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GENERAL COMPLEMENTARITY AND THE DOUBLE-PRISM EXPERIMENT

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Contents

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Abstract. The interesting double–prism experiment of Ghose, Home, and Agarwal 5) has been suggested 5, 8, 16, 7, 6) as a challenge to Bohr's views of complementarity.

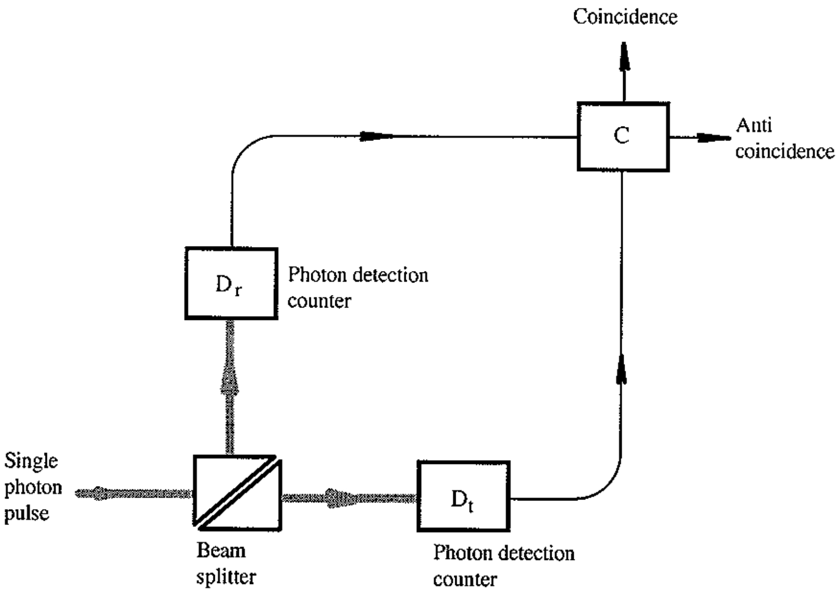
With the *linguistic complementarity* 14) as a general notion of complementarity, we analyze Bohr's views of complementarity. We find that a careful formulation of Bohr's *wave-particle complementarity*, with an explicit characterization of *measurability* as a *particular* low-level kind of quantum theoretical *inferribility*, is in fact not confronted by the double–prism experiment. Much less is the outcome a challenge to Bohr's *primary view of complementarity*, namely as a tension between definability and observability in a quantum mechanical observation language.

This primary view of Bohr, which is visible already in his Como paper 1), is a general formulation of quantum mechanical complementarity. Although it connects well to the subsequent metamathematical development of the wider linguistic complementarity, as a tension between describability and interpretability in a language, it is today surprisingly seldom referred to in quantum mechanical texts. Rather, discussions of the role of complementarity for the *interpretability problem* for quantum theory tend to focus on Bohr's view of complementarity in terms of *phenomena*, which he conceived in his later discussions with Einstein (and which appears less clear than Bohr's primary view of complementarity).

The hierarchical structuring of quantum theory which is suggested in our analysis of the double-prism experiment, and in the analysis of complementarity in terms of phenomena, is of interest also for the general interpretability problem for quantum theory, and for questions about a quantum mechanical reality.

1 *The Double–Prism Experiment and Claims Against the Wave-Particle Complementarity*

In the experiment, a double–prism is used as beam splitter, with a reflection path and a tunnelling path, respectively. A source is used which emits a *single photon state* of light. The prism gap is chosen such that if the beam is transmitted along the tunnelling path, which is indicated by a click in a counter in that path, then it must have wave nature — not preventing a possible simultaneous particle nature (this is an important point to which I will return). In the reflection path, there is another counter. Repeated runs indicate strict anticoincidence between the two counters, supporting the hypothesis that the behaviour of the emitted entities is particle like. The experiment supports the hypothesis that the emitted entities do have *both* wave *and* particle nature.



