Ongoing research at Lund University, Invited talk given at University of Parma, Italy. Orationem Meam.

Ståhle, P.; Bjerkén, Christina; Tryding, Johan; Kao-Walter, Sharon

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UNIVERSITÀ DEGLI STUDI DI PARMA

DIPARTIMENTO DI INGEGNERIA CIVILE, DELL'AMBIENTE, DEL TERRITORIO E ARCHITETTURA

AVVISO SEMINARIO

Mercoledì 21 dicembre 2016, ore 10:30-12:30

(aula 8, Plesso Ingegneria Didattica)

Ongoing research at Lund University (Per Ståhle, Lund University, Sweden)

The presentation will give an overview of ongoing research at Lund University. Four projects will be reviewed. 1) The necking type of fracture that occur in elastic plastic materials with a sub-critical thickness that reduces the fracture process to pure necking is described. Laminate applications will be discussed. 2) Stress corrosion crack growth with a moving mesh is described. A single chemical criterion for removal of matter replaces the crack growth, crack path and crack branching criteria. 3) A phase field model of delayed hydride cracking. The mechanics of surface blister hydrides forming at cold spots on zirconium surfaces and hydrides at crack tips will be presented. 4) Bone growth controlled by stress driven diffusion will be demonstrated. Both a switch of state and an alternative, a set of coupled PDEs derived for a phase field model are used to trace the growth of a bone exposed to mechanical load.

Biosketch

Per Ståhle addresses problems in the theoretical and applied mechanics of solids. His research involves stress corrosion, hydrogen embrittlement and materials testing. He has a focus on the mechanical conditions leading to dissolution of materials, instabilities at material surfaces, growth of precipitates and initiation and growth of cracks. He became honorary doctor and professor at Uppsala University 1986, Professor of Mechanics of Materials at Malmö University and is since 2009 Professor of Solid Mechanics at Lund University.

Per was visiting Professor at Brown university 1989/90, University of California Santa Barbara 1992, Tokyo Institute of Technology and Academy of Science Beijing 1997. From 2004 to 2009 he was the chairman of the National Committee for Theoretical and Applied Mechanics at the Swedish Royal Academy of Science, correspondent of EuroMech, expert for the Swedish Research Council, the NATO Collaborative Research Council and the Royal Society of London. He is presently Swedish representative in The International Union of Theoretical and Applied Mechanics since 2012, the official blogger for the Elsevier publication Engineering Fracture Mechanics and a member of the executive committee of the European Society of Structural Integrity.

Mercoledì 21 dicembre 2016, ore 10:30-11:30 (aula 8, Plesso Ingegneria Didattica)

Ongoing Solid Mechanics Research at Lund University

Per Ståhle, Lund University, Sweden

1) Necking Type of Fracture of Laminates

2) Stress Corrosion Crack Growth - a Moving Boundary Mod.

3) Modelling of delayed hydride cracking

4) Bone Growth Controlled by Stress Driven Diffusion

A Strong Toughening Mechanism in an Elastic Plastic Laminate

P. Ståhle[1], C. Bjerkén[2], J. Tryding[3] and S. Kao-Walter[4]

[1]Lund University, Lund
[2]Malmö University, Malmö
[3]TetraPak AB, Lund
[4]Blekinge Technical University, Karlskrona

Necking vs fracture

Fracture toughness of aluminium is $\approx 24\ \mathrm{MPa\,m}^{1/2}$.
Measured toughness of an 9μm aluminium foil is $3.5\ \mathrm{MPa\,m}^{1/2}$ due to necking.

The stress intensity factor is

$$K \sim \text{sheet thickness}^{1/2}$$

A sheet thickness $> 400\mu$m is needed to restore $K_{\mathrm{I}c}$ fracture control.

The largest load per unit of length

$$P \sim \text{sheet thickness}^{3/2}$$

A sheet thickness $> 32\mu$m is needed to provide the strength of a non necking 9μm aluminium foil.

