LUND UNIVERSITY

Tool Life and Wear Modelling in Metal Cutting, Part 2 – Based on Combining the Archard and the Colding Equations

Ståhl, Jan-Eric; Johansson, Daniel; Schultheiss, Fredrik; Zhou, Jinming; Bushlya, Volodymyr

Published in:
Proceedings of the 6th Swedish Production Symposium

2014

Link to publication

Citation for published version (APA):
Ståhl, J.-E., Johansson, D., Schultheiss, F., Zhou, J., & Bushlya, V. (2014). Tool Life and Wear Modelling in Metal Cutting, Part 2 – Based on Combining the Archard and the Colding Equations. In *Proceedings of the 6th Swedish Production Symposium* The Swedish Production Academy.

Total number of authors:
5

General rights
Unless other specific re-use rights are stated the following general rights apply:
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.
• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: https://creativecommons.org/licenses/

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

TOOL LIFE AND WEAR MODELLING IN METAL CUTTING, PART 2 – BASED ON COMBINING THE ARCHARD AND THE COLDING EQUATIONS

Jan-Eric Ståhl, Daniel Johansson, Fredrik Schultheiss, Jinming Zhou, Volodymyr Bushlya

[1]*Lund University, Division of Production and Materials Engineering, Lund, Sweden*

jan-eric.stahl@iprod.lth.se

Abstract: In this article an analytical and empirical model for describing tool life and tool wear in metal cutting is presented. The model is based on combining the Colding tool life equation and an extended version of the Archard wear function. It is shown that through the combining of these two models a substantial saving of resources can be achieved in terms of the workpiece material required, as well as the manpower and machine time needed for determining the model constants and the optimum cutting data to be employed.

Keywords: Machining, Turning, Tool life, Wear model, Colding, Archard.

1. INTRODUCTION

This publication complements and is partly based on the published study of Johansson *et al.* (2014). In industrial production it is highly important to be able to predict and describe the selected tool lifetime and the optimal cutting data in regard to production costs. Being able to determine optimal cutting data requires use of analytical models describing the relationship between cutting data and the lifetime of the cutting tool. Cutting data refers here primarily to the cutting speed v_c, the feed rate f and the cutting depth a_p. The product of these 3 parameters gives the material removal rate, in cm^3/min for example, referred to here as MRR (Metal Removal Rate). The product of the tool engagement time t_i and MRR is proportional to the direct processing costs. The tool lifetime T is the length of time the tool can be in operation before it is adjudged, with a given level of probability, to no longer function properly. The lifetime of a tool is generally determined by defining a wear criterion which, when reached, means the tool is considered to no longer be sensible to continue using. Various direct or indirect wear criteria can be employed, which of these applies depending upon the area of application of the tool. A direct criterion may, for example, be a certain maximal degree of wear of the cutting edge or of damage to it that the avoidance of tool failure permits. A maximum degree of wear seen as tolerable for a tool's being able to continue to meet some criterion regarding the dimensional, surface or performance characteristics achieved may also be set. An indirect criterion, in turn, may concern the maximum engagement time seen as permissible for achieving a particular level of quality regarding the surface, the dimensions, or the material characteristics (surface integrity) of the workpiece, without direct account being taken of the level of wear on the tool.

A wear model describes the relationship between time t and the level of wear, such as flank wear VB for example, that has taken place, as assessed in terms of the cutting data or cutting conditions involved. Examples of such models are those of Archard (1953) and of Usui and Shirakashi (1984). A tool lifetime model, in contrast, is concerned with the time it takes for some predetermined wear criterion, such as $VB = VB_T = 0.3$ mm, to be reached. Such models can be exemplified by those of Taylor (1906) and of Colding (1959). The wear models of Archard (1953) and Usui and Shirakashi (1984) contain various constants, the values of which can be difficult to determine. The models describe courses of events that are often highly complex and can involve loads of a mechanical, thermal, tribological or chemical character, or some combination of these. Although such models tend to function well under controlled and well-defined conditions, there are considerable difficulties in adapting them to industrial conditions, and definite limitations to their applicability there. The models of Taylor (1906) and of Colding (1959), describing the tool lifetime as a function of the cutting data, require that

experiments pertaining to the model of concern be continued up to the point at which the wear criterion involved is reached. In addition, in order for statistically significant results to be obtained, it is often necessary for experiments to be repeated or for further cases to be considered. The minimum number of cases that need to be investigated in any given experiment is partly a function of the number of constants the model in question contains. The amount of work material needed for determining with sufficient accuracy the values of the constants that the model contains can be considerable. Accordingly, conducting an investigation here can be quite time-consuming. Hägglund (2013), summarizing work making use of Colding's equation, found the equation to show a high degree of validity in a wide variety of applications. In work of his own he reported on, he found that under favourable conditions the cutting speed v_c associated with a given tool lifetime can be determined with an error of less than 1 %.

2. GOALS AND LIMITATIONS

As indicated, there are certain problems associated with each of the two basic types of models referred to above, there being clear limitations to the validity of wear models, and tool lifetime models often requiring that rather extensive investigations be carried out in order to determine the values for the constants that are needed. The present study concerns the possibility of combining Archard's wear function with Colding's equation in such a way that major weaknesses of both models can be overcome, without loss of the major positive characteristics that each of them possesses. A goal closely associated with this is to create a tool life model for cutting tools such that the constants the model contains can be determined on the basis of measurement series concerned with successive degrees of wear, without its being required that the final degree of wear in a given case reach the tool life criterion level.

The study is limited in scope to investigation of the longitudinal turning of alloyed carbon steel, the characteristics and behaviour of which correspond to those of AISI 4340 (SS 2541). Flank wear plays a dominant role in the processing of material of this type, the applicability of the model there depending upon this being the case. The model developed here can be used in many different ways. The present investigation is concerned in particular with illustrating in connection with the model how measurement series in which different levels of wear have been reached can serve as a basis for determining the values of the constants the model employs. The results of the study are compared with results reported by Johansson *et al.* (2014) in which Colding's equation alone and different final levels of wear were involved in determining the constants to be employed. The present model can be used primarily within 4 different areas of application where the present study is located within area IV:

I. Determining and comparing the tool lifetimes of different workpiece materials.
II. Determining and comparing the lifetimes and degrees of wear of tools differing in the material(s) of which they are composed when in the workpiece material involved is the same throughout.
III. Determining tool lifetimes in relation both to variations in the cutting resistance of the workpiece material and to variations in the geometric form of the cutting tool as regards the edge radius r_β and the clearance angle α.
IV. Enabling the constants in Colding's equation to be determined without its being necessary that all the measurement series involved lead to the tool lifetime criterion being reached.

