(\lambda)|d\lambda| \leq \begin{cases} 0 & \mathrm{TE} \\ \infty & \mathrm{TM} \end{cases}$$

It is expected that the limit value tends to zero for TE case, since the absorption bandwidth shrinks at grazing angles for an absorber with finite thickness. In the next section it is shown that the upper bound for TM polarization always overestimates the integral value and consequently it is of no importance that it tends to infinity.

# 4 Numerical Investigation of the Bounds

The applicability of the extracted physical bounds are investigated in this section. For this reason a versatile absorber is considered that can adapt easily to different design goals. The absorber is a single resistive layer capacitive circuit absorber [5] with the schematic shown in Fig. 1. Four different design cases are considered where the absorption takes place for:

- normal angle of incidence, Fig. 2.
- TE polarization at 45°, Fig. 3.
- TM polarization at 45°, Fig. 4.
- both normal and 45° angle of incidence and both polarizations, Fig. 5.

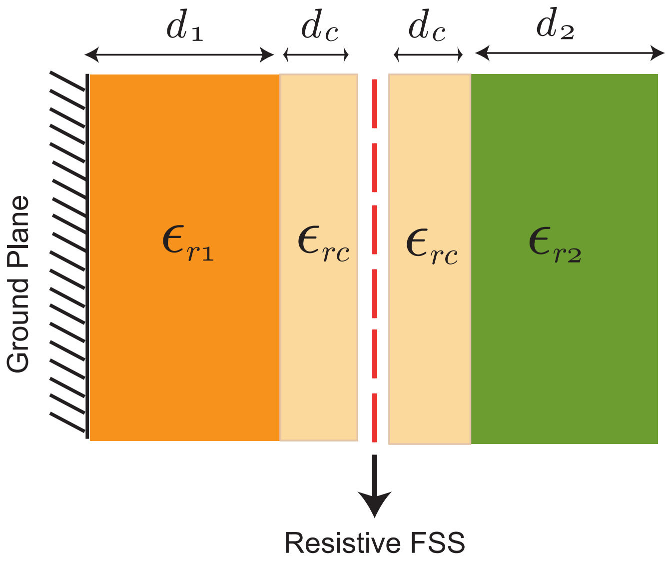


**Figure 1**: The schematic of the single resistive layer absorber. The FSS layer is a square patch periodic array.

| Design ID | $d_1$( mm) | $d_2$( mm) | Thickness | $\epsilon_{r1}$ | $\epsilon_{r2}$ |
|---|---|---|---|---|---|
| Normal | 2.4 | 3.2 | 6 | 1 | 2.5 |
| TE - 45° | 2.4 | 4.7 | 7.5 | 1 | 1.95 |
| TM - 45° | 3.9 | 4.1 | 8.4 | 1.13 | 2.54 |
| All | 3.3 | 5 | 8.7 | 2.9 | 2.1 |

**Table 1**: The thicknesses and permittivity of the dielectric layers for different design goals. ($d_c = 0.2\,\mathrm{mm}$ & $\epsilon_{rc} = 4.4$.)

The corresponding values of the dielectric layers thicknesses and permittivities are tabulated in Table 4 for each different design case. Now for each design case, the numerical value of the integral of the reflectance over wavelength is calculated (frequency interval 0.01–50 GHz) and is compared to the theoretical limits of Eq. (3.5). The calculations are given in Tables 4-4.

From the calculated values it is seen that in all cases, the upper bound for the TM case overestimates the value of the integral. Even in the case that the absorber is designed for TM polarization the ratio between the integral value and the theoretical limit is small ($\approx 0.67$). Therefore, the bound for TM polarization cannot be used

| Reflection Coefficient | $\lvert \int \ln \lvert R(\lambda) \rvert d\lambda \rvert$ | Upper Bound | Ratio |
|---|---|---|---|
| $R_N$ | 107.03 | 118.43 | 0.9 |
| $R_{TE}$ | 76.63 | 83.75 | 0.91 |
| $R_{TM}$ | 47.18 | 114.86 | 0.41 |

**Table 2**: Comparison of the integral of reflectance and the upper bound for the absorber designed for normal angle of incidence.

