Nonmagnetic ultra wideband absorber with optimal thickness

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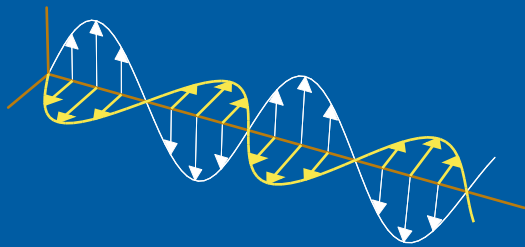
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Abstract

Design of ultra wideband absorbers with bandwidth ratios larger than 10:1 is investigated. It is explained that if there is no constraint on the total thickness of the absorber, achieving large bandwidths is straightforward. The problem becomes challenging when the minimization of the total thickness is considered. It is shown that for a given frequency response, the total thickness of a nonmagnetic absorber cannot be less than a theoretical limit. If a design method can reduce the total thickness to the theoretical limit level, its superiority over other design methods is doubtless. It is demonstrated that the capacitive circuit absorber approach has this unique feature. In order to clarify the design ideas and techniques, the optimal absorber is developed in different stages. It is shown that unequal periods for the low-pass frequency selective surfaces are essential for attaining the optimal performance.

1   Introduction

The simplest way to construct a nonmagnetic absorber is to place a homogenous resistive sheet, a quarter of wavelength, above the perfect conducting ground plane [6]. Such an absorber is called Salisbury screen and possesses a narrow absorption bandwidth (around 26%). The bandwidth can be increased by adding more resistive sheets to the medium, $\lambda/4$ apart from each other, to construct Jaumann absorber [4, 5]. Jaumann absorber broadens the absorption bandwidth in cost of total thickness increase. The bandwidth of a Jaumann absorber can be enlarged significantly by replacing the homogenous resistive sheets by band-stop resonating frequency selective surfaces, resulting in circuit analog absorber [14–16]. Circuit analog absorbers have been considered as the most wideband solution for years. Without any proof, bandwidth in excess of 10:1 is reported in a recent publication as a figure of merit for the approach [16]. The aim of the current paper is to provide a more powerful method of design for ultra wideband absorbers with bandwidth ratios around 11:1 or more. Since there is no access to the classified wideband circuit analog absorbers mentioned in the Munk's paper [16], the superiority of our design method cannot be proved by direct comparison. Fortunately, fundamental laws of physics provide a reliable method of evaluation.

From fundamental concepts such as causality, time invariance, linearity and analytical properties of the reflection coefficient in complex frequency plane, Rozanov has proved that there exists an upper theoretical limit on the ultimate bandwidth of a metal backed absorber [19]. The upper band for a nonmagnetic absorber is dictated by its total thickness. Equivalently, for a given absorption frequency response the total thickness of the absorber cannot fall below a theoretical limit. This minimum thickness provides a fair criteria for evaluating the efficiency of a design method.

By the aid of the capacitive circuit absorber method [9], an excellent absorber is designed with a very large bandwidth ($3.26 - 34.65\,\text{GHZ}$, $f_\text{H}/f_\text{L} \approx 10.6$). This bandwidth is unique among all the designs ever published [2, 3, 8, 10, 14–16]. It is shown that the total thickness of the design ($14.5\,\text{mm}$) is only 0.7% thicker than the

minimum possible thickness dictated by the physical bound. This fact proves that no design can outperform our optimal absorber. To illustrate the design ideas and techniques the final design is developed in three different steps. In each development phase, the absorber satisfies the bandwidth requirement but the total thickness is reduced gradually by proper techniques. It is shown that the resistive periodic layers must have unequal periods in order to achieve the optimal thickness.

2 Minimum Possible Thickness of a Metal Backed Absorber

Consider a flat metallic ground plane coated by an absorber. The absorber is a general stratified medium made of passive, linear, time invariant and causal materials. Consequently the complex permittivity and permeability of the materials obey the Kramers-Kronig relations [7, 17]. If the incident wave has the $\exp(j\omega t)$ time dependence then the reflection coefficient is analytic in the lower half-plane of complex ω [17]. It has been proved by Rozanov that for normal incidence illumination, the infinite integral of natural logarithm of the reflection coefficient is bounded from above [19]:

$$\mid \int_0^\infty \ln |R(\lambda)| \, d\lambda \mid \leq 2\pi^2 \sum_i \mu_{s,i} d_i \tag{2.1}$$

In the above equation R is the reflection coefficient, λ is the wavelength in free space and $d_i, \mu_{s,i}$ are the thickness and the static permeability of the layer i of the multilayered slab.