A single experimental arrangement to display *both classical wave and particle-like propagation* of single photon states of light. (Air gap less than wavelength of light.) [After Ghose and Home 6)]

Notice, however, that what is strictly *measured* by clicks in the tunnelling counter is not that something looks like a wave — the wave nature of the transmitted entity is inferred in a metalanguage to the measurement language for the experiment. The anticoincidence measurements, however, do well support, within the measurement language, the hypothesis that the emitted entities — at every measurement — have a particle like nature.

Although the experiment indicates a *simultaneous wave and particle* nature of the emitted entities, there is no direct confrontation of the "wave–particle complementarity" *if understood as preventing a simultaneous measurement* of characteristic wave and particle properties. Let us see how, in fact, the proposed challenges against the wave–particle complementarity use formulations that are indeed loose in reference to direct measurability.

Ghose and Home 6) formulate the opposition between the double–prism experiment and the wave–particle complementarity as follows:

> "in other words, the experimental data recorded in the three counters [of the above figure] contain both wave-like and particle-like information about the propagation of light pulses. The key element of 'mutual exclusiveness' in Bohr's Complementarity Principle is, thus, contradicted."

Notice that the *information* referred to here in fact results from an *inference* which is *not* neutral with respect to measurability of wave-likeness, but involves a metalevel quantum mechanical knowledge.

Ray and Home [18)] explain mutual exclusiveness as follows:

> "as regards Bohr's wave–particle 'complementarity' which implies mutual exclusiveness between classical wave and particle pictures in the sense that for a given experimental arrangement only one type of behavior, classical wave-like *or* particle-like, will manifest itself. In the context of interference–type experiments, the validity of 'mutual exclusiveness' is guaranteed in the formalism of quantum mechanics ..."

Again, this formulation, that a behaviour will become "manifest" in an experiment, does not refer to strict measurability, but allows more general inferences as well, whereby a non-qualified "complementarity" or "mutual exclusiveness" may be challenged. However, in experiments directly showing (and not only implying) interference, a wave-like property is actually measured.

In another article [8)] on the double–prism experiment, by Home and Gribbin, the authors make the following broad claim against Bohr's views on complementarity:

> "The notion of mutual exclusiveness of classical concepts such as wave and particle, or position and momentum is the key element in complementarity. We have concentrated on wave–particle duality, but if complementarity fails in one case then it fails as an overall description of the quantum world. There are different ways of looking at what complementarity 'really means'. The usual approach is to treat it pragmatically. Physicists exploit either the wave or the particle model of light as the situation demands. But this is simply learning to live with the dilemma, not resolving it."

2 *Bohr Complementarities in the Light of the General Linguistic Complementarity; Compatibility with the Double–Prism Experiment*

The proposed challenges of Bohr's complementarity concepts make it natural to reconsider Bohr's original arguments. With Bohr's interest in the role of language for the measurement problem in mind, we directly turn to the later developed linguistic complementarity, which provides a sound and general basis for an analysis and understanding of Bohr's proposals for complementarity concepts.

2.1 *The Linguistic Complementarity*

I had the opportunity to explain the linguistic complementarity at the previous conference, in 1992, here in Helsinki 14). In summary:

> *Language as a complementaristic phenomenon.* In general, complementarity refers to *wholistic* situations where fragmentation into parts does *not* succeed. In its complementaristic understanding, the phenomenon of language is a whole of description and interpretation processes, yet a whole which has no such parts expressible within itself. This constitutes a paradigm for complementarity, the *linguistic complementarity.* Any other known form of complementarity, from proposals from Bergson to Bohr, have been found 13) reducible to the linguistic complementarity, and the reductions themselves do provide an understanding of the complementarities. There are various related ways of looking at the linguistic complementarity:
>
> (i) as descriptional incompleteness: in no language, its interpretation process can be completely described in the language itself;
>
> (ii) as a tension between describability and interpretability within a language;
>
> (iii) as degrees of partiality of self–reference (introspection) within a language: complete self–reference within a language is impossible;
>
> (iv) as a principle of "nondetachability of language".

The linguistic complementarity has its roots in the role of any language, to admit communication or control. This requires the descriptions to be *finitely representable*, and it is from this condition that the linguistic complementarity stems 13).

