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Clark, R. M.; Crawford, H. L.; Macchiavelli, A. O.; Rudolph, D.; Såmark-Roth, A.; Campbell, C. M.; Cromaz, M.; Fallon, P.; Morse, C.; Santamaria, C.

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R. M. Clark,[1] H. L. Crawford,[1] A. O. Macchiavelli,[1] D. Rudolph,[2] A. Såmark-Roth,[2] C. M. Campbell,[1] M. Cromaz,[1]
P. Fallon,[1] C. Morse,[1] and C. Santamaria[1]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[2]*Department of Physics, Lund University, 22100 Lund, Sweden*

The superfluid tunneling model is applied to the calculation of half-lives of the observed $\alpha$ decays in $N = 84$ isotones. Results of our calculations are compared to experimental data on the ground-state $\alpha$ decays along the isotonic chain from $^{144}$Nd to $^{159}$Re. Good agreement is found. The $\alpha$ decays of the known high-spin isomers in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W are also well reproduced, once a reduction in the pairing strength is taken into account. This includes reproduction of the main features of the recently observed fine structure from $^{155}$Lu($25/2^-$) and $^{156}$Hf($8^+$). Predictions for the $\alpha$-decay fine structure of $^{157}$Ta($25/2^-$) and $^{158}$W($8^+$) high-spin isomers are presented.

## I. INTRODUCTION

A recent paper [1] reported on an experiment to study the $\alpha$-decay fine structure of high-spin isomers in the $N = 84$ isotones $^{155}$Lu and $^{156}$Hf. The isomeric states are denoted as $^{155}$Lu($25/2^-$) and $^{156}$Hf($8^+$), where the spin parity assignment, $J^\pi$, for the isomer, assigned to the isomer is indicated in the parentheses. Three new $\alpha$ decays from $^{155}$Lu($25/2^-$) and two from $^{156}$Hf($8^+$) were identified as populating excited states with proton seniority $s > 1$ in the $N = 82$ isotones $^{151}$Tm and $^{152}$Yb, respectively. This was the first report of such highly excited states being populated via $\alpha$ decay of spin-trap isomers in nuclei below $^{208}$Pb. Indeed, the study of $\alpha$-decay fine structure from such isomers may offer a unique probe of excited states in medium-heavy nuclei near the proton drip line. However, as pointed out in [1] the theoretical description of $\alpha$-decay fine structure is a challenge, particularly since one must disentangle effects that might influence the decay, such as the single-particle structure of initial and final states, and the role of pairing.

For a description of fine structure in $\alpha$ decay, one must consider three major factors: (i) the energies of the states involved—the larger the $Q$ value of the specific $\alpha$ decay, $Q_\alpha$, the shorter the lifetime will be; (ii) the angular momentum of the states involved in the decay—a large difference in angular momentum will give rise to a larger centrifugal barrier resulting in a longer lifetime; and (iii) the role of the odd particle(s) on the blocking of pairing correlations—pairing enhances the decay through a barrier and so a reduction in pairing again leads to a longer lifetime. As described in the next section, the superfluid tunneling model (STM) [2–4] enables us to examine the influence of each of these factors on $\alpha$ decay.

In an initial study [5], it was shown that the STM could be applied to the description of $\alpha$ decay of ground-state and multiquasiparticle states across different regions of the nuclear chart from the neutron-deficient $A \sim 150$ region up

through the heavy actinide region. In another study [6] we applied the STM to compare with the experimental data on all known even-even superheavy nuclei with $100 \leqslant Z \leqslant 118$, i.e., from isotopes of fermium ($Z = 100$) to oganesson ($Z = 118$). Remarkable quantitative agreement, comparable to the fits of recent empirical parametrizations, was found. Notably, we were able to reproduce the features of the observed fine structure in the $\alpha$ decay from the high-K isomer in $^{270}$Ds [6].

In this paper we apply the STM to a systematic investigation of the $\alpha$ decay of the $N = 84$ isotones from $^{144}$Nd to $^{159}$Re. The experimental data on the $\alpha$ decay from the ground states are reproduced to an accuracy that is better than other contemporary approaches, exemplified by the empirical formula of Royer [7]. With respect to the $\alpha$ decay of the known high-spin isomers in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W, we are able to reproduce the main decay properties, including reproduction of the observed fine structure from $^{155}$Lu($25/2^-$) and $^{156}$Hf($8^+$) once a reduction in the pairing strength is taken into account. Predictions for the $\alpha$-decay fine structure of $^{157}$Ta($25/2^-$) and $^{158}$W($8^+$) high-spin isomers are also presented.

