Phase Field Modelling of Bone Growth

What it is and what lead to it? Talk given to the memory of Carl Wilhelm Ossén. Orationem Meam.

Ståhle, P.

2017

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What it is and what lead to it

by

Gustav Lindberg, Per Ståhle, Leslie Banks-Sills, Abdallah Shokry, Wuregul Reheman

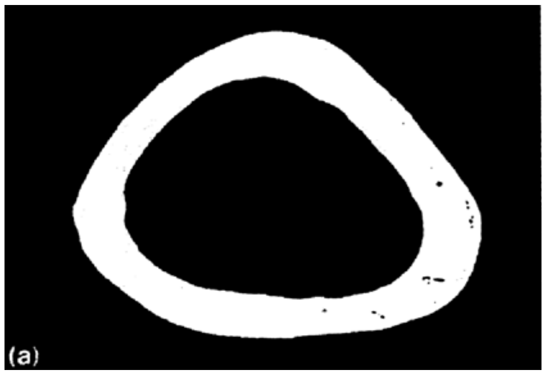
Osteoclasts - dissolving bone
Osteoblasts - remodelling

Periosteum reactions: tearing · breaking · stretching

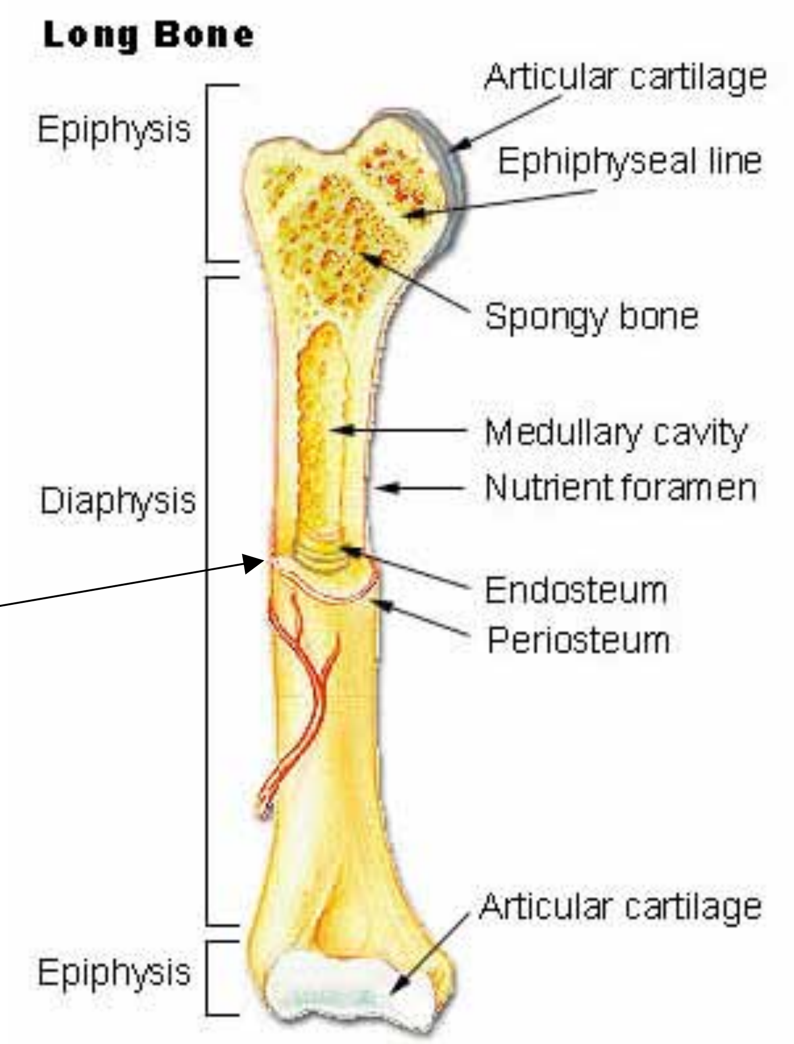
Osteomyelitis · malignant neoplasms · osteosarcoma · chondrosarcoma · fibrosarcoma · lymphoma · metastasis