For the case of our interest, wideband nonmagnetic absorbers, the above equation can be rearranged into the following form:

$$d \geq \frac{\mid \int_0^\infty \ln |R(\lambda)| \, d\lambda \mid}{2\pi^2} \tag{2.2}$$

Equation. (2.2) states that for a given absorption frequency response, the total thickness of the absorber (d) cannot be less than a theoretical limit. This minimum possible thickness can be used as an evaluation tool to judge the efficiency of a design method.

The class of absorbers mentioned above, shows total reflection at low frequencies. This asymptotic behavior has been taken into consideration in the derivation of Eq. (2.1), see [19], and can be expressed in mathematical form as the following limit [1]:

$$\lim_{|\lambda| \to \infty} R(\lambda) = -1 + j4\pi\mu d/\lambda \tag{2.3}$$

In the absorption band the reflection coefficient is designed to be less than a desired value. Finally at higher frequencies, the reflection coefficient is increased from its minimum value in the absorption band and usually tends to total reflection level. Therefore, a piecewise linear approximation of the frequency response in dB scale can be used as a design objective (similar to the Bode plots used in signal processing

or electronics except that the frequency axis is not in logarithmic scale). In this way the desired reflection coefficient can be expressed as:

$$R(f) = \begin{cases} -R_0(\frac{f-f_1}{f_2-f_1}) & f_1 \leq f \leq f_2 \\ -R_0 & f_2 \leq f \leq f_3 \\ R_0(\frac{f-f_4}{f_4-f_3}) & f_3 \leq f \leq f_4 \\ 0 & \text{otherwise} \end{cases} \qquad (2.4)$$

In the above equation, the R_0 is the desired absorption level in the absorption band in dB scale (a positive number). The transition intervals should be selected in a way that they result in practical slopes for the frequency response (less than $80\,\mathrm{dB/decade}$). By mapping the frequency response of Eq. (2.4) into wavelength domain and using Eq. (2.2), the minimum possible total thickness of the absorber can be calculated.

To define a design objective, the case with $f_1 = 1.5\,\mathrm{GHz}$, $f_2 = 3.3\,\mathrm{GHz}$, $f_3 = 34.7\,\mathrm{GHz}$, $f_4 = 40\,\mathrm{GHz}$ and $R_0 = 20\,\mathrm{dB}$ is considered. For these values the minimum possible thickness is calculated to be $14.4\,\mathrm{mm}$. In the next section a design is presented with the total thickness of $14.5\,\mathrm{mm}$ and in good agreement with the desired frequency response. Undoubtedly, it proves that our design is the best possible solution.

3 Ultra Wideband Absorber

Designing an ultra wideband absorber with optimal thickness is a complicated problem. Many restricting difficulties must be overcome to achieve such an optimal performance. The best way to illustrate the design techniques is through gradual development of the optimal design and comparison of the different steps of the development to each other. Therefore, the optimal absorber is evolved in three stages in the followings.

3.1 First Step: The Simplest Solution

Designing a wideband absorber without any constraint on its total thickness is not a difficult task. Jaumann absorber with many resistive layers is a straightforward solution. Especially, since the resistivity of the homogenous resistive sheets are ideally frequency independent, modeling the absorber over a very large frequency interval with simple circuit elements (resistances and transmission lines) is effortless. Jaumann absorbers do not suffer from the harmonic or anti-resonance problems of circuit analog absorbers made of band-stop frequency selective surfaces [9, 15, 16]. An eight resistive sheets Jaumann absorber can satisfy the bandwidth requirement of the design objective. The frequency response of the ultra wideband Jaumann absorber is shown in Fig. 1. The large bandwidth of the absorber $2.95 - 35.2\,\mathrm{GHz}$ leads to bandwidth ratio of $f_\mathrm{H}/f_\mathrm{L} \simeq 11.93$. If the bandwidth ratio is the only important parameter, our Jaumann absorber outperforms the classified circuit analog absorbers claimed by Munk [16]. It is the large thickness of the Jaumann absorber
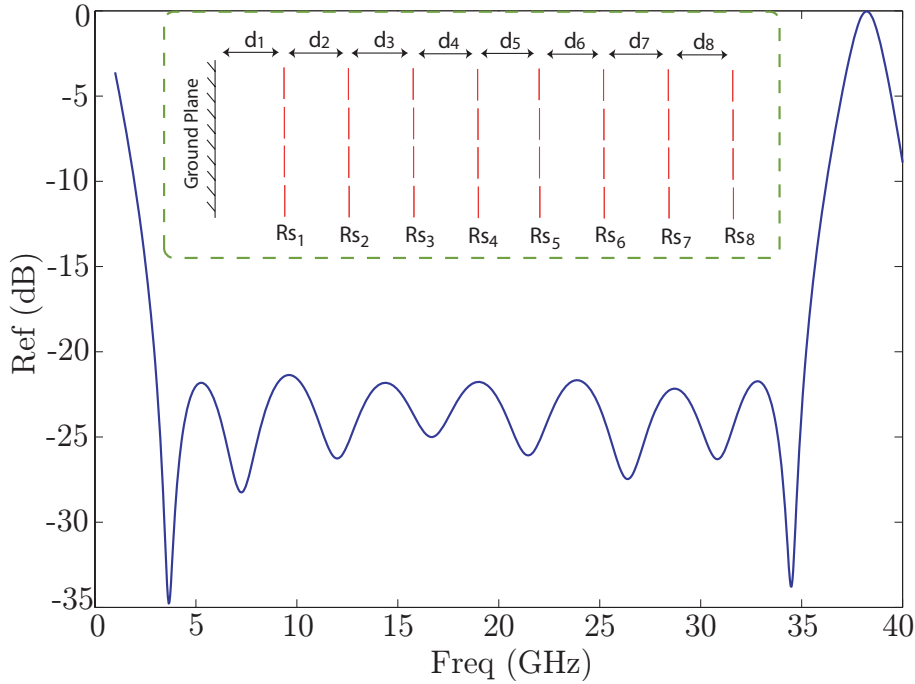
Figure 1: The frequency response and the schematic of a 8 resistive layer Jaumann absorber. The 20 dB absorption bandwidth is $2.95 - 35.2$ GHz, ($f_{\mathrm{H}}/f_{\mathrm{L}} \simeq 11.93$).