In previous papers [5,6] the model has been discussed in detail but, for completeness, we will describe the main features of the STM in Sec. II. In Sec. III we compare the results of our calculations to experimental data on the ground-state $\alpha$ decays for the $N = 84$ isotones. In Sec. IV we focus on calculations of $\alpha$ decays from the known highly excited ($E_x \approx 2$ MeV) high-spin isomers in $N = 84$ nuclei from $^{155}$Lu to $^{158}$W. This will be followed by a short summary.

## II. SUPERFLUID TUNNELING MODEL

In this paper we have used the STM as described in [8], which has been successfully applied previously to calculations of particle emission including $\alpha$ decay and cluster radioactivity [2–6]. The model involves the nucleus evolving

to a clusterlike configuration. In the case of $\alpha$ decay, this comprises a touching configuration of the daughter nucleus and $\alpha$ particle. The subsequent decay process is described in terms of the standard Gamow theory of tunneling through a barrier. The evolution of the parent nucleus to the clusterlike configuration is dominated by pairwise rearrangements of nucleons, which occur under the action of the residual nuclear interaction, dominated by pairing.

The Hamiltonian of the model can be written as

$$\left( \frac{\hbar^2}{2D} \frac{\partial^2}{\partial \xi^2} + V(\xi) \right) \psi(\xi) = E\psi(\xi). \tag{1}$$

$\xi$ is a generalized deformation variable describing the path of the system in the multidimensional space of deformations. In the case of only quadrupole deformation, this would mean that $\xi$ is proportional to the axial deformation parameter, $\beta_2$. The parent nucleus evolves from a configuration with a small deformation, $\xi \approx 0$, to the touching configuration of the daughter plus $\alpha$ particle defined to be at $\xi = 1$.

Equation (1) can be discretized on a mesh of $n$ steps such that for each step $\Delta \xi = 1/n$. One can then derive the expression for the inertial mass parameter as

$$D = -\frac{\hbar^2}{2v} n^2. \tag{2}$$

$v$ is the transition matrix element between two successive steps. For $\alpha$ decay, $n = 4$ is assumed [4,8]. The transition matrix element is governed by the pairing operator and is estimated using the BCS model to be

$$v = -\left( \frac{\Delta_n^2 + \Delta_p^2}{4G} \right). \tag{3}$$

$G = 25/A$ MeV is the standard pairing strength and $\Delta_n = \Delta_p = \Delta = 12A^{-1/2}$ MeV are the pair gap parameters [9]. In addition to the well-known smooth decrease with $A$, the pairing gap is expected to contain a dependence on the neutron excess $(N - Z)/A$ [10]. Since we are investigating a long isotonic chain of nuclei, to investigate this effect we have also considered an expression (again giving $\Delta$ in MeV) of the form

$$\Delta = \left\{ a - b \left[ \frac{(N - Z)}{A} \right]^2 \right\} A^{-1/3}. \tag{4}$$

This equation was originally proposed in [10] with fit parameters of $a = 7.2$ and $b = 44$ (with $\Delta_n = \Delta_p = \Delta$ in MeV). We performed an independent fit yielding parameters of $a = 5.9$ and $b = 11.2$, which have been used in this paper. We also investigated separately fitting the proton and neutron pair gaps ($\Delta_n \neq \Delta_p$), as suggested in [11], but found our results changed little under such an assumption. The nomenclature of $\Delta_{BM}(=12A^{-1/2}$ MeV) and $\Delta_{VJH}$ [from Eq. (4), with our parametrization] is used throughout when it is necessary to distinguish which of the different expressions for the pairing gap has been used.

The decay constant, $\lambda$, can be calculated in terms of the $\alpha$-particle formation probability, $P$, the assault frequency of the particle against the barrier (also known as the knocking frequency), $f$, and the transmission coefficient of the $\alpha$ particle

through the barrier, $T$, such that

$$\lambda = PfT. \tag{5}$$

To calculate $P$ we use the wave function of the ground state of a harmonic oscillator such that $P = |\psi(\xi = 1)|^2$ with

$$\psi(\xi) = \left( \frac{\alpha}{\sqrt{\pi}} \right)^{\frac{1}{2}} e^{-\frac{1}{2}\alpha^2 \xi^2} \tag{6}$$

where

$$\alpha^2 = \sqrt{\frac{C}{2|v|}} n. \tag{7}$$

The potential-energy parameter is $C = 2V(\xi = 1) = 2(V_N + V_C - Q_\alpha)$ with $V_N$ and $V_C$ being the nuclear potential (for which we used the Christensen-Winther potential [12]) and the Coulomb potential, respectively. $Q_\alpha$ is the $Q$ value for the specific $\alpha$-decay transition being considered and is determined from the experimentally measured $\alpha$-decay energy, $E_\alpha$. The details of the potential parameters used can be found in [5]. The assault frequency can then be calculated via the formula $f = \omega/2\pi$, where $\omega = \sqrt{C/D}$.

