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## Fast & Loose: Resource-bounded Reasoning in Wason's Selection Task

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## Introduction

- In Wason's **Selection Task (ST)** [Was68] subjects **do not perform as classical-logical reasoners**.
- If **ST** is treated as a **"loose probabilistic task"** [OC94], observed inferential behavior is **optimal for Bayesian reasoners**.
- [OC94] acknowledge that **Bayesian computations are too complex** for bounded-rational reasoners, but no model so far.
- We obtain **convergence to the Bayesian solution** under assumptions generalizing Bayesian Rationality w/ **severely bounded cognitive resources**.

## Instructions for the Selection Task

"Here are four cards, all of which have a number on one side and a letter on the other, followed by a rule that applies only to them:

A K 4 7

**(Rule) If there is a vowel on the one side (P), then there is an even number on the other side (Q)**

Which card (if any) must be turned in order to decide if the rule is true—without turning unnecessary cards?"

## Components of the Model (Overview)

- Interrogative Model of Learning**: generalizes Bayesian (probabilistic) learning, captures interrogative strategies. [Hin07, GG14].
- Non-classical semantics**: subsumes truth-functional semantics [Was68] and probabilistic semantics proved equivalent to the Bayesian model [Duc09]
- Cooperative Principle** [Gri89]: constraints the expectations based on the instructions (generalize constraints on Bayesian priors).

## Model (1): Interrogative Learning

- Sequential two-player game w/ asymmetric info.**: Inquirer (*I*) vs Nature (*N*).
- Problem= $\langle T, ? \rangle$ :  $T$  = theory;  $?$  = partition of models of  $T$  Mod( $T$ ).
- $N$  chooses a state  $s \in \text{Mod}(T)$  and a set of available answers Ans.
- $I$  must identify  $s$  'up to' inclusion in some cell of  $Q$ :
  - $I$  uses 'instrumental' questions,  $N$  answers **only if in Ans**.
  - $I$  stops asking questions at information fixpoints.

## Model (2): Generalized Adams Hypothesis

- Adams' Hypothesis** captures the Bayesian model of **ST** [Duc09]
 
$$\text{Prob}(\text{If } P \text{ then } Q) = \text{Prob}(Q|P) \text{ if } P > 0, 1 \text{ otherwise (AH)}$$
- Generalized Adams Hypothesis (GAH)**: **X expects If P, then Q** to hold to the same extent that **X expects Q to hold, knowing P**.

## Model (3): Cooperative Principle

Quality Rule is **true**

Quantity+Relation **P** and **Q** are mentioned, **not-Q** is not:

**(Rule\*) If there is a vowel on the one side, then there is an even number on the other side more likely than a non-even one.**

- Rule\*=true in state  $s$  iff **P/Q-occurrences** in  $s$  (denoted  $m_s$ ) **outnumber P/not-Q occurrences** in  $s$  (denoted  $n_s$ ).

## Model (4): Putting it Together

**Modeler's representation:**

$$\text{Mod}(T) = \{(A, y_1), (K, y_2), (x_3, 4), (x_4, 7) : x_i \in \{A, \dots, Z\}, y_i \in \mathbb{R}\} \quad (1)$$

