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Abstract—**Massive MIMO, also known as very-large MIMO or large-scale antenna systems, is a new technique that potentially can offer large network capacities in multi-user scenarios, where the base stations are equipped with a large number of antennas simultaneously serving multiple single-antenna users on the same frequency. However, the radio-frequency (RF) chains associated with the antennas increase the system complexity and hardware cost. Antenna selection is a signal processing technique that can help reduce the number of RF chains, while preserving the system performance at a certain required level. We study the transmit antenna selection in measured massive MIMO channels from several measurement campaigns in the 2.6 GHz frequency range. Convex optimization is used to select the antenna subset that maximizes the dirty-paper coding (DPC) capacity in the downlink. With a certain number of RF chains, we increase the number of base station antennas from the same as the RF chains to a large number, from which we perform the antenna selection. The investigation shows that with more available antennas than RF chains, the antenna selection can significantly improve the system performance, especially for a compact cylindrical array, which without this antenna selection shows lower performance than a physically large linear array with the same number of elements, in the studied scenarios.**

I. INTRODUCTION

Massive MIMO, also known as very-large MIMO or large-scale antenna systems, is an emerging technology in wireless communications. It scales up the conventional MIMO by possibly orders of magnitude. With massive MIMO, we consider multi-user MIMO (MU-MIMO) [1] where a base station is equipped with a large number (say, tens to hundreds) of antennas, and is serving several single-antenna users in the same time-frequency resource.

It has been shown in theory that with a large number of base station antennas such a system can remarkably improve performance in terms of link reliability, data rate and radiated-energy efficiency [2] [3] [4]. However, in practice, the number of antennas at the base station cannot be made arbitrarily large due to physical constraints and complexity of implementing such a system. Moreover, the cost should also be considered. We know that adding more antennas at the base station is usually inexpensive, and the additional digital signal processing units become ever cheaper as well. However, the RF elements, such as radio-frequency (RF) amplifier, mixer and analog-to-digital/digital-to-analog (AD/DA) converter can be relatively expensive. For a massive MIMO system, it can be very expensive to deploy RF chains for all the antennas at the base station. To deal with these, in this paper we consider antenna selection as a powerful signal processing technique

that reduces the system complexity and cost, yet preserves system performance at a certain required level.

Antenna selection has been widely studied for conventional MIMO with a small number of antennas, such as in [5] and [6]. Basically, the "best" N out of M antenna signals are selected, up/down-converted, and then processed. This reduces the number of required RF chains from M to N, thus leads to significant savings, while still keeping most of the benefits from the full MIMO system. The selection criteria can be maximization of channel capacity, signal-to-noise ratio (SNR) at the receiver, or minimization of eigenvalue spread or bit-error-rate (BER). With a certain number of RF chains and more antennas than that, antenna selection improves the system performance by exploiting the spatial selectivity, as the subset of antennas with the best channel conditions is selected and switched to the RF chains. When we have a large number of antennas at the base station, the propagation channel potentially provides much more spatial selectivity, from which the system performance may be greatly improved.

In this paper, we consider transmit antenna selection in the downlink of massive MIMO systems, and the maximization of capacity/sum-rate is used as selection criterion. We assume that the base station has perfect channel state information (CSI) over all the antennas. This is based on the assumption that performing channel estimation on an antenna can be done with a less complex and less costly device than a full transceiver. We apply the transmit antenna selection on measured massive MIMO channels using two types of antenna arrays in the same realistic environment, as reported in [7]. One array is a compact cylindrical array with 128 directive patch antenna elements, while the other is a physically large linear array with the same number of antenna elements but with omni-directional patterns. In [7], we have studied the capacity/sum-rate performance in the downlink using the two large arrays. We showed both the average capacities/sum-rates, as well as the 5%-95% outage regions, for 2000 random selections out of the 128 antennas, when using between 4 and 100 RF chains. We observed that the linear array achieves higher average capacities/sum-rates than the cylindrical array in the studied scenarios, and that the cylindrical array has larger variations in capacity/sum-rate over the 2000 random antenna selections. For the physically large linear array, the variation is due to the large-scale fading over the array [8] [9], while for the compact cylindrical array, it is mainly because of the directivity of its patch antenna elements. These
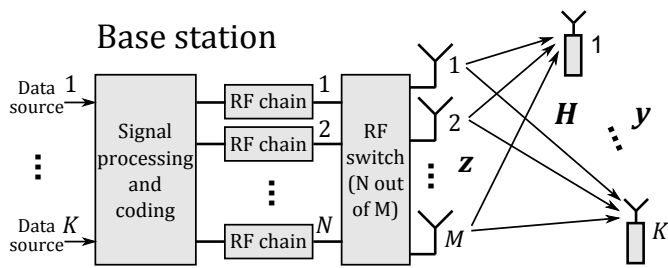
Fig. 1. System model of a MU-MIMO system in the downlink: the base station is equipped with M antennas and N RF chains, and is simultaneously serving K single-antenna users ($K \leq N \leq M$).

