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# TRUE EQUIVALENT CHIP THICKNESS FOR TOOLS WITH A NOSE RADIUS 

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#### Abstract

A majority of the established systems for choice and optimization of cutting data are based on Woxén's equivalent chip thickness, $\mathrm{h}_{\text {ew }}$. In metal cutting theory and models, the equivalent chip thickness is of vital importance when the depth-of-cut $a_{p}$ is in the same order or smaller than the nose radius $r$. Woxén made considerable simplifications in his chip area model, that form the basis for calculations of the equivalent chip thickness. Basic mathematical solutions, e.g. describing the chip area on circular inserts, are lacking. This article describes the geometrical implications when machining with round inserts. The error in Woxén's equivalent chip thickness is largest when the depth-of-cut is less than $1 / 4$ of the nose radius. The calculations of the equivalent chip thickness based on the Woxén model are up to $50 \%$ wrong, for some combinations of cutting data in the finishing range. The presented results explain the difficulties in getting a good validity in the models used to calculate tool life in finishing machining. The error leads to an underrating of the tool load in many machining situations.


Keywords: Woxén, equivalent chip thickness, metal cutting, round inserts.

## 1. INTRODUCTION

When machining with an insert with a nose radius the theoretical chip thickness will vary, from the major cutting edge, along the nose radius, to the minor cutting edge. The theoretical chip thickness $h_{1}$ is together with the cutting speed $\mathrm{v}_{\mathrm{c}}$ the two most important factors that influences the functionality and productivity of the cutting process.
To produce a cut surface with acceptable properties, the cutting tool must have a curved bridging between the major and the minor cutting edge. In many cases a base geometry in the form of a circular arc with a standardized radius is used. Inserts with a pronounced nose radius are most commonly used in turning operations.
Using a constant approach angle and a constant feed, the major cutting edge will cut a chip with a constant theoretical chip thickness. The tool load will vary along the tool nose due to the variation in theoretical chip thickness. Woxén (Woxén 1932) introduced an equivalent chip thickness, $\mathrm{h}_{\mathrm{ew}}$, in 1932, with the purpose to use it as a characteristic parameter describing the mean theoretical chip thickness along the tool nose.
The stresses in a cutting tool are approximately proportional to the theoretical chip thickness $\mathrm{h}_{1}$. This means that $h_{1}$ has a dominant influence on the tool wear and the tool life.

Along the tool nose, $h_{1}$ will vary from its maximum value down to 0 . When machining with an $a_{p}$ less than the size of the nose radius, varying cutting conditions will rule along all of the active cutting edge. Models describing tool wear and tool life will be dramatically simplified if a characteristic or equivalent value of h1 can be introduced.
Woxéns equivalent chip thickness describes a kind of theoretical mean chip thickness, based on the active cutting edge length. Another interpretation of the equivalent chip thickness is that it joins together combinations of significant cutting parameters into one single parameter. Woxéns representation gives a value of the equivalent chip thickness for different choices of feed f , depth-of-cut $\mathrm{a}_{\mathrm{p}}$, approach angle $\kappa$ and nose radius r .
An increased depth-of-cut means that the major cutting edge increases its part of the process energy conversion. This also means that $h_{e w}$ will approach the current value of $h_{1}$ for the major cutting edge. For a smaller depth-of-cut, $h_{\mathrm{ew}}$ will significantly differ form this value.
Most systems for optimization and choice of cutting data use the equivalent chip thickness. Recommended cutting data from tool and material catalogues are often based on $\mathrm{h}_{\mathrm{ew}}$.
2. LIST OF SYMBOLS