Normal Osteoporosis

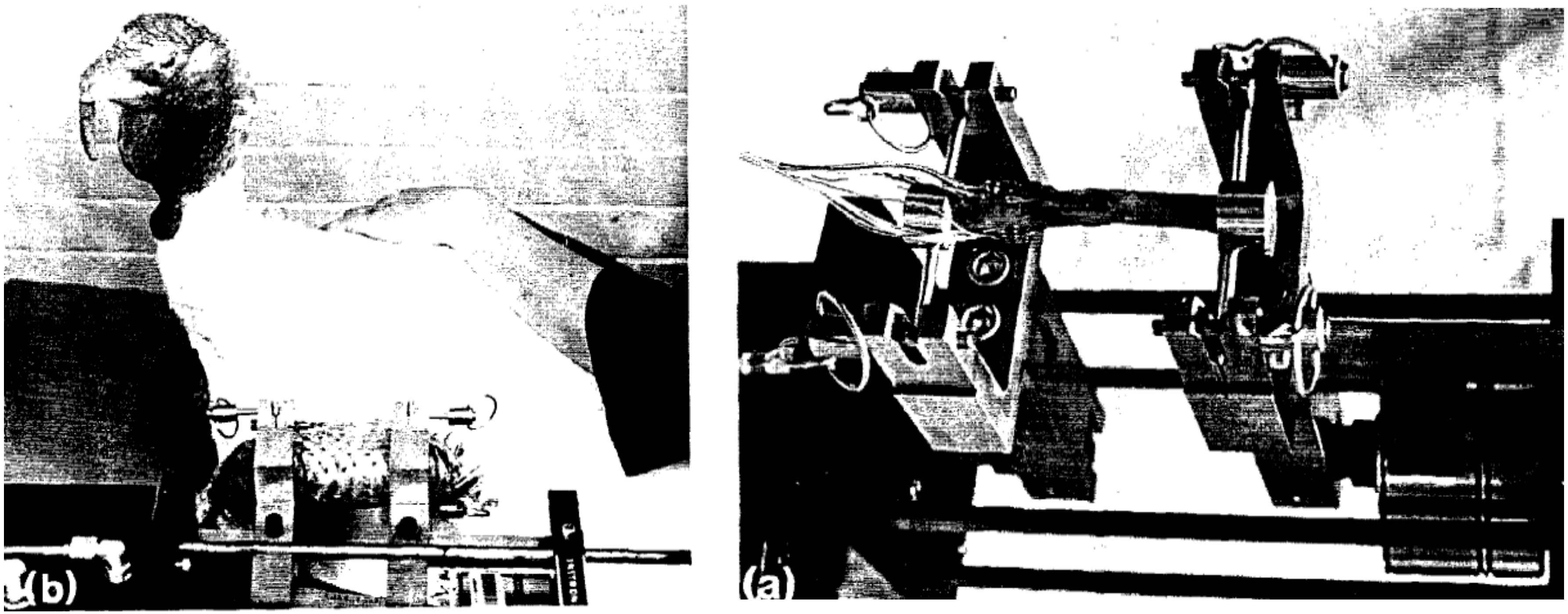
100.000 osteoporosis related fractures yearly in Sweden



