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Reciprocity calibration methods for Massive MIMO based on antenna coupling

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Abstract—In this paper we consider time-division-duplex (TDD) reciprocity calibration of a massive MIMO system. The calibration of a massive MIMO system can be done entirely at the base station (BS) side by sounding the BS antennas one-by-one while receiving with the other BS antennas. With an M antenna BS, this generates $M(M - 1)$ signals that can be used for calibration purposes. In this paper we study several least-squares (LS) based estimators, differing in the number of received signals that are being used. We compare the performance of the estimators, and we conclude that it is possible to accurately calibrate an entire BS antenna array using the mutual coupling between antennas as the main propagation mechanism.

Index Terms—Massive MIMO, TDD reciprocity, antenna calibration, estimation, uplink, downlink

I. INTRODUCTION

Massive MIMO has gained a lot of interest in the later years as it has a potential to increase the energy efficiency significantly of cellular networks compared to current technologies, while still providing a good network capacity and using mobile terminals with limited complexity [1]. In order to realize the true potential of this technology there are several practical challenges that need to be investigated, one of them being the reciprocity calibration problem [2]. Basically, one can not afford to transmit pilot symbols from every antenna in the downlink channel, receive them at the terminal side, and feed back channel state information (CSI) to the BS so that it can calculate suitable pre-coding coefficients. Such a procedure would degrade the spectral efficiency significantly considering the amount of feedback information required, due to the large number of BS antennas. Instead, a common approach is to operate in time division duplex (TDD) mode, and rely on the reciprocity of the channel to compute proper pre-coding coefficients based on uplink CSI.

It is generally agreed in wireless systems that the propagation channel is reciprocal, but the different transceiver radio frequency (RF) chains are not. Hence, in order to use reciprocity and calculate the pre-coding coefficients, we have to know or estimate the differences in the (frequency) responses between the uplink and downlink parts of the hardware chains. Such an estimation procedure is called reciprocity calibration.

Reciprocity calibration was discussed generally in [3]. A calibration scheme was presented where the reciprocity parameters are estimated based on bi-directional channel measurements. This requires feedback from one side of the link,

thus making this approach not suitable in a massive MIMO context.

A novel massive MIMO calibration approach was proposed and implemented in a test bed in [4]. In this setup one of the antenna elements in the base station is used as a reference element, which successively transmits and receives pilot signals to and from all other antennas. The reciprocity calibration weights are simply calculated as the ratio between the forward and reverse radio channels with respect to this reference element. This method works well as long as the reference element has a good channel to all the other antenna elements, but has shown to be sensitive to the exact placement of the reference antenna.

In [5] the authors generalize the method presented in [4] and apply it in a distributed large-scale MIMO setup to calibrate access points. A robust least squares (LS) framework is derived based on successive transmission and reception of pilots solely between these access points. The methodology presented in our paper can be seen as an extension of this framework back to the case of Massive MIMO to calibrate a BS antenna array and its multiple RF-chains. Thus, instead of a random (often Rayleigh distributed) wireless channel between access points we have in our case a deterministic, often strong, component due to the antenna coupling. In this paper we use the mutual coupling between antennas to be able to estimate the reciprocity calibration coefficients.

The remainder of this paper is structured as follows: in Sec. II we introduce the system models used and the reciprocity calibration concept; in Sec. III we present the different calibration methods studied; in Sec. IV the impact of the calibration error in the capacity of a massive MIMO system is analyzed for different precoders; and finally Sec. V wraps up the paper.

II. SYSTEM MODEL

A. Channel Reciprocity

Due to the internal electronics of the BS and the single-antenna mobile stations (MS), the measured uplink/downlink channels are not only determined by the propagation channels, but those are also influenced by the RF chains. Let the uplink and downlink radio channels between the BS and MS be denoted as

$$\begin{aligned} g_{m,k}^U &= r_m^B \tilde{g}_{m,k}^U t_k^M \\ g_{k,m}^D &= r_k^M \tilde{g}_{k,m}^D t_m^B, \end{aligned} \quad (1)$$

where $m \in [0, \dots, M-1]$ is the BS antenna index, $k \in [0, \dots, K-1]$ is the MS antenna index, r^B and r^M represent the BS and MS receiver RF chains, t^B and t^M represent the BS and MS transmitter RF chains, and \tilde{g}^U and \tilde{g}^D are the uplink and the downlink propagation channels, respectively.