| A | True chip area | $\mathrm{mm}^{2}$ |
| :--- | :--- | :--- |
| Aw | Woxéns chip area | $\mathrm{mm}^{2}$ |
| $\mathrm{a}_{\mathrm{p}}$ | Depth-of-cut | mm |
| f | Feed | $\mathrm{mm} / \mathrm{rev}$ |
| $\mathrm{h}_{1}$ | Theoretical chip thickness | mm |
| $\mathrm{h}_{\mathrm{e}}$ | True equivalent chip thickness | mm |
| $\mathrm{h}_{\mathrm{eW}}$ | Woxéns equivalent chip thickness | mm |
| $\mathrm{l}_{\mathrm{c}}$ | Active cutting edge length | mm |
| $\mathrm{l}_{\mathrm{ce}}$ | Equivalent active cutting length | mm |
| $\mathrm{l}_{\mathrm{cW}}$ | Woxéns active cutting length | mm |
| r | Nose radius | mm |
| $\mathrm{x}, \mathrm{y}$ | Help variables | mm |
| $\Delta$ | Error function | - |
| $\delta$ | Angular variable | ${ }^{\circ}$ |
| $\kappa$ | Approach angle | ${ }^{\circ}$ |

## 3. PROBLEM DESCRIPTION

Based on experience, it is very hard to predict tool life in finish machining operations. There are several reasons why rough machining easier can be described in tool life models, than finish machining.
One of the reasons is how the theoretical chip thickness behaves, as a constant when rough machining with large values of $a_{p}$, compared to the significant variation for $a_{p}<r$.
If studied closely, it can be concluded that Woxéns equivalent chip thickness has quantitative imperfections for $a_{p}<r$. Below, a modified version of Woxéns equivalent chip thickness is derived and presented. This modified equivalent chip thickness $h_{e}$ is based on Woxéns fundamental conditions according to equation 1 , where he is given by the ratio between the chip area A and the active cutting edge length $l_{c}$.

$$
\begin{equation*}
h_{e}=\frac{A}{l_{c}} \tag{1}
\end{equation*}
$$

## 4. DELIMITATIONS

In the presented work, a simple circular bridging between the major and minor cutting edge is used. This geometry is the most commonly used in turning. The attempt is based on the condition that the bridging between the major and minor cutting edge is represented by a part of a circular arc. The depth-ofcut is limited to the range less than the nose radius $r$.

## 5. REALIZATION

The mathematical calculations were performed using the mathematical software Mathcad, versions 11 and 14.

## 6. THE EQUIVALENT CHIP THICKNESS $h_{\text {eW }}$

Woxén approximates the chip area A using the product between depth-of-cut $a_{p}$ and feed f . This approximation results in computational errors that are not insignificant in under certain conditions. The equivalent chip thickness according to Woxén can be calculated as:

$$
\begin{align*}
& h_{e W}=\frac{A_{W}}{l_{c W}}= \\
& \frac{a_{p} \cdot f}{\frac{a_{p}-r(1-\cos \kappa)}{\sin \kappa}+\kappa \cdot r+\frac{f}{2}} \tag{2}
\end{align*}
$$

where $A_{W}$ is Woxéns chip area and $\mathrm{l}_{\mathrm{cW}}$ is the length of the cutting edge that is active in the cutting process. The active cutting length is built by 3 parts, a linear part which is the major cutting edge to the tangation point to the tool nose, the nose part which is equivalent to $\kappa \cdot r$, and one final part that is approximated by $\mathrm{f} / 2$, according to Figure 1.
The latter approximation is considered to be acceptable from an accuracy viewpoint. In Woxéns model, the tool nose is straightened out, forming a rectangular area that describes the chip area according to Figure 2.


Figure 1 Representation of the active cutting length into 3 parts, $l_{c I}+l_{c I I}+l_{c I I}$, [3].


Figure 2 Woxéns chip area A with the equivalent chip thickness $\mathrm{h}_{\text {ew }}$.

## 7. TRUE EQIVALENT CHIP THICKNESS

The design of an insert between the major and the minor cutting edge, is normally achieved through a nose radius r . Along the tool nose the theoretical chip thickness will start at its nominal value and successively decrease to a value of 0 over the minor cutting edge, according to Figure 3.


Figure 3. Theoretical chip thickness along the tool nose for $\mathrm{r}=1.2 \mathrm{~mm}, \kappa=90^{\circ}$ and $\mathrm{f}=0.4 \mathrm{~mm} / \mathrm{rev}$. Scales in $\mu \mathrm{m}$.