$d_1(\mathrm{mm})$	$d_2(\mathrm{mm})$	$d_3(\mathrm{mm})$	$d_4(\mathrm{mm})$
3.9	3.9	4	3.9
$d_5(\mathrm{mm})$	$d_6(\mathrm{mm})$	$d_7(\mathrm{mm})$	$d_8(\mathrm{mm})$
3.9	3.9	4	3.9

Table 1: The thicknesses of the dielectric layers for the Jaumann absorber.

that disqualifies the method for practical applications. The details of the parameters of the Jaumann absorber are tabulated in Tables 1 and 2. The total thickness of the wideband Jaumann absorber is 31.4 mm, more than twice the optimal thickness calculated in the previous section (14.4 mm).

3.2 Intermediate Step: Capacitive Circuit Absorber with Single Spatial Periodicity

To improve the performance of the Jaumann absorber, the homogenous resistive sheets are replaced by frequency selective surfaces (FSS) . Band-stop resonating FSS elements have been used in the design of circuit analog absorbers [14–16]. For years these absorbers have been considered as the most wideband solutions. As mentioned earlier, without any evidence bandwidth ratios of 10:1 have been claimed by Munk [16] as a figure of merit for this type of absorbers. A resonating FSS element shows a complicated frequency response over a bandwidth ratio of 10:1. The harmonic of the fundamental resonance occurs in the best case at a frequency

$R_{s1}(\Omega/Sq)$	$R_{s2}(\Omega/Sq)$	$R_{s3}(\Omega/Sq)$	$R_{s4}(\Omega/Sq)$
305	579	873	1266.5
$R_{s5}(\Omega/Sq)$	$R_{s6}(\Omega/Sq)$	$R_{s7}(\Omega/Sq)$	$R_{s8}(\Omega/Sq)$
1796	2480	3724	2067

Table 2: The resistivity of the homogenous resistive sheets for the Jaumann absorber.

about 3 times higher than the main resonant frequency (e.g. the hexagon ring, for the dipole shape elements second harmonic shows up). Before the harmonic frequency the anti-resonance effect takes place [9, 14–16]. Consequently for a bandwidth ratio of 10:1, all these phenomena must be taken into consideration in the circuit model of FSS layers and the matching process of the absorber. This complicated task has never been explained in public literature. Instead of spending time to reveal the secret behind these circuit analog absorbers, a better alternative is presented in this paper.

It has been shown with lots of examples and explanations that wideband low-pass arrays can replace the band-stop resonating elements in FSS based absorbers [9]. Low-pass arrays have no resonant frequency for a very large frequency interval and consequently the harmonic and the anti-resonance effects disappear over the whole absorption bandwidth. Therefore, wideband modeling of the FSS elements and the absorber becomes considerably simpler than the conventional circuit analog absorbers [9]. It has been shown that the bandwidth of the absorber is not affected by this replacement [9]. The fact is further verified by the design examples of the current paper, in particular the optimal design that has a unique performance. By showing that broad bandwidths can be achieved without facing the complexity of the wideband modeling of resonating arrays, the predominance of our method is demonstrated.

The next step in approaching the optimal design is to use low-pass periodic arrays (in our examples, periodic square patch arrays) instead of homogenous resistive sheets. The structure is then a capacitive circuit absorber [9]. Since the bandwidth ratio the of absorber is in order 10:1, it is expected that a few periodic layers are required to attain the large bandwidth. Three different cases might happen for the periodicity of the periodic layers:

- The spatial periods are the same.