A relation between the uplink and downlink radio channels can be established as

$$g_{k,m}^D = b_{m,k} g_{m,k}^U. \quad (2)$$

Here we denote $b_{m,k}$ as the *calibration coefficient* between radios m and k , since if obtained, it allows to compute the downlink channel based on the uplink channel estimate. Assuming perfect reciprocity of the propagation channel, $b_{m,k}$ can be expanded as

$$b_{m,k} = \frac{r_k^M \tilde{g}_{k,m}^D t_m^B}{r_m^B \tilde{g}_{m,k}^U t_k^M} = \frac{r_k^M t_m^B}{r_m^B t_k^M}. \quad (3)$$

Hence, it can be seen that the non-reciprocity between radio channels can be calibrated externally, i.e., by feeding back the downlink channel. Such approach is unfeasible in a massive MIMO context, since for each terminal, the number of channel estimates to feedback to the BS scales with M [2].

B. Internal Calibration

Let us now introduce the channel between two BS radios as

$$h_{\ell,m} = r_\ell^B \tilde{h}_{\ell,m} t_m^B \quad (4)$$

where $\ell \neq m$, $\ell \in [0, \dots, M-1]$, and $\tilde{h}_{\ell,m}$ is the propagation channel between the BS antennas ℓ and m . We introduce the calibration coefficient between BS radios as

$$h_{\ell,m} = b_{m \rightarrow \ell} h_{m,\ell}, \quad (5)$$

which by assuming perfect reciprocity yields¹

$$b_{m \rightarrow \ell} = \frac{h_{\ell,m}}{h_{m,\ell}} = \frac{r_\ell^B t_m^B}{r_m^B t_\ell^B} = \frac{1}{b_{\ell \rightarrow m}}. \quad (6)$$

One of the main contributions from [4] was an internal reciprocity calibration method for a massive MIMO base station. The method has two main points as basis:

1)

$$b_{m,k} = \frac{t_m^B}{r_m^B} \frac{r_k^M}{t_k^M} = \frac{r_n^B t_m^B}{r_m^B t_n^B} \frac{r_k^M t_n^B}{r_n^B t_k^M} = b_{m \rightarrow n} b_{n,k}. \quad (7)$$

i.e., calibration between radios m and k can also be achieved if their forward and reverse channels to another BS radio n are jointly processed. Throughout the paper we set $n = 0$ for convenience and denote this radio as the reference radio.

2) As long as each downlink channel estimate from all BS antennas deviate from the real ones by the same complex factor, the resulting downlink beam pattern shape does not change. Thus, since the transceiver response of any

terminal shows up as a constant factor to all BS antennas, its contribution can be omitted from the calibration procedure.

Combining (2) with the previous two points yields

$$g_{k,m}^D = b_{m,k} g_{m,k}^U \quad (8)$$

$$\stackrel{1)}{=} b_{m \rightarrow 0} b_{0,k} g_{m,k}^U \quad (9)$$

$$\stackrel{2)}{\Leftrightarrow} g_{k,m}^D = b_{m \rightarrow 0} g_{m,k}^U \quad (10)$$

where $g_{k,m}^D$ is a relative downlink channel that absorbs $b_{0,k}$. Thus relative downlink channels can be obtained by multiplying the respective uplink channels with their respective calibration coefficients to a reference radio. The authors in [5] took this approach one step forward in order to calibrate access points of a distributed MIMO network. A novelty in their approach was

$$g_{k,m}^D = b_{m \rightarrow 0} g_{m,k}^U \quad (11)$$

$$\Leftrightarrow g_{k,m}^D = b_m g_{m,k}^U \quad (12)$$

where $b_m = \frac{r_m^B}{t_m^B} = \frac{1}{b_{m \rightarrow 0}} \frac{t_0^B}{r_0^B}$, and $g_{k,m}^D$ is another relative downlink channel. This relative equivalence not relaxes the double-indexing overhead, but allows different calibration coefficients to be treated as mutually independent (!).

