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Phase Field Modelling of Bone Growth

What it is and what lead to it? Talk given to the memory of Carl Wilhelm Ossén. Orationem Meam.

Ståhle, P.

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Phase Field Modelling of Bone Growth

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What it is and what lead to it

by

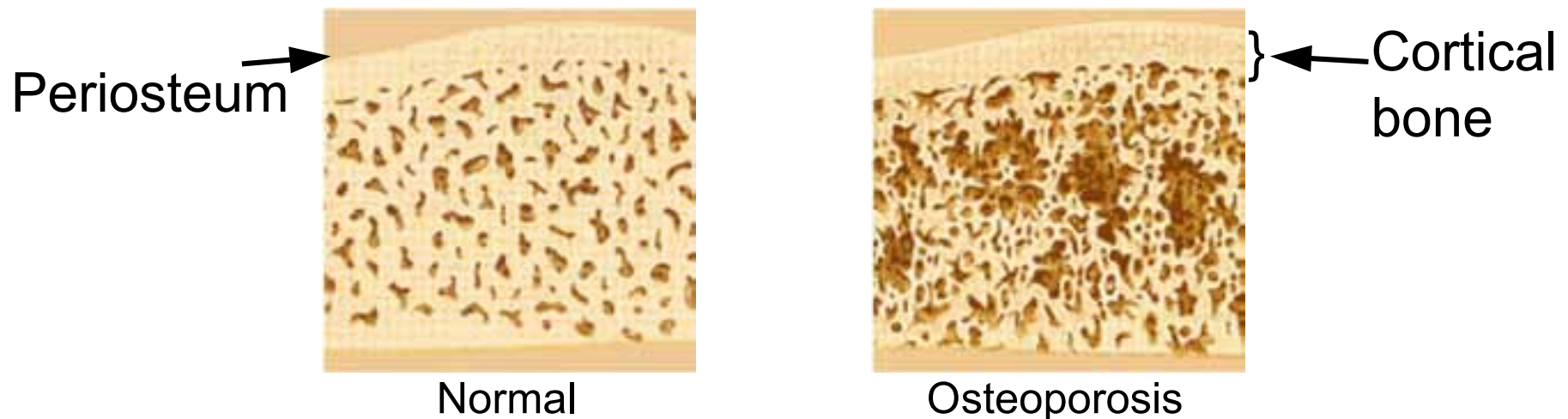
Gustav Lindberg, Per Ståhle, Leslie Banks-Sills, Abdallah Shokry, Wuregul Rehemann

Osteoclasts - dissolving bone

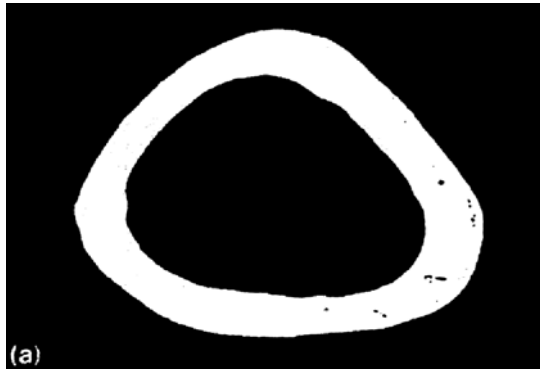
Osteoblasts - remodelling

Periosteum reactions: tearing · breaking · stretching

Osteomyelitis · malignant neoplasms · osteosarcoma ·
chondrosarcoma · fibrosarcoma · lymphoma · metastasis