- The unequal periods of layers are commensurate i.e. the ratio of the periods is a rational number.

- The periodic layers have noncommensurate periodicities.

From the design point of view, the noncommensurate periodicities case is the most convenient choice for the designer, since, as shown in the followings, the values of the required capacitances can be selected and synthesized with the highest degree of freedom. On the contrary, the electromagnetic analysis of the absorber becomes very complicated if not untractable. For the lossless multilayered FSS structures, a

| $d_1(\,\mathrm{mm})$	$d_2(\,\mathrm{mm})$	$d_3(\,\mathrm{mm})$
4.3	3.5	3.6
$d_4(\,\mathrm{mm})$	$d_5(\,\mathrm{mm})$	$d_6(\,\mathrm{mm})$
3.5	0.6	2.3

Table 3: The thicknesses of the dielectric layers for the capacitive circuit absorbers with single spatial periodicity. All the layers are free space except layer No.6 with permittivity 1.6. The dielectric cover embedding the second square patch array is not listed in the table.

$w_1(\,\mathrm{mm})$	$w_2(\,\mathrm{mm})$	$w_3(\,\mathrm{mm})$	$w_4(\,\mathrm{mm})$
6	5.9	5.8	5
$R_{s1}(\Omega/Sq)$	$R_{s2}(\Omega/Sq)$	$R_{s3}(\Omega/Sq)$	$R_{s4}(\Omega/Sq)$
159	292.5	436	385

Table 4: The characteristic of the square patches (width (w) and the resistivity (R_s)) used in the capacitive circuit absorber with single spatial periodicity ($a = 6\,\mathrm{mm}$).

few techniques have been introduced to analyze dissimilar periodicities [11, 12, 18]. As it is explained, both general categories of analysis methods: those that analyze the entire system simultaneously (e.g. method of moment [13]) and those employing cascading approach based on the generalized scattering matrix techniques, become complicated and high memory resource demanding problems for the noncommensurate periodicities case. The simplest situation for electromagnetic analysis of the absorber occurs when the resistive layers have similar periods but it introduces numerous constraints on the absorber design process and consequently cannot result in the optimal thickness. Fortunately, the optimal absorber of our interest can be designed by the moderate level of complexity and acceptable degree of freedom, i.e., the commensurate periods. To illustrate the ideas clearly and explain the required techniques to achieve the optimal performance, the simple case of equal periodicity is considered first. Then, a four resistive layer capacitive circuit absorber satisfies our desired bandwidth. The schematic and the frequency response of the single spatial periodicity capacitive circuit absorber is shown in Fig. 2. The absorption bandwidth is 3.16–34.96 GHz with a bandwidth ratio $f_\mathrm{H}/f_\mathrm{L} \simeq 11.03$. The details of the dielectric layers and the square patch parameters used to synthesize the RC circuits of the equivalent circuit model (periodicity (a), width (w) and sheet resistivity (R_s), see Fig. 3) are tabulated in Tables 3 and 4. The absorber has a total thickness of 18.2 mm which is a considerable reduction in comparison to the 8 resistive layer Jaumann absorber of the first example (31.4 mm) but it is still thicker than the optimal thickness (14.4 mm).

The 26.4% difference from the optimal thickness is due to utilization of similar periods for the square patches. The reason can be explained by considering the circuit model of absorber, see Fig. 2, and the way that the low-pass series RC circuits of this model are synthesized by periodic square patches. The geometry of a periodic
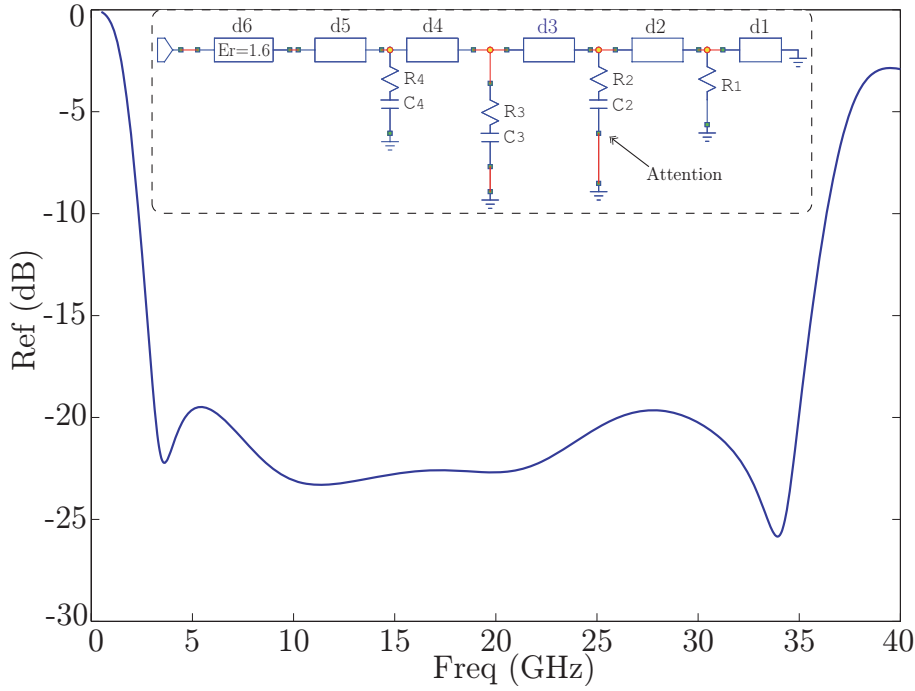
Figure 2: The frequency response of the capacitive circuit absorber with single spatial periodicity and its equivalent circuit model. The second layer shown by the arrow (Attention) is embedded in the middle of a dielectric cover not shown in the figure ($T_c = 0.2\,\text{mm}$ and $\epsilon_r = 2$). The 20 dB absorption bandwidth 3.16–34.96 GHz, $(f_H/f_L \simeq 11.03)$.