Finally, the transmission coefficient, $T_L$, for the $\alpha$ particle to tunnel through the Coulomb barrier starting from the daughter-$\alpha$ touching configuration is given by

$$T_L = \frac{\rho}{F_L^2(\eta, \ \rho) + G_L^2(\eta, \rho)} \tag{8}$$

where $\rho = R_0 k$ with $k = \sqrt{2\mu Q_\alpha}/\hbar$ ($\mu$ is the reduced mass) and $R_0 = 1.2(A_D^{1/3} + A_\alpha^{1/3}) + 0.63$ fm, and $\eta = 1/ka$ where $a = \hbar^2/(e^2 \mu Z_D Z_\alpha)$. Here, $F_L$ and $G_L$ are the regular and irregular Coulomb functions [13], which take into account the additional centrifugal barrier when the orbital angular momentum, $L$, of the emitted $\alpha$ particle is nonzero.

## III. GROUND-STATE $\alpha$ DECAYS FOR THE $N = 84$ ISOTONES

We have used the STM, as described above, to calculate the strongest $\alpha$-decay branches from the ground states of $N = 84$ isotones [14,15]. The lightest $N = 84$ isotone with a known $\alpha$-decay branch is $^{144}$Nd with a ground-state half-life of $T_{1/2,\text{expt}} = 2.29(15) \times 10^{15} y = 7.22(50) \times 10^{22}$ s, while the heaviest is $^{159}$Re [15] with a ground-state half-life of $2.1(4) \times 10^{-5}$ s. The results of our calculations are given in Table I. A comparison between the experimental data and theory is plotted in Fig. 1. Some interesting qualitative features can be seen in Fig. 1. Spanning over 27 orders of magnitude in the half-lives, the agreement between experiment and theory is quite remarkable. On this scale, one sees a smooth behavior for the $N = 84$ isotones with $Z > 64$. There is a deviation from this behavior at $Z = 64$ ($^{148}$Gd), which is not so surprising when one considers that $^{146}$Gd ($Z = 64$, $N = 82$) is regarded as a semi-doubly-magic nucleus and one expects a discontinuity in quantities such as $Q_\alpha$, as well as potential inadequacy of the BCS approximation, near these nucleon numbers. One sees an odd-even staggering in the

TABLE I. The half-lives of the known strongest $\alpha$ decays (taking into account the experimentally measured total half-lives and branching ratios) from the ground states of the $N = 84$ isotones, $T_{1/2,\text{expt}}(\alpha)$ in seconds. The half-lives were also calculated using the superfluid tunneling model with two different treatments of the pairing. In the third column we use the standard pairing expression for the pair gap, $\Delta = \Delta_{\text{BM}} = 12A^{-1/2}$ MeV [9], while in the fourth column we use the expression for the pairing gap given in Eq. (4), $\Delta = \Delta_{\text{VJH}}$ [10], with the parameters from [5], as discussed in the text.

| Nuclide | $T_{1/2,\text{expt}}(\alpha)$ (s) | $T_{1/2,\text{STM}}(\Delta_{\text{BM}})$ (s) | $T_{1/2,\text{STM}}(\Delta_{\text{VJH}})$ (s) |
|---|---|---|---|
| $^{144}$Nd | $7.22(50) \times 10^{22}$ | $6.49 \times 10^{23}$ | $4.04 \times 10^{23}$ |
| $^{145}$Pm | $1.99(5) \times 10^{17}$ | $1.93 \times 10^{18}$ | $1.15 \times 10^{18}$ |
| $^{146}$Sm | $2.1(2) \times 10^{15}$ | $1.41 \times 10^{16}$ | $8.02 \times 10^{15}$ |
| $^{147}$Eu | $9.4(26) \times 10^{10}$ | $6.74 \times 10^{11}$ | $3.72 \times 10^{11}$ |
| $^{148}$Gd | $2.2(4) \times 10^{9}$ | $8.50 \times 10^{9}$ | $4.52 \times 10^{9}$ |
| $^{149}$Tb | $8.9(8) \times 10^{4}$ | $7.04 \times 10^{4}$ | $3.68 \times 10^{4}$ |
| $^{150}$Dy | $1.20(17) \times 10^{3}$ | $3.24 \times 10^{3}$ | $1.64 \times 10^{3}$ |
| $^{151}$Ho | $1.60(22) \times 10^{2}$ | $2.71 \times 10^{2}$ | $1.33 \times 10^{2}$ |
| $^{152}$Er | $1.14(5) \times 10^{1}$ | $3.05 \times 10^{1}$ | $1.46 \times 10^{1}$ |
| $^{153}$Tm | $1.63(5) \times 10^{0}$ | $3.49 \times 10^{0}$ | $1.63 \times 10^{0}$ |
| $^{154}$Yb | $4.42(6) \times 10^{-1}$ | $9.96 \times 10^{-1}$ | $4.53 \times 10^{-1}$ |
| $^{155}$Lu | $7.6(2) \times 10^{-2}$ | $1.30 \times 10^{-1}$ | $5.79 \times 10^{-2}$ |
| $^{156}$Hf | $2.3(3) \times 10^{-2}$ | $4.80 \times 10^{-2}$ | $2.09 \times 10^{-2}$ |
| $^{157}$Ta | $1.05(5) \times 10^{-2}$ | $1.57 \times 10^{-2}$ | $6.70 \times 10^{-3}$ |
| $^{158}$W | $1.25(21) \times 10^{-3}$ | $2.75 \times 10^{-3}$ | $1.16 \times 10^{-3}$ |
| $^{159}$Re | $2.6(14) \times 10^{-4}$ | $5.41 \times 10^{-4}$ | $2.24 \times 10^{-4}$ |