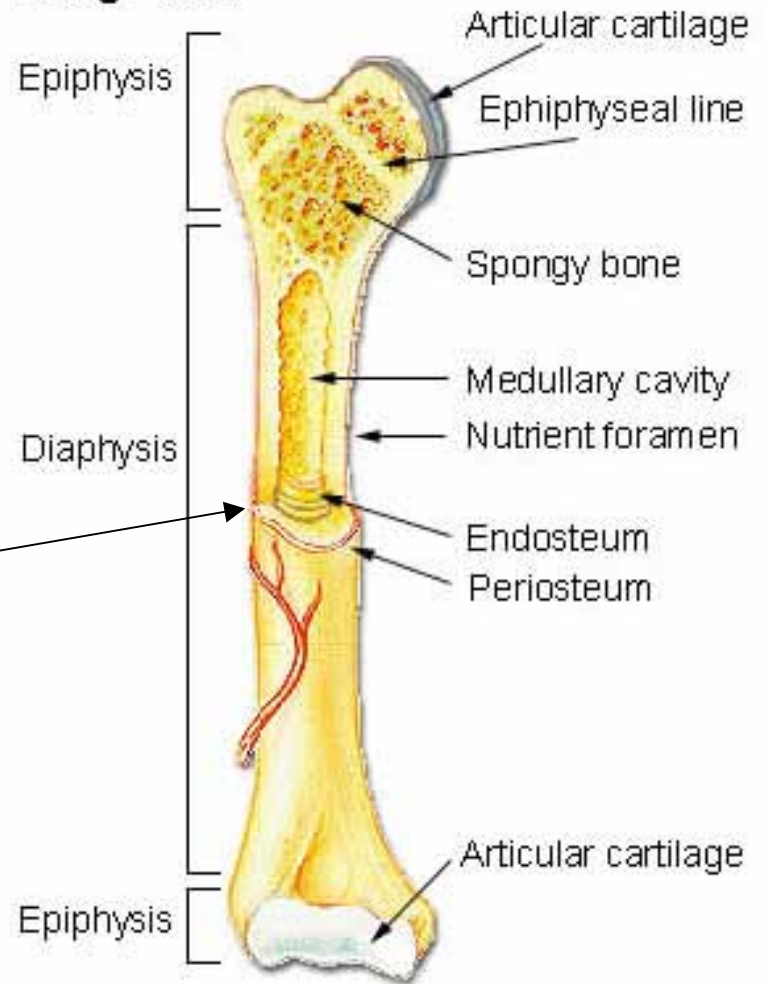


100.000 osteoporosis related fractures yearly in Sweden

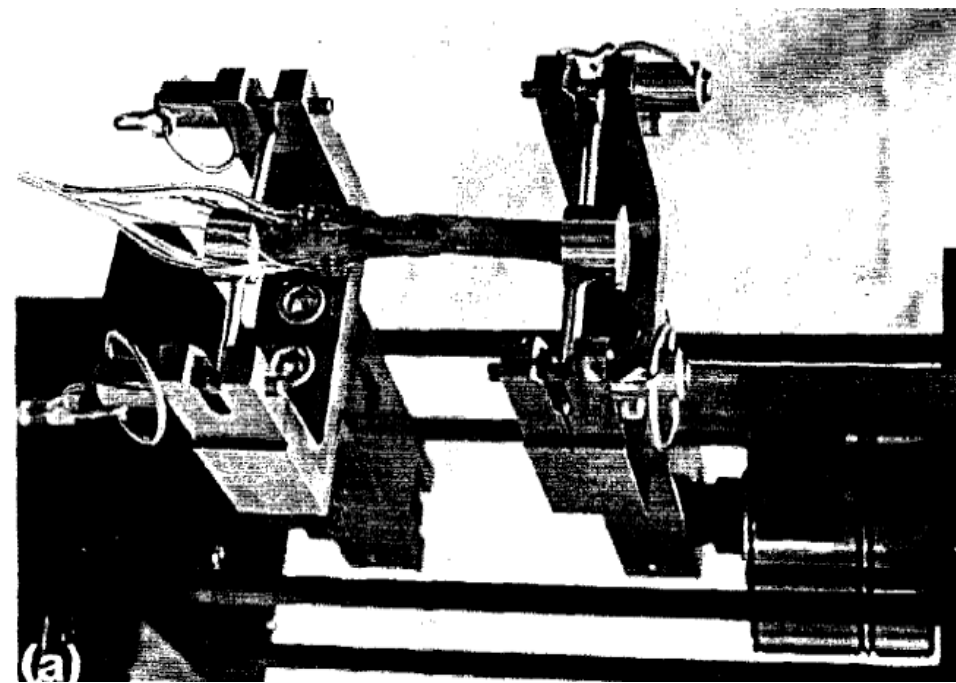
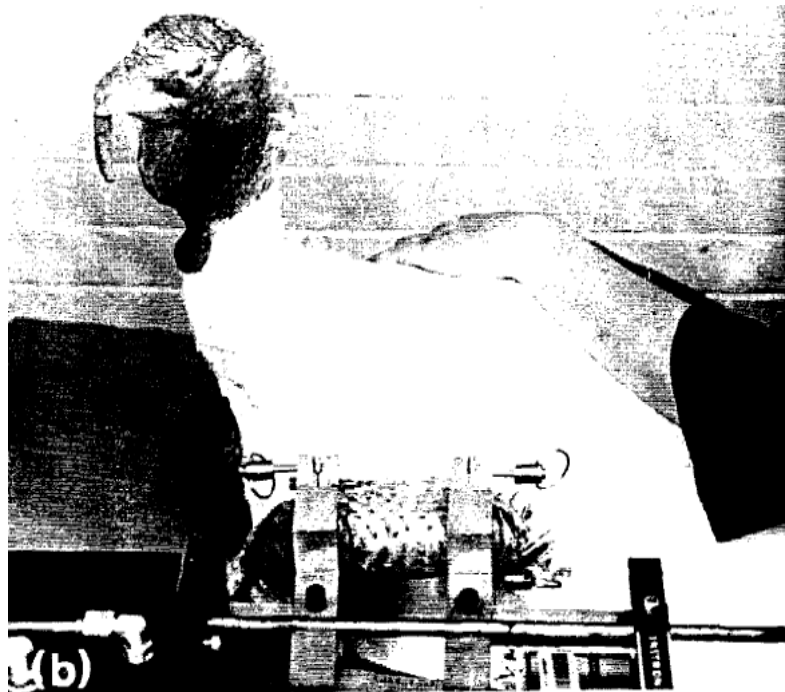