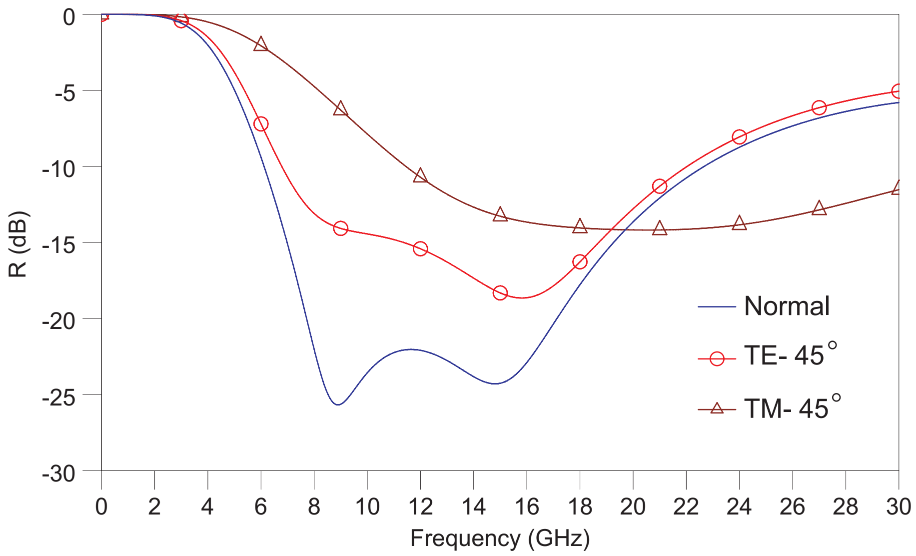


**Figure 2**: The frequency response of the absorber when it is designed for good performance at normal angle of incidence.

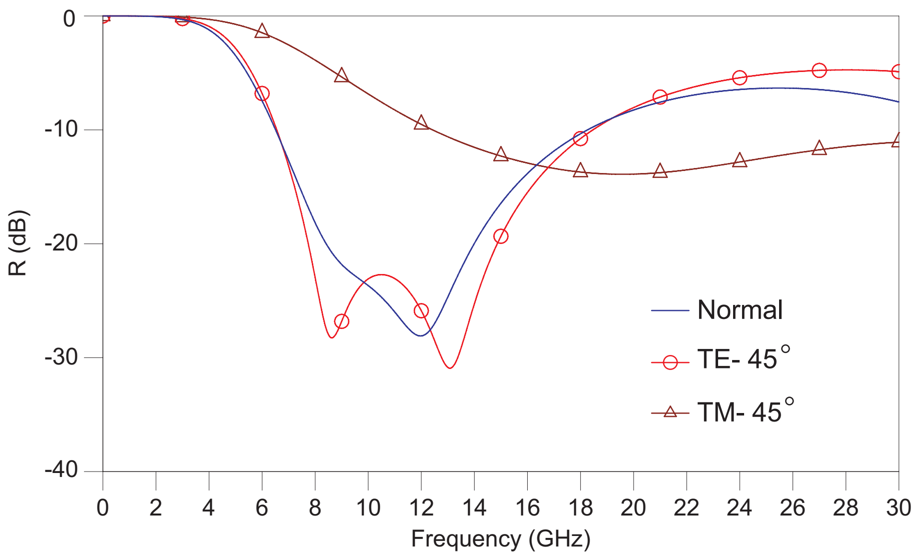


**Figure 3**: The frequency response of the absorber when it is designed for good performance at 45° angle of incidence and TE polarization.

| Reflection Coefficient | $\lvert\int \ln \lvert R(\lambda)\rvert d\lambda\rvert$ | Upper Bound | Ratio |
|---|---|---|---|
| $R_N$ | 90.45 | 148.04 | 0.61 |
| $R_{TE}$ | 94.33 | 104.68 | 0.9 |
| $R_{TM}$ | 41.6 | 140.96 | 0.3 |

**Table 3**: Comparison of the integral of reflectance and the upper bound for the absorber designed for 45° and TE polarization.

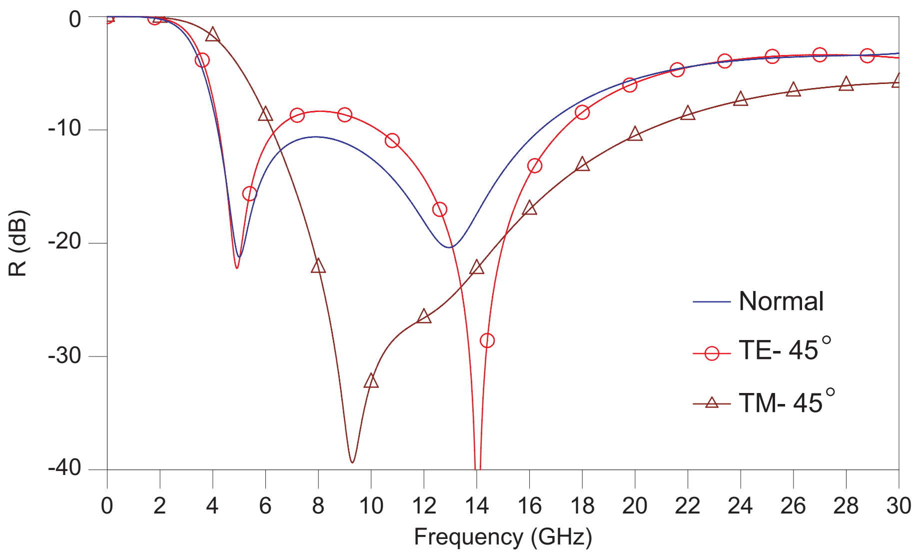


**Figure 4**: The frequency response of the absorber when it is designed for good performance at 45° angle of incidence and TM polarization.

