



LUND UNIVERSITY

Some results on convolutional codes over rings

Johannesson, Rolf; Wan, Zhe-Xian; Wittenmark, Emma

Published in:
[Host publication title missing]

DOI:
[10.1109/ISIT.1997.613204](https://doi.org/10.1109/ISIT.1997.613204)

1997

[Link to publication](#)

Citation for published version (APA):
Johannesson, R., Wan, Z.-X., & Wittenmark, E. (1997). Some results on convolutional codes over rings. In *[Host publication title missing]* (pp. 284) <https://doi.org/10.1109/ISIT.1997.613204>

Total number of authors:
3

General rights

Unless other specific re-use rights are stated the following general rights apply:
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Some Results on Convolutional Codes over Rings

Rolf Johannesson, Zhe-xian Wan, and Emma Wittenmark¹

Department of Information Technology, Information Theory Group
Lund University, S-221 00 LUND, Sweden
Email: emma@it.lth.se

Abstract — Convolutional codes over rings were motivated from phase-modulated signals. Some structural properties of generator matrices of convolutional codes over rings have been studied. Here, a condition for a convolutional code over a ring to be systematic is given and shown to be equivalent to the condition given by Massey and Mittelholzer. Furthermore, the conditions of generator matrices over \mathbb{Z}_{p^e} being catastrophic, basic, and minimal are considered, and the predictable degree property of polynomial generator matrices is considered.

I. SUMMARY

Massey and Mittelholzer [1] introduced convolutional codes over rings together with their motivation from phase-modulated signals. They also showed that convolutional codes over rings behave much differently than convolutional codes over fields. Structural properties of convolutional codes over rings were discussed in [2] [3]. We have here studied some structural properties of convolutional codes over rings in more detail.

Let R be a commutative ring with identity, $R[D]$ be the polynomial ring over R , and $R(D)$ be the ring of rational functions over R in the indeterminate D , such that the trailing coefficient of the denominator polynomials are units in the ring R . Let $R_r(D)$ be the subring of $R(D)$ consisting of those elements (equivalence classes) which contain a representative $\frac{f(D)}{q(D)}$ with $q(0)$ being a unit in R . We call this the ring of realizable functions and the elements in $R_r(D)$ realizable functions. Obviously, we have the relation $R_r(D) \subset R(D)$ between these two rings.

Definition 1 A realizable transfer function matrix $G(D)$ with entries in $R(D)$ is called a generator matrix if its rows are free over $R(D)$.

For convolutional codes over fields, all codes have both systematic and nonsystematic generator matrices. Thus, in the field case, being systematic is an encoder property. However, in the ring case, being systematic is a code property [2]. A convolutional code C over a ring R is defined to be systematic if it has a systematic generator matrix.

Theorem 1 A convolutional code C over a ring R is systematic if and only if it has a generator matrix $G(D)$ that has a $b \times b$ subdeterminant which is a unit in $R_r(D)$.

It is worth noting, that it is not required that every generator matrix of a systematic code C has a $b \times b$ subdeterminant that is a unit in $R_r(D)$. But a generator matrix $G(D)$ that does not have $b \times b$ subdeterminant which is a unit in $R(D)$, cannot generate a systematic code!

Let C_0 be the start module of a rate- b/c convolutional code C over a ring R , i.e., C_0 consists of all c -tuples $\mathbf{v}(0)$ for which $\mathbf{v}(D)$ is a causal codeword in C . In [2], a convolutional code C over a ring R is defined to be proper if one can select b components so that the c -tuples in C_0 , when restricted to these components, form the free module R^b . Then they proved

Proposition 1 [2] A convolutional code is systematic if and only if it is proper.

We proved

Theorem 2 Proposition 1 is equivalent to Theorem 1.

Consider a generator matrix $G(D)$ over \mathbb{Z}_{p^e} . We can write $G(D) = G_0(D) + G_1(D)p + \dots + G_{e-1}(D)p^{e-1}$, where the entries of all $G_i(D)$ are in $\mathbb{Z}_p(D)$. Then $G(D) \bmod p = G_0(D)$. Furthermore,

- (i) $G(D)$ is catastrophic if $G(D) \bmod p$ is catastrophic, and for polynomial generator matrices only if $G(D) \bmod p$ is catastrophic,
- (ii) $G(D)$ is basic if and only if $G(D) \bmod p$ is basic.

From (i) it follows that for a polynomial generator matrix $G(D)$ over \mathbb{Z}_{p^e} , $G(D)$ is catastrophic if and only if $G(D) \bmod p$ is catastrophic, which was announced by Massey and Mittelholzer in [1]. However, for nonpolynomial generator matrices over \mathbb{Z}_{p^e} , the catastrophicity of $G(D)$ does not imply catastrophicity of $G(D) \bmod p$. For minimality of generator matrices, such an easy relation between the generator matrices $G(D)$ and $G(D) \bmod p$ does not exist. For example, the generator matrix $G(D) = (1 + pD)$ over \mathbb{Z}_{p^2} is not minimal, but $G(D) \bmod p = (1)$ is minimal over \mathbb{Z}_p .

Forney defined the predictable degree property of polynomial generator matrices [4]. For convolutional codes over rings we have,

Theorem 3 A polynomial generator matrix $G(D)$ whose row-wise highest degree coefficients are units in R has the predictable degree property if and only if the rows of $[G(D)]_h$ are free over R , where $[G(D)]_h$ consists of the row-wise highest degree coefficients of $G(D)$.

REFERENCES

- [1] J. L. Massey and T. Mittelholzer, "Convolutional Codes over Rings", in *Proc. Fourth Joint Swedish-Soviet Int. Workshop on Information Theory*, Aug. 27 - Sept. 1, pp. 14-18, 1989, Gotland, Sweden.
- [2] J. L. Massey and T. Mittelholzer, "Systematicity and Rotational Invariance of Convolutional Codes over Rings", *Proc. 2nd Int. Workshop on Alg. and Combinatorial Coding Theory*, Leningrad, Sept. 16-22, 1990.
- [3] T. Mittelholzer, "Minimal Encoders for Convolutional Codes over Rings" in *Communications Theory and Applications*, HW Comm. Ltd., pp. 30-36, 1993.
- [4] G. D. Forney Jr., "Convolutional Codes I: Algebraic Structure", *IEEE Trans. on Information Theory*, 16, pp. 720-738, 1970.

¹This work was supported in part by the Swedish Research Council for Engineering Sciences under Grant 94-77