Layers and laminate properties

Metal foil (fully annealed AA1200 aluminium) Stiff and Brittle

$$t_A = 9\mu\text{m},\ E_A = 71\text{GPa},\ \sigma_{bA} = 73\text{MPa},\ \nu_L = 0.33,\ F_A = 12.5\text{N}$$

Polymer (PolyEthene LDPE, LD270) Weak and Soft

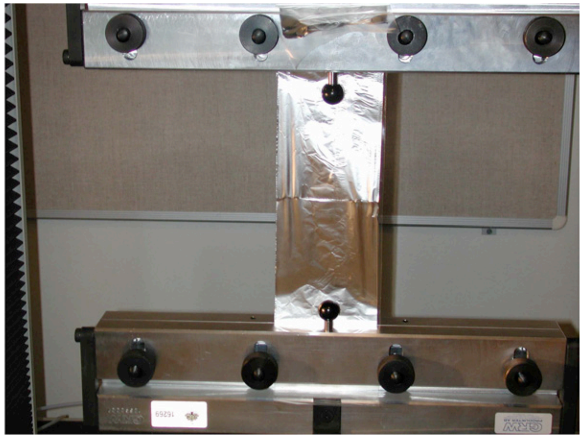
$$t_L = 27\mu\text{m},\ E_L = 126\text{MPa},\ \sigma_{bL} = 8\text{MPa},\ \nu_L = 0.45,\ F_L = 2\text{N}$$

The laminate (homogenized, plane stress) Stiff and Ductile

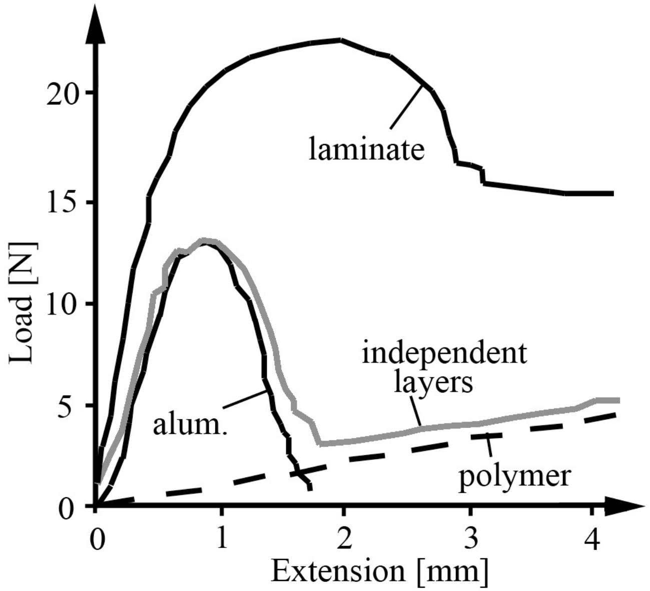
$$t_{lam} = 36\mu\text{m},\ E_{lam} = 18\text{GPa},\ \sigma_{b\,lam} = 27\text{MPa},\ \nu_{lam} = 0.3,\ F_{lam} = 22.5\text{N}$$

Test results

(Kao-Walter *et al.*, 02, Macionczyk and Bruckner, 99)