Note that the absolute reference to the terminals was lost in the derivation step 2), which makes $b_{m \rightarrow 0}$ or b_m valid calibration coefficients up to a complex factor. Thus, downlink pilots still need to be broadcast through the beam to compensate for this uncertainty, as well as for the RF chain responses of the terminals. The overhead of these supplementary pilots is reported as very small [2]. Also note that the calibration coefficients are valid over long periods of time (compared to the channel coherence interval) since BS radios share the same synchronization references.

C. System Model for BS-BS Signals

As shown in Sec. II-B, reciprocity calibration can be carried out without the need of any feedback from the MSs. To estimate the calibration coefficients b_m we sound the M antennas one-by-one by transmitting a pilot symbol from each one and receiving on the other $M-1$ silent antennas. For simplicity, we use a pilot symbol $p = 1$. Let $y_{m,\ell}$ denote the signal received at antenna m when transmitting at antenna ℓ . It follows that the received signals between any pair of antennas can be written as

$$\begin{aligned} \begin{bmatrix} y_{\ell,m} \\ y_{m,\ell} \end{bmatrix} &= \tilde{h}_{\ell,m} \begin{bmatrix} r_\ell^B t_m^B \\ r_m^B t_\ell^B \end{bmatrix} + \begin{bmatrix} n_{\ell,m} \\ n_{m,\ell} \end{bmatrix} \\ &= \alpha_{\ell,m} \begin{bmatrix} b_\ell \\ b_m \end{bmatrix} + \begin{bmatrix} n_{\ell,m} \\ n_{m,\ell} \end{bmatrix}, \end{aligned} \quad (13)$$

where $\alpha_{\ell,m} = t_\ell^B t_m^B \tilde{h}_{\ell,m} = t_\ell^B t_m^B \tilde{h}_{m,\ell}$ due to reciprocity, and $[n_{\ell,m} \ n_{m,\ell}]^T$ is a vector of independent zero-mean circularly symmetric complex Gaussian distributed random variables, each one with variance N_0 .

¹Note that we denote the calibration coefficients between two BS radios using " \rightarrow " to distinguish from the calibration coefficient between a BS radio and an MS which uses ",".

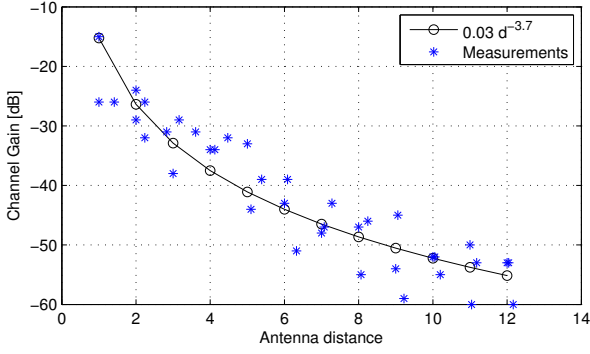


Fig. 1. Measured antenna coupling. The horizontal axis represents the antenna spacing in units of $\frac{\lambda}{2}$ between measured antennas.

D. Statistical Model of BS-BS Channels

We next put forth the statistical models for the channel between antennas that we have used in this work. The channel between two antennas ℓ and m is modeled as

$$\tilde{h}_{\ell,m} = \beta_{\ell,m} \exp(j\phi_{\ell,m}) + w_{\ell,m}, \quad (14)$$

where $\beta_{\ell,m}$ is assumed known and models the channel gain due to antenna coupling, the channel phase $\phi_{\ell,m}$ is uniformly distributed between 0 and 2π , and $w_{\ell,m} \sim \mathcal{CN}(0, N_w)$ models multipath propagation with no dominant component.

To model the antenna coupling $\beta_{\ell,m}$, we measured channel gains between $\frac{\lambda}{2}$ spaced antennas of a 25x4 dual polarized antenna array, a custom made massive MIMO antenna array for our testbed [6], in an anechoic chamber. We averaged the frequency response magnitude over a 20 MHz bandwidth centered at 3.7 GHz which the array was originally designed to operate at. Fig. 1 shows the measured results. Only the E-plane orientation field was measured. This explains the difference between measured channel gains for same measured distances since antenna elements oriented in the E-plane orientation are more strongly coupled than others [7].