decimal logarithm of the $\alpha$-decay half-lives for the isotones with $Z \leqslant 64$ (which is still there but less visibly pronounced for the isotones with $Z > 64$; see discussion below), which indicates the importance of treating pairing in an appropriate manner.

For a quantitative comparison, a common approach is to calculate the average of the absolute values of the differences
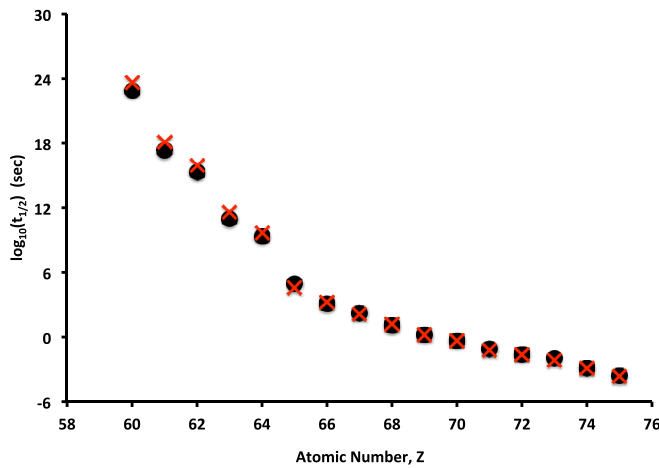


FIG. 1. Decimal logarithm of $\alpha$-decay half-lives (in seconds) for the strongest decay branch from the ground state of each of the $N = 84$ isotones. The filled circles are the experimental data (second column of Table I) and the red crosses are the results from the STM calculations using the pairing gaps $\Delta = \Delta_{\text{VJH}}$ (fourth column of Table I).
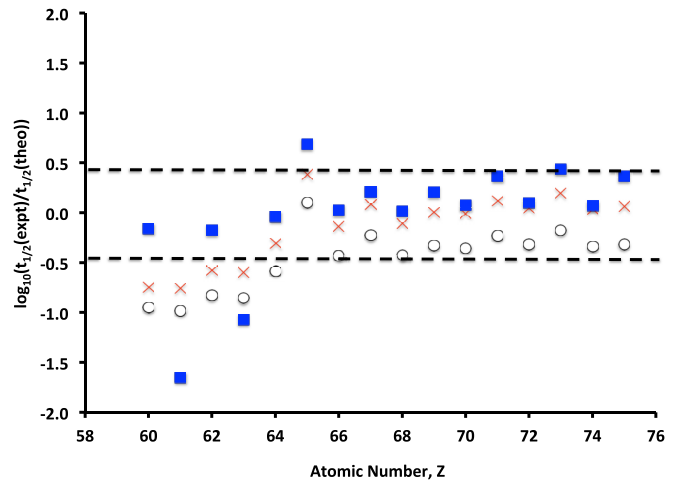


FIG. 2. Decimal logarithms of the ratios between the experimental and theoretical $\alpha$-decay half-lives for the strongest decay branch from the ground state of each of the $N = 84$ isotones. The open circles use the STM calculations with the standard pair gap parametrization $\Delta = \Delta_{\text{BM}}$, while the red crosses use the STM calculations with the pairing gaps calculated using $\Delta = \Delta_{\text{VJH}}$, as discussed in the text. The blue squares are values calculated using the Royer formulas [7]. The dashed horizontal lines indicate values of $\pm 0.477$, corresponding to a factor-3 difference between experimental and theoretical $\alpha$-decay half-lives.

