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## Phase Field Modelling of Bone Growth

What it is and what lead to it? Talk given to the memory of Carl Wilhelm Ossén. Orationem Meam.

Ståhle, P.

2017

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# **Phase Field Modelling of Bone Growth**

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## **What it is and what lead to it**

by

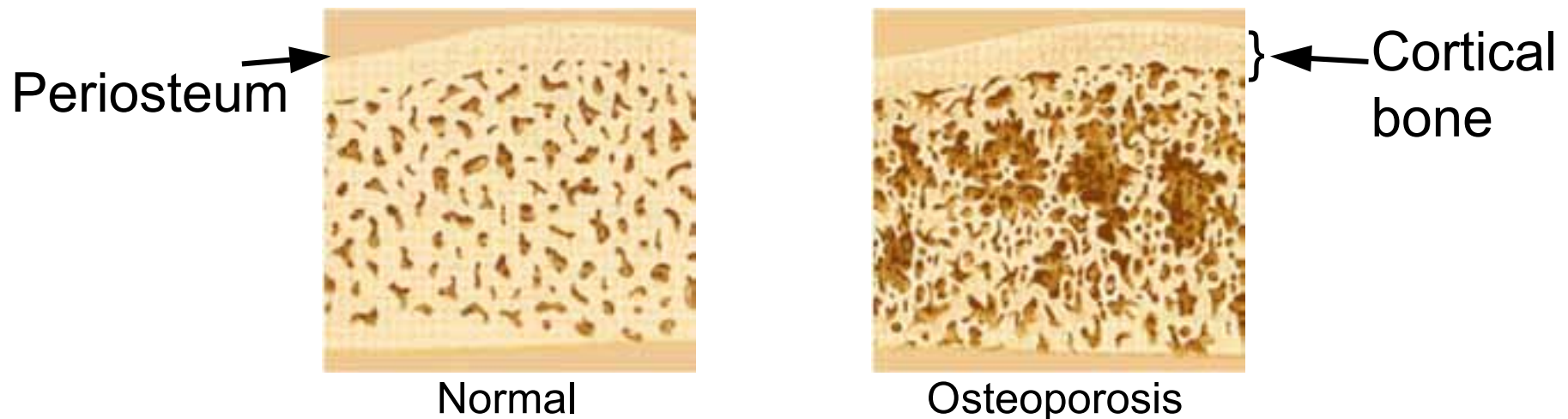
Gustav Lindberg, Per Ståhle, Leslie Banks-Sills, Abdallah Shokry, Wuregul Rehemana

Osteoclasts - dissolving bone

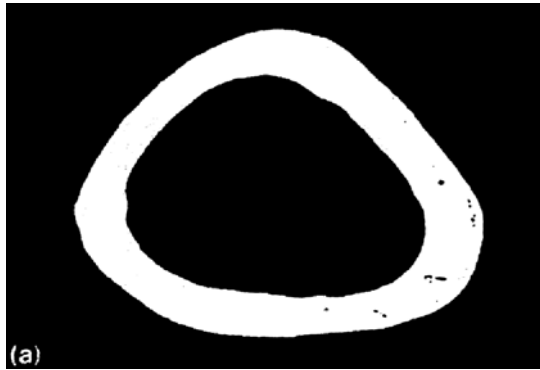
Osteoblasts - remodelling

Periosteum reactions: tearing · breaking · stretching

Osteomyelitis · malignant neoplasms · osteosarcoma ·  
chondrosarcoma · fibrosarcoma · lymphoma · metastasis