3. ARCHARD AND COLDING MODELS

Archard's wear function was developed primarily for describing the wear on a body of lesser hardness that occurs when it glides across a smooth surface which is of greater hardness, as is shown in Fig. 1. In order for the model to be valid, it is necessary that the wear be of abrasive character and be temperature-independent. Colding's equation (Colding, 1959) is based to a considerable degree on a curve adjustment being made in studying the relationship between the lifetime of the tool in question and the cutting data obtained. The resulting equation can also be considered as an extension of Taylor's well-known equation (Taylor, 1906), a matter that can be clearly seen in two studies of Lindström's reformulation of Colding's equation (Hägglund, 2013; Lindström, 1989).

2.1 Archard's wear function

The present author Ståhl (Ståhl and Andersson, 2008; Ståhl, 2012) has developed a version of Archard's wear function that also takes account of the cutting geometry of the tool and of changes in the cutting forces present in the course of wear, as shown in Fig. 2. Archard's (Archard, 1953) modified wear equation can be written as it is in Eq. 1, the geometric interpretations of it being shown are in Fig. 2.

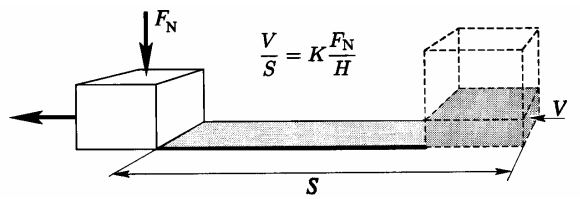

Fig. 1. A schematic presentation of the wear caused by the sliding of one body over another, as viewed in terms of Archard's model, where K is the wear constant, H is the hardness of the material exposed to wear, and V is the wear volume, according to Jacobson and Vedmar (2003).

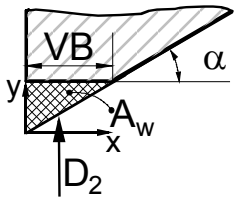

Fig. 2. An idealized view of the cutting geometry for a cutting process, the directions of the coordinates of which are shown, the components of the cutting force VB producing flank wear being indicated.

$$\begin{aligned} V &= k_0 \cdot D_2 \cdot e = k_0 \cdot p \cdot A_c \cdot v_c \cdot t \\ D_2 &= p \cdot A_c = p \cdot VB \cdot b \\ e &= v_c \cdot t \end{aligned} \qquad (1)$$

In Eq. 1 k_0 is the wear function (constant), D_2 is the force component in the growth-of-wear direction, and e is the tool engagement distance, the latter being the product of the cutting speed v and the tool engagement time t. The force component D_2 can be expressed as the mean normal pressure acting on the contact surface A_c. In the 2-dimensional case, the contact surface A_c is obtained as the product of the flank wear VB and the theoretical chip width b. The force component D_2, as shown in Fig. 2, acts perpendicular to the contact surface. It can be expressed as a function of VB, in accordance with Eq. 2. The constants D_{22} and D_{21} are determined on the basis of laboratory tests. The value of the constant D_{22} is given primarily by the micro-geometrical design of the cutting edge (r_β), in combination with the work material selected. The force D_2 can be normalized by dividing it by the current active cutting length b. It can be described with use of small letters then in accordance with Eq. 3. The wear volume V is defined geometrically then in Fig. 2 as Eq. 4 where y/x, dy/dx and y are defined according to Eq. 5 and 6.

$$D_2 = D_{22} + D_{21} \cdot VB \qquad (2)$$

$$d_2 = d_{22} + d_{21} \cdot VB \qquad (3)$$

$$V = A_w \cdot b = \frac{y \cdot x}{2} \cdot b = \frac{x^2}{2} b \cdot \tan\alpha \qquad (4)$$

$$\frac{y}{x} = \tan\alpha \quad \frac{dy}{dx} = \tan\alpha \qquad (5)$$

$$y = \frac{2 \cdot V}{x \cdot b} = \frac{2 \cdot V}{A_c} \qquad (6)$$

A_w is the wear surface, the variable x describes how the flank wear progresses, and the angle α is the tool relief angle. The wear progresses in the direction y. The rate of wear is obtained as Eq. 7. Deriving Archard's modified wear equation enables the following equation to be obtained, Eq. 8.

$$\frac{dy}{dt} = \frac{2}{A_c} \cdot \frac{dV}{dt} \qquad (7)$$

$$\frac{dV}{dt} = k_0 \cdot D_2 \cdot v_c \qquad (8)$$

Combining Eq. 2, 4, 6 and 7 gives Eq. 9. Separation of the variables x and t yields Eq. 10.

$$dy = dx \cdot \tan\alpha = \frac{2k_0}{b \cdot x}(D_{22} + D_{21}x)v_c \cdot dt \qquad (9)$$

$$b\int_{r_\beta}^{VB} \frac{x \cdot dx}{D_{22} + D_{21} \cdot x} = \frac{2k_0}{\tan\alpha}\int_0^t v_c \cdot dt \qquad (10)$$

The separation of variables provided in Eq. 10 is only permissible generally if the wear function k_0 is independent both of the level of flank wear VB and of the variable x, although regardless of whether or not this is the case, it can still be accepted nevertheless in cases in which either the values for the wear criterion VB_T are rather low or VB has only a limited effect on k_0. If the relationship between k_0 and VB is known, there is the possibility of shifting that relationship over to the other side of the equation and then integrating in the usual way. Studies have indicated, however, that doing so introduces considerable complexity, in particular since selection of the constants to be employed can be difficult. Integration of Eq. 10 yields Eq. 11. After insertion of the integration limits, the time t can be obtained as Eq. 12.

$$\left[\frac{x}{D_{21}} - \frac{D_{22}\cdot\ln(D_{22}+D_{21}\cdot x)}{D_{21}^2}\right]_{r_\beta}^{VB} = \frac{2k_0\cdot v_c\cdot t}{b\cdot\tan\alpha} \quad (11) \qquad t = \frac{D_{21}VB + D_{22}\ln\frac{D_{22}+D_{21}r_\beta}{D_{22}+D_{21}\mathrm{VB}} - D_{21}r_\beta}{\frac{D_{21}^2 v_c\cdot 2k_0}{b\cdot\tan\alpha}} \quad (12)$$

Use of Eq. 12 enables the tool lifetime t = T corresponding to a given wear constant VB = VB_T to be determined. For a given engagement time t, when the resulting flank wear VB has been determined, the wear constant and its progress can be calculated as follows, Eq. 13. The wear function k_0 is highly dependent upon the process temperature in the cutting zone.

