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Decoding Procedure Capacities for the Gilbert-Elliott Channel [1]

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Abstract — **Sequential decoding for the Gilbert-Elliott channel is considered. The decoding procedure capacity C_D is defined to be the supremum of the rates for which there exists a code that gives arbitrarily small decoding error probability. For different assumptions of the decoder's knowledge of the channel states expressions for C_D are derived.**

I. Introcuction

Assume that a tree code is used together with sequential decoding to communicate over the Gilbert-Elliott channel. Let $P(\mathcal{E})$ denote the average probability of decoding error over the ensemble of random, infinite depth tree codes. In this paper we address the question: "When will $P(\mathcal{E}) \to 0$?".

Consider the Gilbert-Elliott channel model and denote the error probabilities in the Good and Bad states by e_G and e_B, respectively. Furthermore, let P_G and P_B denote the fraction of time spent in the Good and Bad states, respectively.

II. Decoding procedure capacity

Let us define the *decoding procedure assumptions*, D. The optimistic assumption, $D = o$, assumes that the decoder has a complete knowledge of the channel state, which could be given by a genie. The pessimistic assumption, $D = p$, assumes that the decoder neither is given any channel state information nor tries to make any estimate of it. Given the decoding procedure assumption D and the use of the Gilbert-Elliott channel, let C_D denote the supremum of the rates for which we can guarantee that there exists a code that gives an arbitrarily small decoding error probability $P(\mathcal{E})$. We will call C_D the *decoding procedure capacity.*

We have proved that the decoding procedure capacities are given by

$$C_o = P_G \cdot C_{BSC}(e_G) + P_B \cdot C_{BSC}(e_B)$$

and

$$\begin{aligned} C_p &= P_G \cdot (C_{BSC}(e_G) - h(b)) + P_B \cdot (C_{BSC}(e_B) - h(g)) \\ &= C_o - (P_G \cdot h(b) + P_B \cdot h(g)), \end{aligned}$$

where b and g denote the transition probabilities from Good to Bad and from Bad to Good, respectively, in the channel model.

Theorem 1 *Given the Gilbert-Elliott channel and the decoding procedure assumptions, the use of a rate R random, infinite depth tree code with the stack decoder, then for any rate $R < C_D$ and $\eta \in \mathbb{Z}^+$,*

$$P(N \geq \eta) \to 0 \quad \textit{if } \eta \to \infty,$$

where N is the number of computations in an incorrect *subtree.*

When we wish to transmit over an ordinary Discrete Memoryless Channel at rates (above R_{comp} and) close to its capacity, it is sufficient to allow the number of computations of sequential decoding to go to infinity to be able to guarantee that $P(\mathcal{E})$ can be chosen arbitrarily small. We will show that this is also sufficient for transmission close to rates C_D, which is the motivation why we call these rates "decoding procedure *capacities*".

Theorem 2 *Given the assumptions of Theorem 1, then for any rate $R < C_D$ the average probability of decoding error*

$$P(\mathcal{E}) \to 0,$$

if the number of computations, N, is allowed to go to ∞.

Since the important condition in Theorem 2 is that $R < C_D$, it is clear that the theorem's statement, *given* the decoding procedure assumptions, is equivalent to stating that the maximal transmission rate over the Gilbert-Elliott channel is at least the rate C_D.

In the pessimistic case we can interpret this as follows. For arbitrarily small $P(\mathcal{E})$, there exists a code such that the transmission rate will be (at least) C_p, *even without* any knowledge of the channel state or any attempt to estimate it.

III. Channel capacity

A common method to lowerbound C_{GE} is to calculate $C_{BSC}(\bar{e})$, where $\bar{e} = P_G \cdot e_G + P_B \cdot e_B$, but it turns out that C_p is a better lower bound for channels with a stable behaviour. The optimistic case helps us to find a stronger result:

Theorem 3 *Given that the receiver has a complete channel state knowledge, then the channel capacity for the Gilbert-Elliott channel C_{GE}^{R} is equal to*

$$C_{GE}^{R} = C_o.$$

From the proof of Theorem 3 follows immediately

Corollary 4 *Given that* both *transmitter and receiver have complete knowledge of the channel state* sequence *then for the channel capacity of the Gilbert-Elliott channel C_{GE}^{TR} we have*

$$C_{GE}^{TR} = C_{GE}^{R}.$$

It should be noted that the capacities C_{GE}^{TR} and C_{GE}^{R}, in contradiction to what is the case for C_D, are parameters purely dependent of the channel's properties and that nothing is assumed about the decoding method. In the derivations of C_D we assume sequential decoding, but by deriving them we show that they are achievable rates as such, given the decoding procedure assumptions.

References

[1] Gunilla Bratt: "Sequential Decoding for the Gilbert-Elliott Channel — Strategy and Analysis". Ph.D. Thesis. Dept. of Information Theory, Lund University, Lund, Sweden, 1994.

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