As a rough estimate a $0.03d^{-3.7}$ curve match our measurements well. This simplified fit will be used in our simulations which allows for reproducible results.

III. RECIPROCITY CALIBRATION METHODS

A. Direct-path based LS [4]

Here we estimate $\mathbf{b} = [b_0, b_1, \dots, b_{M-1}]^T$ solely using the signals $y_{0,m}$ and $y_{m,0}$. Since b_m can be estimated up to a multiplicative constant, we set $b_0 = 1$ with no loss of generality and solve for the remaining $[b_1, \dots, b_{M-1}]^T$. A least-squares approach can be pursued which seeks to jointly optimize b_m and $\alpha_{\ell,m}$ according to

$$(\hat{b}_m, \hat{\alpha}) = \arg \min_{b_m, \alpha} \left\| \begin{bmatrix} y_{0,m} \\ y_{m,0} \end{bmatrix} - \alpha_{\ell,m} \begin{bmatrix} 1 \\ b_m \end{bmatrix} \right\|^2. \quad (15)$$

It is easy to verify that the solution to (15) is given by

$$\hat{b}_m = \frac{y_{0,m}}{y_{m,0}} \quad \text{and} \quad \alpha_{\ell,m} = y_{0,m}. \quad (16)$$

Note that this ratio has unbounded second moment.

B. Generalized LS [5]

This approach generalizes the *Direct-path based LS* estimator by considering the full set of signals in (13). An LS cost function can be formulated as

$$J(\mathbf{b})_{\text{LS}} = \sum_{m, \ell \neq m} |b_m y_{m,\ell} - b_\ell y_{\ell,m}|^2. \quad (17)$$

To minimize (17) one can set its gradient $\nabla J(\mathbf{b})$ to zero and solve for \mathbf{b} . To exclude the trivial solution $\mathbf{b} = \mathbf{0}$, we set $b_0 = 1$ as previously mentioned. This yields

$$\hat{\mathbf{b}} = -(\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{a}_1 b_0 \quad (18)$$

where $\mathbf{A} = (\mathbf{a}_1 \mathbf{A}_1)$ (i.e., \mathbf{a}_1 is the first column of \mathbf{A} , \mathbf{A}_1 is a matrix made of the $M-1$ last columns of \mathbf{A}) and \mathbf{A} is structured as

$$A_{m,\ell} = \begin{cases} \sum_{i=1}^M |y_{m,\ell}|^2, & m = \ell \\ -y_{m,\ell}^* y_{\ell,m} & m \neq \ell \end{cases}. \quad (19)$$

C. Generalized weighted LS

All sets of double directional measurements are given the same weights in (17). If one still maintains an LS formulation, it is intuitive that the estimator's performance can be improved if any statistical information of $\alpha_{\ell,m} = t_\ell^B t_m^B (\beta_{\ell,m} \exp(j\phi_{\ell,m}) + w_{\ell,m})$ is known. In a practical (massive MIMO) antenna array, knowledge of the coupling gains $\beta_{m,\ell}$ is indeed at hand, see Sec. II-D. Thus the cost function can be empirically re-defined to

$$J(\mathbf{b})_{\text{WLS}} = \sum_{m,\ell} |\beta_{m,\ell} b_m y_{m,\ell} - \beta_{\ell,m} b_\ell y_{\ell,m}|^2. \quad (20)$$

It can be shown that weighting the cost function with the complex coupling gains $\beta_{m,\ell} \exp(j\phi_{m,\ell})$ yields the same estimator as (20), thus making phase information irrelevant for the current problem formulation.