$$k_0 = \frac{D_{21}\cdot VB + D_{22}\ln\frac{D_{22}+D_{21}\cdot r_\beta}{D_{22}+D_{21}\cdot VB} - D_{21}\cdot r_\beta}{\frac{D_{21}^2\cdot v_c\cdot 2\cdot t}{b\cdot\tan\alpha}} \quad (13)$$

3.2 Colding equation

Colding formulated the relationship presented in Eq. 14 which, for a given combination of a cutting tool and a workpiece, describes the relationship between the tool life T of the cutting tool, the cutting speed v_c and the equivalent chip thickness h_e.

$$v_c = e^{\left[K - \frac{(\ln(h_e)-H)^2}{4\cdot M} - (N_0 - L\cdot\ln(h_e)\cdot\ln(T))\right]} \quad (14) \qquad T = e^{\left[-\frac{H^2 - 2\cdot H\cdot\ln(h_e) + \ln(h_e)^2 - 4\cdot K\cdot M + 4\cdot M\cdot\ln(v_c)}{4\cdot M\cdot(N_0 - L\cdot\ln(h_e))}\right]} \quad (15)$$

The equation is based, according to Colding (2006), solely on the curve adjustment of the measurement points involved, without it's having any clear relationship to deterioration of the tool. Studies by the present authors are underway that appears to indicate, however, that values obtained for the Colding constants can be linked to mechanisms and loads that contribute to destructive processes that affect the functioning of the tool, primarily in the form of wear. Woxén (1932) introduced an equivalent chip thickness h_e in 1932 with the aim of using it as a characteristic parameter for describing the mean theoretical chip thickness along the tool nose with respect to the length of the active edge line l_c, Eq. 16. Additional relationships for calculating the Woxéns equivalent chip thickness h_e have also been reported by among others Bus *et al.* (1971), Hodgson and Trendler (1981), and Carlsson and Stjernstoft (2001). The equivalent chip thickness as computed in line with Woxén is an approximation that results in a computational error when low values, numerically similar, for the feeding rate f and the cutting depth a_p are involved. Ståhl and Schultheiss (2012) have published the results of exact computations that also apply to cases of finishing in which the feeding rate f and the cutting depth a_p can assume values of similar size.

The Colding equation involved needs to be used with considerable caution outside the cutting-data interval in which the measurement points are located. Sizeable errors can be made in estimates of tool lifetime based on extrapolations from the cutting data that are available. Difficulties can come about there in the form of numerical problems in the computation of certain values to be used in the Colding equation when the numerator in Eq 15 is very low in value. A singularity related to this is obtained then in regard to the value for the chip thickness h_e = h_{es} computed using Eq. 17.

$$h_e = \frac{A}{l_c} \approx \frac{a_p\cdot f}{\frac{a_p - r(1-\cos\kappa)}{\sin\kappa} + \kappa\cdot r + \frac{f}{2}} \quad (16) \qquad h_{es} = e^{\frac{N0}{L}} \quad (17)$$

Since Colding's equation contains 5 constants, where C_i = (K, H, M, N0, L), at least 5 separate investigations are needed to determine the values that should be assigned to them. Experience according to (Colding, 2006) shows that each investigation should involve measurements being made on at least 2 cutting tools that yield the same equivalent chip thickness he and for which the same set of cutting speeds are employed. It can be sensible to select the measurement points in such a way that they are representative of the cutting data in question. Colding's constants are best measured with the help of curve adjustment and use of methods aimed at keeping measurement errors at a minimum. The uses of Colding equation and its adaptation to different machining applications have been further investigated by Hägglund (2013).

4. COMBINING OF THE ARCHARD AND COLDING MODELS

Setting the tool engagement time t in Eq. 12 equal to the tool lifetime T in Eq. 15 allows Archard's wear function k_0 to be expressed as a function both of Colding's constants C_i and of the remaining parameters p_i = (d_{21}, d_{22}, VB, r_β, α), variables v_c and h_e being defined as in Eq. 18, so that $k_0 = k_0(C_i,p_i,v_c,h_e)$. In Eq. 18 the normalized cutting force components d_{22} and d_{21} have been inserted into Eq. 3 presented earlier.

$$
\begin{aligned}
k_{0AC} &= \frac{\tan\left(\frac{\pi\cdot\alpha_0}{180}\right)\cdot\left(d_{21,0}(VB_0 - r_{\beta 0}) + d_{22}\ln\frac{d_{22,0}+d_{21,0}\cdot r_{\beta,0}}{d_{22,0}+d_{21,0}\cdot VB_0}\right)}{120\cdot d_{21,0}^2\cdot v_c}\cdot e^A \\
A &= \frac{H^2 - 2H\cdot\ln(h_e) + \ln(h_e)^2 - 4K\cdot M + 4M\cdot\ln(60\cdot v_c)}{4M\cdot(N0 - L\cdot\ln(h_e))} \\
k_{0AC} &= k_{0AC}(v_c, h_e, C_i, p_i)
\end{aligned}
\tag{18}
$$

Reinserting k_0 obtained using Eq. 18 into Eq. 12 allows the tool lifetime T_{AC} to be based on Colding's constants (C_i), in line with Eq. 19. The parameters included in Eq. 17, together with p_{i0} = (d_{210},d_{220},VB_0,$r_{\beta 0}$,α_0), thus become the reference parameters, their yielding the index 0. Eq. 19 provides the possibility of adjusting the lifetime of the tool T_{AC} for those parameters the values of which deviate from the corresponding reference values.

Through use of Eq. 18 or Eq. 19, Colding's constants C_i can be determined for specific levels of wear VB = VB_T in relation to a reference value VB_{T0}, such as VB = 0.30 mm.

$$
\begin{aligned}
T_{AC} &= \frac{d_{21}(VB_T - r_\beta) + d_{22}\ln\frac{d_{22}+d_{21}\cdot r_\beta}{d_{22}+d_{21}\cdot VB_T}}{\frac{120\cdot d_{21}^2 v_c\cdot k_{0AC}}{\tan\left(\frac{\pi\cdot\alpha}{180}\right)}} \\
T_{AC} &= T_{AC}(v_c, h_e, VB_T, r_\beta, \alpha, d_{21}, d_{22})
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
v_c &= \frac{d_{21}(VB_T - r_\beta) + d_{22}\ln\frac{d_{22}+d_{21}\cdot r_\beta}{d_{22}+d_{21}\cdot VB_T}}{\frac{T_{AC}\cdot 120\cdot d_{21}^2\cdot k_{0AC}}{\tan\left(\frac{\pi\cdot\alpha}{180}\right)}} \\
v_C &= v_C(T_{AC}, h_e, VB_T, r_\beta, \alpha, d_{21}, d_{22})
\end{aligned}
\tag{20}
$$

Those constants that have been determined C_{i0} can be used for computing the tool lifetimes associated with use of other wear criteria than the reference criterion.