square patch embedded in the middle of a dielectric cover is shown in Fig. 3. The low-pass frequency behavior of the square patch array can be modeled by a series RC circuit [9]. The capacitance of the equivalent circuit is a complex function of periodicity (a), the width to periodicity ratio (w/a), the permittivity of the dielectric cover (ϵ_c) and its thickness (T_c). The equivalent capacitance is proportional to the periodicity of the array for fixed values of width to periodicity ratio (w/a) and the dielectric cover parameters. The fact is illustrated in Fig. 4 by considering two different width to periodicity ratios for a lossless (PEC) square patch array embedded in a dielectric cover. This proportionality helps the designer to obtain different values of capacitances during the synthesis process. When this freedom is taken away by considering a single periodicity for all the layers, the desired capacitance values must be achieved by the other parameters, i.e., the width to periodicity ratio and the permittivity of the dielectric cover. The capacitance increases exponentially withe the width to periodicity ratio and becomes singular at values close to unity. The behavior is shown in Fig. 5 for a fixed periodicity. The exponential dependence makes the synthesis process difficult because a small change in the width to periodicity ratio changes the capacitance value significantly. The capacitances of the circuit model of the absorber range from large values for the layers close to ground plane to smaller ones for layers close to free space (see the values in Table 5). If the periodicity is fixed, the values of achievable capacitances do not have the required diversity to
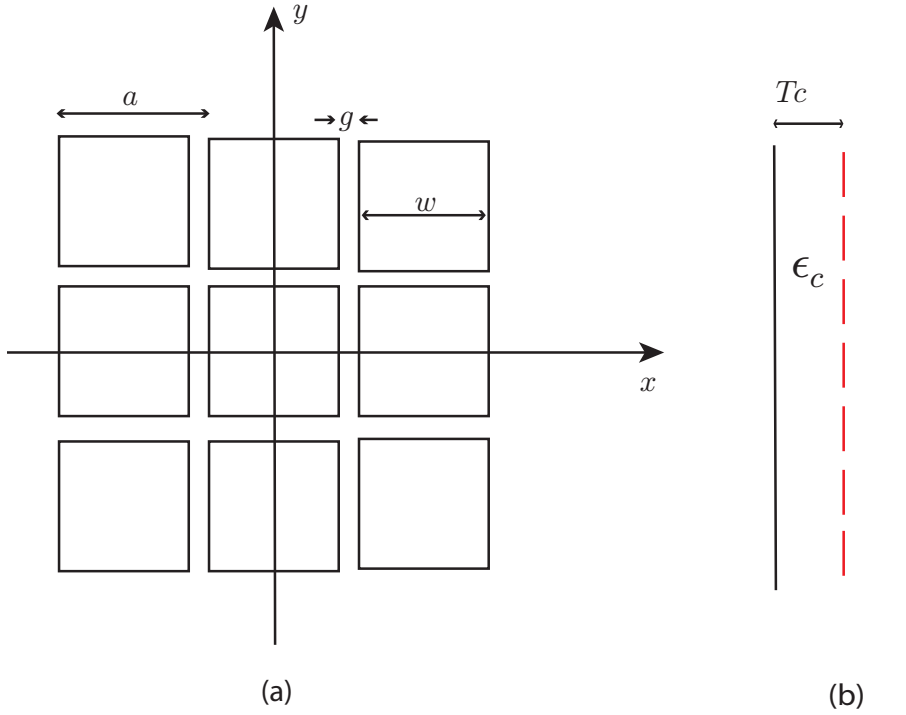
Figure 3: The geometry of a periodic square patch embedded in the middle of a dielectric cover. (a) Front view. (b) Side view.

achieve the optimal performance. Therefore, some clever choices must be made to make the absorber as good as possible. For this reasons, the first resistive layer is selected to be a homogenous resistive sheet in the circuit model of Fig. 2, in this way the equivalent capacitance becomes virtually infinite. The second resistive layer has been embedded in a dielectric cover (with $T_c = 0.2\,\text{mm}$ and $\epsilon_r = 2$) to make the synthesis of its large capacitance value possible (see the values for the intermediate design in Table 5). It is a simple way to achieve large capacitances when large spatial periods can not be used. In the next design example it is shown that the performance of the absorber can be improved by better selection of the values of the capacitances, in particular by replacing the homogenous resistive sheet of the fist layer by a proper periodic square patch array. Then, different spatial periods for the layers are essential.