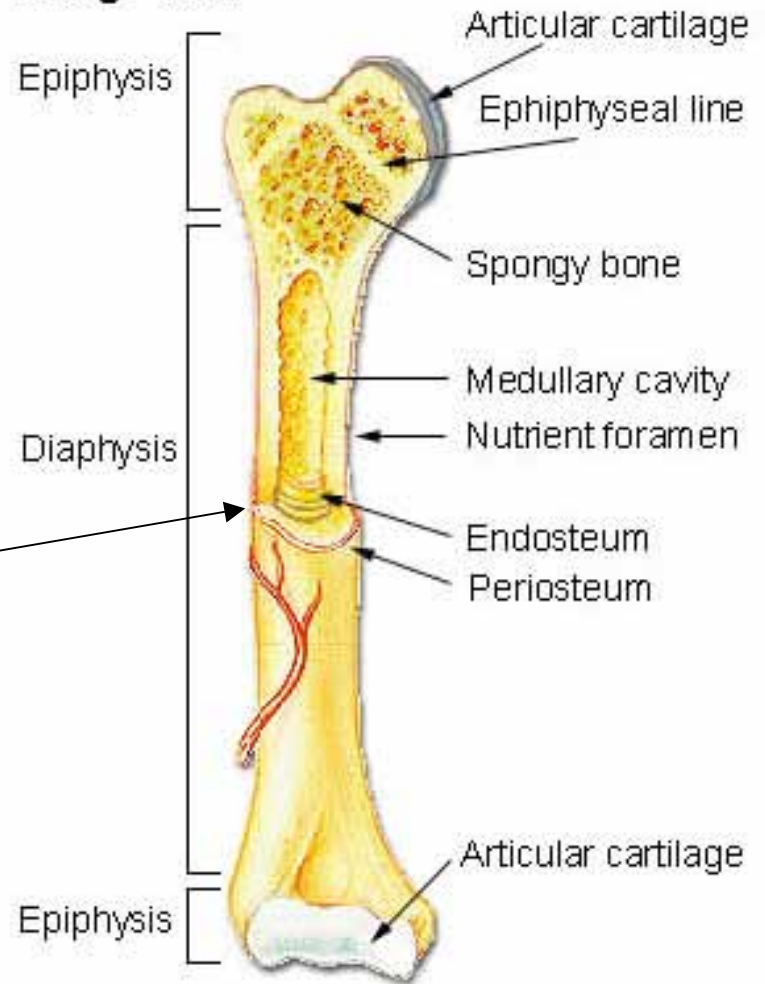


100.000 osteoporosis related fractures yearly in Sweden

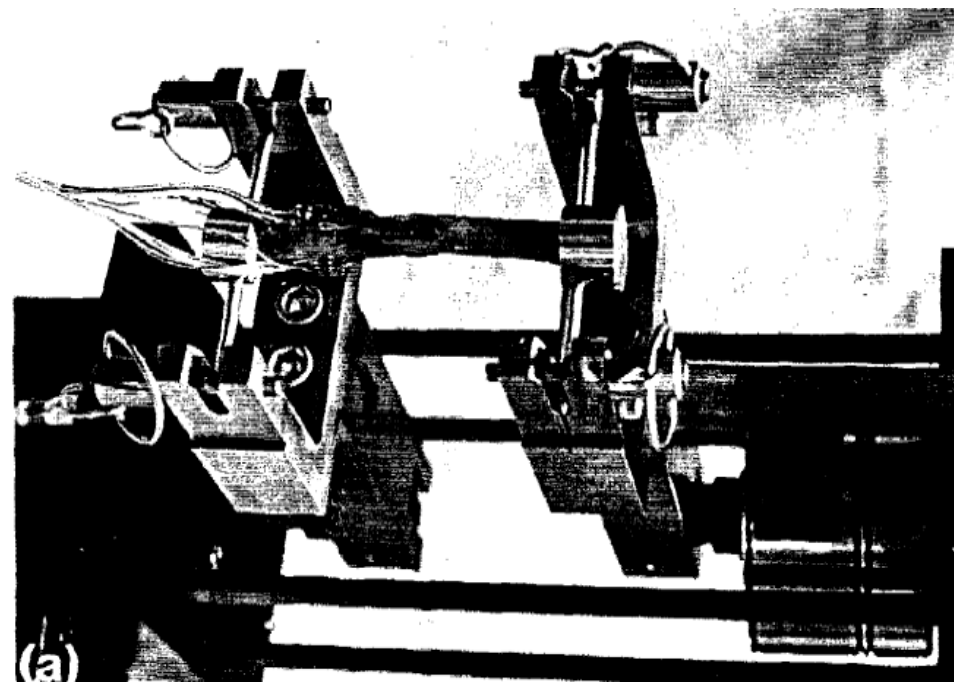
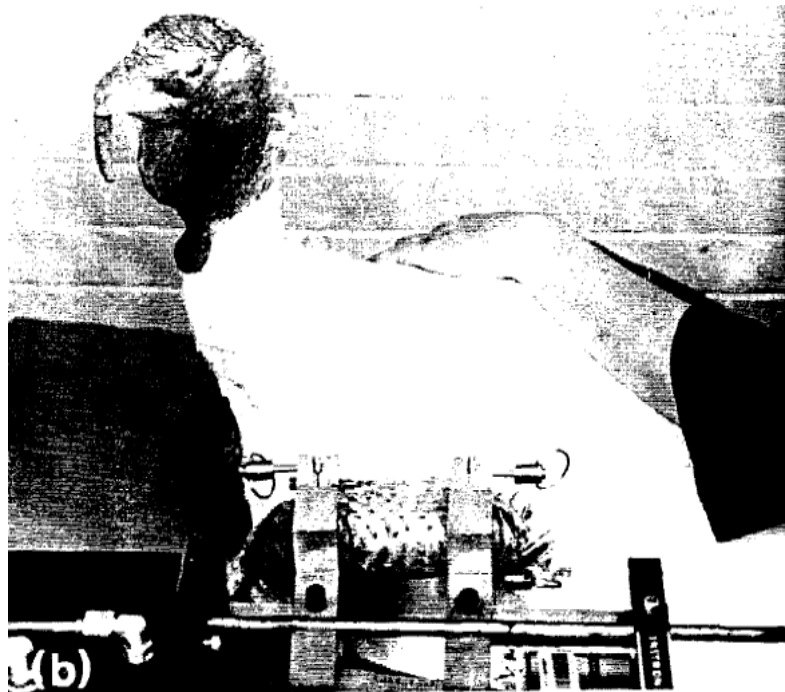