By way of illustration of the tension view (ii), which will prove essential for the following comparative development, consider a programming language, where descriptions are programs for a "universal" Turing machine and interpretations are the corresponding computational behaviours (computation of partial recursive functions). Let us *increase* the *interpretability* by mowing from the partial recursive functions to the total recursive functions, which, unlike the partial recursive functions, are understandable according to a classical function concept, which makes the objects more easily interpretable. Then *describability* is *decreased* in the sense that we cannot any longer describe within the language (in terms of syntactic conditions on the programs) which programs will be interpretable (as total recursive functions).

The tension between describability and interpretability has a continuous nature for languages with a full denumerable stock of symbols. This is a consequence of a metamathematical result, an interpolation theorem, which I argue elsewhere 15).

View (iv), "the nondetachability of language", is another way of looking at view (iii). If the language in which we communicate and conceive could be isolated (detached) as a well described object, then it would be completely introspective. According to view (iii), this cannot be the case, and the nondetachability follows. The following quotation from Petersen 17) illustrates an early insight into the nondetachability aspect:

> "When it was objected that reality is more fundamental than language and lies beneath language, Bohr answered, 'We are suspended in language in such a way that we cannot say what is up and what is down'."

2.2 *Bohr's Primary View of Complementarity*

When Bohr first introduces the concept of complementarity in quantum physics, in his Como paper 1), he does so in a way which is perhaps more general than what the subsequent discussions of it seem to reflect and appreciate. I am thinking of his primary view of complementarity as a tension between definability and observability:

> "This situation [Planck's quantum of action] has far-reaching consequences. **[1]** On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. **[2]** But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time loose their immediate sense. **[3]** On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word.
>
> **[4]** The very nature of the quantum theory thus forces us to regard the space–time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively.
>
> **[5]** "Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a 'complementarity' theory the consistency of which can be judged only by weighing the possibilities of definition and observation."
>
> [The enumeration of Bohr's sentences is provided by us.]

Let us comment on these formulations from a general linguistic perspective.

[1] recognizes that definition (as does also description) requires fragmentation, or isolation, of an object to be defined (described) in the sense that the definition (description) process does not interfere with the object (cf the classical requirement of a definition to be circle–free).

[2] is the conclusion that the isolation requirement behind definability makes observation impossible because observation is associated with a quantum of action, i.e., an interaction which prevents isolation. Hence the concepts of space and time, which are associated with observation (we observe, or measure, in terms of space and time), loose sense.

[3] is essentially equivalent to [1] plus [2] by contraposition: observation prevents full definability. Since definition (as well as description) requires regularities (we always define and describe in terms regularities), nondefinability is associated with nonregularity, which Bohr qualifies as noncausality.

[4] states that space-time coordination, which is associated with observability, and the claim of causality, which is associated with definability, are complementary features "symbolizing the idealization of observation and definition respectively".

[5] is a clear and explicit quest for a complementarity theory, which explains the tension (possibility of weighing) between definability and observability (within an observation language) when the quantum postulate prevails.

As seen later on in the Como paper, Bohr's use of the expression 'weighing', in "weighing the possibilities of definition and observation", has an unmistakable continuous gradation. Compare page 582 of the Como paper 1).

With definability a special case of describability, and with observability a special case of interpretability, I have elsewhere 13, 14) argued that Bohr's primary view of complementarity (tension between definability and observability) is a particular case of the linguistic complementarity (tension between describability and interpretability within a language). Namely, for a quantum mechanical measurement language.

Whereas the linguistic complementarity is based on the general requirement of finite representability of descriptions, Bohr argues the tension aspect within the measurement language from the quantum of action. The nature of this quantum is such that it prevents introspective description (definition) of what really happens *within* the quantum jump.

However, the measurement language may well be embedded in in a *less constructive* language which allows inferences, like inductive ones, which are less secured in direct measurements. In such a language we may increase the introspectivity above that which is permitted when we remain within the measurement language. However, also the embedding language is subject to the linguistic complementarity, which means that the increase of descriptive introspection goes at the price of decreased interpretability. As we are about to see in section 3, this view is central for an understanding of the double–prism experiment.