D. Generalized Neighbor LS

In Sec. III-B and Sec. III-C we addressed performance improvements to (15) by jointly processing $M(M-1)$ signals. In this subsection we investigate if an entire BS antenna array can be accurately calibrated solely based on signals to/from neighbor antennas, thus using less than $4M$ signals for the case of a planar array. The cost function in this case is given by

$$J(\mathbf{b})_{\text{NLS}} = \sum_m \sum_{\ell \in \mathcal{A}_m} |b_m y_{m,\ell} - b_\ell y_{\ell,m}|^2. \quad (21)$$

where \mathcal{A}_m is the set of indexes of adjacent antennas to antenna m . Besides the obvious reduced number of multiplications needed to generate \mathbf{A}_1 , the advantages of such neighbor based calibration are manifold: (i) with proper antenna indexing, the final estimator inversion $(\mathbf{A}_1^H \mathbf{A}_1)^{-1}$ is potentially performed faster since $\mathbf{A}_1^H \mathbf{A}_1$ can be arranged as an L -banded Hermitian matrix with $L \ll M$ [8]; (ii) the received signal power level is approximately the same for all neighbor receiving antennas. This simplifies post-compensation due to hardware

adaptations, e.g., automatic gain control (AGC), or non-linear dependencies, e.g., amplifiers; (iii) it allows distant antennas to measure their neighbor channel simultaneously with (almost) no interference, speeding up the calibration process.

E. Simulated calibration accuracy

We simulated reciprocity calibration for the case of a 5x20 planar patch array. We used the antenna coupling loss model established in Sec. II-D and set the variance of the channel Rayleigh component to $N_w = -50$ dB. One of the center antenna elements of the array was defined as the reference. For the general case, modeling the statistics of RF chains responses is a hard task, thus we follow the same approach as [5], where both transmitter and receiver (i.e., t_m^B and r_m^B) have uniformly distributed phase between $[-\pi, \pi[$ and uniformly distributed magnitude between $[1 - \epsilon, 1 + \epsilon]$ with ϵ such that $\sqrt{\mathbb{E}\{|t_m^B| - 1\}^2} = \sqrt{\mathbb{E}\{|r_m^B| - 1\}^2} = 0.1$.

We focus on the distinct cases of neighbor antennas and furthest away antennas from the reference one. The latter are positioned at the array edges where coupling to the reference is practically null, thus being the the hardest calibration case. Results for others antennas should, in principle, fall within these bounds.

For all approaches, we choose to normalize all results with respect to the (calibration) signal-to-noise ratio SNR_{Cal} of the neighbor antenna channel. With this normalization it is straightforward to see how different calibration methods “close the gap” between the best and worst calibration scenarios.

At low SNR_{Cal} values, its visible from Fig. 2 that the direct-path (DP) based estimator do not possess finite second moment, i.e., the simulated MSE do not converge as the number of simulation runs increases. As for the generalized estimators, the LS estimator (Sec. III-B) shows the worst performance at low SNR_{Cal} . This is justified by the weak received signals being equally weighted in the cost function. The weighted LS estimator (Sec. III-C) compensates for this, but has worst performance at high SNR_{Cal} (by a small margin) since weights are not optimized in an MSE sense. Overall, the neighbor LS (Sec. III-D) scheme works fairly well.

A rough estimate of the calibration SNR_{Cal} regime where a massive MIMO basestation as our testbed [6] operates is given by

$$\text{SNR}_{\text{cal}} = P_{RX} - N \approx 80\text{dB}, \quad (22)$$

where $P_{RX} = -15$ dBm is the maximum allowed receive power per RF-chain, $N = 10 \log_{10}(kBT_0) + N_F + G \approx -95$ dBm is the receiver noise power, k is Boltzmann’s constant, $B = 20$ MHz is the channel bandwidth, $T_0 = 290$ K is the standardized room temperature, $N_F = 6$ dB is the noise figure of the receiver chain, and $G = 0$ dB is a normalized amplifier gain. In practice, hardware limitations as ADC resolution and frequency harmonics will degrade the calibration performance. However, a margin of tens of dBs is still available to compensate for such impairments while still achieving acceptable performance for the applications we target, as will be discussed in further detail in Sec. IV.