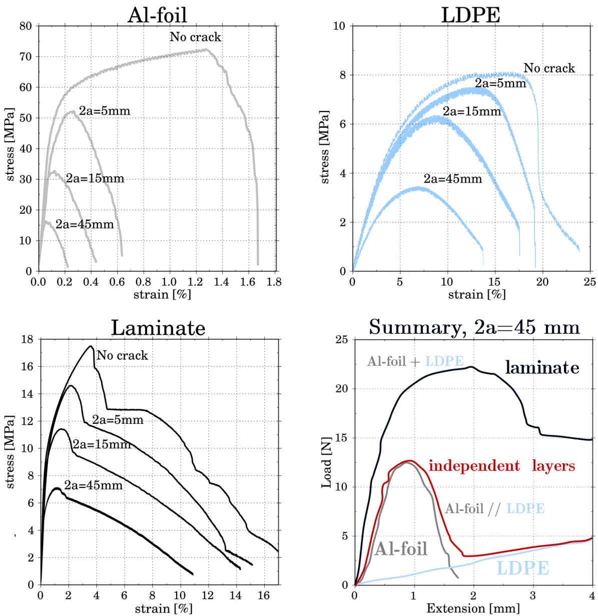


Fracture mechanical test



Load displacement curves

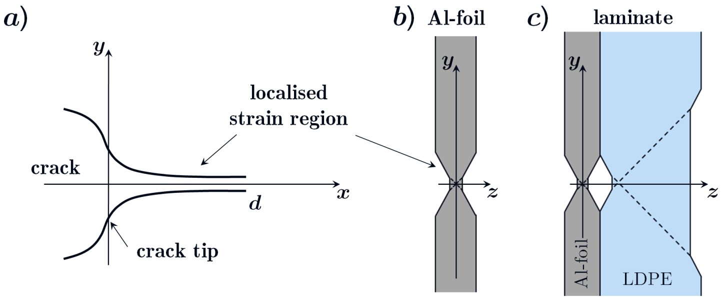
Test results



Stress vs. strain for tensile tests
a) Al-foil, b) Polymer

a) Stress vs. strain for the laminate. b) Load vs. extension. Summary of the aluminium, polymer and laminate results. Crack length 45mm (Kao-Walter et al., 2002), (Hutchinson, 2013)