2.3 *Bohrs View of Complementarity in Terms of Phenomena*

In his later writings, Bohr uses the concept of *phenomenon* to refer to a *systemic whole* of an atomic process in interaction with a measuring apparatus. This is for example seen in the following quotation from Bohr 2) (page 4).

> “While, within the scope of classical physics, the interaction between object and apparatus can be neglected or, if necessary, compensated for, in quantum physics this interaction thus forms an inseparable part of the phenomenon. Accordingly, the unambiguous account of proper quantum phenomena must, in principle, include a description of all relevant features of the experimental arrangement."

For example, in quantum physics a photon *per se* is not an appropriate object. But a “photon–investigated–in–an–interference–experiment” is, as a phenomenon (showing a wave-like feature), an appropriate quantum physical object. Again, a “photon–investigated–in–a–which–path–experiment” is to be regarded *another* phenomenon (showing a particle-like feature).

From a general point of view, Bohr’s procedure here, namely to resolve a problem in terms of a proposed indivisibility of a phenomenon, is natural and sound. It has been called a *complementaristic resolution* by Lindenberg and Oppenheim 9). It is hierarchical in nature, and depends on the establishment of the *unsolvability* of a problem within a certain domain (an *intradomain* problem), which is made do disappear in mowing to a wholistic conceptualisation.

We have a particular case of the complementaristic resolution in the double–slit experiment, without and with modifications for “which path” measurements. Here Bohr argued the impossibility of a simultaneous measuring of both wave and particle properties of a quantum object, the *wave–particle complementarity*, with reference to the Heisenberg uncertainty relation for position and momentum. Also de Broglie, although somewhat reluctant to Bohr’s concept of wave–particle complementarity, once argued 3) (page 83) an unsolvable intradomain problem with reference to Heisenberg’s uncertainty relations.

A main difference between Bohr’s primary view of complementarity, and his view of complementarity in terms of phenomena, is that the latter requires some specific demonstration of an unsolvable intradomain problem. Whereas the former is built around the quantum postulate as a generally accepted quantum mechanical principle. By way of example, the wave–particle complementarity may easily be criticized the way a particular cause of the required intradomain unsolvability can be criticized on general grounds.

Bohr’s reasoning around his concepts of complementarity does involve a conceptualization of language. Compare his plea for describing experimental results in “plain language” in order to secure “unambiguous description”. Today, when we know more of the complementaristic phenomenon of language, we can argue that not even for plain language, even with a physical vocabulary referring to classical physical experimentability, nonambiguity can be upheld.

Bohr’s plea for founding the understandings of quantum mechanics on direct inferences from measurements, may seem to reflect a physicist’s desire for clarity about what the quantum mechanical formalism means. This may be compared with a mathematician's desire for clarity about the meaning of a mathematical formalism, which is visible in constructive branches of mathematics like intuitionistic mathematics. In both cases, however, one finds that what really can be understood well this way (only allowing measurable inferences; only allowing constructive inferences) does not cover certain natural questions for understandability which can only be answered less constructively. Like questions of a quantum mechanical *reality*, and like questions into the mathematical *foundations*.

3 *Wave-And-Particle Hypotheses; Hierarchical Views*

Let us examine the way the double-prism experiment *supports* the hypothesis that the stimulating light has both wave like and particle like properties. We introduce the following notations, where T, R, W, P denote predicates which are formalizable in a quantum theory with the intended interpretations:

Tr: at run r the tunnelling counter clicks,
Rr: at run r the reflection counter clicks,
Wr: at run r the stimulating light has a wave-like property,
Pr: at run r the stimulating light has a particle-like property.

The quantum theory, denoted T, is supposed to be formulated on a sufficiently high level, above that of strict measurability, to contain the following theorems:

$$\vdash_T \; \forall r : Tr \Rightarrow Wr \qquad \text{(tunnelling is a wave phenomenon)} \tag{1}$$

$$\vdash_T \; \forall r : Rr \Rightarrow \; Wr \vee Pr \qquad \text{(reflection of waves as well as particles)} \tag{2}$$

$$\vdash_T \; \forall r : Rr \equiv \neg Tr \qquad \text{(anticoincidence)} \tag{3}$$

$$\vdash_T \; \forall r : Pr \qquad \text{(particle property at each run)} \tag{4}$$

$$\vdash_T \; \exists r \exists s : Rr \;\&\; Ts \qquad \text{(there are runs exhibiting reflection, and there are runs exhibiting tunnelling).} \tag{5}$$

Consider the hypothesis:

$$H_1 \;\equiv\; \forall r \, Wr \& Pr, \tag{6}$$

that the light, at every run of the experiment, has both wave-like and particle-like properties. Since $\not\vdash_T \forall r Wr$, we understand that H_1 is indeed *hypothetical* in T.