Bone from examined turkey

### Long Bone

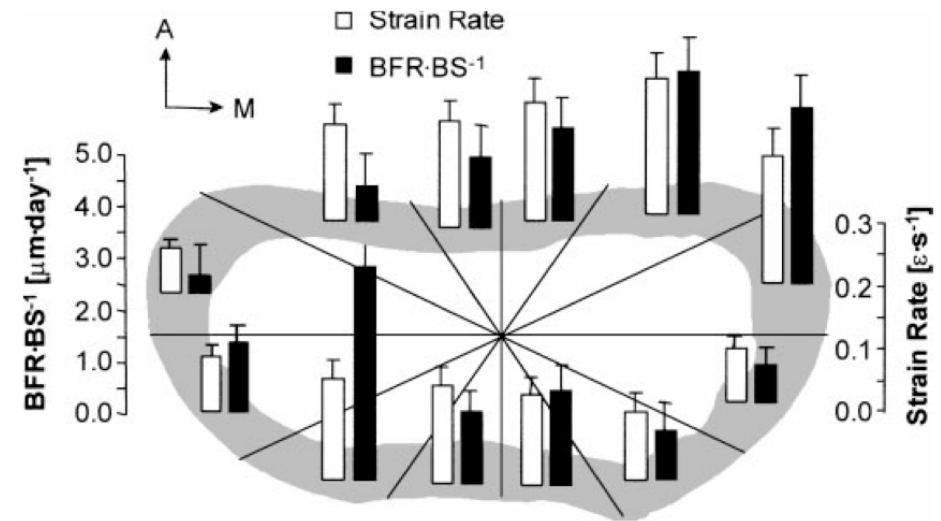
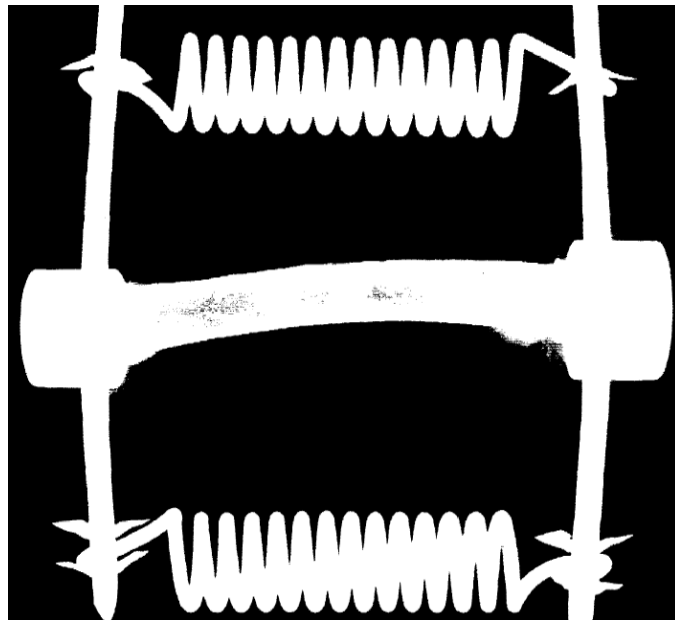