How the tool lifetime is affected by variations in machinability can be determined on the basis of the parameters d_{22} and d_{21}. Similarly, one can note how the tool lifetime is affected by geometric variations in the edge radii r_β and in the clearance angle α, i.e. the tolerances and the attachments of the cutting tool. In this respect, the edge radii r_β and the indirect parameters d_{22} have a double effect on the tool lifetime, both a geometric one in altering the integration boundaries in accordance with Eq. 10, and a force-related one through the normal forces directed at the tool flank, as described in Eq. 3, being dependent upon the initial level of flank wear VB = r_β for t = 0.
Since the values for these parameters are identical with the reference values involved, Eq. 19 gradually becomes identical with Colding's equation, i.e. with Eq. 15. Under these conditions, the procedure employed becomes, in mathematical terms, a kind of circular proof, since the reintroduction of k_0 in Eq. 12, in line with Eq. 18, results in what amounts to Colding's equation.

5. EXPERIMENTS AND CALCULATION OF CONSTANTS

The experiments carried out, each in the form of a measurement series concerned with successive degrees of wear of a cutting tool, involved use of longitudinal turning. There were 8 experiments of this sort that were complete in the sense of the wear criterion being reached. Five of these formed the basis for the modelling reported on, the 3 remaining experiments that were complete being excluded from further consideration, either because of plastic deformation (PD) having taken place or because of the combination of cutting data there creating numerical problems connected with Colding´s singularity, as it is called; see Fig. 11 and Fig. 12. In the course of the experiments, more than 50000 cm^3 of chips were produced, based on use of about 400 kg of work material. The total tool engagement time involved amounted to 8 hours and 50 minutes, the total engagement distance being 83.5 km in length. The cutting data for each of the completed experiments is shown in Table 1. Further conditions that applied during the experiments are reported in Table 2. The values for the constants other than Colding's constants C_i are reported in Table 3.

Table 1. Presentation of the cutting data for each of the experiments, the approach angle involved being $\kappa = 93^o$ and the nose radius r = 0.8 mm.

No.	Cutting speed v_c [m/min]	Feed rate f [mm/rev]	DoC a_p [mm]	ECT h_e [mm]	Com.
1	220	0.20	2.5	0.164	Excluded
2	150	0.45	2.5	0.354	
3	100	0.45	2.5	0.354	Excluded
4	150	0.20	2.5	0.164	Excluded
5	200	0.30	2.5	0.241	
6	300	0.25	2.5	0.203	
7	250	0.30	2.5	0.241	
8	300	0.20	2.5	0.164	

Table 2. Further conditions that applied during the experiments.

Workpiece material:	SS 2541, AISI 4340	
Workpiece geometry:	Length 900 mm, Diameter Ø200 → Φ80 mm	
Tool, insert:	CNMG120408-M6 TP 1500, r_β = 40 - 45 µm	SECO TOOLS
Machine, lathe:	SMT 500	SMT
Condition:	Dry machining	

During the experiments that were carried out, the degree of wear VB that took place was registered at regular intervals as a function of the tool engagement time, whenever this took place the cutting tool being photographed in 3D to ensure that no plastic deformation had occurred. The measurements were carried out with use of a white-light microscope of the brand Alicona InfiniteFocus (2013). During the experiments, the 3 cutting-force components were registered continuously, this making it possible to assess what tool engagement intervals between the successive geometric measurements that were carried out were most appropriate. The wear development that occurred in the course of each of the 5 measurement series that were involved – 2, 5, 6, 7 and 8 – is reported in Fig. 3. The wear data was interpolated ahead to create equal distances between the separate stages of wear considered (ΔVB). Details of this procedure are reported by Johansson *et al.* (2014).

The values for the model constants d_{22} and d_{21} were determined by linear regression of the cutting forces measured in the respective measurement series, as reported by Ståhl and Andersson (2008). The successive mean values obtained served as the basis for the model reported on.

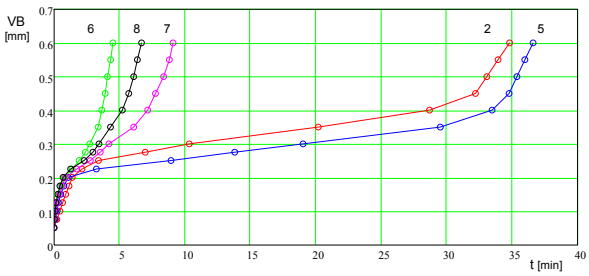

Fig. 3. Development of flank wear (VB) shown as a function of time t in minutes for each of the 5 cases considered.

The assessed values employed in the model are the average linear error discrepancies ε_{err} in %, determined by use of Eq. 21, between the actual cutting speed $v_{c,exp}$ and the modelled cutting speed $v_{c,mod}$ for the respective measurement series.

Table 3. Additional model constants.

Edge radius	r_β = 0.45 µm	$r_{\beta 0}$ = 0.45 µm
Clearance angle	α = 6°	α_0 = 6°
Force component	d_{22} = 550 N	$d_{22,0}$ = 550 N
Force component	d_{21} = 1750 N/mm	$d_{21,0}$ = 1750 N/mm
Reference wear	VB_0= 0.30 mm	-

$$\varepsilon_{err} = \frac{100}{n} \cdot \sum_{j=1}^{j=n} \left| \frac{v_{c,\exp_j} - v_{c,\mathrm{mod}_j}}{v_{c,\exp_j}} \right| \qquad (21)$$

Determination of the Colding constants was carried out by use of the least squares method, which is a built-in function in the Mathcad 15 computer program. It is based on an algorithm developed by Levenberg-Marquardt (Levenberg, 1944; Marquat, 1963). The tool lifetime, computed by use of Colding's equation (Eq. 15), is based on the corresponding tool lifetime criterion $VB_T = 0.10 \rightarrow 0.60$ [mm], together with Colding's constants as given in Table 4. Computation of the tool lifetime with use of the combined model presented in Eq. 18 takes as its point of departure an average level of wear of $VB_{Tm} = 0.10 \rightarrow 0.60$ [mm] as obtained for the 5 measurement series taken account of here.

