



# LUND UNIVERSITY

## Active distances and cascaded convolutional codes

Höst, Stefan; Johannesson, Rolf; Zigangirov, Kamil; Zyablov, Viktor V.

*Published in:*  
[Host publication title missing]

*DOI:*  
[10.1109/ISIT.1997.613022](https://doi.org/10.1109/ISIT.1997.613022)

1997

[Link to publication](#)

*Citation for published version (APA):*  
Höst, S., Johannesson, R., Zigangirov, K., & Zyablov, V. V. (1997). Active distances and cascaded convolutional codes. In [Host publication title missing] (pp. 107) <https://doi.org/10.1109/ISIT.1997.613022>

*Total number of authors:*  
4

### General rights

Unless other specific re-use rights are stated the following general rights apply:  
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# Active Distances and Cascaded Convolutional Codes<sup>1</sup>

Stefan Höst<sup>(1)</sup>, Rolf Johannesson<sup>(1)</sup>, Kamil Sh. Zigangirov<sup>(1)</sup>, and Viktor V. Zyablov<sup>(2)</sup>

<sup>(1)</sup> Dept. of Information Technology

Lund University

P.O. Box 118

S-221 00 Lund, Sweden

stefanh@it.lth.se, rolf@it.lth.se, kamil@it.lth.se

<sup>(2)</sup> Inst. for Problems of Information Transmission

of the Russian Academy of Science

B. Karetnyi per., 19, GSP-4

Moscow, 101447 Russia

zyablov@ippi.ac.msk.su

**Abstract** — A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.

## I. INTRODUCTION

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories  $m \geq 1$  and call them *active distances* since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

## II. ACTIVE DISTANCES

Consider the ensemble of binary, rate  $R = b/c$ , periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory  $m$  and period  $T$ ,

$$G = \begin{pmatrix} G_0(t) & \cdots & G_m(t+m) \\ & G_0(t+1) & \cdots & G_m(t+m+1) \\ & & \ddots & \\ & & & G_0(t+T-1) & \cdots & G_m(t+T-1+m) \end{pmatrix} \quad (1)$$

in which each digit in each of the matrices  $G_i(t+T)$  for  $0 \leq i \leq m$  and  $0 \leq t \leq T-1$ , is chosen independently and equally likely to be 0 and 1.

Let  $\mathcal{U}_{[t_1-m, t_2+m]}^r$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2+m}$  such that the first  $m$  and the last  $m$  subblocks are zero and they do not contain  $m+1$  consecutive zero subblocks.

Let  $\mathcal{U}_{[t_1-m, t_2]}^c$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2}$  such that the first  $m$  subblocks are zero and they do not contain  $m+1$  consecutive zero subblocks.

Let  $\mathcal{U}_{[t_1-m, t_2]}^s$  be the set of information sequences  $u_{t_1-m} \dots u_{t_2}$  such that at least one subblock is nonzero and they do not contain  $m+1$  consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

$$G_{[t, t+j]} = \begin{pmatrix} G_m(t) & & \\ \vdots & \ddots & \\ G_0(t) & & G_m(t+j) \\ & \ddots & \vdots \\ & & G_0(t+j) \end{pmatrix}. \quad (2)$$

**Definition 1** Let  $C$  be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the  $j$ th order active row distance is

$$a_j^r \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j+m]}^r} w_H(u_{[t-m, t+j+m]} G_{[t, t+j+m]}), \quad (3)$$

the  $j$ th order active column distance is

$$a_j^c \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^c} w_H(u_{[t-m, t+j]} G_{[t, t+j]}), \quad (4)$$

and the  $j$ th order active segment distance is

$$a_j^s \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}_{[t-m, t+j]}^s} w_H(u_{[t-m, t+j]} G_{[t, t+j]}). \quad (5)$$

For a convolutional code encoded by a time-varying, non-catastrophic, polynomial generator matrix we define its free distance as  $d_{\text{free}} \stackrel{\text{def}}{=} \min_j a_j^r$ .

## III. CASCADED CODES

Consider a scheme with two convolutional codes in cascade.

**Theorem 1** There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies

$$\delta_l^r \stackrel{\text{def}}{=} \frac{a_j^r}{mc} \geq (l+1)h^{-1} \left(1 - \frac{l}{l+1}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (6)$$

for  $l \geq l_0^r = O(\frac{1}{m})$ ,

$$\delta_l^c \stackrel{\text{def}}{=} \frac{a_j^c}{mc} \geq lh^{-1}(1-R) - O\left(\frac{\log_2 m}{m}\right) \quad (7)$$

for  $l \geq l_0^c = O(\frac{\log_2 m}{m})$ , and

$$\delta_l^s \stackrel{\text{def}}{=} \frac{a_j^s}{mc} \geq lh^{-1} \left(1 - \frac{l+1}{l}R\right) - O\left(\frac{\log_2 m}{m}\right) \quad (8)$$

for  $l \geq l_0^s = \frac{R}{1-R} + O(\frac{\log_2 m}{m})$ .

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz.,  $\frac{R}{-\log_2(2^{1-R}-1)}$ .

## REFERENCES

- [1] C. Thommesen and J. Justesen, *Bounds on distances and error exponents of unit-memory codes*, IEEE Trans. Inform. Theory, vol. IT-29, pp. 637–649, July 1983.
- [2] S. Höst, R. Johannesson, K. Sh. Zigangirov, and V. V. Zyablov, *Active distances for convolutional codes*, Submitted to IEEE Trans. Inform. Theory, Dec 1996.

<sup>1</sup>This research was supported in part by the Royal Swedish Academy of Sciences in cooperation with the Russian Academy of Sciences and in part by the Swedish Research Council for Engineering Sciences under Grant 94–83.