variations over antenna subsets motivate us to study antenna selection. With the "best" antenna subset, better performance can be achieved than the average for both the cylindrical and linear arrays. We would like to investigate the performance gain we can harvest with antenna selection in the measured channels. Specifically, with a certain number of RF chains, we increase the number of antennas from the same as the RF chains to a large number, and study how much we can gain by having more antennas than the number of RF chains using both the cylindrical and linear arrays. In this work, the transmit antenna selection problem is solved through convex optimization. Although our solution does not ensure a global optimum, we know that the obtained gains from the antenna selection are achievable.

The rest of the paper is organized as follows. In Sec. II, we describe the system model and formulate the antenna selection as a convex optimization problem. In Sec. III, we continue with the description of the measured channels using the two large arrays, for which we perform the antenna selection. Then in Sec. IV, we present the antenna selection results in the measured channels, and discuss how much we can gain by having more antennas than RF chains using the two types of arrays at the base station. Finally we summarize this work and draw conclusions in Sec. V.

II. SYSTEM DESCRIPTION

The system description is divided into two parts. First the signal model of a MU-MIMO system with transmit antenna selection is outlined. Then we define the addressed antenna selection, and formulate it as a convex optimization problem.

A. System model

We consider the downlink of a single-cell MU-MIMO system: the base station is equipped with M antennas and N RF chains, and is simultaneously serving K single-antenna users ($K \leq N \leq M$), as shown in Fig. 1. Perfect CSI over all the antennas is assumed to be known here. Base on the CSI, the "best" N are selected out of the M antennas according to some criterion. These N antennas are then connected to the N RF chains through the RF switch.

Our signal model of the narrow-band downlink channel is,

$$\boldsymbol{y}_f = \sqrt{\frac{\rho K}{N}} \boldsymbol{H}_f^{(N)} \boldsymbol{z}_f + \boldsymbol{n}_f, \qquad (1)$$

where $\boldsymbol{H}_f^{(N)}$ is the normalized channel matrix from the selected N antennas to the K users at a center frequency indicated by f, while superscript (N) denotes that N columns are selected out of M in the full propagation matrix $\boldsymbol{H}_f$, $\boldsymbol{z}_f$ is the transmit vector across the selected N antennas, and $\boldsymbol{n}_f$ is a zero-mean complex Gaussian noise vector with unit variance elements. The variable ρ contains the transmit energy, assuming that $\boldsymbol{z}_f$ satisfies $\mathbb{E}\left\{\|\boldsymbol{z}_f\|^2\right\} = 1$. As can be seen from the term $\rho K/N$, we increase the transmit power with the number of users and reduce it as the number of RF chains/selected antennas grows. As K increases, we keep the same transmit power per user. With increasing N, the array gain increases and we choose to harvest this gain as reduced transmit power instead of increased receive SNR at the users. As discussed in [7], ρ is the interference-free SNR at each user.

The channel normalization is performed in such a way that the global attenuation in the channels are normalized, the small-scale and large-scale fading are remaining. This means that we retain the power variations over frequencies and base station antennas, and the channel matrix is normalized to have unit average energy in its entries. The spatial properties of the propagation channels are kept, especially the spatial selectivity among the base station antennas, which we rely on to harvest the performance gain by doing antenna selection.