The values for Colding's constants computed for use in Archard's wear function k_{0AC} are reported in Table 5. The set of indata of the sort needed for determining the constants to be used in Colding's equation and in the combined Archard-Coldings equation is exemplified in Fig. 4 and Fig. 5.

$$h_e := \begin{pmatrix} 0.354 \\ 0.241 \\ 0.203 \\ 0.241 \\ 0.164 \end{pmatrix} \quad v_c := \begin{pmatrix} 150 \\ 200 \\ 300 \\ 250 \\ 300 \end{pmatrix} \cdot \frac{1}{60} \quad T_{20} := \begin{pmatrix} 1.40 \\ 1.08 \\ 0.78 \\ 0.87 \\ 0.70 \end{pmatrix} \quad VB_{20} := \begin{pmatrix} 0.20 \\ 0.20 \\ 0.20 \\ 0.20 \\ 0.20 \end{pmatrix}$$

a b c d

Fig. 4. Example of a set of indata based on 5 measurement series for determination of Colding's constants, where T_{20} (column c) is the time needed to reach the wear criterion VB = 0.20 mm (shown in column d) in each of the measurement series.

$$h_e := \begin{pmatrix} 0.354 \\ 0.241 \\ 0.203 \\ 0.241 \\ 0.164 \end{pmatrix} \quad v_c := \begin{pmatrix} 150 \\ 200 \\ 300 \\ 250 \\ 300 \end{pmatrix} \cdot \frac{1}{60} \quad T_{20m} := \begin{pmatrix} 1.4 \\ 8.96 \\ 0.13 \\ 4.21 \\ 0.70 \end{pmatrix} \quad VB_{20m} := \begin{pmatrix} 0.20 \\ 0.25 \\ 0.10 \\ 0.30 \\ 0.20 \end{pmatrix}$$

a b c d

Fig. 5. Example of a set of indata based on 5 measurement series for determination of the Archard-Colding constants, where T_{20m} (column c) is the time needed to reach the wear criterion VB according to column d, where $VB_{Tm} = 0.21$ mm.

Table 4. Values obtained for the constants in Colding's equation through use of Eq. 13, shown for different wear criteria for flank wear VB = 0.10 → 0.60 mm, Johansson *et al.* (2014).

		Colding constants					
	VB_T	K	H	M	N0	L	ε_{err}
	0.10	5.542	-3.3664	1.5466	0.6055	-0.2013	5.0
	0.15	5.298	-3.0083	2.7518	0.5059	-0.0383	3.5
	0.20	5.417	-1.806	1.5517	1.2741	-0.2139	2.9
	0.25	5.810	-1.6983	0.2404	0.5242	-0.2296	0.4
Ref.	0.30	5.815	-1.6918	0.4568	0.4704	-0.2164	1.0
	0.35	5.841	-1.5145	0.9089	0.5020	-0.2364	0.9
	0.40	5.851	-1.3151	1.9241	0.4919	-0.2348	0.9
	0.45	5.888	-0.9259	4.4606	0.4981	-0.2363	0.8
	0.50	5.911	-0.8677	3.1123	0.5043	-02381	0.6
	0.55	5.922	-0.9616	2.5709	0.5016	-0.2340	0.6
	0.60	5.941	-0.8291	2.8579	0.5069	-0.2366	0.6

Table 5. Values obtained for the constants in the Archard-Coldings equation through use of Eq. 19, shown for different wear criteria for flank wear VB = 0.10 → 0.60 mm.

		Archard-Colding constants					
	VB_{Tm}	K	H	M	N0	L	ε_{err}
	0.10	5.6875	-1.1228	1.0004	0.5461	-0.0134	3.8
	0.16	5.8772	-2.5298	1.2858	0.6435	-0.2185	6.7
	0.21	5.7294	-1.6178	0.2172	0.7739	-0.4417	2.0
	0.25	5.7445	-1.5946	0.1892	0.7799	-0.4535	1.1
Ref.	0.30	5.8290	-1.5184	0.2367	0.8291	0.4389	0.9
	0.35	5.7817	-1.5198	0.5683	0.6318	-0.3325	0.5
	0.40	5.7736	-1.5830	1.6703	0.5331	-0.2576	1.4
	0.46	5.9561	-2.2802	1.1335	0.3133	-0.0813	1.6
	0.48	5.8452	-1.2465	1.3685	0.5899	-0.2699	0.4
	0.57	5.9098	-2.0899	1.0446	0.3574	-0.1419	0.4
	0.60	5.8083	-2.0872	1.1217	0.3869	-0.1555	0.3

Table 6. Values for the Archard-Colding constants obtained for different average wear levels VB_{Tm} = 0.10 → 0.35 mm used for computing the cutting speed for the constant wear criterion VB_{Ref} = 0.30 mm and a lifetime T = 12 min, where $\varepsilon_{err,0.3}$ is the model error in %.

	Archard-Coldings constants						
	VB_{Tm}	K	H	M	N0	L	$\varepsilon_{err,0.3}$
	0.10	5.6875	-1.1228	1.0004	0.5461	-0.0134	199
	0.16	5.8772	-2.5298	1.2858	0.6435	-0.2185	53
	0.21	5.7294	-1.6178	0.2172	0.7739	-0.4417	11.5
	0.25	5.7445	-1.5946	0.1892	0.7799	-0.4535	8.1
Ref.	0.30	5.8290	-1.5184	0.2367	0.8291	0.4389	6.5
	0.35	5.7817	-1.5198	0.5683	0.6318	-0.3325	3.3

6. RESULTS AND DISCUSSION

There are difficulties in the modelling of cutting-tool wear when use is made of the wear criterion VB_T as a parameter. The difficulties are caused by the fact that the cutting tool coating varies in the degree to which it affects cutting-tool wear, depending upon the value that the wear criterion is assigned. When some particular level of flank wear has been reached, the wear that has occurred will have penetrated the cutting tool coating at some point, whereupon wear of the substrate begins to occur. This results in an interaction between the coating and the substrate, leading to an increase in the effects on the ongoing process that the characteristics of the substrate have as the amount of the coating that covers the substrate declines, as shown in Fig. 6. The relationship between the degree to which the substrate is covered by the coating and the values Colding's constant assumes has been discussed earlier by Johansson *et al.* (2014).

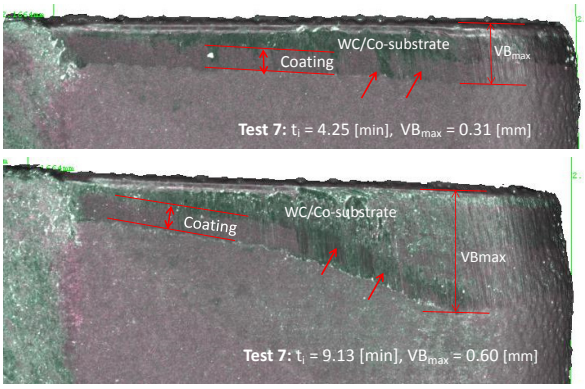

Fig. 6. Cutting tool wear in the form of flank wear for VB = 0.31 mm and VB = 0.60 mm, respectively.