in the decimal logarithms given as

$$\delta = \frac{1}{N} \sum_{k=1}^{N} \left| \log_{10}\left( \frac{T_{1/2,\text{expt},k}}{T_{1/2,\text{theo},k}} \right) \right|. \tag{9}$$

In Fig. 2, we plot the decimal logarithms of the ratios between the experimental and theoretical half-lives as a function of $Z$ for three different model approaches. Note that a value of 0.477 (the dashed horizontal lines in Fig. 2) corresponds to a factor-3 difference between experiment and theory. We have used the STM with the pairing gaps as either $\Delta = \Delta_{\text{BM}}$ or $\Delta = \Delta_{\text{VJH}}$, which are represented by the open circles and red crosses, respectively, in Fig. 2. We also show the results of an empirical-fitting method by Royer [7], which has become a widely used approach for the prediction of $\alpha$-decay half-lives. They are given by the blue squares in Fig. 2. Over the full range of the experimental data ($60 \leqslant Z \leqslant 75$) the values of $\delta$ are $\delta_{\text{BM}} = 0.467$, $\delta_{\text{VJH}} = 0.261$, and $\delta_{\text{Royer}} = 0.353$. One can see from Fig. 2 and Table I that the biggest deviations for each of the models occur for $Z < 64$. Above $Z = 64$, we find the values of $\delta$ are $\delta_{\text{BM}} = 0.296$, $\delta_{\text{VJH}} = 0.106$, and $\delta_{\text{Royer}} = 0.231$.

We can conclude that the STM is able to reproduce the experimental data on the $\alpha$ decay of $N = 84$ isotones to a better level of accuracy than contemporary empirical formulas such as that of Royer [7]. The influence of the $Z = 64$ subshell closure is seen in the data. Above $Z = 64$, taking into account the symmetry-energy-like dependence of the pairing gaps ($\Delta = \Delta_{\text{VJH}}$) yields an even better reproduction of the experimental data. In Fig. 2, one sees that there remains some small odd-even staggering, which could be reduced further

TABLE II. The experimentally known $\alpha$-decay branches from the high-spin isomers in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W. The first column gives the nuclide and the spin parity assigned to the isomeric state. The second column is the energy of the $\alpha$-decay line, $E_\alpha$. The third column indicates the spin-parity assignment for the final states, $J_f^\pi$. The fourth, fifth, and sixth columns give the lifetimes of the $L = 8$ $\alpha$ transitions from the isomer as seen experimentally and calculated using the full pair gap, $\Delta = \Delta_{\text{VJH}}$, and the reduced pair gap, $\Delta = 0.6 \times \Delta_{\text{VJH}}$, respectively. The seventh and eighth columns give the experimental and theoretical values of the branching strength, $b_\alpha^{\text{expt}}$ and $b_\alpha^{\text{STM}}$, respectively, as a percentage of the total observed decay from the state.

| Nuclide | $E_\alpha$(keV) | $J_f^\pi$ | $T_{1/2,\text{expt}}(\alpha)$ (ms) | $T_{1/2,\text{STM}}(\Delta_{\text{VJH}})$ (ms) | $T_{1/2,\text{STM}}(0.6 \times \Delta_{\text{VJH}})$ (ms) | $b_\alpha^{\text{expt}}$(%) | $b_\alpha^{\text{STM}}$(%) |
|---|---|---|---|---|---|---|---|
| $^{155}$Lu($25/2^-$) | 7383(4) | $11/2^-$ | 2.71(3) | 0.038 | 1.73 | 99.964(6) | 99.944 |
|  | 5937(15) | $15/2^-$ |  |  |  | $2.4(13) \times 10^{-3}$ | $1.4 \times 10^{-2}$ |
|  | 5928(5) | $(15/2^+)$ |  |  |  | $2.8(6) \times 10^{-2}$ | $3.8 \times 10^{-2}$ |
|  | 5521(8) | $(19/2^+)$ |  |  |  | $5.8(16) \times 10^{-3}$ | $3.8 \times 10^{-3}$ |
| $^{157}$Ta($25/2^-$) | 7744(8) | $11/2^-$ | 1.7(1) | 0.021 | 0.95 | 100 |  |
| $^{156}$Hf($8^+$) | 7775(5) | $0^+$ | 0.52(1) | 0.007 | 0.32 | 99.990(4) | 99.979 |
|  | 6274(15) | $2^+$ |  |  |  | $6.4(30) \times 10^{-3}$ | $1.9 \times 10^{-2}$ |
|  | 5942(15) | $3^-$ |  |  |  | $3.8(23) \times 10^{-3}$ | $2.7 \times 10^{-3}$ |
| $^{158}$W($8^+$) | 8.286(7) | $0^+$ | 0.143(19) | 0.002 | 0.078 | 100 |  |