Bone from examined turkey

Long Bone

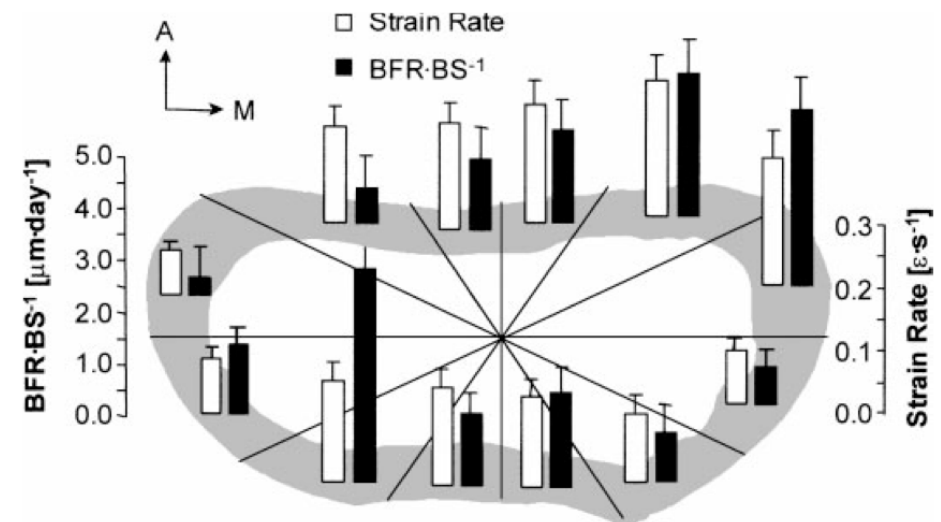
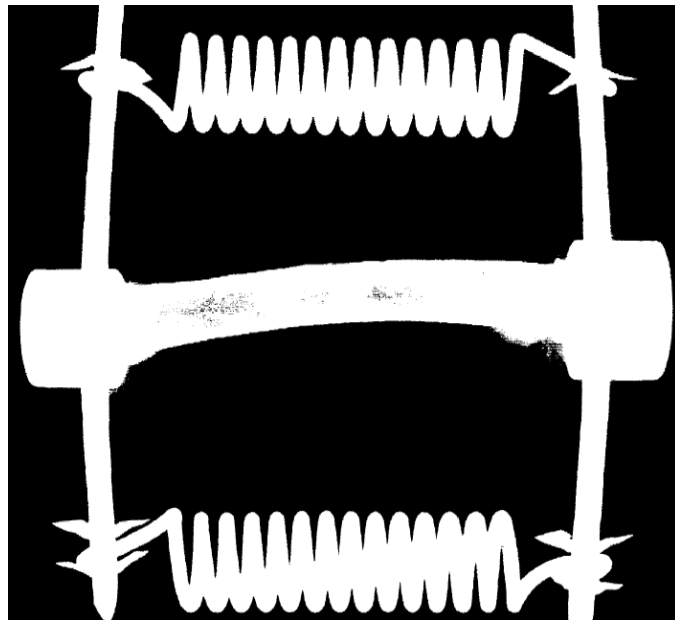