The capacity in this downlink channel at a certain frequency is given as [10],

$$C_{\text{DPC},f} = \log_2 \det\left(\boldsymbol{I} + \frac{\rho K}{N}\left(\boldsymbol{H}_f^{(N)}\right)^H \boldsymbol{P}_f \boldsymbol{H}_f^{(N)}\right), \quad (2)$$

which can be achieved by the non-linear dirty-paper coding (DPC) technique [11]. The diagonal matrix $\boldsymbol{P}_f$ with $P_{f,i}, i = 1, 2, ..., K$, on its diagonal allocates the power among the K user channels. The capacity is found by optimizing over $\boldsymbol{P}_f$ under the total power constraint $\sum_{i=1}^{K} P_{f,i} = 1$. This optimization can be done by the sum power iterative waterfilling algorithm in [12].

For antenna selection, we introduce an $M \times M$ diagonal matrix $\boldsymbol{\Delta}$ with the diagonal elements $\Delta_i, i = 1, 2, ..., M$, being binary variables, and indicating the ith antenna is selected or not,

$$\Delta_i = \begin{cases} 1, & \text{selected} \\ 0, & \text{otherwise,} \end{cases} \qquad (3)$$

satisfying $\sum_{i=1}^{M} \Delta_i = N$. According to Sylvester's determinant theorem, $\det(\boldsymbol{I} + \boldsymbol{A}\boldsymbol{B}) = \det(\boldsymbol{I} + \boldsymbol{B}\boldsymbol{A})$, and introducing the antenna selection matrix $\boldsymbol{\Delta}$, we can write Eq. (2) as,

$$\begin{aligned} C_{\text{DPC},f} &= \log_2 \det\left(\boldsymbol{I} + \frac{\rho K}{N} \boldsymbol{P}_f \boldsymbol{H}_f^{(N)}\left(\boldsymbol{H}_f^{(N)}\right)^H\right) \\ &= \log_2 \det\left(\boldsymbol{I} + \frac{\rho K}{N} \boldsymbol{P}_f \boldsymbol{H}_f \boldsymbol{\Delta} \boldsymbol{H}_f^H\right), \qquad (4) \end{aligned}$$

where $\boldsymbol{H}_f$ is the full propagation matrix from all the M base station antennas to the K users. We select the N antennas that maximize the average DPC capacity over the frequencies, and

thus the optimal $\boldsymbol{\Delta}$ is found by,

$$\boldsymbol{\Delta}_{\mathrm{opt}} = \arg\max_{\boldsymbol{\Delta}} \; \mathbb{E}_f \left\{ \log_2 \det \left(\boldsymbol{I} + \frac{\rho K}{N} \boldsymbol{P}_f \boldsymbol{H}_f \boldsymbol{\Delta} \boldsymbol{H}_f^H \right) \right\}. \tag{5}$$

Now we turn our attention to this optimization problem at hand.

B. Convex optimization problem

To maximize the average DPC capacity over frequencies, we need to optimize over the power allocation $\boldsymbol{P}_f$ among the users, and the antenna selection $\boldsymbol{\Delta}$. It is difficult to optimize over these two at the same time. Therefore, we divide the optimization into two steps: 1) we assume equal power allocation among the users, i.e., $P_{f,i} = 1/K$, and select the N antennas that maximize the average capacity; 2) with the selected antennas, we optimize over the user power allocation $\boldsymbol{P}_f$ and thus find the maximum average capacity. This simplification does not ensure a global optimum, however, it gives us a hint how much capacity gain can be achieved by using antenna selection.