Since one and the same model is used to describe the wear both of the coating and of the substrate (the cutting tool material itself), the modelling error increases if the constants that the model contains are described on the basis of results of studies that differ in terms of the level of wear involved, as can be seen in Fig. 7 and Fig. 8. In the cases in question, the modelling error increases as the value of the wear criterion decreases. In the studies carried out, the coating was found to show a higher degree of variation in its characteristics than the substrate did, this meaning that the modelling error was greater when the wear criterion selected was lower. The difference between the coating and the substrate in the degree of variation in the respective characteristics can be thought to have had three major causes:

1. That the coating is more brittle and less tough than the substrate, this leading to some extent to a non-wear-related breakdown of the tool.
2. That the coating is more susceptible to flaking in the vicinity of the cutting edge than elsewhere; see the arrows in Fig. 6.
3. In combination with this (2 above), the wear volume is least (see Fig. 2) and the local pressure directed at the cutting tool greatest, in the vicinity of the cutting edge close to the stagnation point.

For the case at hand, one can note in Fig. 7 that the effect the coating has on the modelling error tends to be marginal for values of VB_T greater than 0.25 mm in size. As long as levels of wear lower than the wear criterion, such as a value of VB_T = 0.3 mm, are used as a basis for determining the constants to be employed in the Archard-Colding equation, the modelling error increases as the average level of wear VB_{Tm} decreases. Use of a rather low value for VB_{Tm} in determining the constants to be employed in the model can result a considerable saving in terms both of material and of time, in testing the cutting tool as well as in getting production underway.

An important factor is that of how large a model error occurs and can be accepted with use of a given model. It is usually required of a model to be employed industrially that it has a model error of less than 10 %. In practice this means that the cutting speed computed for a given tool lifetime, or for the manufacture of some given number of parts or components, needs to be predicted with an error of no greater than 10 %. Use of a cutting speed some 10 % lower than the model prescribes can be expected to provide for the robust manufacture of the parts or components in question, in terms of the tool lifetime it provides or the tool wear it results in.

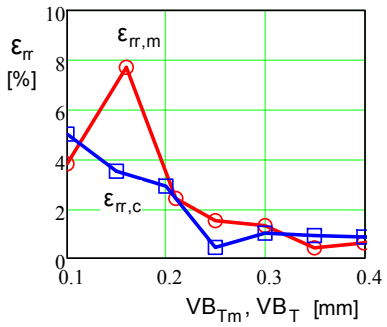

Fig. 7. Average modelling error $\varepsilon_{rr,c}$ for Colding's equation with use of different wear criteria VB_T = 0.1 → 0.4 [mm] (□), and an average modelling error of $\varepsilon_{rr,m}$ for the Archard. Colding equation at different levels of average wear VB_{Tm} = 0.1 → 0.4 [mm] (○).

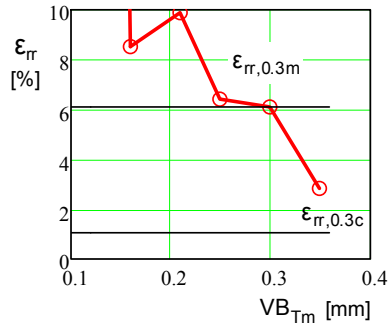

Fig. 8. Average modelling error $\varepsilon_{rr,0.3m}$ obtained with use of the Archard-Colding equation for different levels of average wear VB_{Tm} = 0.1 → 0.35 [mm] (○) for modelling cutting data while employing a wear criterion of VB_T = 0.30 mm, a comparison also being provided with the model error $\varepsilon_{rr,0.3c}$ connected with use of the wear criterion VB_T = 0.3 [mm] obtained with use of Colding's equation.

In Fig. 9 an example is provided of the modelled lifetime of a particular cutting tool, shown as a function of the cutting speed, for various average levels of wear VB_{Tm}. This can be used in determining the constants to be employed in the Archard-Colding equation. Note that the average levels of wear VB_{Tm} 0.10 mm and 0.15 mm provide no adequate basis for extrapolation of a wear criterion of VB_T = 0.30 mm, a fact which is consistent with the model error computed for Fig. 8. Two measurements points that were employed here are shown as squares in the diagram (□).

An example of a presentation of the Archard-Colding equation at what is termed the Colding plane for a tool lifetime of T = 12 minutes and a tool wear criterion VB_T = 0.30 mm is shown in Fig. 10. The model constructs employed were determined on the basis of average wear levels of VB_{Tm} = 0.20, 0.25 and 0.30 mm, as well as the case of the final result in each of the 5 measurement series reaching the level of the wear criterion (red curve). Under the ideal conditions of use of a perfect model and of a completely homogeneous tool material, i.e. a tool material without a coating or any gradients in its composition, all of the curves in Fig. 10 would coincide completely.

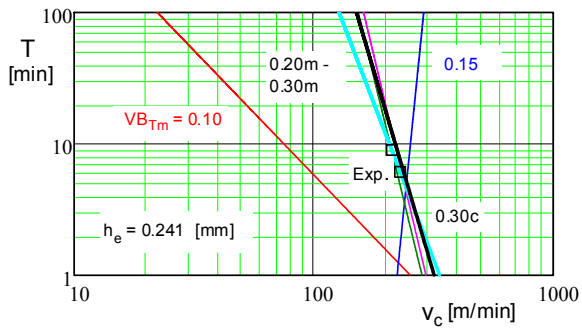

Fig. 9. The Archard-Colding equation, described at the Taylor plane, used for obtaining an equivalent chip thickness of h_e = 0.241 mm for VB_T = 0.30 mm, the respective curves being based on results for measurement series differing in the average level of wear VB_{Tm} involved.

Whereas at the Colding plane the tool lifetime parameter T displays a mathematical singularity l, the singularity becomes extended to form a singularity line when the values for the wear criterion VB_T vary at the same time as the tool lifetime level T remains constant. This is exemplified most clearly in Fig. 11, and is shown in Fig. 12

and in Fig. 13 as a vertical iso-cutting-speed line there. According to Eq. 17, the value that Colding's similarity h_{es} has is dependent upon the wear criterion VB_T (VB_{Tm}) selected, as can be seen in Fig. 11, this being an effect of the values of the Colding constants being a function of VB_T, as Johansson *et al.* (2014) have shown. In Colding's equation in the form in which Johansson *et al.* (2014) and Hägglund (2013) present it, the original singularity point is replaced by a similarity line when the tool lifetime criterion VB_T varies. In the Archard-Colding equation, a singularity point is obtained for the set of model constants involved, as shown in Fig. 11.