Observations of exercise stimulated bone growth




(Lanyon and Rubin, 84)

Observations of exercise stimulated bone growth



(Lanyon and Rubin, 84, Judex, Zernicke, 00)

Svante Arrhenius




Born Svante August Arrhenius
19 February 1859
Wik Castle, Sweden

Died 2 October 1927 (aged 68)
Stockholm, Sweden

Nationality Swedish

Alma mater Uppsala University
Stockholm University

Wilhelm Ostwald




Born Friedrich Wilhelm Ostwald
2 September 1853
Riga, Governorate of Livonia,
Russian Empire (now Latvia)

Died 4 April 1932 (aged 78)
Großbothen near Leipzig,
Germany

Nationality Baltic German by birth. Prussian,
German (after 1871)

Alma mater University of Dorpat

Lev Landau



Född 1908^[1]
Baku^[2]

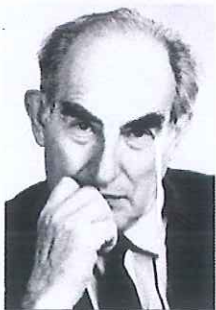
Död 1 april 1968^{[3][4]}
Moskva^[5]

Begravd Novodevitjkyrkogården

Nationalitet Kejsardömet Ryssland och
Sovjetunionen

Alma mater Sankt Petersburgs universitet,
Ioffe Institute och Baku State
University

Vitaly Ginzburg




Born Vitaly Lazarevich Ginzburg
4 October 1916
Moscow, Russian Empire

Died 8 November 2009 (aged 93)
Moscow, Russia

Nationality Russia

Alma mater Moscow State University

Lars Onsager



Born November 27, 1903
Kristiania (Oslo), Norway

Died October 5, 1976 (aged 72)
Coral Gables, Florida, U.S.

Residence United States

Nationality Norway, U.S.

Alma mater Yale University
Norwegian University of Science
and Technology

Erwin Schrödinger



Erwin Schrödinger det år han tilldelades Nobelpriset

Född 12 augusti 1887
Wien, Österrike-Ungern

Död 4 januari 1961 (73 år)
Wien, Österrike

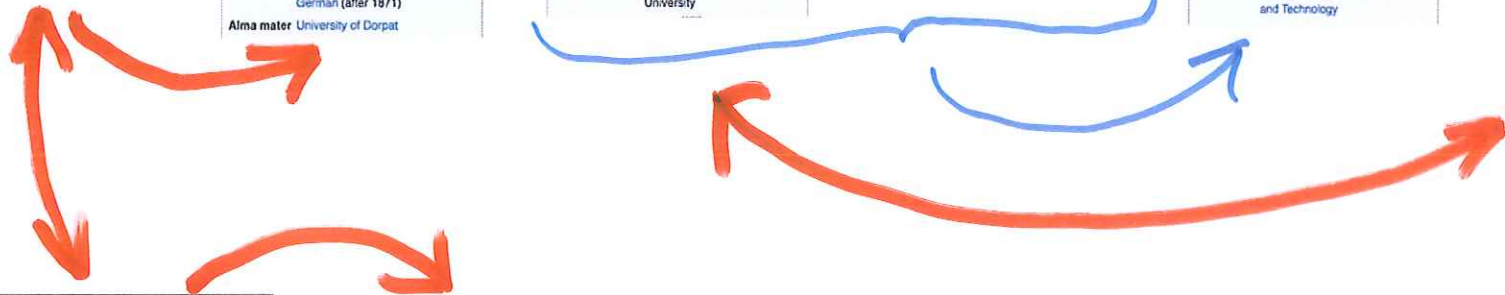
Medborgarskap Österrike, Tyskland, Irland

Nationalitet Österrikisk

Forskningsområde Fysik

Institutioner Breslaus universitet
Zürichs universitet
Humboldt-Universität zu Berlin
Oxfords universitet
Universität Graz
Dublin Institute for Advanced
Studies
Universität i Gent

Alma mater Wiens universitet



Carl Wilhelm Oseen



C.W. Oseen 1909 vid utnämningen till professor vid Uppsala universitet.