Through a geometrical observation according to Figure 4, presented by (Brammertz 1960) among others, the relations between significant parameters can be identified.

The relations between theoretical chip thickness $h_{1}(\delta)$, feed f , and nose radius r can be identified in Figure 4 and calculated according to the equation system according to equation 3 .


Figure 4. Graphical representation of the chip area end the variation in theoretical chip thickness along the tool nose, for $\mathrm{r}=1.2 \mathrm{~mm}, \mathrm{~K}=90^{\circ}$ and $\mathrm{f}=0.4$ $\mathrm{mm} / \mathrm{rev}$. Scales in $\mu \mathrm{m}$.

The relations can be drawn by studying the two rightangled triangles in the figure, where x and y are help variables and $\delta$ the tool nose angular variable. For $\delta=$ $90^{\circ}, h_{1}=f \cdot \sin (\kappa)$. Including the help variables $x$ and y , there are 3 unknowns and 3 equations.

$$
\begin{align*}
& r^{2}=(f+x)^{2}+(r-y)^{2} \\
& x=(r-y) \cdot \tan \delta  \tag{3}\\
& \cos \delta=\frac{r-y}{r-h_{1}}
\end{align*}
$$

where $h_{1}$ is a function of the angle $\delta$. There exist several solutions to the equitation system. The only valid solution gives the theoretical chip thickness, depending on the angle $\delta$ (i.a.) as:

$$
\begin{align*}
& h_{1}(\delta, r, f)=f \cdot \sin \delta+r- \\
& \sqrt{f^{2} \cdot \sin ^{2}(\delta)+r^{2}-f^{2}} \tag{4}
\end{align*}
$$

Figure 5 illustrates equation 4 , where the angle $\delta$ is variable.


Figure 5 Theoretical chip thickness $\mathrm{h}_{1}(\delta)$ along the tool nose for feeds $\mathrm{f}=0.2,0.4$ and $0.6 \mathrm{~mm} / \mathrm{rev}$, nose radius $r=1.2 \mathrm{~mm}$ and approach angle $\kappa=90^{\circ}$.

The angular position for $h_{1}=0$ in Figure 5 can be calculated out of equation 4 by inserting $h_{1}(\delta)=0$, after which the angle $\delta_{0}$ can be obtained through equitation 5.

$$
\begin{equation*}
\delta_{0}=-\sin ^{-1}\left(\frac{f}{2 \cdot r}\right) \tag{5}
\end{equation*}
$$

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An equation describing the angular function $\delta_{a p}$ can be formulated according to equation 6 , by studying the large triangle in Figure 6.


Figure 6. The appearance of the chip area for machining cases where $\mathrm{a}_{\mathrm{p}}<\mathrm{r}$.

Figure 6 and Figure 7 illustrate the chip area when $\mathrm{a}_{\mathrm{p}}<\mathrm{r}$. The chip area can be identified as the sum of 3 different surface elements, the surface A according to equitation 7 with integration limits ( $\delta_{0}, \delta_{\text {ap }}$ ), the triangular surface $A_{t}$, and the segmental surface $A_{c}$. The segmental surface $A_{c}$ can be calculated by using the chordal formula, $\mathrm{A}=0.5 \cdot \mathrm{r}^{2} \cdot(\theta-\sin (\theta))$. The integration limits $\delta_{0}$ och $\delta_{\text {ap }}$ are calculated by using equitation 5.