In Step 1, the optimization problem of antenna selection can be described as,

$$\begin{aligned}
\text{maximize} \quad & \mathbb{E}_f \left\{ \log_2 \det \left(\boldsymbol{I} + \frac{\rho}{N} \boldsymbol{H}_f \boldsymbol{\Delta} \boldsymbol{H}_f^H \right) \right\} \\
\text{subject to} \quad & \Delta_i \in \{0, 1\} \\
& \sum_{i=1}^{M} \Delta_i = N.
\end{aligned} \tag{6}$$

The optimal selection algorithm is an exhaustive search over all possible antenna combinations. However, for massive MIMO where M can be more than one hundred, the exhaustive search can hardly be done due to an extremely large number of possible antenna combinations. We therefore need to find a more tractable optimization strategy. As can be seen from (6), the objective function of average capacity is concave in $\boldsymbol{\Delta}$, since $\log_2 \det (\boldsymbol{X})$ is concave if $\boldsymbol{X}$ is a positive-definite matrix, and the concavity of a function is preserved under affine transformation [13]. Since the variables Δ_i are binary integer variables, it makes the optimization problem NP-hard. In order to solve this optimization problem, we use the concept of linear programming relaxation. According to [14] and [15], the original problem in (6) can be relaxed to the following,

$$\begin{aligned}
\text{maximize} \quad & \mathbb{E}_f \left\{ \log_2 \det \left(\boldsymbol{I} + \frac{\rho}{N} \boldsymbol{H}_f \boldsymbol{\Delta} \boldsymbol{H}_f^H \right) \right\} \\
\text{subject to} \quad & 0 \leq \Delta_i \leq 1 \\
& \sum_{i=1}^{M} \Delta_i = N,
\end{aligned} \tag{7}$$

where we have relaxed the constraint that each Δ_i must be a binary variable to a weaker constraint that each Δ_i is a real number in the interval $[0, 1]$. The original problem thus becomes a convex optimization problem solvable in polynomial time. This relaxation yields a fractional solution of Δ_i, from which the N largest ones are selected, and their indices represent the selected N antennas at the base station.

In [14] and [15], simulation results showed that the performance of this problem with relaxation solved by convex optimization techniques is very close to the optimal one based on exhaustive search. To verify this, we simulated the antenna selection for the theoretical independent and identically distributed (i.i.d.) complex Gaussian channels at one frequency point. We compared the scheme of selecting the largest N Δ_i's with an exhaustive search over all the Δ_i's that are greater than zero. We found that the two schemes give the very close results, except for the case where we select a small number of antennas out of a large number of available antennas, i.e., $N \ll M$. However, the performance drop of the maximized capacity is less than 5% in this case. Therefore, we consider that the relaxation to a convex optimization problem is reliable in this antenna selection work. The only thing that might reduce the effectiveness of antenna selection is frequency selectivity in the channels. This is why we performed the simulations in i.i.d. channels with only one frequency point. The problem with frequency-selective channels in antenna selection has been raised and discussed in [5]. Basically, different antenna subsets are optimum for different (uncorrelated) frequency bands. In the limit that the system bandwidth is much larger than the coherence bandwidth of the channel, and if the number of resolvable multipath components is large, all possible antenna subsets become equivalent. Despite all this, our measured massive MIMO channels have a signal bandwidth of 50 MHz and is at a center frequency of 2.6 GHz, where we observed a large coherence bandwidth, i.e., more than 25 MHz in line-of-sight (LOS) scenarios and 5 MHz in rich scattering scenarios. This allows the effective use of antenna selection. The measured channels are described in more detail in the next section.

The largest N Δ_i's are rounded up to 1, and the rest are set to 0, the near-optimal antenna selection matrix $\tilde{\boldsymbol{\Delta}}_{\mathrm{opt}}$ is therefore obtained. Finally, the average capacity is maximized by optimizing power allocation among users at each frequency as below,

$$C_{\mathrm{DPC}} = \mathbb{E}_f \left\{ \log_2 \det \left(\boldsymbol{I} + \frac{\rho K}{N} \boldsymbol{P}_f \boldsymbol{H}_f \tilde{\boldsymbol{\Delta}}_{\mathrm{opt}} \boldsymbol{H}_f^H \right) \right\}. \tag{8}$$

With our system model and optimization strategy defined above, we now move on to the channel measurements used in the analysis.