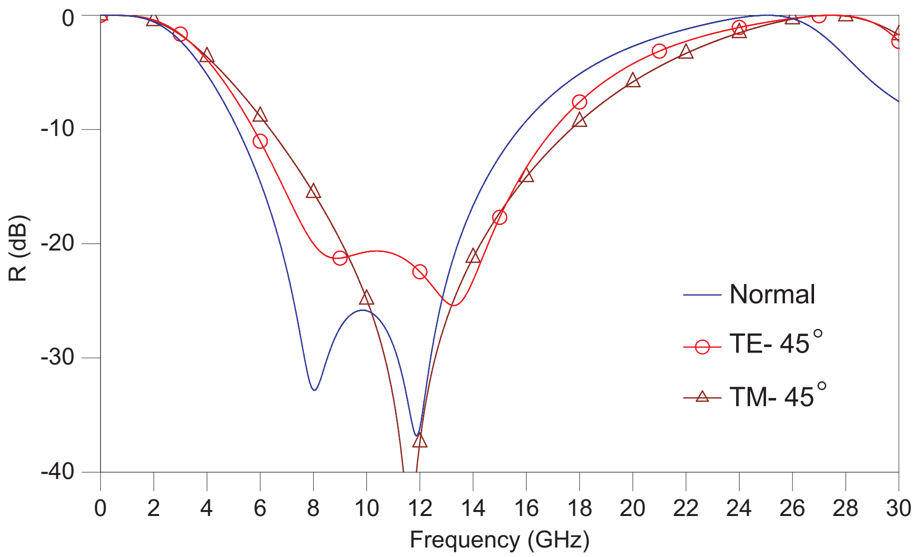


**Figure 5**: The frequency response of the absorber when it is designed for both normal and 45° angles of incidence and both polarizations.

| Reflection Coefficient | $\lvert \int \ln \lvert R(\lambda) \rvert d\lambda \rvert$ | Upper Bound | Ratio |
|---|---|---|---|
| $R_N$ | 115.54 | 165.81 | 0.7 |
| $R_{TE}$ | 107.87 | 117.24 | 0.92 |
| $R_{TM}$ | 109.09 | 162.52 | 0.67 |

**Table 4**: Comparison of the integral of reflectance and the upper bound for the absorber designed for 45° and TM polarization.

| Reflection Coefficient | $\lvert \int \ln \lvert R(\lambda) \rvert d\lambda \rvert$ | Upper Bound | Ratio |
|---|---|---|---|
| $R_N$ | 140.65 | 171.73 | 0.82 |
| $R_{TE}$ | 110.24 | 121.43 | 0.91 |
| $R_{TM}$ | 112.78 | 192.48 | 0.58 |

**Table 5**: Comparison of the integral of reflectance and the upper bound for the absorber designed for both normal and 45° angles of incidence and both polarizations.

for estimation of the optimal thickness of a nonmagnetic absorber. Fortunately, the opposite happens for the bound in case of TE polarizations. In almost every design case, it results in fair approximation of the integral values with ratios larger than 0.9 (the ideal case is 1). Consequently, the Eq. (2.3) can be generalized easily to arbitrary angle of incidence for a nonmagnetic absorber as the following:

$$d \geq \frac{\lvert \int_0^\infty \ln \lvert R(\lambda) \rvert \, d\lambda \rvert}{2\pi^2 \cos \alpha_0} \tag{4.1}$$

where $\alpha_0$ is the angle of incidence.

# 5 Conclusion

The physical bound of a flat metal backed absorber is generalized to arbitrary angle of incidence. To achieve this goal, the asymptotic behavior of the reflection coefficient at low frequency is studied. It is shown that at oblique angles of incidence the theoretical limit becomes polarization dependent. Design examples and numerical calculations are provided to examine the applicability of the derived bounds. It is shown that the theoretical limit for the TM polarization is not useful in estimating the optimal thickness of a nonmagnetic absorber. In contrary, the TE polarization is able to perform the task efficiently. Simple relation is suggested for calculating the optimal thickness of a nonmagnetic absorber for a desired frequency response at an arbitrary angle of incidence.

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