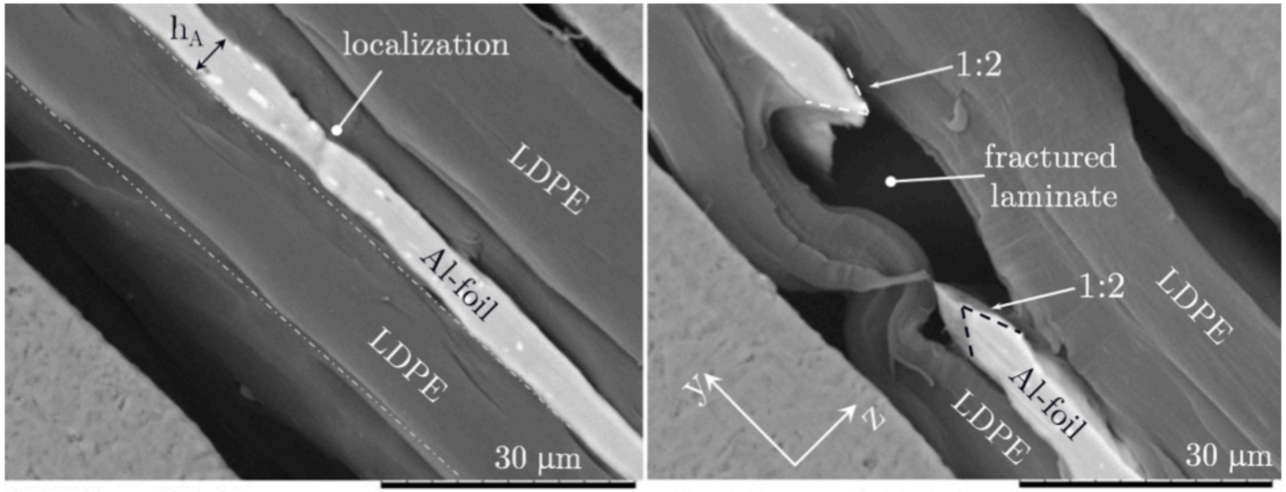
Work of failure

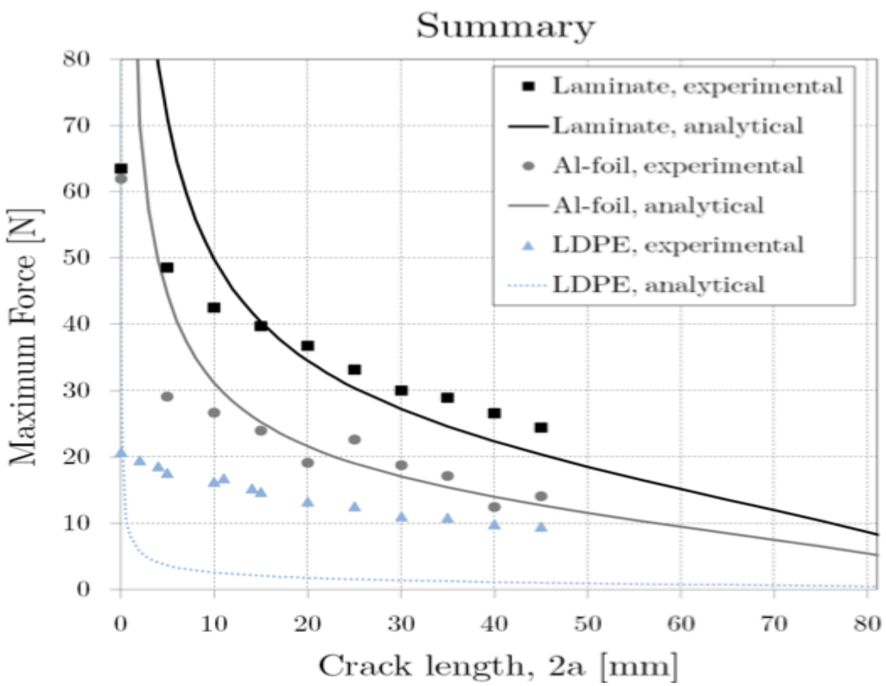


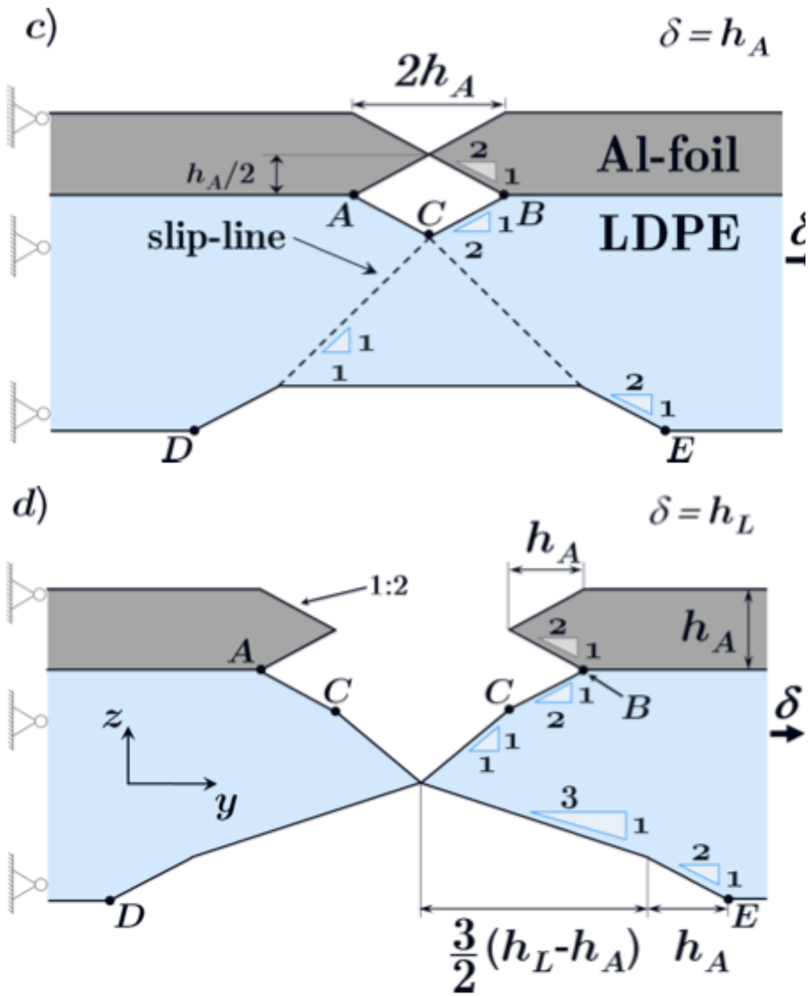
Strip yield zone ahead a crack tip. a) the crack geometry in the plane $z = 0$. b and c) the slip region as seen in a plane $x = \text{const}$.