Bone from examined turkey

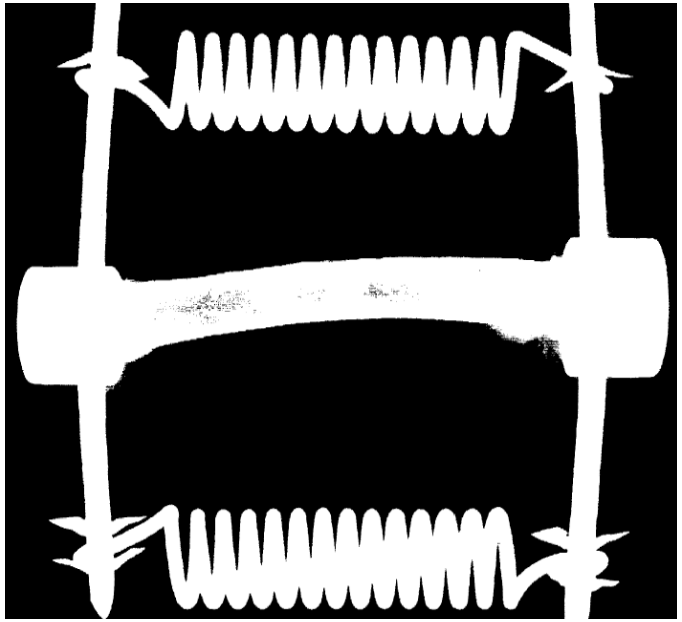


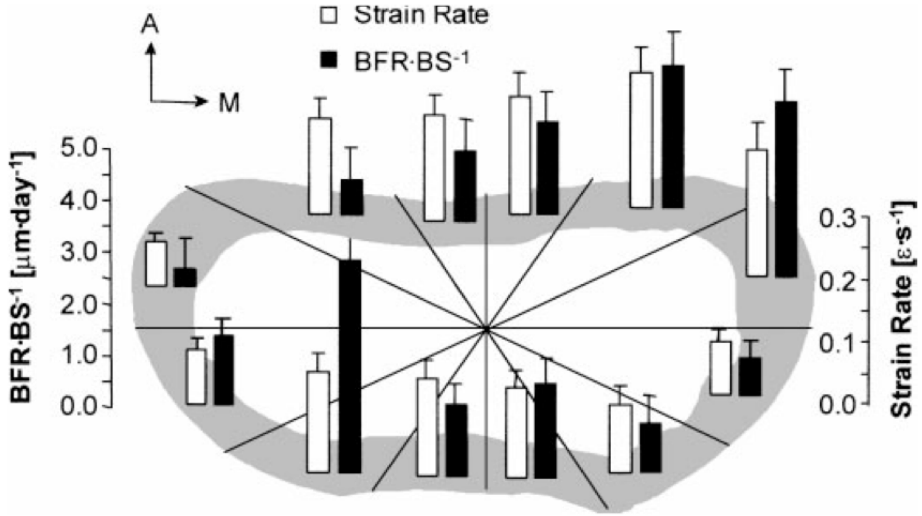
Observations of exercise stimulated bone growth



(Lanyon and Rubin, 84)

Observations of exercise stimulated bone growth





(Lanyon and Rubin, 84, Judex, Zernicke, 00)

Svante Arrhenius

Born	Svante August Arrhenius 19 February 1859 Wik Castle, Sweden
Died	2 October 1927 (aged 68) Stockholm, Sweden
Nationality	Swedish
Alma mater	Uppsala University Stockholm University

Wilhelm Ostwald

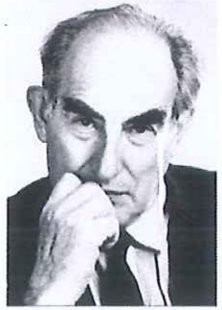
Wilhelm Ostwald by Nicola Perscheid

Born	Friedrich Wilhelm Ostwald 2 September 1853 Riga, Governorate of Livonia, Russian Empire (now Latvia)
Died	4 April 1932 (aged 78) Großbothen near Leipzig, Germany
Nationality	Baltic German by birth. Prussian, German (after 1871)
Alma mater	University of Dorpat

Lev Landau

Född	1908[1] Baku[2]
Död	1 april 1968[3][1][4] Moskva[5]
Begravd	Novodevitjekyrkogården
Nationalitet	Kejsardömet Ryssland och Sovjetunionen
Alma mater	Sankt Petersburgs universitet, Ioffe Institute och Baku State University

Vitaly Ginzburg



Born	Vitaly Lazarevich Ginzburg 4 October 1916 Moscow, Russian Empire
Died	8 November 2009 (aged 93) Moscow, Russia
Nationality	Russia
Alma mater	Moscow State University

