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Brief Paper

A Robust Sampled Regulator for Stable Systems with Monotone Step Responses*

K. J. ÅSTRÖM†

Key Words—Digital control; sampled data systems; system order reduction; stability; robustness.

Abstract—It is shown that a discrete time integrating regulator can be designed based on a strongly simplified model. The regulator has only two tuning parameters which are easy to choose from the step response of the open loop system. The problem is a prototype for a general robustness problem.

1. *Introduction*

It is useful to have simple robust methods for solving simple problems. This paper gives a very simple way of designing a robust digital regulator for stable linear single-input single-output systems with positive impulse responses. The basic idea is that a regulator is designed for a strongly simplified model of the process. The regulator obtained has two tuning parameters, a gain and the sampling period. It is shown that the regulator works for a given class of problems provided that a simple condition is satisfied. Linear time-invariant systems with monotone step responses occur commonly in the process industries. Typical examples are systems which describe flows, levels, concentrations, and temperatures. See e.g. Åström (1976). The regulator obtained has integral action. The settling time of the closed loop system is larger than the settling time of the open loop system.

2. *The algorithm*

Consider a stable system with a monotone step response H. See Fig. 1. The system is approximated with the following crude model

$$y(t) = bu(t-T),\ b>0. \tag{1}$$

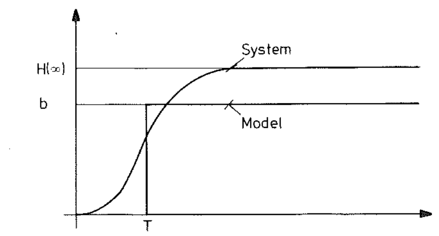

FIG. 1. Step responses of the system and the approximating model.

This model is characterized by two parameters, the gain b and the time delay T. There are many ways to design a simple regulator for the system (1). One possibility is to use a sampled dead-beat regulator with integral action. Such a regulator is obtained as follows. Choose T as the sampling period and let y_k denote the value of the signal y at time kT.

Equation (1) then gives

$$y_k - y_{k-1} = b[u_{k-1} - u_{k-2}]$$

Require that y should equal the command signal y^0 after a delay of one sampling period. The following control law is then obtained

$$u_k = [y_k^0 - y_k]/b + u_{k-1}. \tag{2}$$

This control law has two tuning parameters b and T. If the control (2) is applied to the system (1) it follows that

$$y_k = y_{k-1}^0$$

3. *Analysis*

It will now be investigated what happens when the sampled regulator (2), which is designed using the simplified model (1), is applied to a real system. The following result holds.

Theorem 1. Consider a stable time invariant linear system with monotone nonnegative step response. The closed loop obtained with the regulator (2) is always stable if

$$2H(T) > H(\infty) \tag{3}$$

and

$$2b > H(\infty). \tag{4}$$

Proof. Let h be the impulse response of the system. Then

$$y(t) = \int_0^\infty h(s)u(t-s)\mathrm{d}s.$$

Since the control signal u is constant over the sampling periods, it follows that

$$y_k = \left(\int_0^T h(s)\mathrm{d}s\right)u_{k-1} + \left(\int_T^{2T} h(s)\mathrm{d}s\right)u_{k-2} + \dots \tag{5}$$

The closed loop system is thus characterized by equations (2) and (5). Elimination of y between these equations gives

$$\sum_{n=1}^{\infty} a_n u_{k-n} = y_k^0/b$$

where

$$a_n = \begin{cases} 1 & n=0 \\ [H(T)-b]/b & n=1 \\ [H(nT)-H(nT-T)]/b & n>1. \end{cases}$$

It follows from a theorem of Wiener (1933) that the closed loop system is asymptotically stable if the function

$$A(z) = \sum_{n=0}^{\infty} a_n z^{-n}$$

has the property

$$\inf|A(z)| - 1 \quad \text{for } |z| \geq 1.$$

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See also Desoer and Vidyasagar (1975). Since

$$\left|\sum_{n=1}^{\infty} a_n z^{-n}\right| \leqq \sum_{n=1}^{\infty} |a_n| \quad \text{for} \quad |z| \geqq 1$$

the system is thus stable if

$$\sum_{n=1}^{\infty} |a_n| < 1.$$

Consider

$$\sum_{1}^{\infty} |a_n| = \left|\frac{H(T)-b}{b}\right| + \frac{H(2T)-H(T)}{b} + \frac{H(3T)-H(2T)}{b} + \dots$$

$$= \left|\frac{H(T)-b}{b}\right| + \frac{H(\infty)-H(T)}{b}.$$

Two different cases are considered separately. First assume that $H(T) \geqq b$. It then follows from (4) that

$$\sum_{1}^{\infty} |a_n| = \frac{H(\infty)-b}{b} < 1.$$

Next, assume that $H(T) < b$. Then it follows from (3) that

$$\sum_{1}^{\infty} |a_n| = 1 + \frac{H(\infty)-2H(T)}{b} < 1$$

and the theorem is proven.