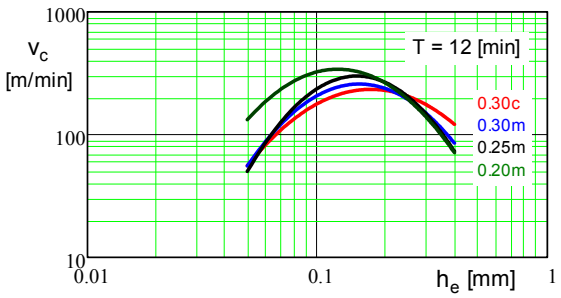

Fig. 10. The Archard-Colding equation described at the Colding plane for a constant tool lifetime T = 12 min, the model constants being determined at different average wear levels VB_{Tm}, for computing the combination of v_c and h_e for VB_T = 0.30 mm.

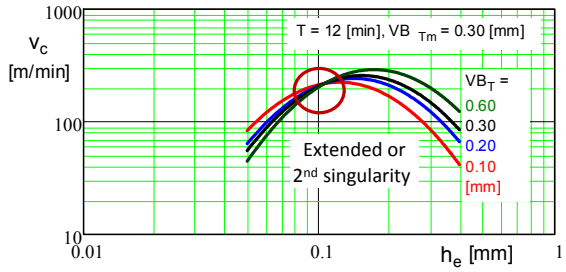

Fig. 11. The Archard-Colding equation described at the Colding plane for a tool lifetime of T = 12 min for different wear criteria VB_T for which the second singularity is identified.

When the model constants are determined on the basis varying wear levels, and wear-level values of less than 0.20 mm are involved, the coatings that are employed can affect the results appreciably, as can be seen in Fig. 9. When higher wear-value levels, in excess of 0.20 mm, are involved, the effects of the tool coating decline, as can be noted in comparing Fig. 12 with Fig. 13 and illustrated in Fig. 16. Use of the Archard-Colding equation as presented in Eq. 18, and of the Colding equation as presented in Eq. 14, in connection with different tool lifetime criteria VB_T, provides the possibility of recreating the wear curves applying to different measurement series in the manner exemplified in Fig. 15.

In Fig. 15, the modelled wear-process curve pertaining to measurement series 8 is compared with the corresponding curve based on the experimental data obtained for that series. For the data in the study as a whole that was considered, there was found to be a modelling error of about 1 % based on averaging the ε_{rr} values presented in Table 4 and Table 5. Through use of Colding's equation, the recreated tool wear curve, based on the modelling of each of the wear criteria considered, could be obtained. Use of the combined model based on the Archard-Colding equation made it possible to recreate graphically the combined wear curve involved through successively following the different VB_{Tm} curves, VB then being set equal to VB_{Tm} (●). The dark green point (●) represents the case in which Colding's constants are determined on the basis of measurement series in which the wear criterion VB_T = 0.30 mm is reached in each of them. There, the Archard-Colding equation and Colding's equation for VB_{Ref} become equivalent to one another in the result they provide.

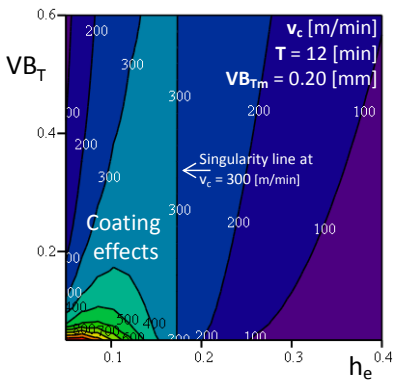

Fig. 12. The Archard-Colding equation described at the VB_T-h_e plane, the cutting speed serving as a parameter for the tool lifetime T = 12 min, based on VB_T = 0.20 mm.

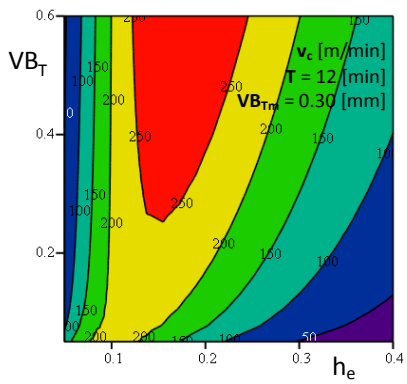

Fig. 13. The Archard-Colding equation described at the VB_T-h_e plane, the cutting speed v_c serving as a parameter for the tool lifetime T = 12 min, based on VB_T = 0.30 mm.

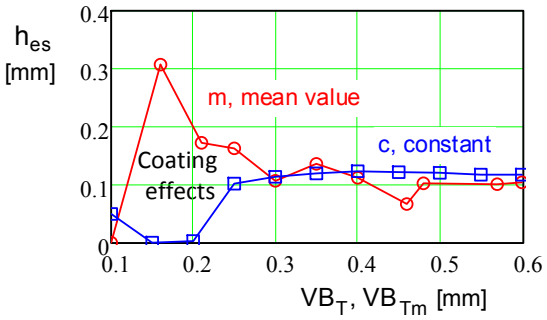

Fig. 14. Colding's singularity shown as a function of the wear criterion VB_T that is selected and of the average wear level VB_{Tm}.

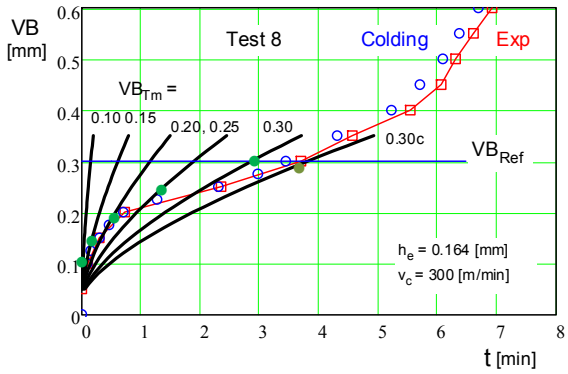

Fig. 15 The recreated wear curve VB shown as a function of the tool engagement time t as based on Colding's equation (○), the Archard-Colding equation (the unbroken lines, ● and ●) and experimental data (□).