# Observations of exercise stimulated bone growth




(Lanyon and Rubin, 84)

# Observations of exercise stimulated bone growth



(Lanyon and Rubin, 84, Judex, Zernicke, 00)

**Svante Arrhenius**




**Born** Svante August Arrhenius  
19 February 1859  
Wik Castle, Sweden

**Died** 2 October 1927 (aged 68)  
Stockholm, Sweden

**Nationality** Swedish

**Alma mater** Uppsala University  
Stockholm University

**Wilhelm Ostwald**




**Born** Friedrich Wilhelm Ostwald  
2 September 1853  
Riga, Governorate of Livonia,  
Russian Empire (now Latvia)

**Died** 4 April 1932 (aged 78)  
Großbothen near Leipzig,  
Germany

**Nationality** Baltic German by birth. Prussian,  
German (after 1871)

**Alma mater** University of Dorpat

**Lev Landau**



**Född** 1908<sup>[1]</sup>  
Baku<sup>[2]</sup>

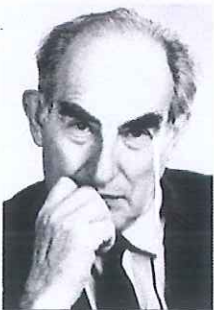
**Död** 1 april 1968<sup>[3][4]</sup>  
Moskva<sup>[5]</sup>

**Begravd** Novodevitjkyrkogården

**Nationalitet** Kejsardömet Ryssland och  
Sovjetunionen

**Alma mater** Sankt Petersburgs universitet,  
Ioffe Institute och Baku State  
University

**Vitaly Ginzburg**




**Born** Vitaly Lazarevich Ginzburg  
4 October 1916  
Moscow, Russian Empire

**Died** 8 November 2009 (aged 93)  
Moscow, Russia

**Nationality** Russia

**Alma mater** Moscow State University

**Lars Onsager**



**Born** November 27, 1903  
Kristiania (Oslo), Norway

**Died** October 5, 1976 (aged 72)  
Coral Gables, Florida, U.S.

**Residence** United States

**Nationality** Norway, U.S.

**Alma mater** Yale University  
Norwegian University of Science  
and Technology

**Erwin Schrödinger**



Erwin Schrödinger det år han tilldelades Nobelpriset

**Född** 12 augusti 1887  
Wien, Österrike-Ungern

**Död** 4 januari 1961 (73 år)  
Wien, Österrike

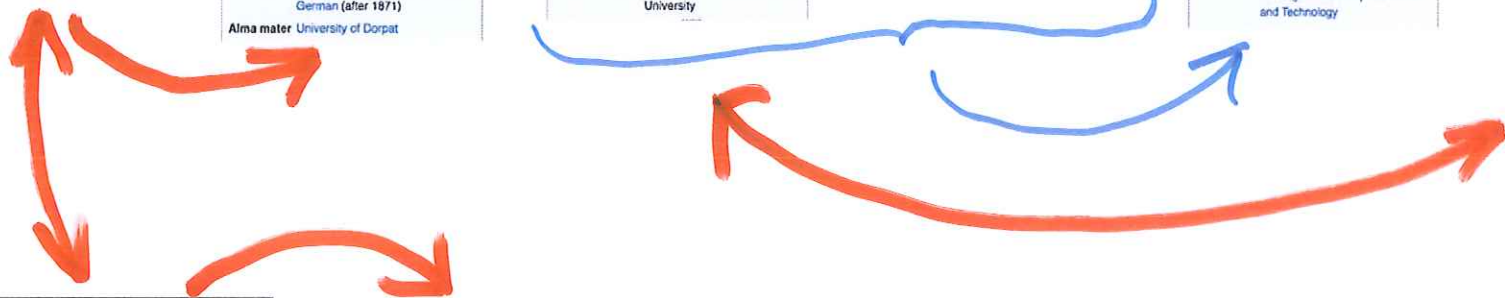
**Medborgarskap** Österrike, Tyskland, Irland

**Nationalitet** Österrikisk

**Forskningsområde** Fysik

**Institutioner** Breslaus universitet  
Zürichs universitet  
Humboldt-Universität zu Berlin  
Oxfords universitet  
Universität Graz  
Dublin Institute for Advanced  
Studies  
Universität i Gent

**Alma mater** Wiens universitet



**Carl Wilhelm Oseen**



C.W. Oseen 1909 vid utnämningen till professor vid Uppsala universitet.