Figure 7. Enlargement of the chip area for the case where $a_{p}<r$.

$$
\begin{align*}
& A=A_{n}+A_{t}+A_{c} \\
& A_{n}=\int_{\delta_{0}}^{\delta_{a p}} f \cdot \sin (\delta)+r-\sqrt{r^{2}-f^{2} \cdot \cos ^{2}\left(\delta_{a p}\right)} d \delta  \tag{7}\\
& A_{t}=\frac{f \cdot h_{1}^{*} \cdot \cos \left(\delta_{a p}\right)}{2} \\
& A_{c}=\frac{r^{2}}{2}\left(\frac{f \cdot \cos ^{-1}\left(\delta_{a p}\right)}{r}-\frac{f \cdot \cos \left(\delta_{a p}\right)}{r}\right)
\end{align*}
$$

Each surface can be calculated by using equation 5 and 6, respectively. The surfaces $A_{n}$ och $A_{t}$ are dominant in size, the surface $\mathrm{A}_{\mathrm{c}}$ is insignificant. Unfortunately, equation 6 is without any analytical solution, due to the fact that the derivate of the chip area as a function of the angular coordinate lacks a primitive function.
For depths-of-cut $a_{p}>r$ the true equivalent chip thickness he can be calculated by adding the area corresponding to the major cutting edge, as $\mathrm{f} \cdot\left(\mathrm{a}_{\mathrm{p}}\right.$ $r) \cdot \sin (\kappa)$, (Ståhl 2007). Furthermore, $h_{e}$ can be calculated for any arbitrary value of approach angle $\kappa$ by setting the upper integration limit to $\kappa$ in equation 6 . Figure 8 illustrates the appearance of the chip area $A$ as a function of $a_{p}$.


Figure 8. The chip area A and its components, as a function of $\mathrm{a}_{\mathrm{p}}, \mathrm{f}=0.4 \mathrm{~mm} / \mathrm{rev}$.

The length of the active cutting edge $1_{c}$ is built by 3 parts, according to equation 8 , for $a_{p}<r$.

$$
\begin{equation*}
l_{c}=r \cdot\left(-\delta_{0}+\delta_{a p}+\delta_{c}\right) \tag{8}
\end{equation*}
$$

Where $\delta_{\mathrm{c}}$ can be calculated as:

$$
\begin{equation*}
\delta_{c}=2 \cdot \sin ^{-1}\left(\frac{f \cdot \cos \left(\delta_{a p}\right)}{2 \cdot r}\right) \tag{9}
\end{equation*}
$$

The equivalent chip thickness he can be calculated by forming the ratio between the chip area A (according to equation 7) and the active cutting length $1_{c}$ (according to equation 8). By also including the previously presented equations, the equivalent chip thickness he can be calculated as:
$h_{e}=\frac{A}{l_{c}}=$

$$
\begin{aligned}
& \frac{\int_{\delta_{0}}^{\delta_{a p}} f \sin (\delta)+r-\sqrt{r^{2}-f^{2} \cos ^{2}\left(\delta_{a p}\right)} d \delta}{r \cdot\left(\sin ^{-1}\left(\frac{f}{2 \cdot r}\right)+\tan ^{-1}\left(\frac{-f+\sqrt{2 \cdot r \cdot a_{p}-a_{p}^{2}}}{r-a_{p}}\right)+\sin ^{-1}\left(\frac{f \cdot \cos \left(\delta_{a p}\right)}{r}\right)\right)}+ \\
& \frac{f \cdot h_{1} \cos \left(\delta_{a p}\right)}{2}+\frac{r^{2}}{2}\left(\frac{f \cos ^{-1}\left(\delta_{a p}\right)}{r}-\frac{f \cos \left(\delta_{a p}\right)}{r}\right) \\
& r \cdot\left(\sin ^{-1}\left(\frac{f}{2 \cdot r}\right)+\tan ^{-1}\left(\frac{-f+\sqrt{2 \cdot r \cdot a_{p}-a_{p}^{2}}}{r-a_{p}}\right)+\sin ^{-1}\left(\frac{f \cdot \cos \left(\delta_{a p}\right)}{r}\right)\right)
\end{aligned}
$$

In Figure 9, $h_{e}$ for $a_{p}<r$ and different feeds is presented.


Figure 9. The equivalent chip thickness $h_{e}$ as a function of depth-of-cut $a_{p}<r, f=0.1 \mathrm{~mm} / \mathrm{rev}$ (lower black curve), $\mathrm{f}=0.2 \mathrm{~mm} / \mathrm{rev}$ (middle blue curve) and $\mathrm{f}=0.4 \mathrm{~mm} / \mathrm{rev}$ (upper red curve) and for nose radius $\mathrm{r}=1.2 \mathrm{~mm}$.

