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## How to understand the J-integral when multiple cracks are growing at different rates - Discussion of fracture paper #14

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## Discussion of fracture paper #14 - How to understand the J-integral when multiple cracks are growing at different rates

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A nice demonstration of toughening by introducing multiple secondary cracking of planes parallel with the primary crack is found in the paper: "Fracture resistance enhancement of layered structures by multiple cracks" by Stergios Goutianos and Bent F. Sørensen in *Engineering Fracture Mechanics*, 151 (2016) 92-108.

The 14th paper belong to the category innovative ideas leading to improved composites. We already know of combinations of hard/soft, stiff/weak or brittle/ductile materials that are used to obtain some desired properties. The results are not at all limited to what is set by the pure materials themselves. It has been shown that cracks intersecting soft material layers are exposed to elevated fracture resistances (see eg. the paper 9 blog). Differences in stiffness can be used to improve fatigue and fracture mechanical properties as found in studies by Suresh, Sou, Cominou, He, Hutchinson, and others. Weak interfaces can be used to diverge or split a crack on an intersecting path. A retardation is caused by the additional energy consumed for the extended crack surface area or caused by smaller crack tip driving forces of diverging crack branches. A primary crack is confined to grow in a weak layer. The crack tip that is modelled with a cohesive zone remains stationary until the full load carrying capacity of the cohesive forces is reached. Meanwhile the increasing stress across an even weaker adjacent layer also develops a cohesive zone that takes its share of the energy released from the surrounding elastic material. At some point the cohesive capacity is exhausted also here and a secondary crack is initiated. Both cracks are confined to different crack planes and will never coalesce. The continuation may follow different scenarios depending on the distance between the two planes, the relative cohesive properties like cohesive stress, critical crack tip opening, the behaviour at closure etc. of the second layer. All these aspects are studied and discussed in the paper. The investigators have successfully found a model for how to design the cohesive properties to obtain structures with optimal fracture resistance. Parameters that are manageable in a production process are the ratio of the cohesive properties of the different crack planes and the distance between the them. A theoretical model is formulated. With it they are

able to predict whether or not the toughness of a layered structure can be increased by introducing weak layers as described.

Their results coincide well with the experimental results by Rask and Sørensen (2012) and they have found a model for how to design the cohesive properties to obtain a structure with optimal fracture resistance. Parameters that are manageable in a production process are the ratio of the cohesive properties of the different crack planes and the distance between the them.

The part that I would like to discuss concerns an estimation of an upper bound of the enhancement of the fracture toughness. The derived theoretical model is based on the J integral taken along a path that ensures path independence. Two different paths are evaluated and compared. Along a remote path the J-value is given as a function of external load and deformation. The structural stiffness is reduced as the crack advances in the direction of the primary crack. In the linear elastic case the J-value is half of the work done by the external load during a unit of crack growth. In an evaluation taken along a local path, J receive contributions from the primary crack tip and the two crack tips of the secondary crack. All three tips are supposed to move a unit of length in the direction of the extending primary crack.

As observed by the authors the secondary crack does not contribute to the energy release rate while what is dissipated at the propagating foremost crack tip is to the same amount produced at the healing trailing crack tip. Both crack tips propagate in the same direction so that the crack length does not change.

An observation from the experimental study was that all crack tips have different growth rates and especially the trailing tip of the secondary crack was found to be stationary. Therefore the contribution from that crack tip to the local energy release rate is annulated which leaves less available to the primary crack. To me this seems right. However, when the two remaining advancing crack tips grow does not the respective contributions to J have to be reassessed to reflect their different growth rates? If we assume that the secondary crack grow faster than the primary crack then the enhancing effect is underestimated by the J-integral. Upper bound or lower bound - I can't decide. I would say that it is a fair estimate of where the fracture resistance will end up.

In conjunction with the evaluation of the work done by the external load during a "unit of crack growth" it seems to be an intricate problem to correlate the unit of crack growth with the different crack tip speeds. Some kind of average perhaps.

Any contribution to the blog is gratefully acknowledged.

Per Ståhle