3.3   Final Step: The Optimal Absorber

The next step in the development of the optimal absorber is the reduction of the thickness $(18.2\,\text{mm})$ of the second design to values very close to the theoretical limit $(14.4\,\text{mm})$. The solution in the circuit model of the absorber is straightforward. The single resistance element of the first layer (see the schematic of the absorber in Fig. 2) must be replaced by a proper series RC circuit (the adjustment of the R & C values of other resistive layers are also required). The idea might seem trivial and too simple to implement but synthesizing the required capacitance values are
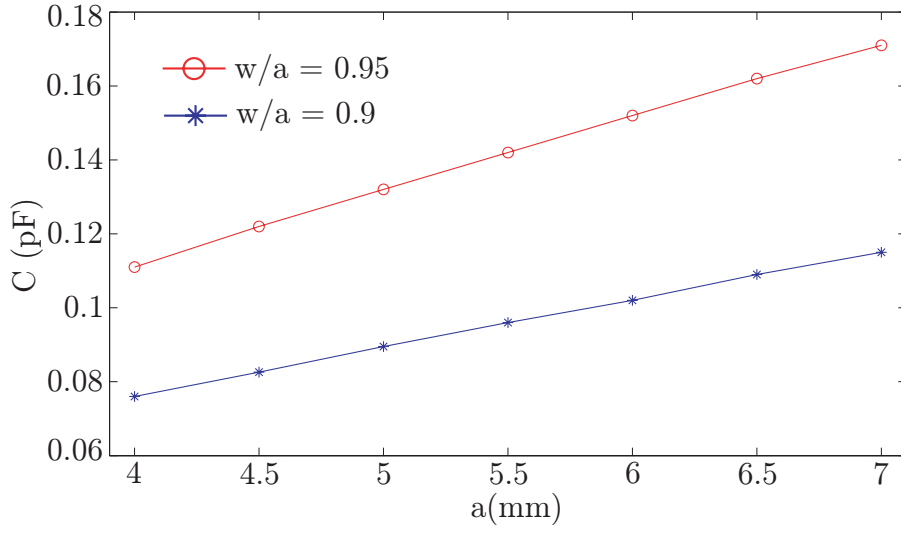
Figure 4: The capacitance of the equivalent circuit as a function of periodicity for fixed values of width to periodicity ratios. The square patch is lossless and $\epsilon_c = 2.3$, $T_c = 0.2\,\text{mm}$.
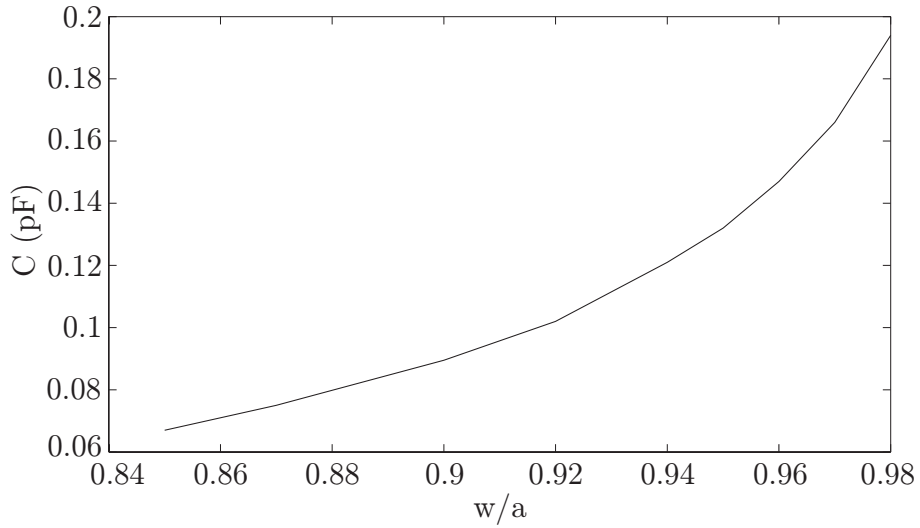
Figure 5: The capacitance of the equivalent circuit as a function of width to periodicity ratios. The square patch is lossless, the periodicity is $a = 5\,\text{mm}$ and $\epsilon_c = 2.3$, $T_c = 0.2\,\text{mm}$.

| Design ID	$C_1(\mathrm{pF})$	$C_2(\mathrm{pF})$	$C_3(\mathrm{pF})$	$C_4(\mathrm{pF})$
Intermediate	–	0.213	0.103	0.04
The Optimal	0.5	0.083	0.0316	0.024

Table 5: The capacitances of the series RC circuits used in the circuit models of the capacitive circuit absorbers.