## 8. COMPARISON BETWEEN $h_{e w}$ AND $h_{e}$

In Figure 10, a comparison between calculated values of the Woxén chip equivalent (equation 2) and the new solution (equation 9) is illustrated. The deviation between the models varies depending on the range of initial conditions.


Figure 10. Comparison between true chip equivalent $h_{e}$ (solid curves) and Woxéns chip equivalent $h_{\text {ew }}$ (broken curves), as a function of depth-of-cut $a_{p}$, $\mathrm{r}=1.2 \mathrm{~mm}, \mathrm{f}=0.1,0.2$ and $0.4 \mathrm{~mm} / \mathrm{rev}$.

An error analysis of Woxéns chip equivalent $h_{\text {ew }}$ can be performed by formulating a function $\Delta$ according to equation 10 , which describes the relative deviance in $\%$ between $h_{e w}$ and $h_{e}$.

$$
\begin{equation*}
\Delta=\frac{h_{e}\left(f, r, a_{p}\right)-h_{e W}\left(f, r, a_{p}\right)}{h_{e}\left(f, r, a_{p}\right)} \cdot 100 \tag{10}
\end{equation*}
$$

In Figure 11 the error function $\Delta$ is presented in the form of contour diagrams, for the nose radii $r=1.2 \mathrm{~mm}$ and $\mathrm{r}=1.6 \mathrm{~mm}$. It is evident that the error can be both positive and negative for the presented cases, where the error lies between -20 to $50 \%$ within the finish machining area. It can also be concluded that combinations of $f$ and $a_{p}$ generates the same value of the equivalent chip thickness along the 0 -line.

## 9. CONCLUSIONS AND DISCUSSION

Without any closer examination it can be deduced that tool life models and systems for choice of cutting data based on Woxéns chip equivalent, provides a very limited precision within the finishing area. The largest error in the calculations of Woxéns chip equivalent is obtained with a depth-of-cut less than $1 / 4$ of the nose radius, which in the presented cases corresponds to a depth-of-cut $a_{p}$ between 0.3 och 0.4 $\mathrm{mm} / \mathrm{rev}$.
A systematic fault in the determination of model constants and the subsequent application of given cutting data recommendations, can to a certain degree limit the effects of the errors in Woxén's approximation. This is due to the fact that the same cutting data combinations are used both to determine the model constants and in later production and metal cutting.
The basic idea with the chip equivalent is precisely that it is equivalent, meaning all combinations of depth-of-cut and feed that generates the same chip equivalent should also generate the same tool life, under similar conditions. In Figure 12 and Figure 13, contour diagrams illustrate how different combinations of feed and depth-of-cut generate the same chip equivalent. Figure 12 shows the results based on $h_{e w}$, Figure 13 shows the results based on $h_{e}$.


$\Delta_{16}$
Figure 11. Deviations in \% between true chip equivalent $h_{e}$ and Woxéns chip equivalent $h_{e w}$ as a function of feed ( x -axis) and depth-of-cut ( y -axis). Upper diagram: nose radius $\mathrm{r}=1.2 \mathrm{~mm}$, lower diagram: $\mathrm{r}=1.6 \mathrm{~mm}$.


Figure 12. Woxéns chip equivalent $h_{e w}$ for different feed $f$ ( $x$-axis) and depth-of-cut $a_{p}$ ( $y$-axis). Nose radius $\mathrm{r}=1.2 \mathrm{~mm}$.


Figure 13. True chip equivalent $h_{e}$ for different feed $f$ (x-axis) and depth-of-cut $a_{p}$ (y-axis). Nose radius $\mathrm{r}=1.2 \mathrm{~mm}$.

By using the developed models and equations to determine the chip equivalent, better conditions to predict tool life end tool wear in metal cutting are created. The model can be adapted to other types of inserts, not having the circular bridging between major and minor cutting edge.

## 10. ACKNOWLEDGEMENTS

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