Född 17 april 1879
Lund

Död 7 november 1944 (65 år)
Uppsala^[1]

Begravd Uppsala gamla kyrkogård^[2]

Nationalitet Svensk

Alma mater Lunds universitet

Waloddi Weibull



Ingenjör

Ernst Hjalmar Waloddi Weibull, född 18 juni 1887 i Vittskövle, Kristianstads län, död 12 oktober 1979, var en svensk ingenjör, naturvetenskapsman och matematiker. Wikipedia

Född: 18 juni 1887, Sverige

Död: 12 oktober 1979, Annecy, Frankrike

Utbildning: Uppsala universitet

Vårt museibibl.
Handl. f. första Lundahållfaren

Lars Onsager



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Born November 27, 1903
Kristiania (Oslo), Norway

Died October 5, 1976 (aged 72)
Coral Gables, Florida, U.S.

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and Technology



Contributions to the free energy

$$\mathcal{F} = \mathcal{F}_{ch} + \mathcal{F}_{gr} + \mathcal{F}_{el} \quad (+\mathcal{F}_{heat} + \mathcal{F}_{grav} + \dots)$$

Elastic energy $\mathcal{F}_{el} = \int \sigma_{ij} d\epsilon_{ij}$

Chemical energy $\mathcal{F}_{ch} = U(\psi)$

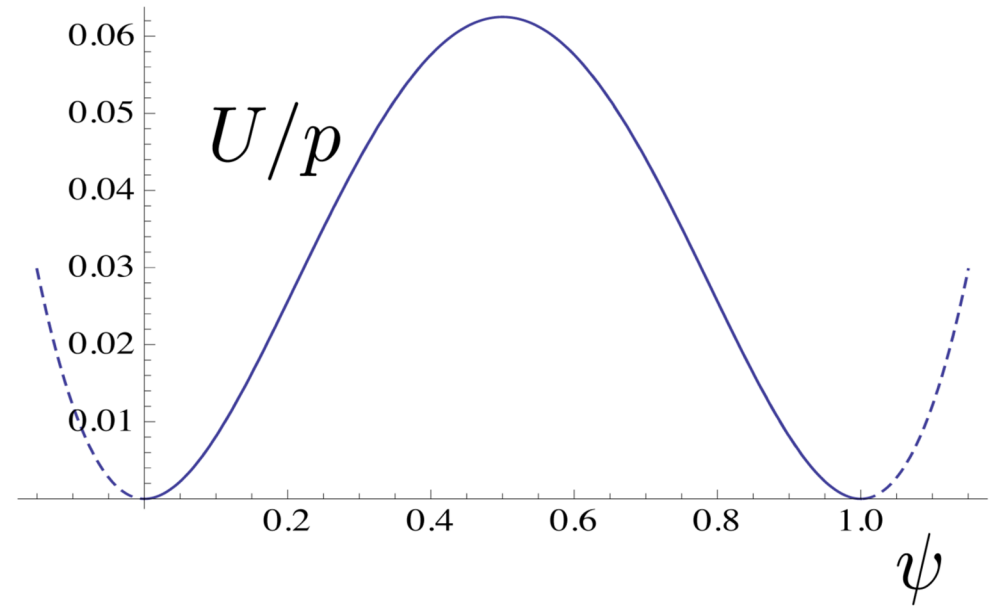
Gradient energy $\mathcal{F}_{gr} = \frac{g_r}{2} (\psi_{,i})^2$