**Född** 17 april 1879  
Lund

**Död** 7 november 1944 (65 år)  
Uppsala<sup>[1]</sup>

**Begravd** Uppsala gamla kyrkogård<sup>[2]</sup>

**Nationalitet** Svensk

**Alma mater** Lunds universitet

**Waloddi Weibull**



Ingenjör

Ernst Hjalmar Waloddi Weibull, född 18 juni 1887 i Vittskövle, Kristianstads län, död 12 oktober 1979, var en svensk ingenjör, naturvetenskapsman och matematiker. Wikipedia

**Född:** 18 juni 1887, Sverige

**Död:** 12 oktober 1979, Annecy, Frankrike

**Utbildning:** Uppsala universitet

Vårt museibibl.  
Handl. f. första Lundahållfaren

### Lars Onsager



Lars Onsager

**Born** November 27, 1903  
Kristiania (Oslo), Norway

**Died** October 5, 1976 (aged 72)  
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Norwegian University of Science  
and Technology



## Contributions to the free energy

$$\mathcal{F} = \mathcal{F}_{ch} + \mathcal{F}_{gr} + \mathcal{F}_{el} \quad (+\mathcal{F}_{heat} + \mathcal{F}_{grav} + \dots)$$

Elastic energy  $\mathcal{F}_{el} = \int \sigma_{ij} d\epsilon_{ij}$

Chemical energy  $\mathcal{F}_{ch} = U(\psi)$

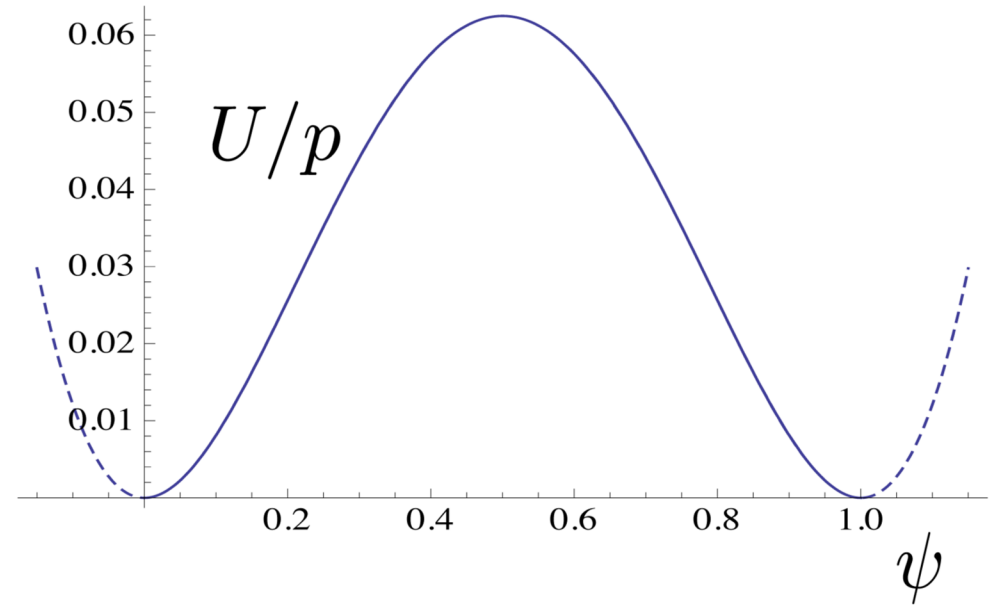
Gradient energy  $\mathcal{F}_{gr} = \frac{g_r}{2} (\psi_{,i})^2$