We now need reference to a theory 11) of *supports*. This theory is a strictly deductive part of the general knowledge of *inductive* processes. It contains the following definition.

> **Definition.** An observation B *supports* the hypothesis H in the theory T when:
>
> $$(\emptyset \neq) \;\; \mathrm{Inf}_{TB} H \subset \mathrm{Inf}_T H.$$

That is, support occurs when the *hypothetical content*, $\mathrm{Inf}_T H$, of H is *diminished* upon augmenting the background knowledge T to TB. (The hypothetical content of an hypothesis H is the linguistic information [15], $\mathrm{Inf}_T H$, generated by H with reference to T if H is regarded as a given piece of certain knowledge rather than as an hypothesis.)

> **Theorem.** $\exists s\, Rs \& Ws$ supports H_1 in T (meaning: a run s, in which the reflection counter clicks and the light has a wave-like property, gives a nonzero, positive support to the hypothesis H_1 in T). However, this support, although fulfilling Popper's requirement for observability, that of being existentially quantified, is not measurable in the double–prism experiment.

This is indicates a somewhat limited capacity of the double–prism experiment to provide *measurable* evidence for a wave–and–particle nature of the stimulating light. A stronger conclusion is obtained by considering the weaker wave–and–particle statement:

$$\exists s \quad Ws \& Ps. \tag{7}$$

In fact, (7) is really weak. It is not hypothetical at all. It is a theorem of T, namely a corollary of the T–theorems: $\exists s Ts$, the implication $Ts \Rightarrow Ws$ and $\forall r Pr$.

Let us consider the question whether the theorem (7) can be deduced in T strictly on the level of *measurable* inferences. I want to argue that (7) is not directly (constructively) inferrible from the T– and R–measurements in the experiment, but uses less constructive T–inferences. Thereby the experiment does not contradict Bohr's wave–particle complementarity in its *constructivist* formulation, preventing a simultaneous *strict measurement* of characteristic wave and particle properties.

Let us examine the above theorem (1), $\forall r : Tr \Rightarrow Wr$, which is crucial in the derivation of theorem (7). It says that tunnelling in the double–prism experiment is a wave phenomenon. How is this to be understood? Notice that Ghose, Home, and Agarwal [7] consider tunnelling an "exclusive" wave property:

> "The interesting aspect of this experiment is that although tunnelling is exclusively a wave phenomenon, single photon states can also tunnel. At the same time, perfect anticoincidence between the two detectors implies particle-like propagation."

That must obviously not be understood so that Tr implies Wr and not Pr, for if it did, we would not have the T–theorems $\forall r : Pr$ and $\exists s : Ws \& Ps$, and there would be no argument against the wave–particle complementarity in the first place.

Rather, I want to understand theorem (1) as follows. There are indeed wave-explanations of tunnelling, like the classical explanation in the Feynman Lectures [4], and the quantum mechanical explanation by Ghose, Home, and Agarwal 5). But wave–explanations do not rule out the possibility of also other types of explanations, unless we argue that *every* explanation (model) of tunnelling is a wave model (not forbidding simultaneous particle properties). Such, usually inductive, arguments may well be given, but are on a level *above* strict measurability. The argument 7) that tunnelling disappears when the prism gap is larger than the wavelength of the light, is only a *support* to the hypothesis that tunnelling necessitates wave appearance. That hypothesis may well be inductively confirmed as a theorem on the higher levels of T. (Compare the support–definition; from the hypothesis that tunnelling is a wave phenomenon, we can conclude that the prism gap must be smaller than the wavelength; this observable fact thus supports the hypothesis which is then *inductively* established as a theorem in T.)