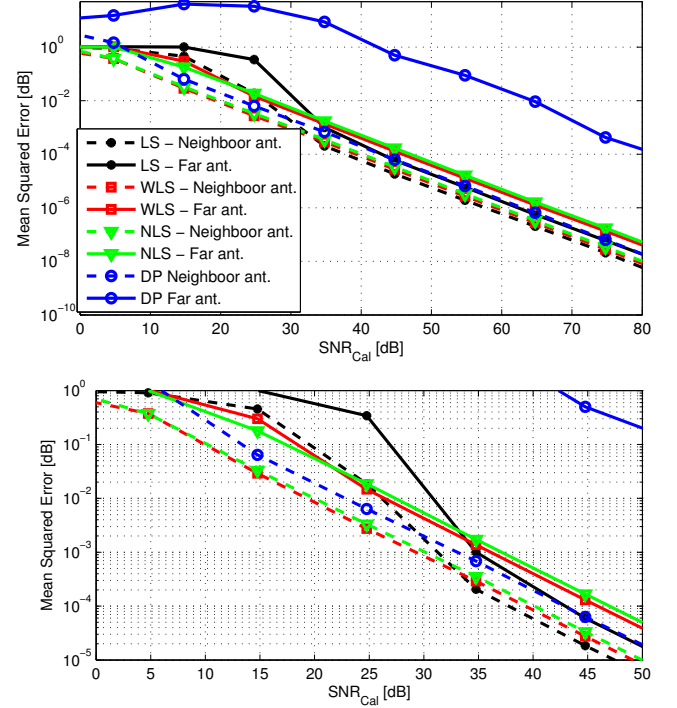


Fig. 2. Mean squared error (MSE) of the calibration coefficients computed for the neighbor and the furthestmost antenna from the reference.

IV. PERFORMANCE ANALYSIS OF A RECIPROCITY CALIBRATED MASSIVE MIMO SYSTEM

In this section we verify the impact of the reciprocity calibration error on the capacity/sum-rate of a massive MIMO downlink transmission with perfect (uplink) channel state information (CSI). We generated the set of calibration signals $[y_{\ell,m} \ y_{m,\ell}]^T$ according to Sec. III-E and used the neighbor based calibration approach (i.e., see Sec. III-D) to estimate the calibration coefficients.

The BS is equipped with $M = \{100, 400\}$ antennas and serves $K = 10$ single antenna mobile users in the same time/frequency resource. The composite received symbol vector at the user side for the case of a narrow-band MIMO channel is described as

$$\mathbf{y} = \sqrt{\frac{\rho}{K}} \mathbf{H} \mathbf{s} + \mathbf{n}, \quad (23)$$

where \mathbf{H} and \mathbf{n} are the $K \times M$ channel matrix and the $K \times 1$ noise vector, respectively, with i.i.d. unit-norm zero-mean circularly symmetric complex Gaussian distributed random elements, $\mathbf{s} = f(\mathbf{x})$ subject to $\mathbb{E}\{|\mathbf{s}|^2\} = 1$ is the transmit precoded version of \mathbf{x} with calibration errors, and ρ/K is the transmit power.

Fig. 3 shows the calibration error-free capacities/sum-rates of three precoders, i.e., maximum-ratio transmission (MRT), zero-forcing (ZF) and dirty paper coding (DPC) scheme. In the high SNR regime, inter-user interference with total power I , upper-bounds the MRT precoder sum-rate to $C_{\text{MRT}}^{\text{UP}} = K \log_2(1 + \frac{M\rho}{KI})$, while the ZF sum-rate and DPC capacity converge to the interference-free case $C_{\text{IF}} = K \log_2(1 + \frac{M\rho}{KN})$.

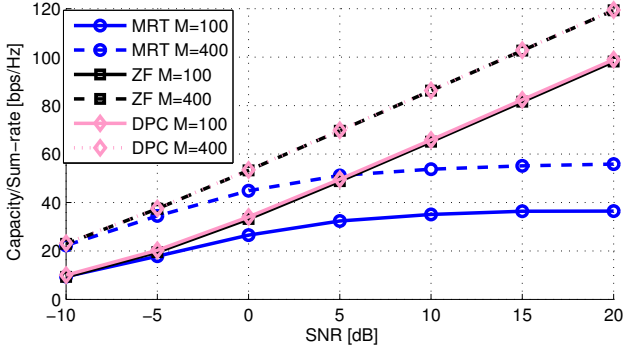


Fig. 3. Capacity/sum-rates for different precoders in i.i.d. massive MIMO downlink channels.