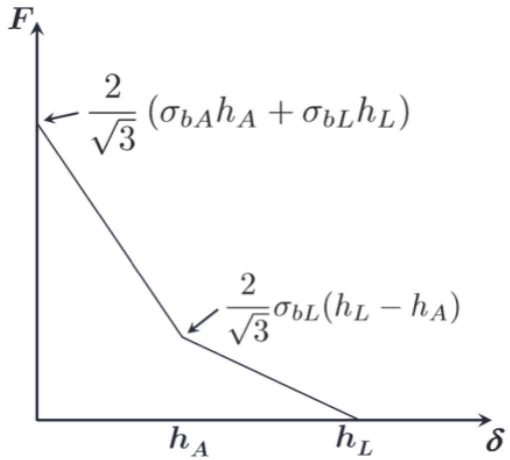
Super-tough Thin Film Laminates

Sharon Kao-Walter, Rickard Hägglund, Eskil Andreasson



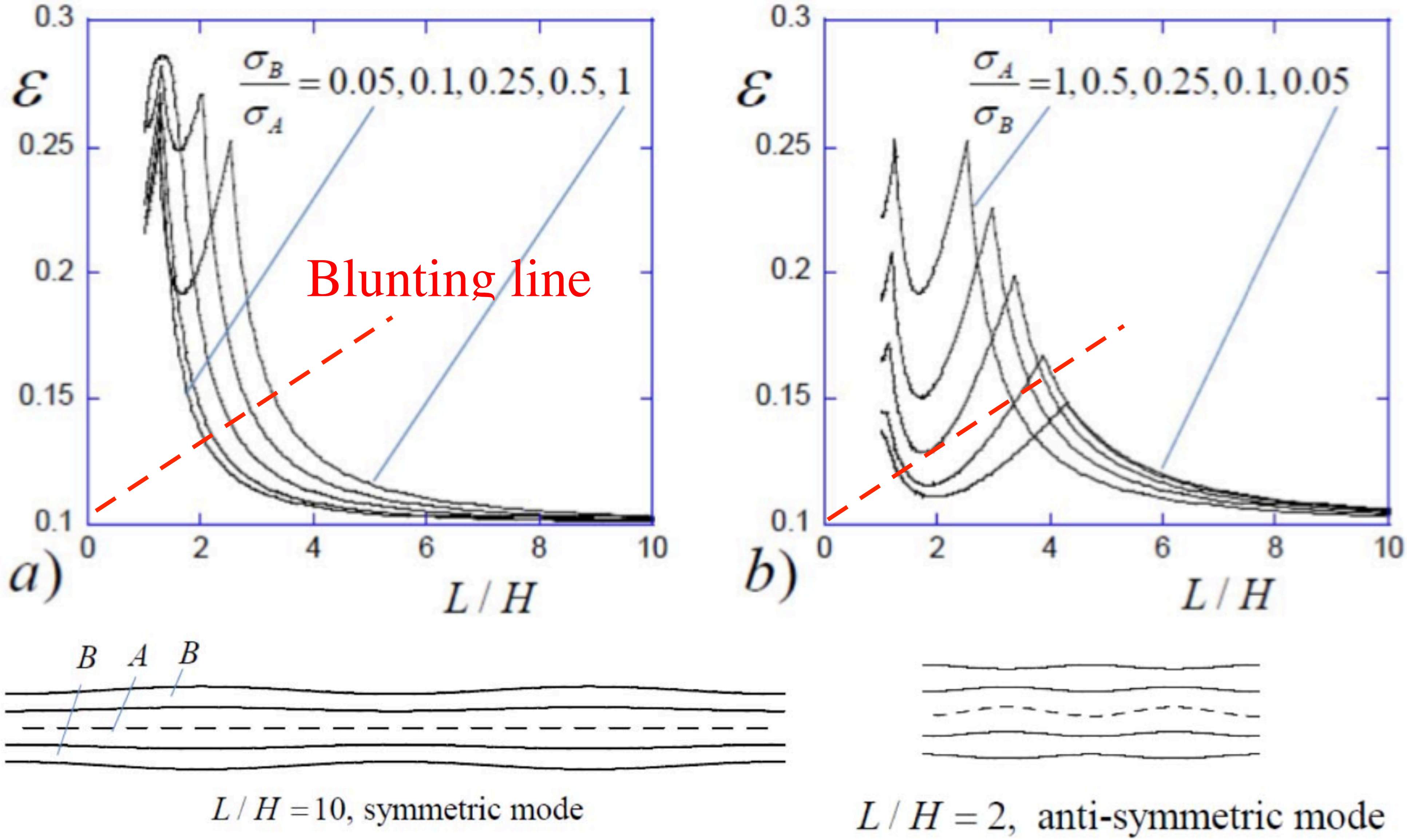






$N_A = N_B = 0.1$ and $h_A/h_B = 2$ (Hutchinson, 2013)

$\sigma_B/\sigma_A = 0.002$ or $\sigma_A/\sigma_B = 0.002$