In inferences on a given level of constructivity, like the deductive inferences on the level (ν) of measurability, information never increases 15):

$$A \vdash_{T^\nu} B \quad \Rightarrow \quad \mathrm{Inf}_{T^\nu} B \subseteq \mathrm{Inf}_{T^\nu} A.$$

Only in jumping from one level to another, the information of a given sentence may jump too.

Since theorem (1) is not available on the level of strict measurability within T, the information of a single click in the tunnelling counter is here too small to yield the larger information needed to characterize a wave phenomenon.

The conclusion is that the double–prism experiment does not challenge the wave–particle complementarity in its constructivist understanding in terms of measurements. Rather, the experiment is interesting in that it calls for a levelled approach to quantum theory.

In general, being a wave, and being a particle, are very complex notions for which we may have intuitive ideas but not complete descriptions. To be a particle, in the sense of being a whole which is not fragmentable in parts, is of course difficult to describe, because we do describe in terms of fragmentation. Much less is it measurable (measurability is a certain constructive restriction on describability).

But specific properties of being a particle, and of being a wave, may well be individually measurable. Compare for example the photon number operator, where the eigenvalue unity measures a kind of individuality associated with being a particle. In order to establish a "wave–particle complementarity" one must not focus on too weak wave and particle properties, for which no unsolvable intradomain problem may exist. Again, one must not make the considered properties too strong. That might prevent also their individual measurability.

4 *Linguistic Understanding of Wave–and–Particle Phenomena*

Previously $^{14)}$ we have argued that quantum theory, with its characteristic inclusion of the measurement process within its domain, has linguistic models. The reason is simply that measurement, or observation, *is* a linguistic phenomenon, just like any other phenomenon of *reference*, or even self–reference.

Let us see to what extent the linguistic perspective is also suggestive for how to understand a wave–and–particle phenomenon. In particular we then refer to the tension view (ii) of the linguistic complementarity.

In an experiment that emphasizes high describability (definability) at the price of low interpretability (observability), a *wave-like* manifestation of a linguistic (wave–and–particle) object may appear. Like an uninterpreted description, or a description that cannot be interpreted in the actual language. Compare the ease with which a wave phenomenon can be mathematically described, and the difficulty we experience to understand what the formulas really mean.

In an experiment that emphasizes high interpretability (observability) at the price of low describability (definability), a *particle-like* manifestation of a linguistic (wave–and–particle) object may appear. Like a nondescribed interpretation, or an object which has no description in the language. Compare the difficulty of describing a particle (if considered a whole without parts), and the ease with which its individuality can be observed (in terms of anticoincidence measurements in a beam splitting experiment).

Of course, there is nothing in the linguistic view which suggest just these physical manifestations (wave; particle) of the extremal appearances of a linguistic object (description with low interpretability; interpretation with low describability). The other way around, the linguistic complementarity, as a general form of complementarity, may well provide a fundamental insight into the nature of wave–and–particle objects as existing beyond constructive (measurable) accessibility.

The possibilities of increasing linguistic introspection (cf view iii of the linguistic complementarity) are suggestive for how to extend (not necessarily monotonically) a theory to describe further parts of its intended interpretation. In particular, I tend to understand Vigier's impressive study $^{19)}$ as such an attempt at high describability of wave-and-particle objects. By the linguistic complementarity, the increased syntactical complexity, however, generates new (nondescribed) interpretation problems.

Laurikainen's approach (compare this volume) to increase interpretability at the price of describability, by involving nondescribed psychological processes for the interpretation of quantum theory, is also of interest as a case of the general linguistic view of the interpretability problem for quantum mechanics.

The hierarchical structuring of quantum theory which we have suggested in our analysis of the double-prism experiment, and in the analysis of complementarity in terms of phenomena, is of interest also for the general interpretability problem for quantum theory, and for questions about a quantum mechanical reality. We look upon the approach of this paper as a first step towards a general understanding of how hierarchical views are being generated from general linguistic reduction concepts (compare the way degrees of noncomputability are generated from Turing-reduction; compare our wider study 10, 12) of reduction concepts).

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