Lars Onsager

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Born	November 27, 1903 Kristiania (Oslo), Norway
Died	October 5, 1976 (aged 72) Coral Gables, Florida, U.S.
Residence	United States
Nationality	Norway, U.S.
Alma mater	Yale University Norwegian University of Science and Technology

Erwin Schrödinger

Erwin Schrödinger det år han tilldelades Nobelpriset

Född	12 augusti 1887 Wien, Österrike-Ungern
Död	4 januari 1961 (73 år) Wien, Österrike
Medborgarskap	Österrike, Tyskland, Irland
Nationalitet	Österrikisk
Forskningsområde	Fysik
Institutioner	Breslaus universitet Zürichs universitet Humboldt-Universität zu Berlin Oxfords universitet Universität Graz Dublin Institute for Advanced Studies Universitet i Gent
Alma mater	Wiens universitet

Carl Wilhelm Oseen

C.W. Oseen 1909 vid utnämningen till professor vid Uppsala universitet.

Född	17 april 1879 Lund
Död	7 november 1944 (65 år) Uppsala[1]
Begravd	Uppsala gamla kyrkogård[2]
Nationalitet	Svensk
Alma mater	Lunds universitet

Waloddi Weibull

Ingenjör

Ernst Hjalmar Waloddi Weibull, född 18 juni 1887 i Vittskövle, Kristianstads län, död 12 oktober 1979, var en svensk ingenjör, naturvetenskapsman och matematiker. Wikipedia

Född: 18 juni 1887, Sverige

Död: 12 oktober 1979, Annecy, Frankrike

Utbildning: Uppsala universitet

Vårt museibibl.
Handl. f. första Lundahållfaren

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Contributions to the free energy

$$\mathcal{F} = \mathcal{F}_{ch} + \mathcal{F}_{gr} \mp \mathcal{F}_{el} \quad (+\mathcal{F}_{heat} + \mathcal{F}_{grav} + \ldots)$$

Elastic energy $\quad \mathcal{F}_{el} = \int \sigma_{ij} \mathrm{d}\epsilon_{ij}$

Chemical energy $\quad \mathcal{F}_{ch} = U(\psi)$

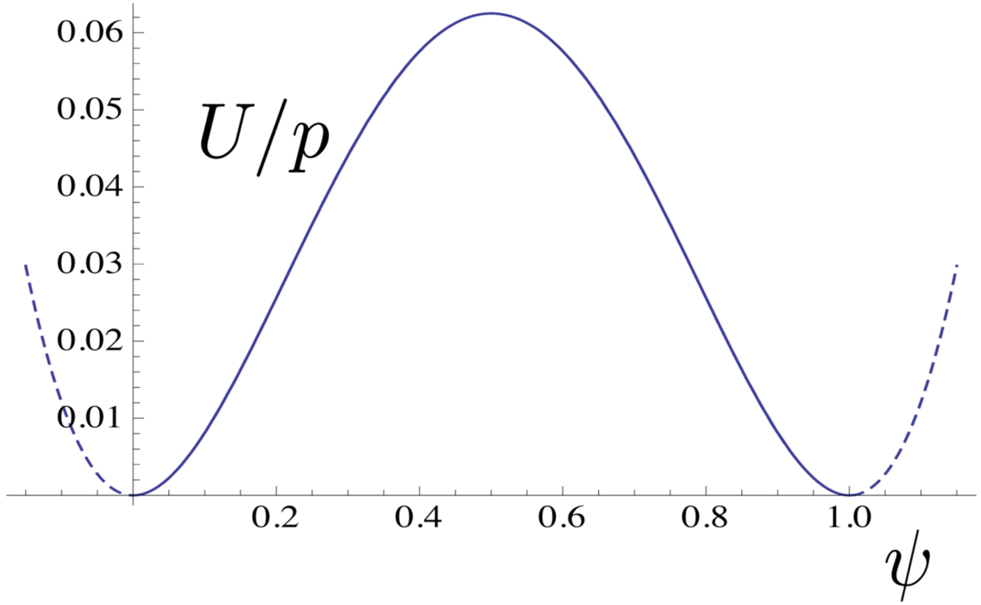
Gradient energy $\quad \mathcal{F}_{gr} = \frac{g_r}{2} \left(\psi_{,i}\right)^2$