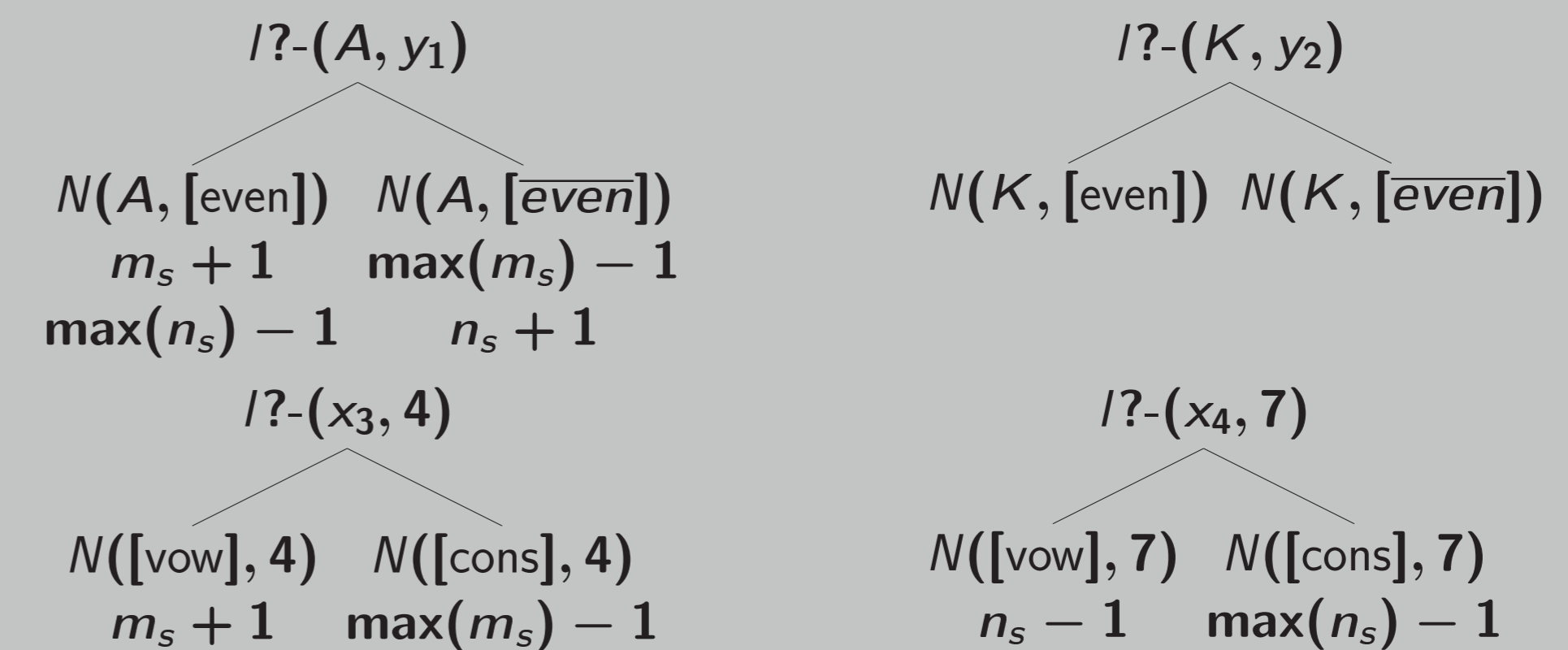
$$? = \{\text{Yes}, \text{No}\} \text{ where } \begin{cases} s \in \text{Yes} & \text{if } m_s > n_s \\ s \in \text{No} & \text{otherwise} \end{cases} \quad (2)$$

$$\forall s \in \text{Mod}(T), \text{Ans}(s) = \{a_1, a_2, a_3, a_4\} \ \& \ a_1 = y_1, a_2 = y_2, a_3 = x_3, a_4 = x_4 \quad (3)$$

- Inquirer's representation**: linguistic competence & categorizing letters as vowels or consonants (vow/cons) & numbers as even or not (even/*even*).

## A 4-card Poll (1): Relevance of Answers

**Relevance 1** Answers affect **estimates for  $m_s$  &  $n_s$** ; questions can be considered independently of one-another. ( $I/N$ : Inquirer's/Nature's move)