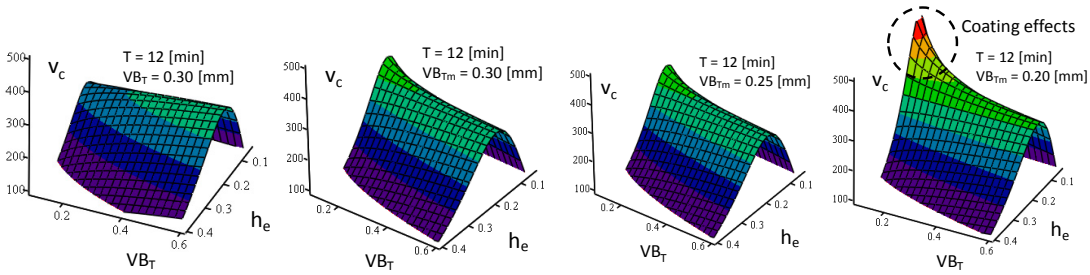

Fig. 16 The Archard-Colding equation for a tool lifetime of T = 12 min and the effects of the tool coating for low values of the wear criterion VB_T.

7. CONCLUSIONS AND FURTHER WORK

Colding's equation functions well in most respects in its use for determining the lifetime of a cutting tool. At the same time, it has the clear disadvantage of at least 5 complete measurement series in which the exact point at which the wear criterion is located being reached in order for the constants that are involved to be determined in a completely adequate way. In cases in which one fails to reach the wear criterion or passes it, an extrapolation or interpolation of the engagement times obtained is needed in order for these to be of use, a procedure that can produce a certain degree of error. Employing the approach instead of combining Archard's equation with Colding's equation makes it possible for different wear criteria to serve as a basis for determining the model constants. The combined model thus produced enables one to take account of variations in the cutting resistance of the work material (the material's characteristics) and variations in the tool geometry in terms of the cutting-edge radius r_β and the clearance angle α. This new model can also be employed in connection with different flank wear criteria, although the model's very considerable flexibility in comparison to Colding's model has the disadvantage of providing less precision for low values of the wear criteria, the characteristics of the coating of the tool having a much stronger effect there.

A number of results obtained in addition to those just summarized are the following:

- It is fully possible to combine Archard's equation with Colding's equation (to form the, Archard-Colding Eq., A-C.) so as to take advantage of what both have to offer in modelling tool lifetimes on the basis of different sets of cutting data.
- Use of this A-C Eq. also makes it possible to recreate, as a function of the tool engagement time, the wear development that occurs.
- A model based on the A-C Eq. can be used in studying the effect the coating of the tool has, the functions it fulfils and the wear development process it is subjected to.
- Having introduced the wear criterion VB_T into the A-C Eq. as an additional variable is a decided gain through its providing the model a further singularity (a singularity line), increasing its usefulness.

A variety of directions that additional development of the Archard-Colding equation could take can be seen. In particular the following:

- Studies of variations in the workpiece material and of the effects on tool lifetime these can have.
- Studies contributing to an understanding of the singularity inherent in the Colding and the Archard-Colding equation and its significance in connection with both equations in metal-cutting terms.
- Application of the Archard-Colding equation to workpiece materials of other types and within many other areas of application.
- Carrying out studies similar to the present one concerning cutting media of other type´s.

8. ACKNOWLEDGEMENT

This study was carried out as a part of the strategic research program SPI (Sustainable Production Initiative), a program involving close cooperation between Lund University, Chalmers and SECO TOOLS, Fagersta.

REFERENCES

Archard, J. F. (1953). Contact and rubbing of flat surfaces, *Journal of Applied Physics*, **24**, 981-988.

Bus, C., N. A. L. Touwen, P. C. Veenstra and A. C. H van der Wolf (1971). On the significance of equivalent chip thickness, *Annals of the CIRP*, **XVIV**, 121-124.

Carlsson, T. and T. Stjernstoft (2001). A model for calculation of the geometrical shape of the cutting tool – work piece interface, *CIRP Annals – Manufacturing Technology*, **50**, 41-44.

Colding, B. (1959). A wear relationship for turning, milling and grinding, Doctoral Thesis, *KTH*, Stockholm.

Colding B. (2006). Colding Seminar in Lund, Open discussion at Technical Faculty Lund University, 16-17 February.

Hodgson T. and P. H. H. Trendler (1981). Turning hardened tool steels with cubic boron nitride inserts, *Annals of the CIRP*, **30**, 63-66.

Hägglund, S. (2013). Methods and Models for Cutting Data Optimization, Doctoral Thesis, *Chalmers University of Technology*, Gothenburg, Sweden.

Jacobson, B., L. and L. Vedmar (2003). Tribologi AK, Maskinelement LTH, *Lunds universitet*, Lund, (In Swedish).

Johansson, D., F. Schultheiss, V. Bushlya, J.M. Zhou and J.-E. Ståhl (2014). Tool Life and Wear Modelling in Metal Cutting, Part 1 – Coding Equation with different Tool Life Criteria, *Proceedings of the 6th International Swedish Production Symposium, Gothenburg, Sweden*.

Levenberg, K. (1944). A method for the solution of certain nonlinear problems in least squares, *Quarterly of Applied Mathematics*, **2**, 164-168.

Lindström, B. (1989). Cutting Data Field Analysis and Predictions – Part 1: Straight Taylor Slopes, *Annals of the CIRP*, **38**, 103-106.

Marquardt, D. W. (1963). An algorithm for least squares estimation of non-linear parameters, *Journal of the Society for Industrial and Applied Mathematics*, **11**, 431-441.

Ståhl, J.-E. (2012). Metal Cutting – Theories and models, *Division of Production and Materials Engineering, Lund University in cooperation with Seco Tools*, ISBN 978-91-637-1336-1, Fagersta Sweden.

Ståhl, J.-E. and M. Andersson (2008). Modelling of metal cutting tool wear based on Archards wear equation, *Proceedings of the 2nd Swedish Production Symposium*, Stockholm, Sweden.

Ståhl, J.-E. and F. Schultheiss (2012). Analytical Calculation of the True Equivalent Chip Thickness for Cutting Tools and its Influence on the Calculated Tool Life. *Advanced Materials Research*, **576**, 80-86.

Taylor, F. W. (1906). On the Art of Cutting Metals, *The American Society of Mechanical Engineers*, New York, USA.

Woxén, R. (1932). Theory and an Equation for the Life of Lathe Tools, Handling 119, *Ingenjörsvetenskapsakademin*, Stockholm. (In Swedish)

Usui, E. and T. Shirakashi (1984). Analytical Prediction of Cutting Tool Wear, *Wear*, **100**, 129-151.