III. Measured Channels

The measured channels were obtained from two measurement campaigns performed with two different large antenna arrays at the base station. Both arrays contain 128 antenna elements and have an adjacent element spacing of half a wavelength at 2.6 GHz. Fig. 2a shows the cylindrical array, having 16 dual-polarized directional patch antennas in each circle and 4 such circles stacked on top of each other, which gives a total of 128 antenna ports. This large antenna array is physically compact with both diameter and height of about 30 cm. Fig. 2b shows the virtual linear array with an omni-directional antenna moving along a rail, in 128 equidistant

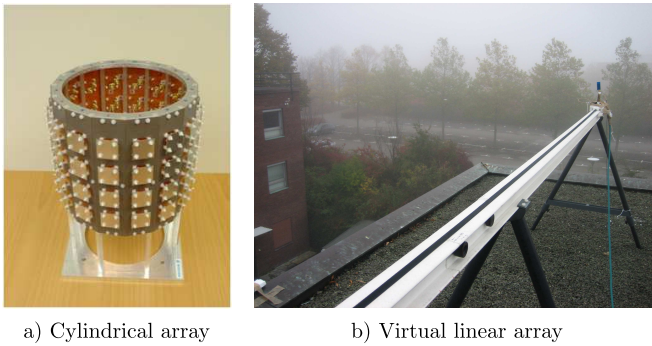a) Cylindrical array   b) Virtual linear array

Fig. 2. Two large antenna arrays at the base station side: a) a cylindrical array with 128 patch antenna elements and b) a virtual linear array with 128 omni-directional antenna positions.
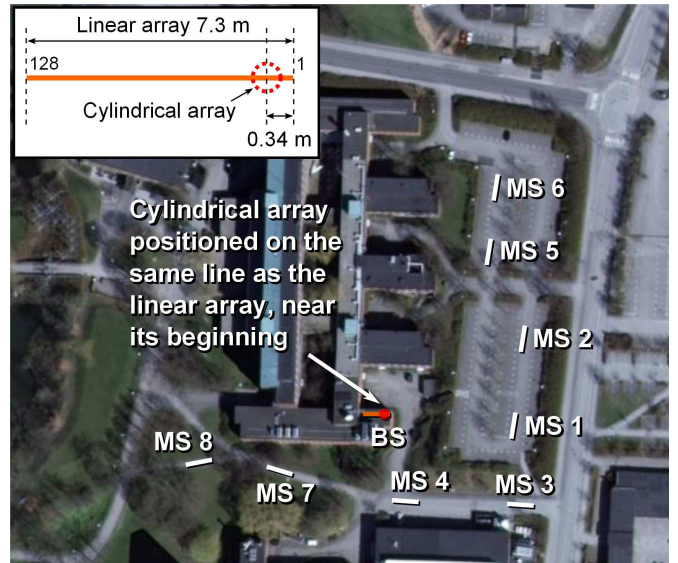

Fig. 3. Overview of the measurement area at the campus of the Faculty of Engineering (LTH), Lund University, Sweden. The two base station antenna arrays were placed on the same roof of the E-building during two measurement campaigns. 8 user sites around the E-building were measured.

positions. In comparison, the linear array is physically large and spans 7.3 m in space. At the user side, an omni-directional antenna was used in both measurement campaigns. The measurement data were recorded at a center frequency of 2.6 GHz and a signal bandwidth of 50 MHz.

Both channel measurements, using the cylindrical array and the linear array, were carried out outdoors at the E-building of the Faculty of Engineering (LTH), Lund University, Sweden. Fig. 3 shows an overview of the semi-urban measurement area. The two base station antenna arrays were placed on the roof of the E-building during the two measurement campaigns. More precisely, the cylindrical array was positioned on the same line as the linear array, near its beginning, as illustrated in Fig. 3. At the user side, the omni-directional antenna was moved around at 8 measurement sites (MS) acting as single-antenna users (see Fig. 3). Among these sites, three (MS 1-3) have line-of-sight (LOS) conditions, and four (MS 5-8) have non-line-of-sight (NLOS) conditions, while one (MS 4) has LOS for the cylindrical array, but the LOS component is blocked by the edge of the roof for the linear array.