Heat  $\mathcal{F}_{heat} = \kappa \rho T$

Gravitation  $\mathcal{F}_{grav} = \rho g u_2$

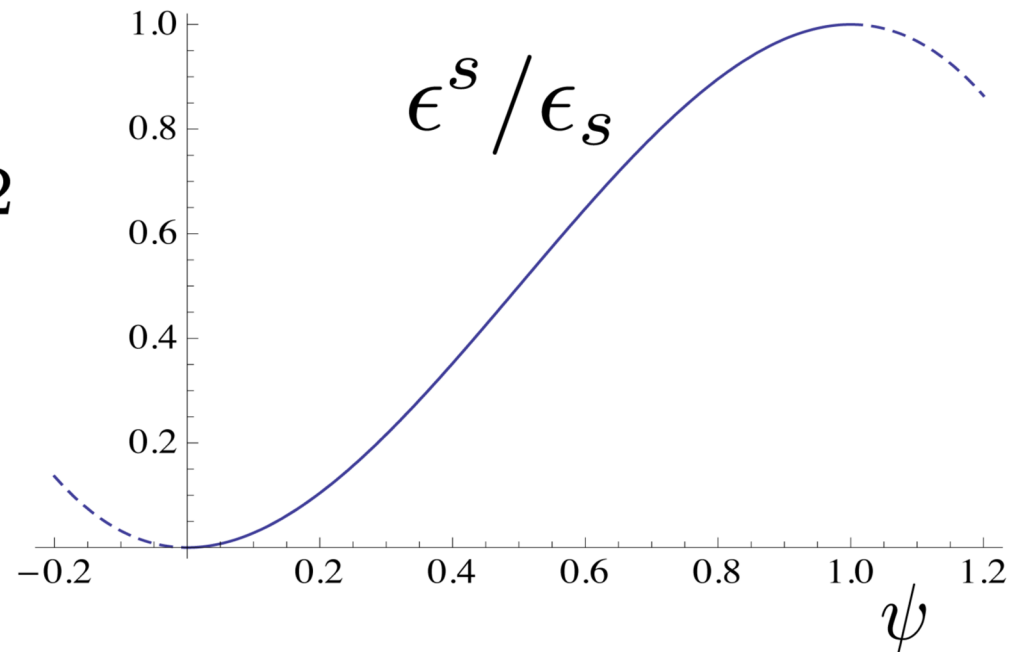
Double-well  
chemical potential

$$U(\psi) = p \psi^2 (1 - \psi)^2$$



Expansion

$$\epsilon^s(\psi) = \epsilon_s (3 - 2\psi) \psi^2$$



Unknown:  $\psi, u_1, u_2, u_3$

$$\text{Phase: } \frac{\partial \psi}{\partial t} = -L_\psi \left( \frac{\partial \mathcal{F}}{\partial \psi} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla \psi)} \right)$$

$$\text{Displ.: } \frac{\partial u_i}{\partial t} = -L_{u_i} \left( \frac{\partial \mathcal{F}}{\partial u_i} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla u_i)} \right)$$

Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \{3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1 - 2\psi)\} (1 - \psi)\psi$$

Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1 - 2\nu} \tilde{u}_{j,ij} = 2\tilde{\epsilon}_{ij,j}^p + \tilde{\epsilon}_{,i}^s$$

In analogy with a fully coupled thermal-stress

# Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993),  
Srolovitz (1989), Freund (1995), Kung-Suk (2000)

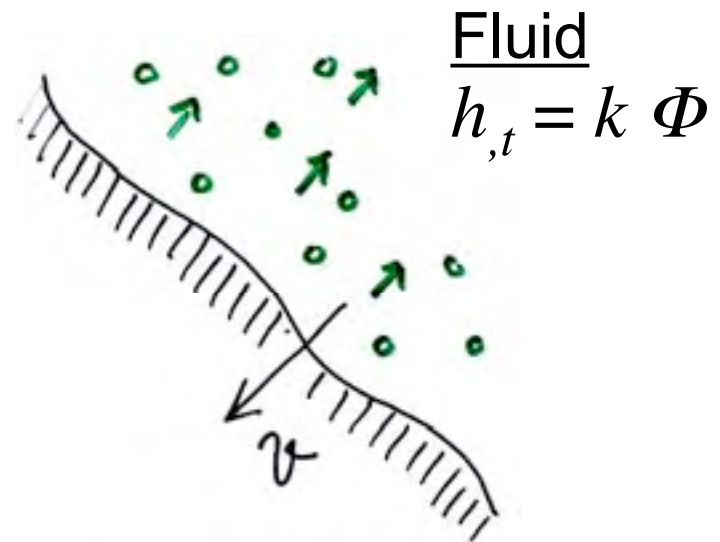
Chemical potential,

$$\Phi = U_c + U$$

$U_c$  surface energy,  $U$  elastic strain energy

Surface diffusion

$$h_{,t} = D \nabla^2 \Phi$$



Governing equations:

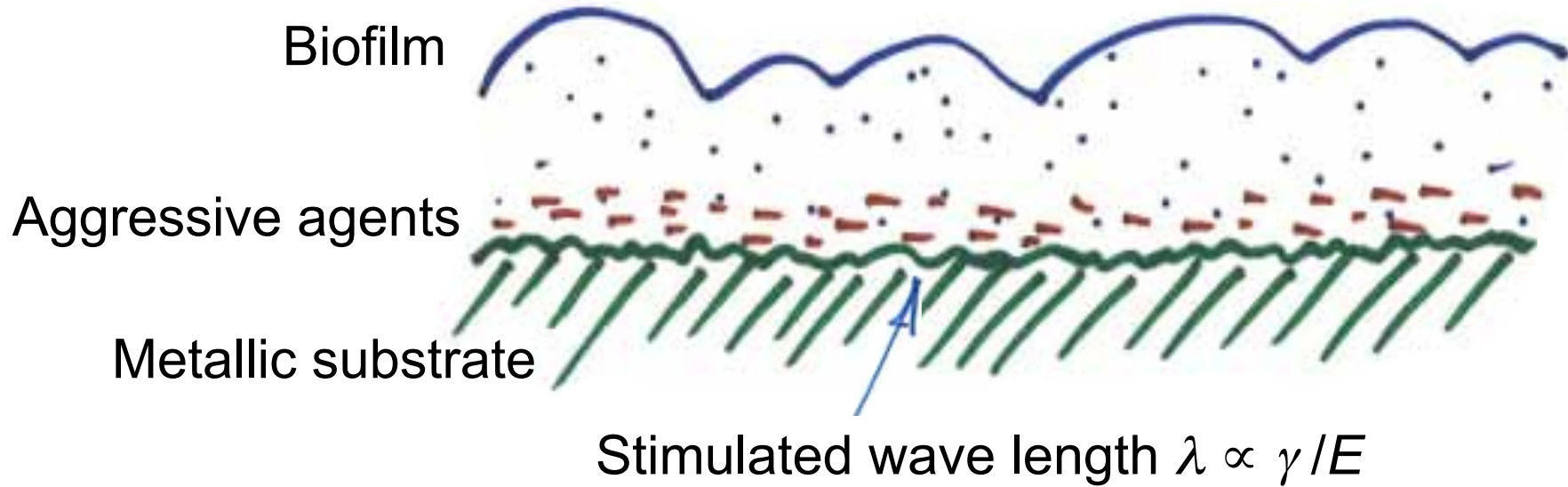
Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left( \gamma \frac{\partial^2 h}{\partial x^2} - \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

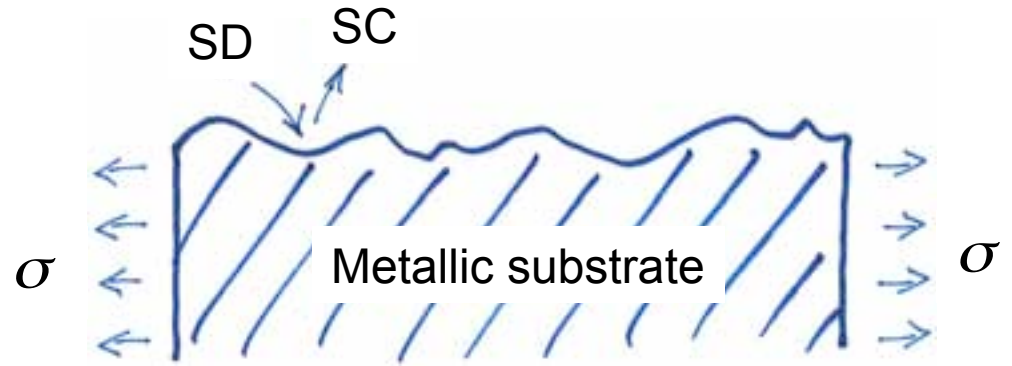
or surface diffusion

$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left( -\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

