Design of Optimal Low-Order Feedforward Controllers for Disturbance Rejection

Hast, Martin; Hägglund, Tore

2012

Abstract

Design of Optimal Low-Order Feedforward Controllers for Disturbance Rejection

Martin Hast, Tore Hägglund
Dept. of Automatic Control, Lund University,

Feedforward is an efficient method to reduce control errors both for reference tracking and disturbance rejection, given that the disturbances acting on the system are measurable. This paper treats the subject of disturbance rejection and presents an analytic solution to the problem of designing a feedforward lead-lag filter which minimizes the integrated square error when the system is subject to a measurable step disturbance, d. The resulting feedforward controller is optimal in an open-loop setting, see Figure 1, for first-order plants with time delays, P_i.

In general, due to for instance modeling errors or non-measurable disturbances, feedforward alone is not sufficient in order to reject disturbances in a satisfying fashion. The structure depicted in Figure 2 with $H = 0$ is a common control structure that utilizes both a feedforward and a feedback controller. However, using this structure, the feedback controller C will interact with, or even counteract the feedforward controller. Hence, the feedforward and feedback controllers should be designed jointly. An alternative feedforward scheme was presented in [1]. By choosing $H = P_2P_3 - P_2P_1G_{ff}$, the controller interaction can be eliminated. Furthermore, the design of the feedforward controller reduces to the equivalent problem in the open-loop case.

Along with the presented optimal design rule, examples that illustrate the behavior of the resulting feedforward controller is presented. The performance of the obtained optimal feedforward controllers have also been compared with other available design methods e.g., [2].

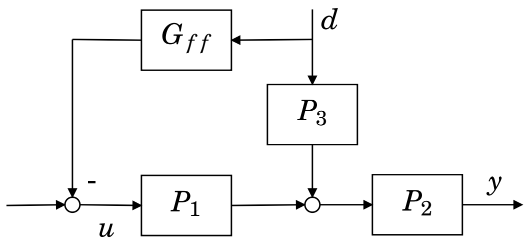

Figure 1 Open-loop structure.

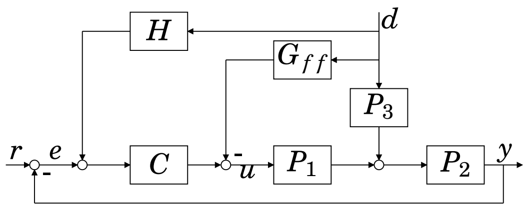

Figure 2 Closed-loop feedforward structure.

References

[1] C. Brosilow and B. Joseph. *Techniques of Model-Based Control*. Prentice Hall PTR, 2002.

[2] José Luis Guzmán and Tore Hägglund. Simple tuning rules for feedforward compensators. *Journal of Process Control*, 21(1):92–102, January 2011.

Design of Optimal Low-Order Feedforward Controllers for Disturbance Rejection

LUND UNIVERSITY

Martin Hast Tore Hägglund

Department of Automatic Control, Lund University

martin.hast@control.lth.se tore.hagglund@control.lth.se

LUND UNIVERSITY

Problem Formulation

Feedforward is an effective method for measurable load-disturbance attenuation. The main contribution of this work is tuning rules that minimize the integrated-squared error (ISE) for step disturbances. It utilizes a decoupling control structure that enables the feedforward controller to be tuned without taking the feedback loop into consideration.

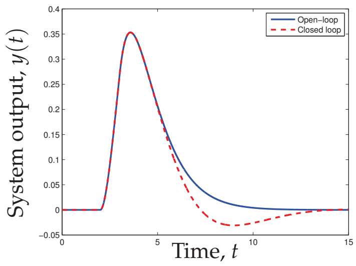

FIGURE 1: Open and closed-loop responses to a unit step disturbance.

Feedforward Structure - Controller Decoupling

Consider the setup in Fig. 2. The transfer function from the disturbance d to the output y is given by

$$G_{yd} = P_2(P_3 - P_1 G_{ff}). \tag{1}$$

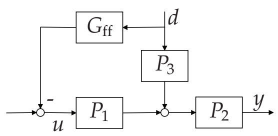

FIGURE 2: Open-loop feedforward structure.