IV. RESULTS AND DISCUSSION

Based on the measured channels, we study the performance of antenna selection in massive MIMO systems. We fix the number of RF chains and increase the base station antennas to a large number. We compare the performance of having equal number of antennas and RF chains with that of having more antennas, for both the linear and the cylindrical arrays. The performance of having equal number of antennas and RF chains ($M = N$) depends on the positions of these antennas in the measured environment, thus the average performance can be obtained by averaging over the random selections out of the 128 antennas, for both the linear and the cylindrical array, as in [7]. When having more antennas than RF chains ($M > N$), the performance depends on the positions of the M antennas through which we make the selection of the N antennas. Therefore, we randomly select M antennas out of the 128, and from each selection, we perform antenna selection to obtain the N that maximizes the DPC capacity, as described in Sec. II. The average performance of having more antennas

than RF chains is thus obtained by averaging over the random selections of the M antennas.

For 20 and 40 RF chains ($N = 20$ and $N = 40$), we use between 20 and 120 base station antennas ($20 \leq M \leq 120$). Note that we start with 40 antennas for 40 RF chains. We choose three typical propagation scenarios to study, as in [7], for which we present the performance results with antenna selections. Each scenario has four users ($K = 4$). In two of the scenarios, the four users are placed close to each other having 1.5-2 m spacing, while in the third, the four users are well separated having larger than 10 m spacing. Combining with the line-of-sight (LOS) condition, the three scenarios are as follows:

1) the four users are close to each other at MS 2, all having LOS condition to the base station;
2) the four users are close to each other at MS 7, without LOS condition;
3) the four users are well separated, at MS 1-4, respectively, all having channels with LOS characteristics.

In all three scenarios, we select the interference-free SNR $\rho = 10$ dB.

The performance results of antenna selection are shown in Fig. 4 to Fig. 6, for the three scenarios, respectively. In Fig. 4 where the users are co-located with LOS conditions, we observe that the cylindrical array has significantly lower performance than the linear array when using the same number of antennas and RF chains. However, as the number antennas increases, by using the antenna selection, the performance of cylindrical array is greatly improved, about 45% and 30%, for 20 and 40 RF chains, respectively, when using 120 antennas. For 40 RF chains, the improvement in performance is less than that of 20 RF chains. This is influenced by the fact
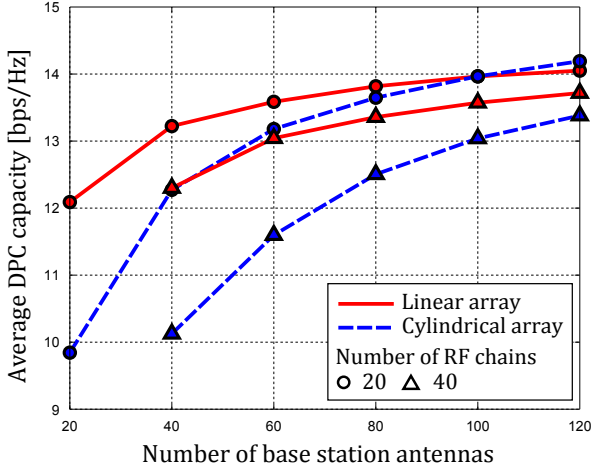
Fig. 4. Average performance of having more base station antennas than RF chains, using the antenna selection technique. The four users are close to each other at MS 2, with LOS to the base station antenna arrays.
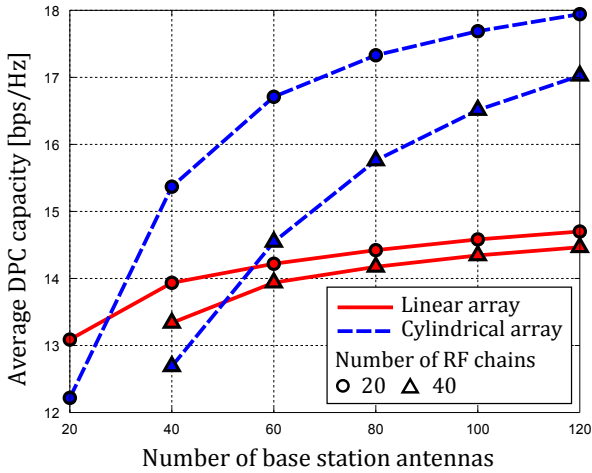

Fig. 5. Average performance of having more base station antennas than RF chains, using the antenna selection technique. The four users are close to each other at MS 7, without LOS to the base station antenna arrays.