# Biocorrosion



**Competing mechanisms**  
SD - surface diffusion  
SC - stress corrosion



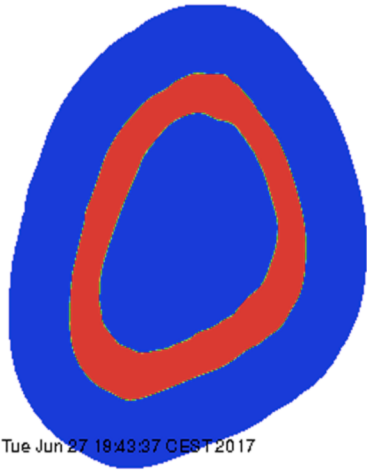
Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \{3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1 - 2\psi)\} (1 - \psi)\psi$$

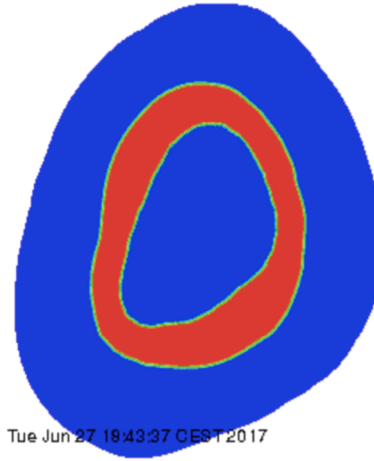
Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1 - 2\nu} \tilde{u}_{j,ij} = 2\tilde{\epsilon}_{ij,j}^p + \tilde{\epsilon}_{,i}^s$$

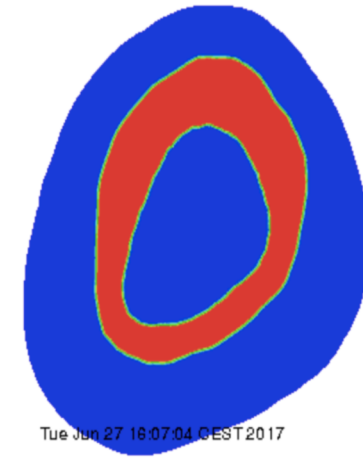
In analogy with a fully coupled thermal-stress



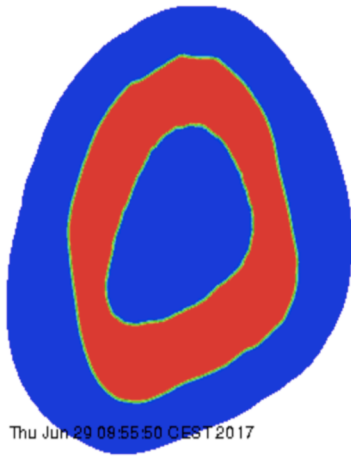
Original cross section



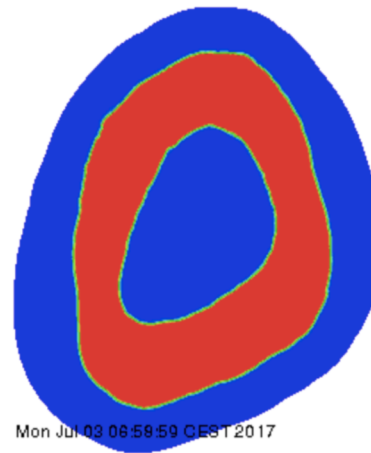
Static load, 21 days



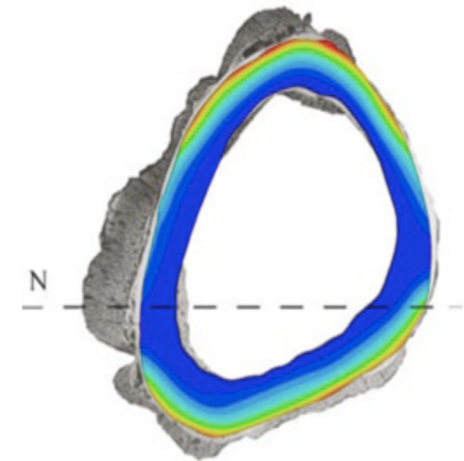
Small cyclic load, 21 days



Medium cyclic load



Large cyclic load



Bone/cartilage experiment