Heat $\mathcal{F}_{heat} = \kappa \rho T$

Gravitation $\mathcal{F}_{grav} = \rho g u_2$

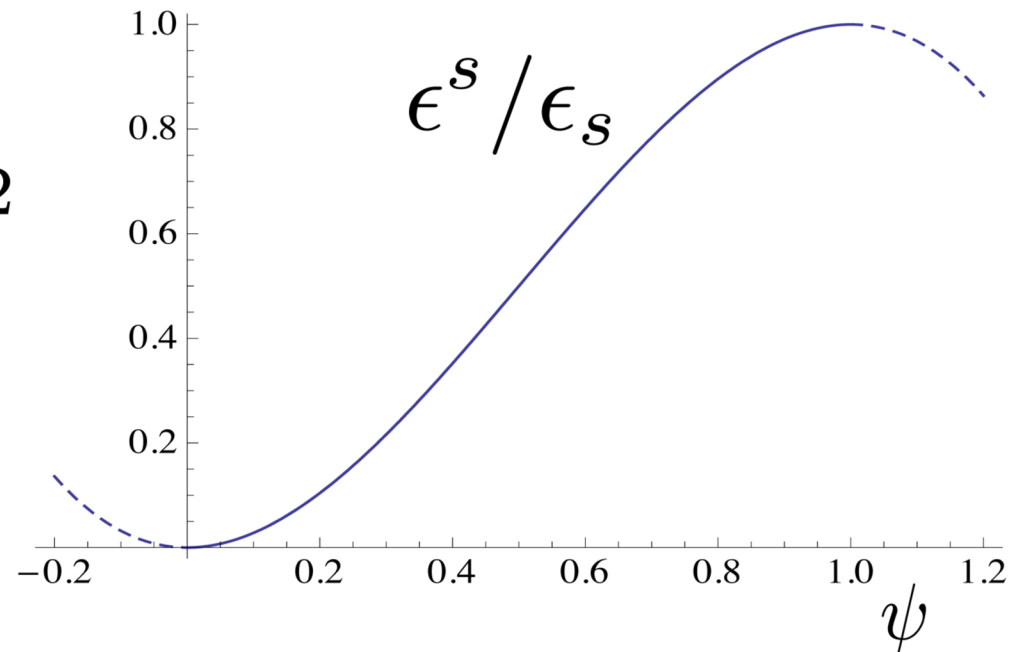
Double-well
chemical potential

$$U(\psi) = p \psi^2 (1 - \psi)^2$$



Expansion

$$\epsilon^s(\psi) = \epsilon_s (3 - 2\psi) \psi^2$$



Unknown: ψ, u_1, u_2, u_3

$$\text{Phase: } \frac{\partial \psi}{\partial t} = -L_\psi \left(\frac{\partial \mathcal{F}}{\partial \psi} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla \psi)} \right)$$

$$\text{Displ.: } \frac{\partial u_i}{\partial t} = -L_{u_i} \left(\frac{\partial \mathcal{F}}{\partial u_i} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla u_i)} \right)$$

Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \{3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1 - 2\psi)\} (1 - \psi)\psi$$

Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1 - 2\nu} \tilde{u}_{j,ij} = 2\tilde{\epsilon}_{ij,j}^p + \tilde{\epsilon}_{,i}^s$$

In analogy with a fully coupled thermal-stress

Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993),
Srolovitz (1989), Freund (1995), Kung-Suk (2000)

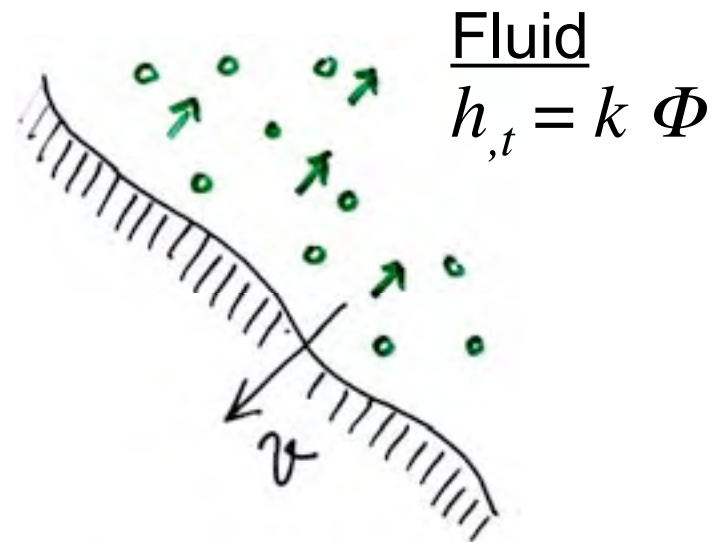
Chemical potential,

$$\Phi = U_c + U$$

U_c surface energy, U elastic strain energy

Surface diffusion

$$h_{,t} = D \nabla^2 \Phi$$



Governing equations:

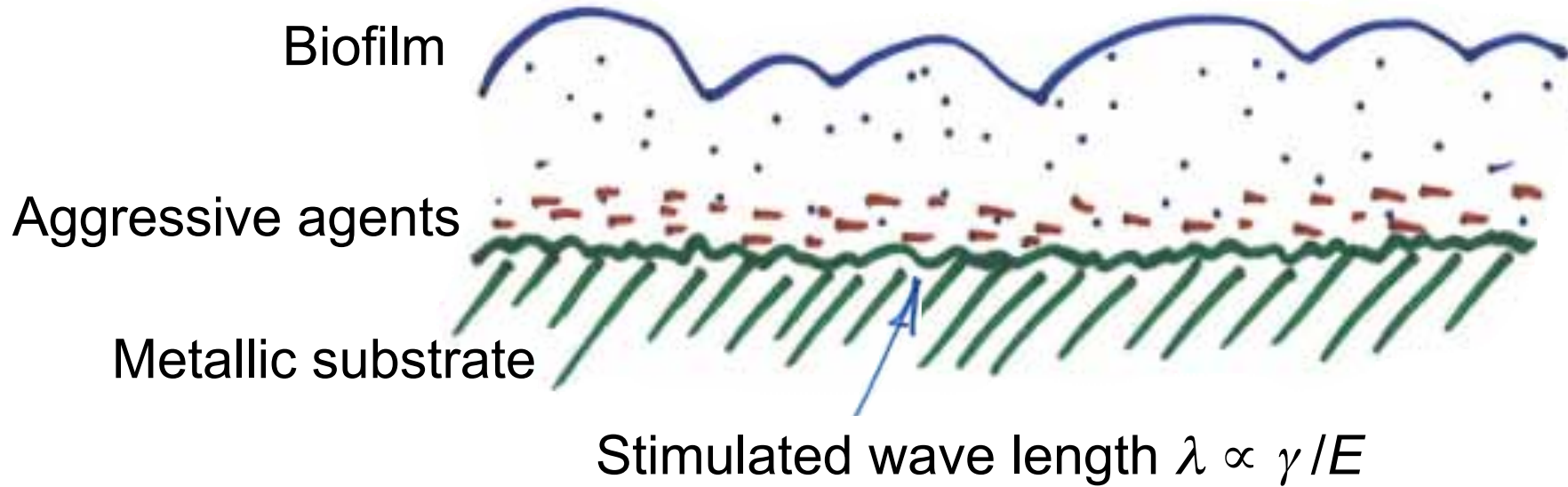
Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left(\gamma \frac{\partial^2 h}{\partial x^2} - \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

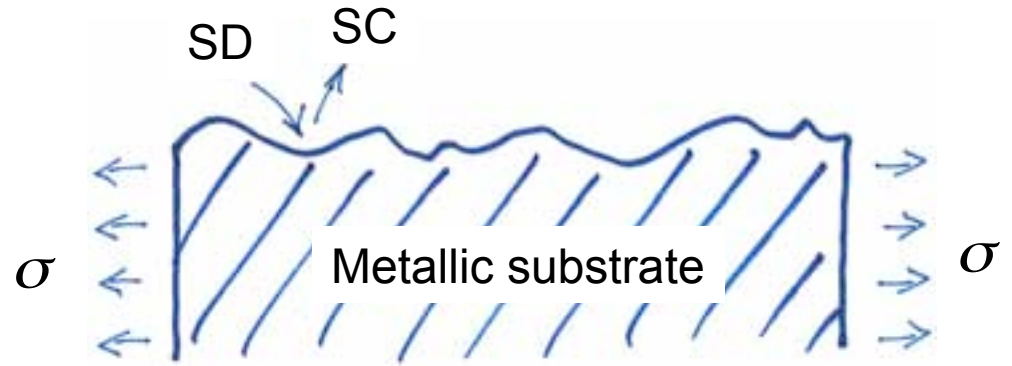
or surface diffusion

$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left(-\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