Heat $\quad \mathcal{F}_{heat} = \kappa \rho T$

Gravitation $\quad \mathcal{F}_{grav} = \rho g u_2$

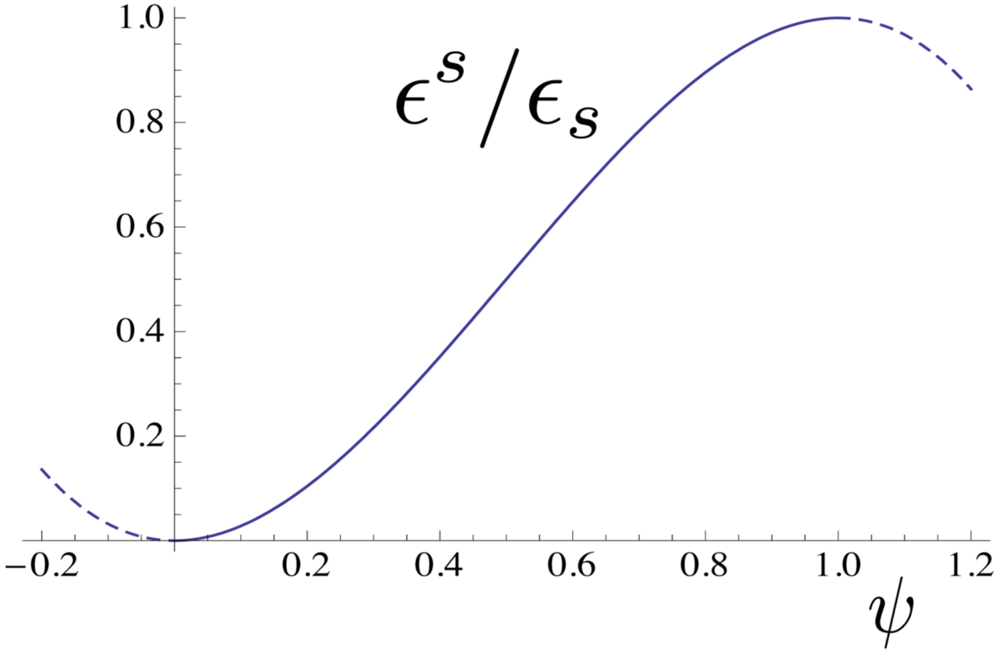
Double-well chemical potential

$$U(\psi) = p\,\psi^2(1-\psi)^2$$



Expansion

$$\epsilon^s(\psi) = \epsilon_s(3-2\psi)\psi^2$$



Unknown: ψ, u_1, u_2, u_3

Phase: $$\frac{\partial \psi}{\partial t} = -L_\psi \left(\frac{\partial \mathcal{F}}{\partial \psi} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla \psi)} \right)$$

Displ.: $$\frac{\partial u_i}{\partial t} = -L_{u_i} \left(\frac{\partial \mathcal{F}}{\partial u_i} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla u_i)} \right)$$

Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \left\{ 3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1-2\psi) \right\} (1-\psi)\psi$$

Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1-2\nu} \tilde{u}_{j,ij} = 2\tilde{\epsilon}^{p}_{ij,j} + \tilde{\epsilon}^{s}_{,i}$$

In analogy with a fully coupled thermal-stress

Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993),
Srolovitz (1989), Freund (1995), Kung-Suk (2000)

