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PROPAGATION INSIDE A BIANISOTROPIC WAVEGUIDE AS AN EVOLUTION PROBLEM

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ABSTRACT

The free source Maxwell system for the bianisotropic medium, in a fixed frequency $\omega \geq 0$ and with time convention $e^{-i\omega t}$, is represented by the equation

$$(0.1) \quad \nabla \times \mathbf{J}\mathbf{e} = i\omega\mathbf{M}\mathbf{e}$$

where $\mathbf{e} := (\mathbf{E}, \mathbf{H})^T$ is the electromagnetic (E/M) field; it is defined in a domain $\Omega \subset \mathbb{R}^3$, depend on ω and take values in \mathbb{C}^6 . We denote

$$\mathbf{J} := \begin{bmatrix} 0 & -I_3 \\ I_3 & 0 \end{bmatrix}$$

The matrix

$$\mathbf{M} := \begin{bmatrix} \boldsymbol{\varepsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{bmatrix}$$

characterizes the medium inside Ω and its entries are complex functions of the frequency ω and the position $\mathbf{r} \in \Omega$. The Gauss law implies that

$$(0.2) \quad \nabla \cdot \mathbf{M}\mathbf{e} = 0$$

Assume that the boundary $\Gamma := \partial\Omega$ is smooth enough; usually Lipschitz is sufficient for most of the applications. Let $\hat{\mathbf{n}}$ be the exterior normal to Γ . For a wide class of boundaries, metallic for example, the perfect electric conductor (PEC) boundary condition for the electric field, $\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{0}$ on Γ , applies.

Let now $\mathbf{A} = (A_x, A_y, A_z)^T$ be a vector field in Ω ; it can be represented as $\mathbf{A} = (\mathbf{A}_\perp, A_z)^T$ where $\mathbf{A}_\perp := A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$ is the transverse and A_z the longitudinal part. It is easily seen that the *curl* operator reads

$$(0.3) \quad \nabla \times \mathbf{A} = \begin{bmatrix} W & 0 \\ 0 & 0 \end{bmatrix} \partial_z \begin{pmatrix} \mathbf{A}_\perp \\ A_z \end{pmatrix} - \begin{bmatrix} 0 & W\nabla_\perp \\ \nabla_\perp \cdot W & 0 \end{bmatrix} \begin{pmatrix} \mathbf{A}_\perp \\ A_z \end{pmatrix}$$

where

$$W := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \hat{\mathbf{z}} \times I_3$$

and $\nabla_\perp := \partial_x\hat{\mathbf{x}} + \partial_y\hat{\mathbf{y}}$ is the formal transverse gradient.

An infinite waveguide is a cylinder

$$\Omega = \Omega_\perp \times \mathbb{R}$$

where $\Omega_\perp \subset \mathbb{R}^2$ is a domain with Γ_\perp . Observe that the wall of the waveguide is $\Gamma = \Gamma_\perp \times \mathbb{R}$ and $\hat{\mathbf{n}}$ coincides with its transverse part and is the exterior normal

to Γ_\perp , whereas $\hat{\boldsymbol{\tau}} := W\hat{\boldsymbol{\nu}}$ is the tangent vector. The PEC boundary condition now reads

$$(0.4) \quad \hat{\boldsymbol{\tau}} \cdot \mathbf{E}_\perp = 0 \quad , \quad E_z = 0 \quad \text{on } \Gamma$$

The fact that the longitudinal variable z runs \mathbb{R} allows us to formulate the Maxwell system as an evolution equation with respect to this variable. Indeed, letting

$$C := \begin{bmatrix} 0 & W\nabla_\perp \\ \nabla_\perp \cdot W & 0 \end{bmatrix}$$

the Maxwell system is written

$$\partial_z \mathbf{V}e = (\mathbf{A}_0 + i\omega\mathbf{M})e$$

where $\mathbf{V} := \hat{\mathbf{z}} \times \mathbf{J}$ and $\mathbf{A}_0 := C\mathbf{J}$. Define now a Hilbert space \mathcal{X} of functions of the transverse variables and consider the E/M field \mathbf{e} as vector-valued a function

$$\mathbf{e} : \mathbb{R} \ni z \mapsto \mathbf{e}(\cdot, \cdot, z) \in \mathcal{X}$$

Then \mathbf{A}_0 can be realized as an unbounded operator in \mathcal{X} and the PEC conditions are incorporated in the domain of \mathbf{A}_0 . Actually, if we separate $\mathbf{u} \in \mathcal{X}$ into “electric” and “magnetic” part

$$\mathbf{u} =: \begin{pmatrix} \mathbf{u}^e \\ \mathbf{u}^h \end{pmatrix},$$

then \mathbf{A}_0 is given explicitly by

$$(0.5) \quad \mathbf{A}_0 \mathbf{u} = \begin{pmatrix} -W\nabla_\perp \mathbf{u}_z^h \\ -\nabla_\perp \cdot W\mathbf{u}_\perp^h \\ W\nabla_\perp \mathbf{u}_z^e \\ \nabla_\perp \cdot W\mathbf{u}_\perp^e \end{pmatrix}$$

The first step is to prove that \mathbf{A}_0 is the generator of a strongly continuous group in \mathcal{X} . The second is to realize Maxwell system as a perturbed abstract degenerate evolution problem

$$(0.6) \quad \mathbf{V}e'(z) = (\mathbf{A}_0 + i\omega\mathbf{M}(\omega))e(z)$$

and apply relevant perturbation arguments in order to establish well-posedness. The research presented here implements exactly this program.

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