by explicitly accounting for the role of blocking by the odd proton on the pairing correlations through a small reduction in the pairing gap used for the odd-$Z$ nuclei (we found that a $\approx 5\%$ reduction of $\Delta$ could account for the observed odd-even staggering). However, such small effects will not alter any conclusions we might draw for the investigation of the $\alpha$ decay of the isomeric states as discussed in the next section. The main point is to show that the STM seems to contain all the necessary physical ingredients to reproduce the major features of the $\alpha$ decay in this region.

### IV. $\alpha$ DECAY FROM THE HIGH-SPIN ISOMERS

We now turn to applying the model to the case of the known $\alpha$-decaying isomers [1,16–19] in the $N = 84$ isotones $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W. In particular, the recent observation [1] of fine structure in the $\alpha$ decay of $^{155}$Lu($25/2^-$) and $^{156}$Hf($8^+$), as discussed in the introduction, represents an interesting challenge to theory. The salient experimental information is summarized in Table II. Four $\alpha$-decay lines are seen from $^{155}$Lu($25/2^-$), with the dominant transition being that to the $11/2^-$ ground state of $^{151}$Tm. The three other transitions are much weaker and decay into seniority $s = 3$ states at high excitation energy ($>1.5$ MeV) in $^{151}$Tm. Three $\alpha$-decay lines are seen from $^{156}$Hf($8^+$), with the dominant transition to the $0^+$ ground state in $^{152}$Yb ($s = 0$) and the two other lines to highly excited $s = 2$ states. To date, no $\alpha$-decay fine structure has been observed from the known high-spin isomers in either $^{157}$Ta or $^{158}$W, with the existence of the former isomer having yet to be confirmed independently.

We performed STM calculations using $\Delta = \Delta_{\text{VJH}}$, which gave the best reproduction of the $N = 84$ ground-state $\alpha$ decays. Assuming the pairing gap parameter is the same as that for the ground state, and accounting for the angular momentum, $L$, of the transition by assuming that it takes the lowest value given by the selection rules $|I_i - I_f| \leqslant L \leqslant I_i + I_f$ and $\pi_i = (-1)^L \pi_f$, we predict the lifetimes of the $L = 8$ $\alpha$-decay lines to be around two orders of magnitude faster than observed experimentally (fifth column of Table II and Fig. 3). When the pairing gap is reduced by 40%, such

that $\Delta = 0.6 \times \Delta_{\text{VJH}}$, we find that the STM calculations give a much better reproduction of the main $L = 8$ $\alpha$-decay lines from the isomers (sixth column of Table II and Fig. 3). This is the same pairing reduction factor that was used in [5,6] to reproduce data on known $\alpha$-decaying multiquasiparticle high-spin isomers in different mass regions. It is also similar to the reduction factor for seniority $s = 2$ or 3 states estimated in [20].

While we expect pairing to be a dominant component of the residual interaction, there may be additional nuclear structure effects that are being effectively compensated for by the reduction of $\Delta$. However, the overall effect must be to reduce the transition matrix element, $v$, of Eq. (2), in order to reproduce the data. Examining the fine structure in more detail may reveal structural dependencies since we are dealing with different final states that have different configurations. The results of our calculations of the branching ratios are compared to experiment in Table II and Fig. 4. We have
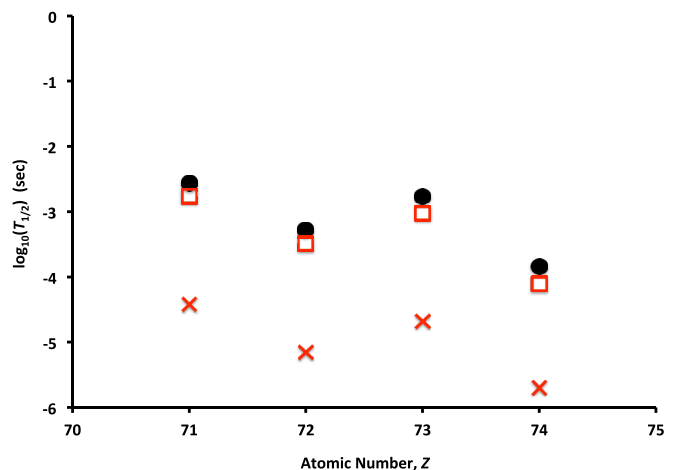