that for more RF chains and the same number of available antennas, we have more antennas in common for any pair of selections on average. Therefore, there is less to gain with a higher number of RF chains. Besides, the performance of 40 RF chains is lower than that of 20 RF chains, this is due to that we reduce the transmit power as the number of RF chains grows. The improvement of the linear array with antenna selection is significantly smaller compared to the cylindrical array, about 15% and 10% at 120 antennas, for 20 and 40 RF chains, respectively. We notice that when using 20 RF chains and 120 antennas, the performance of cylindrical array already surpasses that of the linear array, while for 40 RF chains, the performance of cylindrical array is quite close to the linear array.

In Fig. 5, we have the four users close to each other without LOS conditions. When using the same number of antennas and
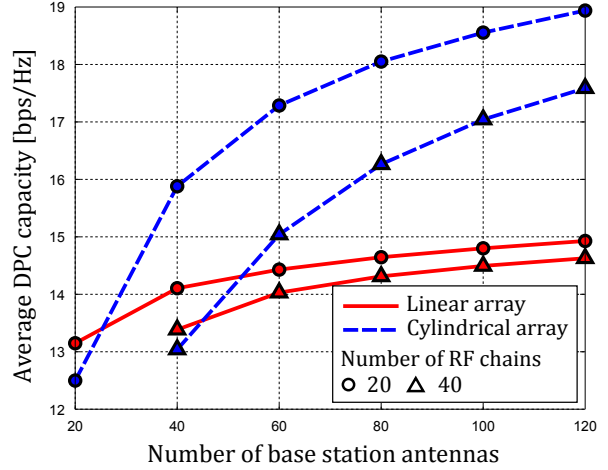

Fig. 6. Average performance of having more base station antennas than RF chains, using the antenna selection technique. The four users are well separated at MS 1-4, each has LOS characteristics to the base station antenna arrays.

RF chains, the performance gap between cylindrical and linear arrays becomes smaller, as compared to the previous scenario. When having more antennas, similarly to the previous scenario, the performance improvement is significant for the cylindrical array, about 45% and 35% at 120 antennas, for 20 and 40 RF chains, respectilvey, while for the linear array, the improvement is around 10%. In this scenario, the cylindrical array achieves higher performance than the linear array already at 40 and 60 antennas for 20 and 40 RF chains, respectively. As the number of antennas increases, the performance of cylindrical array becomes significantly higher than that of the linear array.

In Fig. 6 where the four users are well separated and have LOS characteristics, as the number of antennas increases to 120, the performance of cylindrical array is improved by around 50% and 35%, for 20 and 40 RF chains, respectively, while for the linear array, the increase is around 15% and 10%. Also, in this scenario, the cylindrical array achieves much higher performance than the linear array through antenna selection.

In all three scenarios, we can see that the cylindrical array has much more potential gain to harvest by selecting the "best" antenna subset. This is due to its directive patch antenna elements, which are pointing in different directions. It also makes the cylindrical array experience more spatial selectivity in the propagation channels, as compared to the linear array with omni-directional antenna elements. Thus, by selecting the antennas pointing in the right directions, we can boost the performance of the cylindrical array, and obtain even higher performance than the linear array which shows better performance without this antenna selection.

V. SUMMARY AND CONCLUSIONS

In this paper, we have studied the performance of transmit antenna selection in the measured massive MIMO channels

using two types of large antenna arrays. Convex optimization is used for selecting the antenna subset that maximizes the DPC capacity in the downlink.

The presented investigation shows that in the studied scenarios with more available antennas than the RF chains, the antenna selection can greatly improve the system performance, by exploiting the spatial selectivity in the propagation channels, especially for the cylindrical array. The average DPC capacity can be increased as high as 50% and 35%, for the cylindrical array with 20 and 40 RF chains, respectively, when having 120 available antennas at the base station. With the antenna selection, we can significantly boost the performance of cylindrical array, which without this antenna selection shows lower performance than the linear array. Therefore, it provides an opportunity for the compact cylindrical array to achieve better performance than the physically large linear array, which has a very high angular resolution but is relatively less practical to deploy due to its large size.

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