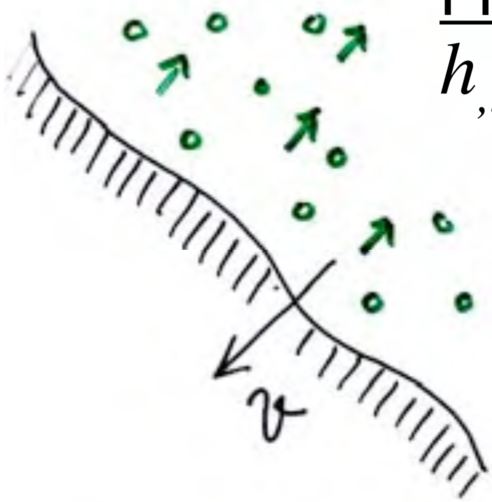
Chemical potential,

$$\Phi = U_c + U$$

U_c surface energy, U elastic strain energy

Surface diffusion

$$h_{,t} = D \nabla^2 \Phi$$



Fluid

$$h_{,t} = k\ \Phi$$

Governing equations:

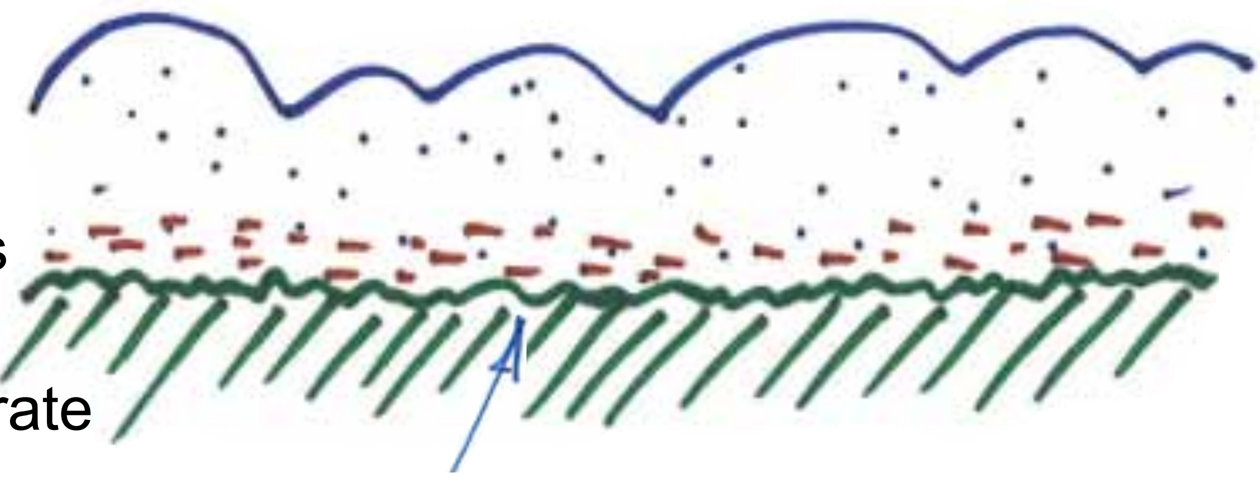
Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left(\gamma \frac{\partial^2 h}{\partial x^2} - \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

or surface diffusion

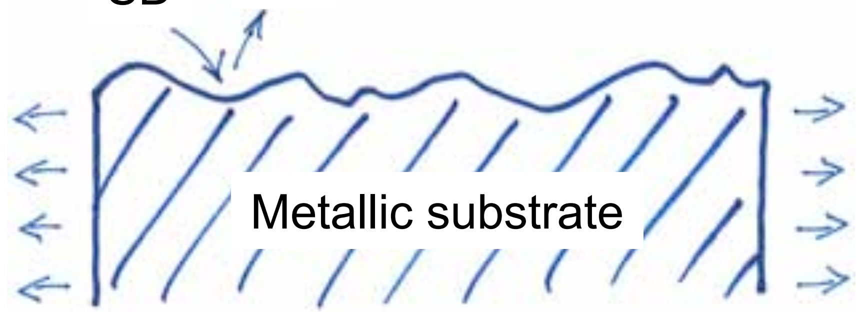
$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left(-\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

Biocorrosion



Competing mechanisms

SD - surface diffusion
SC - stress corrosion



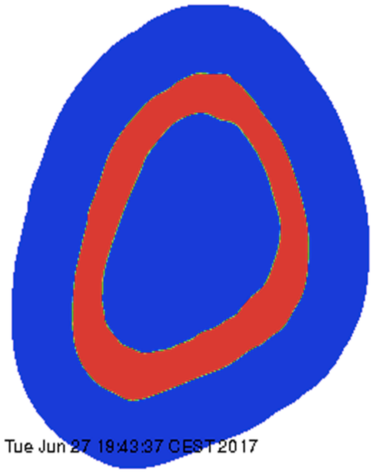
Evolution of the phase.