FIG. 3. Decimal logarithm of the $\alpha$-decay half-lives (in seconds) for the $L = 8$ transition from the high-spin isomers in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W. The black filled circles are the experimental data while the red crosses (red open squares) are the results from the STM calculations using the pairing gaps $\Delta = \Delta_{\text{VJH}}$ ($\Delta = 0.6 \times \Delta_{\text{VJH}}$).
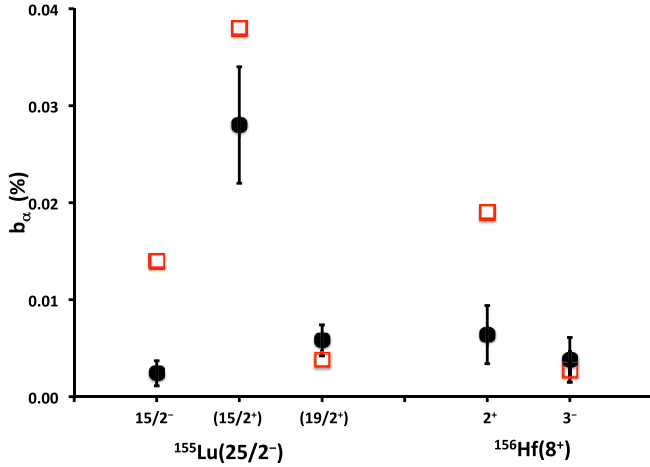
FIG. 4. The $\alpha$-decay branches from the isomers $^{155}$Lu(25/2$^-$) (data points to the left of the plot) and $^{156}$Hf(8$^+$) (data points to the right), given as a percentage of the total $\alpha$ decay from those states, $b_\alpha$. The horizontal axis is marked by the spin parity, $J^\pi$, of the state in the daughter nucleus to which the $\alpha$ line decays. The black filled circles are the experimental data while the red open squares are the results from the STM calculations using the pairing gaps $\Delta = 0.6 \times \Delta_{\text{VJH}}$.

accounted for different angular momenta, $L$, and different decay energies, $E_\alpha$, and used the same $\Delta = 0.6 \times \Delta_{\text{VJH}}$ reduction factor for the pairing in each case. The agreement between theory and experiment is very good with the overall magnitude and relative strengths of the branching reproduced for the fine structures of the decays from both of the isomers $^{155}$Lu(25/2$^-$) and $^{156}$Hf(8$^+$). This implies that the calculations and/or experimental data are essentially insensitive to any additional factors beyond the main factors in the $\alpha$ decays, which are the energy, angular momentum, and pairing, as discussed earlier.

We have also performed calculations to predict as yet unobserved $\alpha$-decay fine structure from states in $^{157}$Ta and $^{158}$W. For completeness, in Table III, we present calculations of all the $\alpha$-decay lines that may possibly be observed from states in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W, whether originating from states at low-excitation energy (such as the 1/2$^+$ and 11/2$^-$ levels in $^{155}$Lu [14,21] and $^{157}$Ta [14]) or from the high-spin isomers, which have been the main focus of this paper. It is interesting to note that there are unobserved branches from $^{155}$Lu(25/2$^-$) which have been calculated to be even stronger than those that have been found already. However, they remain unobserved in the experiment since there was no efficient means to select the daughter states at low excitation in $^{151}$Tm unlike the $\gamma$-ray tagging technique used to select the states at high excitation. Another point worth noting is that the predictions of the fine structure for $^{157}$Ta(25/2$^-$) and $^{158}$W(8$^+$) suggest that several branches should be observable using the techniques described in [1], if similar event statistics can be collected for these cases.

## V. SUMMARY AND CONCLUSIONS

In this paper we have applied the STM to a systematic investigation of the $\alpha$ decay of the $N = 84$ isotones from

TABLE III. Predicted branches from states in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W. The initial states and final states are indicated by the spin-parity assignment, $J^\pi$ (first column). Also given are the energy of the $\alpha$ line, $E_\alpha$ in MeV, the angular momentum change involved in the transition, $L$, and the branching strength as a percentage of the total $\alpha$ decay from the state. Only branches with $b_\alpha > 10^{-3}\%$ are given in the table. Note that for $^{156}$Hf and $^{158}$W the $0^+ \rightarrow 0^+$ ground-state decays are essentially 100% branches.