$d_1(\mathrm{mm})$	$d_2(\mathrm{mm})$	$d_3(\mathrm{mm})$	$d_4(\mathrm{mm})$
4.2	3.4	3.2	3.3

Table 6: The thicknesses of the dielectric layers (permittivity equals one for all) for the optimal absorbers with different spatial periods (The thickness of the dielectric cover is not listed).

extremely difficult. The values of the capacitances for the circuit model of the optimal absorber are tabulated in Table 5 together with the values for the previous design, to facilitate comparison. As seen from Table 5 the capacitance values for the optimal design vary significantly from layer to layer. The contrast between the large and small values is very high ($C_1/C_4 \approx 20.8$) and it is impossible to synthesize such noticeably different values of capacitances with a single spatial periodicity. The only way to proceed is to utilize different periods for the square patch layers. As explained earlier, it is impractical to have arbitrary values of spatial periodicity for the square patches, since the noncommensurate periods problem is very difficult to analysis. Fortunately, it is possible to achieve the optimal performance by the aid of periodic arrays with commensurate periods. Then it is possible to define a finite global periodicity for the whole absorber structure. This global periodicity is used as the dimension of the unit cell in the simulation of the electromagnetic analysis of the absorber.

The frequency response and the schematic of the optimal design are shown in Fig. 6. The absorption bandwidth of the absorber covers the frequency interval 3.26–34.65 GHz with a relative bandwidth of $f_\mathrm{H}/f_\mathrm{L} \simeq 10.63$. The total thickness of the absorber is 14.5 mm, which is only 0.1 mm (0.7%) larger than the minimum possible thickness. The thickness of the dielectric layers and the dimensions of the periodic square patch elements and the resistivity of the sheets that they are made of, are all tabulated in Tables 6 and 7. The values of the spatial periods of Table 7 show that the periodicity of the fourth layer ($a_4 = 1.7$ mm) is half of the third layer periodicity ($a_3 = 3.4$ mm) and similarly the spatial period of third square patch layer is half of the periods of the two first layers ($a_2 = a_1 = 6.8$ mm). In this way the periodicity of the first two layers ($a_2 = a_1 = 6.8$ mm) becomes the global period of the whole absorber and can be used as the dimension of the unit cell for the simulation of the absorber. In the simulation with a commercial software functioning based on unit cell concept (such as CTS Microwave Studio which is used throughout this paper), the absorber looks like a multi-scale structure. The geometry of different periodic layers of the square patch arrays are shown in Fig. 7.
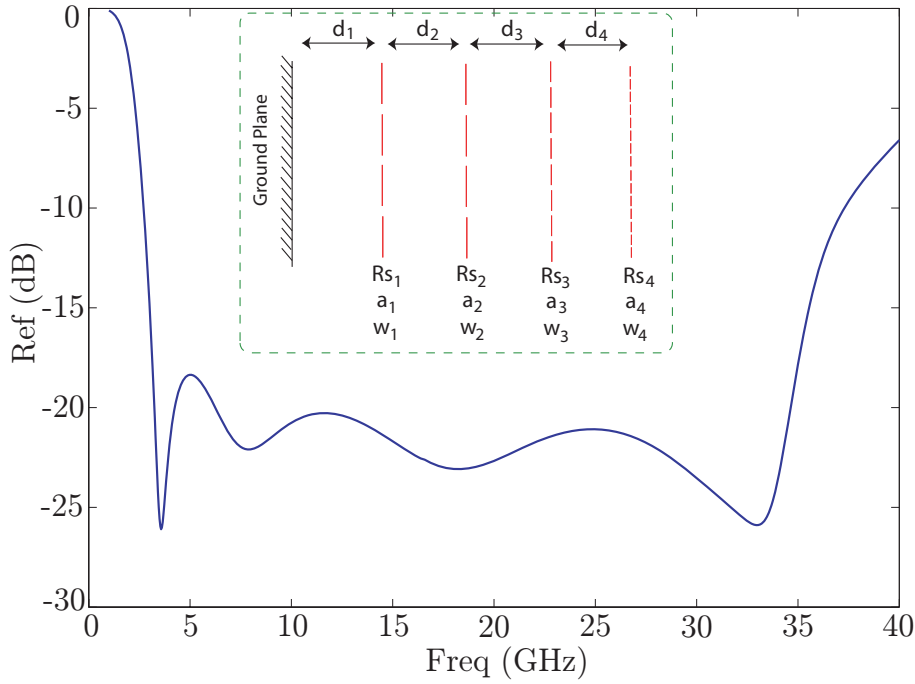
Figure 6: The frequency response and the schematic of the ultra wideband absorber with optimal thickness. The resistive layers are periodic square patches with periodicity (a), width (w) and sheet resistivity (R_s). The 20 dB absorption bandwidth 3.26–34.65 GHz, ($f_H/f_L \simeq 10.63$). The first resistive layer is embedded in the middle of a dielectric cover not shown in the figure ($T_c = 0.2$ mm and $\epsilon_r = 4.4$).

