Stephens' and Mottelson's models for rotation-aligned odd-particle motion

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Published in:
The Villars conference

1974

The Villars conference, Jan 21-25, 1974, invited lecture.
(For the conference report, not for publication)

Stephens' and Mottelson's models for rotation-aligned odd-particle motion

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Recently a large new family of yrast states for odd-A nuclei in transitional regions have been encountered experimentally. At first glance they appear to form normal rotational bands. However, every other spin member seems to be missing. Secondly, the associated inertia appears large by almost an order of magnitude compared to the apparent inertia value of the even-even neighbour nuclei. The associated coupling scheme is now rather well understood as a realignment of the single-particle orbital from the nuclear deformation axis to the nuclear rotation axis.

The rotor model [1)] describing the coupling of an odd particle to a rotating body, is represented by the following Hamiltonian

$$(1) \qquad H = H_{sph} + k\beta Y_{20}(\theta') + A\vec{R}^2$$

where $\vec{R}$ is the rotor angular momentum and where θ' refers to the polar angle of the deformed field measured relative to the rotor symmetry axis, the 3-axis. We furthermore limit ourselves to the so-called unique-parity case, when the single-particle may be assigned a relatively pure j-orbital with $j >> 1$. We furthermore have $\vec{I} = \vec{R} + \vec{j}$ and $R_3 = 0$, the latter relation a consequence of the fact that no rotation of the rotor is permitted around its symmetry axis. The usual adiabatic base wave functions are then given as

$$(2) \qquad \Phi(I,j,\Omega) = \chi(j,\Omega)\, D^{I}_{M\Omega}$$

The diagonal energies of (1) in terms of the base (2) may be written as (we neglect the case of $\Omega = 1/2$)

(3) $E(I,j,\Omega) = e_j + A\{I(I+1) + j(j+1)\} - \frac{1}{3}K'j(j+1) + K\Omega^2$

In this expression K comes in part from the splitting of the quadrupole field, in part from the $\vec{R}^2$ term

(4) $K = K' - 2A$

where

(5) $K' = k\beta \cdot \sqrt{45/4\pi}\,\left[j(j+1)\right]^{-1}$

and

(6) $\langle j\Omega\,|k\beta Y_{20}|\,j\Omega\rangle = K'\left[\Omega^2 - \frac{1}{3}j(j+1)\right]$

The off-diagonal elements of $A\bar{R}^2$, or of

(7) $H_{Cor} = A(I_-j_+ + I_+j_-)$

couple states with $\Omega' = \Omega \pm 1$. They can be written

(8) $\langle I,j,\Omega+1\,|\,H_{Cor}\,|\,I,j,\Omega\rangle = A\sqrt{\{I(I+1) - \Omega(\Omega+1)\}\{j(j+1) - \Omega(\Omega+1)\}}$

The inclusion of this coupling gives rise to Coriolis mixed wave functions of the type

(9) $\Psi(I,j) = \sum_{\Omega} C(I,\Omega)\,\phi(I,j,\Omega)$

where the coefficients $C(I,\Omega)$ may be obtained from a direct diagonalisation within the (I,j,Ω)-space. Approximate analytical expressions for $C(I,\Omega)$ have been derived by F. Stephens[2)] on the one hand and B. Mottelson[3)] on the other, which both cast interesting light on the recoupling (rotation-alignment) implied by the Coriolis mixed wave functions.

The one scheme that most directly goes over into the pure adiabatic limit is the approximation developed by Mottelson[2)]. He approximates the expression (8) by

(10) $A' = A(I + 1/2)(j + 1/2)$

The diagonalisation problem

(11) $(H - E) \sum_{\Omega} C(I,\Omega)\, \phi(I,j,\Omega) = 0$

may be rewritten as a recursion relation for the C-coefficients

(12) $(K\Omega^2 + \lambda - E)C(\Omega) - A'\left[C(\Omega + 1) + C(\Omega - 1)\right] = 0$

where

(13) $\lambda = A\left[\{I(I + 1) + j(j + 1)\}\right] - \frac{1}{3} Kj(j + 1)$

Mottelson now goes on to consider Ω a continuous variable and approximates the last bracket in (12) by $C(\Omega) + \frac{1}{2} \frac{\partial^2 C(\Omega)}{\partial\Omega^2}$. In this way (12) becomes a second-order differential equation for $C(\Omega)$ of the harmonic oscillator type in Ω-space.