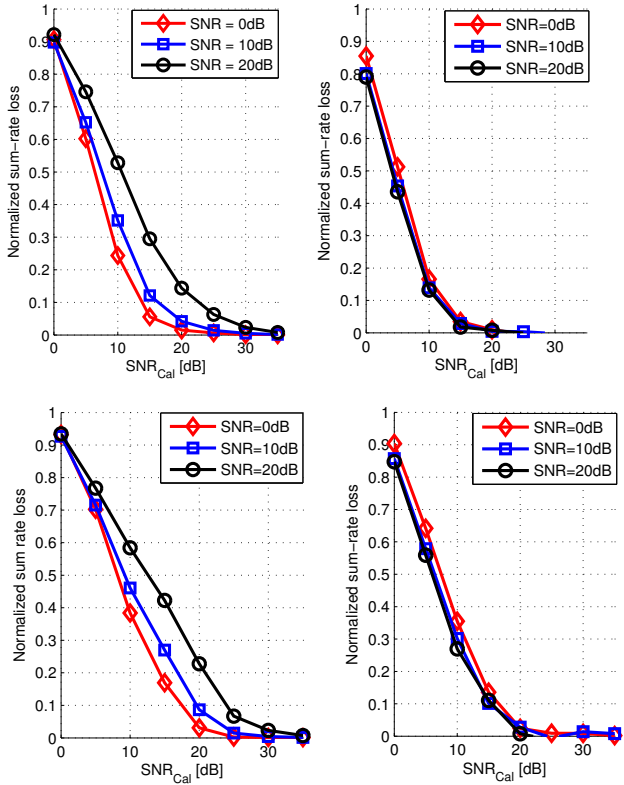


Fig. 4. Sum-rate loss due to calibration errors for different precoders using the neighbor LS calibration method at different communications SNRs. Up) $M=100$; Down) $M=400$ left) Zero-forcing precoder; right) Maximum ratio transmission.

Fig. 4 shows the downlink sum-rate loss due to reciprocity calibration errors for MRT and ZF precoders using the neighbor LS estimator to compute the calibration coefficients. For a given precoder, the sum-rate loss was obtained by normalizing the obtained sum-rates by their respective error-free ones (Fig. 3). Overall, the sum-rate loss using the MRT precoding scheme shows to be more robust to calibration errors compared to the ZF case: (i) less calibration SNR_{Cal} is needed to achieve similar capacity losses, (ii) capacity losses are less sensitive to the current communications SNR. Significant capacity losses happen for SNR_{Cal} values smaller than 35dB and 20dB for the ZF and MRT precoders, respectively. These

SNR_{Cal} values provide reference levels for achieving “good enough” calibration performance. Noticeably, the extended estimators introduced in Sec. III-C and Sec. III-D, improve the calibration performance within $0\text{dB} < \text{SNR}_{\text{Cal}} < 35\text{dB}$ compared to current state-of-art methods [5], where significant capacity losses occur due to calibration errors, see Fig. 2. Note that, for the considered channel model between BS antennas, calibration accuracy is reduced as the number of BS antennas M grows.

V. CONCLUSIONS

In this paper we extended a reciprocity calibration framework which was originally developed for calibrating the access points of a distributed MIMO system, in order to calibrate a massive MIMO BS antenna array.

Inter-BS antenna channels exhibit strong deterministic characteristics which can be incorporated in the calibration model to enhance performance. The performance of the studied estimators indicates that is possible to calibrate an entire massive MIMO BS antenna array using antenna coupling as the main propagation mechanism. From the specifications of a massive MIMO base station testbed as [6], we verified a calibration accuracy margin of tens of dBs better than a calibration accuracy leading to significant capacity losses. The downlink capacity loss of a massive MIMO system using the MRT precoding scheme was shown to be more robust to calibration errors compared to the ZF case.

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