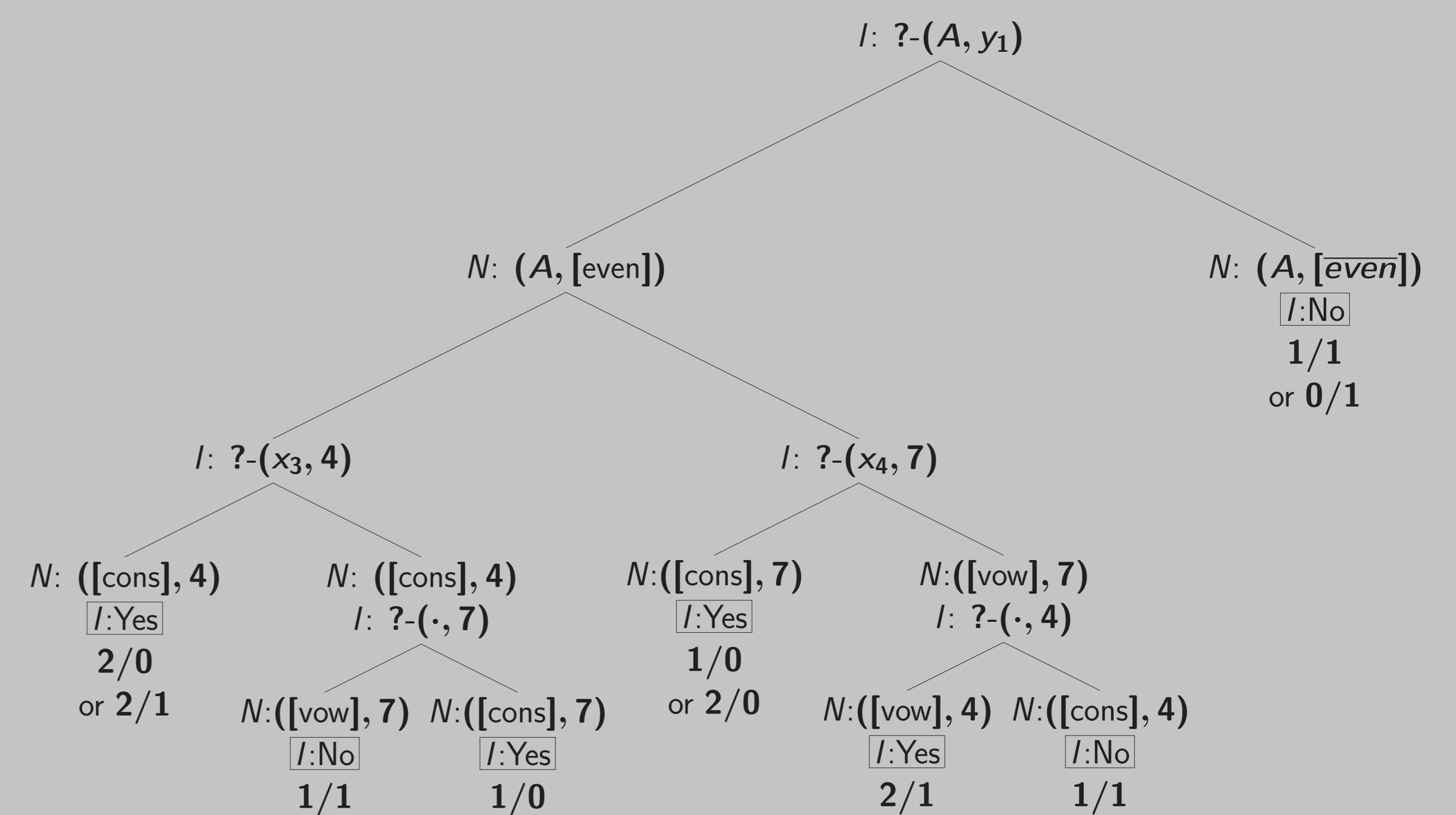
**Relevance 2**  $(A, y_1)$  is always relevant (for estimating  $m_s$  or  $n_s$ );  $(x_3, 4)$  and  $(x_4, 7)$  are possibly relevant (for  $m_s$  and  $n_s$ , resp.) or irrelevant;  $(y_2)$  is always irrelevant.

**Relevance 3** Questions can be **ordered by relevance**:

$$?-(A, y_1) \succ ?-(x_3, 4) \sim ?-(x_4, 7) \succ ?-(K, y_2)$$

## A 4-card Poll (2): General Learning Strategy

- Relevant cards only**, most relevant first.
- $I$  **stops at fixpoint** when she can solve  $\langle T, ? \rangle$  based on the ratio  $m_s/n_s$ , i.e. output Yes or No.



## A 4-card Poll (3): Tie-Breakers

- Under indifference vis-à-vis Rule**, **no single best** sequence:
 
$$?-(A, \cdot) \wedge ?-(\cdot, 4) \wedge ?-(\cdot, 7) \sim ?-(A, \cdot) \wedge ?-(\cdot, 7) \wedge ?-(\cdot, 4)$$
- Under assumptions of [OC94]**:
  - Initially estimates Rule=false: **not-Q** is **at least as likely as Q**, given **P**.
  - Questions aim at **most surprising first** (potential P/Q cases).
 
$$?-(A, \cdot) \wedge ?-(\cdot, 4) \wedge ?-(\cdot, 7) \succ ?-(A, \cdot) \wedge ?-(\cdot, 7) \wedge ?-(\cdot, 4)$$

## Conclusion: Low-Cost Rational Analysis

- With **CP** and pragmatic inferences, **ST** is solvable **without computation of probabilities/utilities or elimination of dominated strategies** which are major contributors to computational costs.
- Local reasoning is cognitively less taxing, but **converges towards optimal data selection** without approximation or heuristics.
- Variations in initial attitude towards Rule/Rule\* and criterion for picking questions can **account for relevance-based selection reversals** from [GKSH01] (future work!).

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