Stored Electromagnetic Energy in Bi-isotropic media

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Causal Natural Modes, Retro-causal Natural Modes and Minimum Radiation Modes

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Abstract— In this talk, we discuss three kinds of modes that are crucial to the understanding of a time domain electromagnetic (EM) response. The first targeted mode is called "causal" natural mode. This mode and its corresponding "causal" natural frequency are widely used in the analysis of the transient response of a radiating structure, for example, in the Singularity Expansion Method [1]. They are solutions to the non-standard eigenvalue problem of, e.g., for a microwave antenna, a surface integral equation. In this case, the integral equation only involves the retarded scalar and vector potentials and as a result the electric field at a given time instant t is solely due to an oscillating current in the past. Moreover, since the EM field equations are invariant under time-reversal symmetry, we can reverse the time axis and construct a retro-causal problem for the conventional integral equation. Then, similar to the "causal" natural mode, retro-causal natural modes can be defined when the integral equation is set free of driving force. Last but not the least, we can combine the causal and retro-causal cases to build a new "radiation field" operator. It will be proven that this operator is closely related to the concept of radiated energy [2, 3], and its modes may be defined as "minimum radiation" modes, i.e., where radiated energy and absorbed energy perfectly cancel each other.

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Stored Electromagnetic Energy in Bi-isotropic Media

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Abstract— Characterizing stored electromagnetic energy is essential in antenna Q calculation [1]. Understanding stored electromagnetic energy as a physical quantity is thus instrumental in characterizing antenna limitations. There exists several expressions for stored electromagnetic energy. In general these expressions give physical results in loss less isotropic media. However, in more complex background materials, such as dispersive or lossy media, these expressions break down, predicting negative energies [2].

This contribution investigates the results of existing stored energy expressions when applied to bi-isotropic media. This is done by implementing the bi-isotropic greens function [3] in the electrical field integral equation in a method of moments code. Stored energy in different types of small antennas will be examined and compared to the stored energy in their equivalent circuit models. The circuit models are generated by Brune synthesis.

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On the Properties of Stored Electromagnetic Energy

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Abstract— A consistent definition of stored electromagnetic energy for non-stationary fields is one of the last fundamental and still unsolved problems of classical electrodynamics. One of the key issues is the potentially infinite total energy within a time-harmonic steady state [1]. There is an ill-defined separation of the total energy into radiated energy and stored energy, where it is assumed that the bearer of the infinity is the radiation [2].

The radiation energy is commonly subtracted separately in the region exterior and interior to the radiator, the boundary being defined by the smallest circumscribing sphere. Many methods are known for determining the radiated (stored) energy outside the circumscribing sphere. All these methods yield qualitatively similar results, although quantitative differences can be found. The situation is more severe in the interior region, since only a few methods are able to take effectively into account the actual shape of the radiator [3, 4]. The differences between the predictions of these methods are moreover qualitative [5].

Besides subtraction of the radiated energy, there are also other techniques for estimating the stored energy, the vast majority of which are inspired by circuit theory [6–8]. However, no matter what concept is used, several fundamental principles should always be kept in mind: e.g., the energy must be positively semi-definite, must be coordinate-independent, must be gauge invariant, and it should be possible to define it locally. Clearly, all the concepts of the stored energy for a dynamic field must also be coherent with those for a static field.

In our talk, we will review all the available concepts that have attempted to determine stored energy. It will be pointed out that all these concepts fail in at least one of the "must have" properties mentioned above. Finally, it will be concluded that no fully consistent definition of stored electromagnetic energy is yet known. This of course raises the question whether the very idea of stored (and radiated) energy is well-posed [9].

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A Surface Integral Expression for the Electromagnetic Energy in a Microwave Cavity

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Abstract— Today, all high energy particle accelerators use microwave cavities for particle acceleration by means of eigenfields, excited by external sources. In the design process of such cavities it is important to evaluate these eigenfields and the corresponding resonance frequencies. Numerical accuracy is crucial since even small errors in the evaluations may cause a deterioration of the performance of the accelerator. It is convenient, and sometimes necessary, to use normalized electric eigenfields $\mathbf{E}_n(\mathbf{r})$ such that

$$\int_V |\mathbf{E}_n(\mathbf{r})|^2 \, \mathrm{d}V = 1 \tag{1}$$

In [1] we developed a high-order convergent Fourier-Nyström scheme for the magnetic field integral equation (MFIE) that can determine the eigenfields and their resonance frequencies with very high accuracy. Since MFIE is a surface integral equation it is very costly to evaluate the volume integral in (1). For this reason we derived a surface integral expression for the normalization integral that is less expensive to evaluate. The integral is expressed in terms of the magnetic vector potential and the scalar electric potential. In this contribution we present an efficient numerical method for the numerical evaluation of the expression for some different types of cavities. We also apply the surface integral expression to the exterior problems of radiation from antennas and scattering of waves from perfectly conducting objects and give an interpretation of the energy it then corresponds to.

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Estimating Antenna Q-factor from the MoM Impedance Matrix

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Abstract— The antenna Q-factor is a useful figure of merit that estimates the antennas bandwidth [1, 2]. One difficult part of determining the antenna Q-factor is to calculate the stored energy of an antenna. A method that can be applied to any small antenna is presented in [3–6]. Where the Q-factor is computed using an in-house solver based on a custom method of moments (MoM) implementation. As this requires writing custom code, it is practical to take advantage of the widely available commercial electromagnetic software that have already implemented the MoM and have additional variety of post-processing capabilities.

In this paper the antenna Q-factor is computed from the numerical frequency derivative of the MoM impedance matrix. The impedance matrix is obtained from the commercial software FEKO [7]. Different Q-factor formulations are presented and used to estimate the Q-factor of antennas. These are then compared with the Q-factor calculated from the derivative of the input impedance [8], and the stored energies.

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