$a_1(\text{mm})$	$a_2(\text{mm})$	$a_3(\text{mm})$	$a_4(\text{mm})$
6.8	6.8	3.4	1.7
$w_1(\text{mm})$	$w_2(\text{mm})$	$w_3(\text{mm})$	$w_4(\text{mm})$
6.7	6.3	3	1.6
$R_{s1}(\Omega/Sq)$	$R_{s2}(\Omega/Sq)$	$R_{s3}(\Omega/Sq)$	$R_{s4}(\Omega/Sq)$
112.5	212	387.2	823.9

Table 7: The characteristic of the square patches (periodicity (a), width (w) and the resistivity (R_s)) used in the optimal absorber.
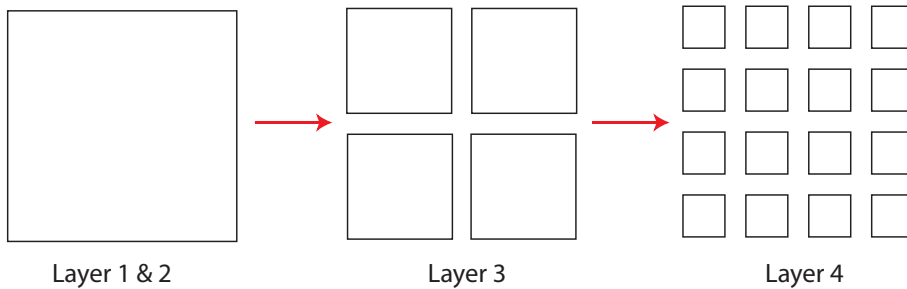
Figure 7: The multi-scale pattern of the periodic layers used in the unit cell of the optimal absorber. Global period is 6.8 mm.

4 Conclusion

The design of ultra wideband absorber with optimal thickness is investigated. It has been claimed by Munk that bandwidth ratios of 10:1 are possible with circuit analog absorber approach without any extra information about the designs [16]. It is explained that achieving such bandwidths with circuit analog absorber method is too complicated and do not result in optimal designs even if one can overcome all the difficulties of working with band-stop resonating FSS elements. Resonating frequency selective surfaces show complicated frequency responses on a bandwidth scale of 10:1 due to the harmonic and anti-resonance frequencies. Providing a circuit model for such periodic arrays considering all these effects is complicated. This paper shows the possibility of achieving the optimal performance with a simpler approach i.e. the capacitive circuit absorber [9].

It is shown that if there is no constraint on the total thickness of the absorber achieving large bandwidths is not a difficult task. The challenge is to design an absorber with optimal thickness for an ultra wideband frequency interval. For a nonmagnetic absorber that consists of linear, time invariant, causal and passive materials the total thickness of the absorber cannot be less than a theoretical limit for a given frequency response. This minimum possible thickness is the best criteria to judge the efficiency of a design method.

An ultra wideband absorber with optimal thickness is designed in three different steps. In this way, the design techniques required to achieve the optimal solution can be illustrated more easily by comparison of the designs steps. It is shown that a Jaumann absorber with many resistive sheets is the simplest way of obtaining large bandwidths. The only problem is that the absorber total thickness is more than twice of the optimal thickness. The next step in the development of the optimal absorber is to replace homogenous resistive sheets of the Jaumann absorber with low-pass periodic square patch arrays. For the intermediate level, similar spatial periodicity is considered for the periodic arrays. It is shown that with fewer resistive layers, compared to the Jaumann absorber, the desired absorption bandwidth can be achieved. The total thickness is reduced significantly but is still far from the optimal value (26.4% larger). It is explained that the only way to achieve the optimal absorber is to utilize different spatial periods for the low-pass FSS layers. It is impractical to have arbitrary periods for the layers because the analysis of the absorber becomes very complicated if not untractable. Fortunately, the optimal absorber can be designed by the practical case of commensurate periods. An optimal absorber is designed with 20 dB absorption bandwidth 3.26–34.65 GHz ($f_H/f_L \simeq$ 10.63) and total thickness of 14.5 mm which is only 0.7% thicker than the theoretical limit.

Acknowledgment

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