| $J_f^\pi$ | $E_\alpha$ (MeV) | $L$ | $b_\alpha^{\text{STM}}$(%) |
|---|---|---|---|
| $^{155}$Lu(11/2$^-$) | | | |
| 11/2$^-$ | 5.661 | 0 | 96.7 |
| 1/2$^+$ | 5.565 | 5 | 2.4 |
| 3/2$^+$ | 5.457 | 5 | 0.8 |
| 5/2$^+$ | 4.982 | 3 | $2.5 \times 10^{-2}$ |
| $^{155}$Lu(1/2$^+$) | | | |
| 1/2$^+$ | 5.586 | 0 | 73.5 |
| 11/2$^-$ | 5.682 | 5 | 11.9 |
| 3/2$^+$ | 5.478 | 2 | 14.5 |
| 5/2$^+$ | 5.003 | 2 | $8.8 \times 10^{-2}$ |
| $^{155}$Lu(25/2$^-$) | | | |
| 11/2$^-$ | 7.383 | 8 | 99.7 |
| 15/2$^-$ | 5.937 | 6 | $1.4 \times 10^{-2}$ |
| 15/2$^+$ | 5.928 | 5 | $3.8 \times 10^{-2}$ |
| 19/2$^+$ | 5.521 | 3 | $3.8 \times 10^{-3}$ |
| 1/2$^+$ | 7.287 | 13 | $9.0 \times 10^{-3}$ |
| 3/2$^+$ | 7.179 | 11 | 0.19 |
| 5/2$^+$ | 6.704 | 11 | $5.3 \times 10^{-3}$ |
| 7/2$^+$ | 6.184 | 9 | $2.1 \times 10^{-3}$ |
| $^{156}$Hf(8$^+$) | | | |
| 0$^+$ | 7.775 | 8 | 99.98 |
| 2$^+$ | 6.274 | 6 | $1.9 \times 10^{-2}$ |
| 3$^-$ | 5.942 | 5 | $2.7 \times 10^{-3}$ |
| $^{157}$Ta(1/2$^+$) | | | |
| 1/2$^+$ | 6.117 | 0 | 87.7 |
| 11/2$^-$ | 6.197 | 5 | 12.3 |
| $^{157}$Ta(11/2$^-$) | | | |
| 11/2$^-$ | 6.214 | 0 | 96.6 |
| 1/2$^+$ | 6.134 | 5 | 3.4 |
| $^{157}$Ta(25/2$^-$) | | | |
| 11/2$^-$ | 7.744 | 8 | 99.9 |
| 1/2$^+$ | 7.664 | 13 | $3.1 \times 10^{-3}$ |
| 15/2$^-$ | 6.132 | 6 | $2.7 \times 10^{-2}$ |
| 15/2$^+$ | 6.138 | 5 | $1.7 \times 10^{-2}$ |
| 19/2$^+$ | 5.921 | 3 | $1.1 \times 10^{-2}$ |
| $^{158}$W(8$^+$) | | | |
| 0$^+$ | 8.286 | 8 | 99.96 |
| 2$^+$ | 6.773 | 6 | $3.7 \times 10^{-2}$ |
| 3$^-$ | 6.275 | 5 | $1.6 \times 10^{-3}$ |
| 5$^-$ | 6.140 | 3 | $2.3 \times 10^{-3}$ |

$^{144}$Nd to $^{159}$Re. The experimental data on the $\alpha$ decay from the ground states are reproduced to an accuracy that is better than other contemporary approaches, exemplified by the empirical formula of Royer [7]. Furthermore, for the $\alpha$ decay of the known high-spin isomers in $^{155}$Lu, $^{156}$Hf, $^{157}$Ta, and $^{158}$W,

we are able to reproduce the main decay properties, including reproduction of the observed fine structure from $^{155}$Lu$(25/2^-)$ and $^{156}$Hf$(8^+)$ once a reduction in the pairing strength ($\Delta \approx 0.6 \times \Delta_{\text{VJH}}$) is taken into account. Predictions for the $\alpha$-decay fine structure of $^{157}$Ta$(25/2^-)$ and $^{158}$W$(8^+)$ high-spin isomers are presented. An interesting point is that the similarity of the decay properties of the main $L = 8$ branches from the isomers $^{156}$Hf$(8^+)$ and $^{157}$Ta$(25/2^-)$ has made it difficult to confirm the existence of the latter state. One alternative for clear identification of the isomer is to search for $\alpha$-decay lines in the fine structure from $^{157}$Ta$(25/2^-)$ to excited states in $^{153}$Lu. Our calculations suggest that decay branches as strong as those recently observed from the $^{155}$Lu$(25/2^-)$

isomer should occur. The $\alpha$-decay fine structure from such spin-trap isomers may also offer a unique structural probe of excited states in medium-heavy nuclei near the proton drip line. We hope that investigations such as ours will contribute to the interpretation of such future experiments.

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