The bounds (3) and (4) can not be improved as is seen by the following counterexamples. Consider a system described by

$$y(t) = b_0 u(t - T_0).$$

To satisfy (3) choose $T_0 \leqq T$. At the sampling instants the closed loop system is then described by

$$u_k = (1 - b_0/b) u_{k-1}.$$

Since $b_0 = H(\infty)$ this equation becomes unstable if (4) is violated. Similarly, choose $b < b_0 < 2b$, which clearly satisfies (4). Assume that $T \leqq T_0 < 2T$, which implies that (3) does not hold. At the sampling instants the closed loop system is described by

$$u_k - u_{k-1} + (b_0/b) u_{k-2} = 0$$

which is unstable.

Based on the theorem the following simple design rule can be obtained for the regulator (2). Choose a sampling period such that $H(T) > H(\infty)/2$. Choose the integrator gain $1/b$ smaller than $2/H(\infty)$. It can be seen that the response time of the closed loop system will in general be slower than the response time of the open loop system. This is often acceptable in simple process control application.

The control law (2) can be interpreted as a special case of model algorithmic control (MAC). See Richalet *et al.* (1978) and Mehra and Rouani (1979). Theorem 1 can therefore also be interpreted as a robustness result for MAC. Another approach to robustness for MAC is given in Mehra and colleagues (1979). In MAC the process is modeled by an impulse response. It is therefore of interest to interpret the conditions of Theorem 1 in terms of impulse responses. The assumption that the step response is monotone implies that the impulse response is nonnegative. The model (1) implies that the process is modeled by an impulse at $t = T$. The inequality (3) implies that T is larger than the median of the impulse response and the inequality (4) implies that the area of the model impulse response is larger than half the area under the impulse response of the system.

4. *An example*

The design procedure is illustrated by an example. Consider a system with the transfer function

$$G(s) = \frac{1}{(s+1)^6}.$$

A step response of the process is shown in Fig. 2. It is seen from this step response that the condition (3) is satisfied for the sampling period $T = 7.5$. The parameter b was chosen to $b = 1.5$.

The response of the closed loop system to a step command signal and a step load disturbance at time 100 is shown in Fig. 3.

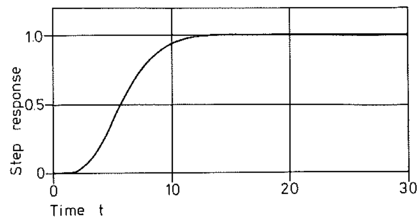

FIG. 2. Step response of the system.

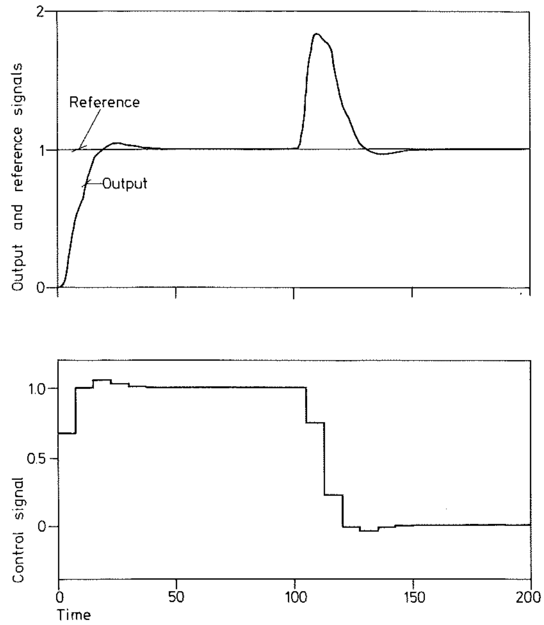

FIG. 3. Response of a system to a step change in the command signal and a unit step load disturbance at time 100.

5. *Conclusions*

It has been demonstrated in a simple case that good control can be obtained by designing a regulator for a simplified process model. It would be of interest to extend this result to other design procedures and other model classes. Results in this direction are given in Åström (1980).

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