Biocorrosion



Competing mechanisms
SD - surface diffusion
SC - stress corrosion



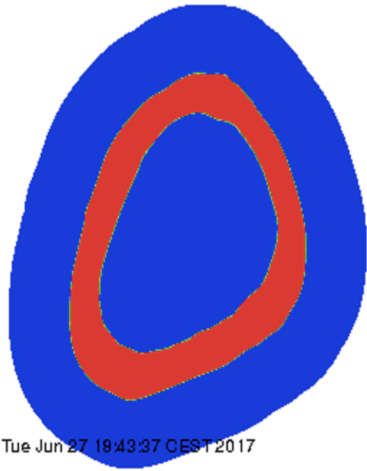
Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \{3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1 - 2\psi)\} (1 - \psi)\psi$$

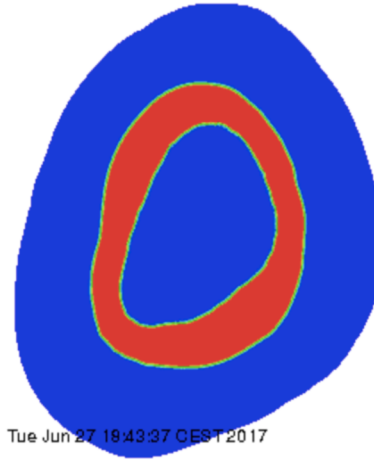
Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1 - 2\nu} \tilde{u}_{j,ij} = 2\tilde{\epsilon}_{ij,j}^p + \tilde{\epsilon}_{,i}^s$$

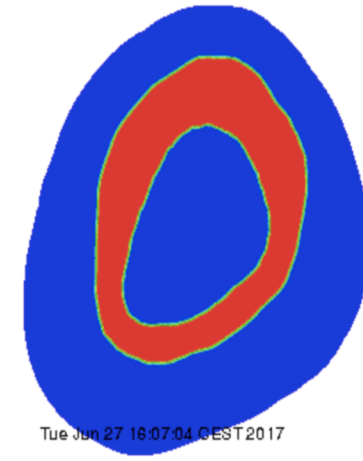
In analogy with a fully coupled thermal-stress



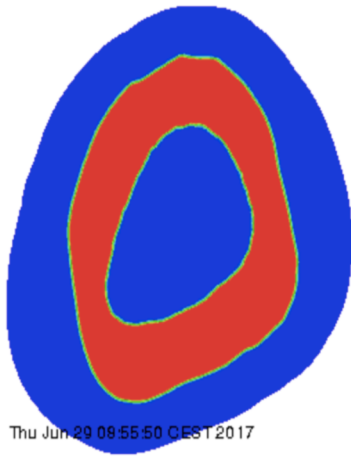
Original cross section



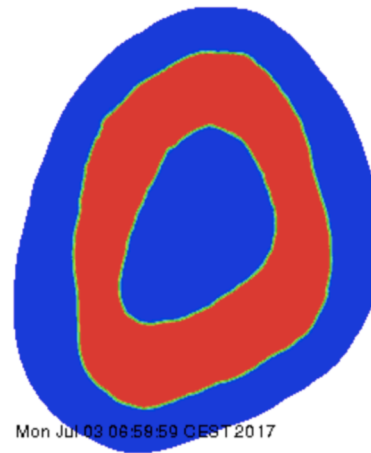
Static load, 21 days



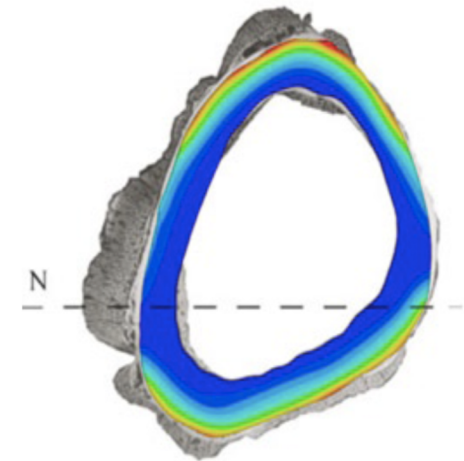
Small cyclic load, 21 days



Medium cyclic load



Large cyclic load



Bone/cartilage experiment