Feedforward control from a measurable disturbance is often implemented as displayed in Fig. 3 with $H = 0$. The transfer function from d to y is then given by

$$G_{yd} = \frac{P_2(P_3 - P_1 G_{ff})}{1 + P_2 P_1 C}. \tag{2}$$

If perfect feedforward is realizable, i.e., $G_{ff} = P_3 P_1^{-1}$, no control error will arise. However, if P_1 is not invertible, the feedforward and feedback controllers will interact. The problem of interacting controllers can be handled by retuning the feedforward controller [2], or by introduction of a filter H. If $H = P_2 P_3 - P_2 P_1 G_{ff}$ [1], it can be shown that the transfer function from disturbance to system output is given by

$$G_{yd} = \frac{P_2 P_3 + P_2 P_1 (CH - G_{ff})}{1 + P_2 P_1 C} = P_2(P_3 - P_1 G_{ff}). \tag{3}$$

The closed-loop behavior is thus the same as the open-loop behavior and the controllers do not interact, which simplifies the tuning procedure for the feedforward controller. The feedback controller can therefore be tuned to handle unmeasurable disturbances and the model mismatch introduced in the filter H.

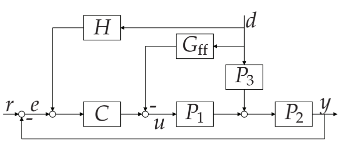

FIGURE 3: Decoupling feedforward structure.

ISE - Optimal Feedforward Design

Consider the setup in Fig. 2 with the feedforward controller

$$G_{ff}(s) = K_{ff} \frac{1 + s\,T_z}{1 + s\,T_p} e^{-sL_{ff}}, \tag{4}$$

and the plant models

$$P_1(s) = \frac{K_1}{1 + sT_1} e^{-sL_1}, \quad P_2(s) = 1, \quad P_3(s) = \frac{K_3}{1 + sT_3} e^{-sL_3}. \tag{5}$$

The objective is to minimize the ISE when the system is subject to a step load-disturbance.

$$\min_{\substack{K_{ff}, T_z \\ L_{ff}, T_p}} \int_0^\infty y^2(t)\,dt \qquad \text{s.t.} \quad \begin{cases} L_{ff} \geq 0 \\ T_p \geq 0 \end{cases} \tag{6}$$

An analytical solution to this problem was presented in [3] and the design procedure is summarized below:

1. $K_{ff} = \dfrac{K_3}{K_1}$.
2. $L_{ff} = \max(0, -L)$, $L = L_1 - L_3$.
3. Introduce $a = \dfrac{T_1}{T_3}$ and $b = a\,(a+1)\,e^{L/T_3}$
   - If $a > 1$ and $b < 4a^2 - 2a$,
   $$T_p = \frac{3a - 1 - b + \sqrt{(a-1)^2(1+4b)}}{b-2} T_3.$$
   - If $a < 1$ and $b < \sqrt{a} + a$,
   $$T_p = \frac{3a - 1 - b - \sqrt{(a-1)^2(1+4b)}}{b-2} T_3.$$
   - Else, $T_p = 0$.
4. $T_z = (T_p + T_1)\left(1 - \dfrac{2T_3^2}{(T_1 + T_3)(T_3 + T_p)\,\mathrm{e}^{L/T_3}}\right)$

Example and Comparison

Using the ISE-optimal tuning rules with the decoupling structure the performance can be significantly improved.

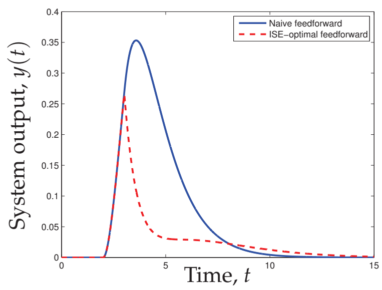

FIGURE 4: Closed-loop system responses for naive and optimal feedforward controllers using the structure displayed in Fig. 3.

Conclusions

In this work an analytical solution to the optimal control problem of minimizing the ISE using a lead-lag feedforward controller is presented. The resulting controller parameters are optimal for a system of first-order plants disturbed by a step. Feedforward to both the controller-input and controller-output has been used in order to avoid controller interaction and facilitate the tuning of the feedforward controller.

References

[1] C. Brosilow, B. Joseph *Techniques of Model-Based Control* Prentice Hall PTR, 2002.

[2] José Luis Guzmán and Tore Hägglund *Simple Tuning Rules for Feedforward Compensators* In *Journal of Process Control* 21(1):92-102, January, 2011.

[3] Martin Hast and Tore Hägglund *Design of Optimal Low-Order Feedforward Controllers. IFAC Conference on Advances in PID Control*, Brescia, March 2012 (Accepted for publication)

Nordic Process Control Workshop, Kgs Lyngby, Denmark, 2012