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \left\{ 3\epsilon_{ii}^{el}\tilde{\epsilon}_s + 2(1-2\psi) \right\} (1-\psi)\psi$$

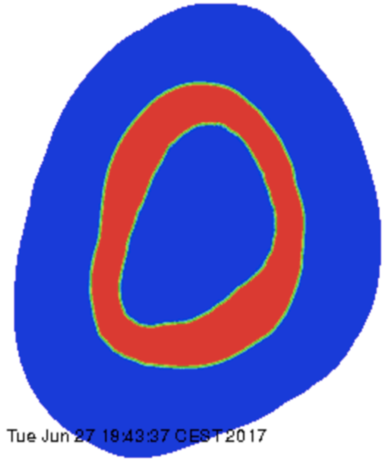
Mechanical equilibrium with expansion

$$\tilde{u}_{i,jj} + \frac{1}{1-2\nu}\tilde{u}_{j,ij} = 2\tilde{\epsilon}^p_{ij,j} + \tilde{\epsilon}^s_{,i}$$

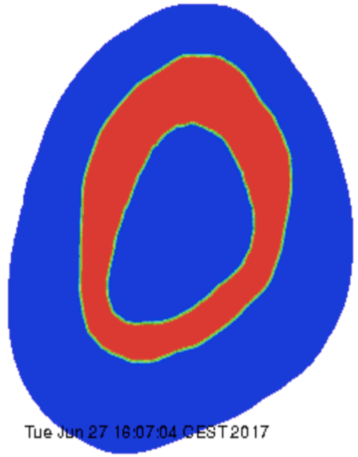
In analogy with a fully coupled thermal-stress



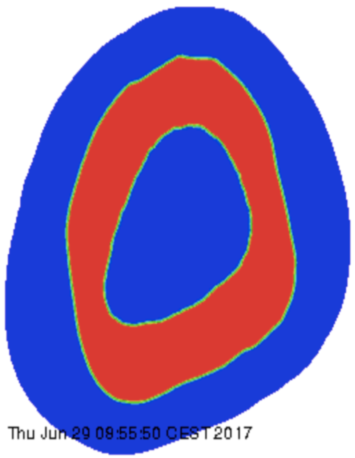
Original cross section



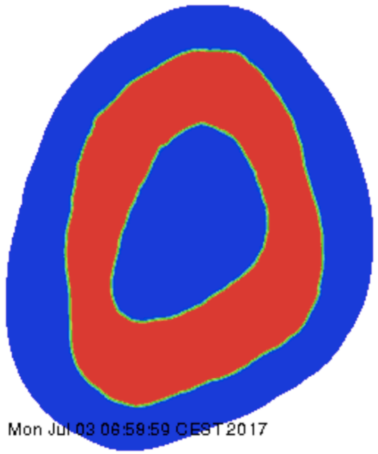
Static load, 21 days



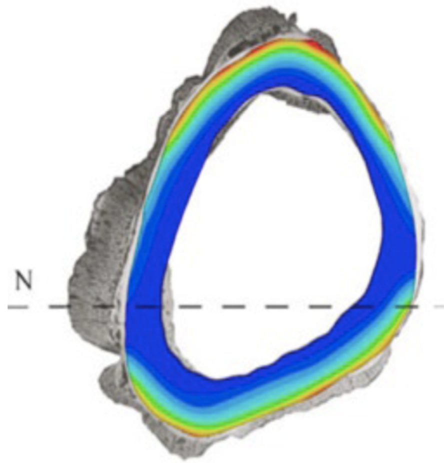
Small cyclic load, 21 days



Medium cyclic load



Large cyclic load



Bone/cartilage experiment