(14) $\frac{\hbar^2}{2\mu} \frac{\partial^2 C}{\partial\Omega^2} + \frac{\mu}{2}\omega^2\Omega^2 C = e_n C$

where

(15) $\frac{\hbar^2}{2\mu} = A' = A(I + \frac{1}{2})(j + 1/2)$

(16) $\frac{1}{2} \mu\omega^2 = K$

and thus

(17) $\hbar\omega = \sqrt{2KA'} = \sqrt{2A(I + 1/2)(j + 1/2)(K' - 2A)}$

The lowest "vibrational" state in Ω-space has a width in Ω-space denoted Ω_0, where

(18) $\Omega_0 = \sqrt{2A'/K} = \sqrt{2A(I + 1/2)(j + 1/2)/(K' - 2A)}$

The corresponding expression for $C(\Omega)$ is then

(19) $C(\Omega) \sim \exp\left[- 1/2\ \Omega^2/\Omega_0^{\ 2}\right]$

which thus implies that for n = 0 all $\Phi(I,j,\Omega)$ components enter with the same sign and that (for n = 0) the amplitude is largest for $\Omega = \pm 1/2$ and then falls off exponentially with $(\Omega/\Omega_0)^2$ in Ω-space. As Ω_0 is I-dependent, the spread over different Ω-components becomes larger for large I-values.

The associated energy according to Mottelson may be written

(20) $E(I,j,n) = \lambda - 2A' + e_n = A\left[I(I+1) + j(j+1) - 2(I+1/2)(j+1/2)\right]$

$+ (n + 1/2)\ \hbar\omega$

Neglecting terms independent of I,j,Ω we may rewrite this as

(21) $E(I,j,n) = A(I - j)^2 + (n + 1/2)\ \hbar\omega$

To ensure the condition of reflection symmetry with respect to the plane perpendicular to the rotor deformation axis the following condition must hold between I,j, and n

(22) $I - j + n = \text{even}$

For n = 0 one thus obtains an yrast spectrum in terms of $R' = |I - j|$

(23) $E = AR'^2$

for $I = j, j + 2, j + 4$ etc. The $j - 2, j - 4, j - 6$ states form a similar spectrum (terminating quickly), slightly displaced relative to the yrast band due to the j-dependence of the $\hbar\omega$-factor.

An alternative way to derive the same type of rotational spectrum has been developed by Stephens and coworkers[24] simultaneously with Mottelson's work.

The starting point is the situation for which $K' = 2A$. At this point the diagonal splitting due to the quadrupole field exactly cancels the splitting due to the Coriolis force. In this case one may exploit the degeneracy to form states that approximately diagonalise the H_{Cor} term. The corresponding wave function is obtained by first considering the intrinsic wave function aligned along the rotation axis with j-projection α , obtained from $\chi(j,\Omega)$ by a rotation through an angle $\pi/2$ from the 3- to the 1-axis:

$$(24) \qquad \chi(j,\alpha) = \sum_{\Omega} d^{j}_{\alpha\Omega}(\pi/2)\,\chi(j,\Omega)$$

A total state that includes the rotor degrees of freedom is taken to be the following

$$(25) \qquad \psi(I,j,\alpha) = \sum_{\Omega} d^{j}_{\alpha\Omega}(\pi/2)\,\chi(j,\Omega)\,D^{I}_{M\Omega}$$

One should note that this is <u>not</u> a product of $\chi(j,\alpha)$ with a D-function. The multiplication with different D-functions, each with a good I_3-value for each Ω-component, is required to assure the rotational symmetry of the problem with reference to the rotor 3-axis.

Up to terms of order $(\Omega/I)^2$ one can prove the following relation

$$(26) \qquad (I_+j_-+I_-j_+)\,\Phi(I,j,\alpha) \simeq 2\sqrt{I(I+1)}\alpha\,\Phi(I,j,\alpha) \simeq 2(I+1/2)\alpha\,\phi(I,j,\alpha)$$

We identity the product of the length of the I-vector and the component of j on the 1-axis.

In this way one obtains

$$(27) \qquad E(I,j,\alpha) = e_j + A\left[I(I+1) + j(j+1) - 2(I+1/2)\alpha\right]$$

The choice of α that gives rise to the lowest total energy for given I obviously corresponds to $\alpha = j$

$$(28) \qquad E(I,j,j) = e'_j + A(I-j)(I-j+1) = e'_j + AR'(R'+1)$$

an expression similar to that obtained by Mottelson apart from the R'te

From the condition of reflexion symmetry one furthermore obtains the following relation, valid in the general α-case

(29) $I - \alpha$ = even

It then follows that for the case of maximal alignment only even values of R' (or $I - j$) are permitted.

A study bears out that the Mottelson and Stephens models give more or less the same results and reproduce equally well a rapidly accumulating number of nuclear data.

References

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