Material parameters

Comparison of structural and material parameters for the different test speciemens.

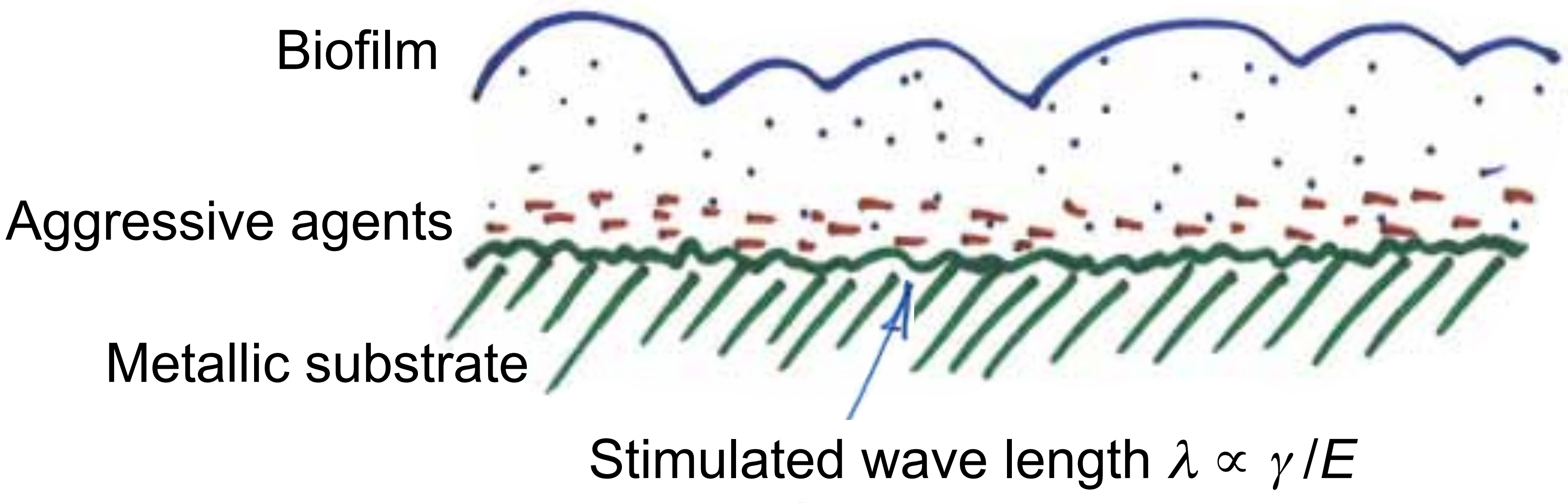
	Al-foil	LDPE	laminate
$h[\mu m]$	9,0	27	36
E $[GPa]$	71,0	0.126	17.9
$\upsilon[-]$	0.33	0.45	0.30
σ_b $[MPa]$	73.0	8.0	26.6
J_c $[N/m]$	188	82.6	109
F_{max} $[N]$ $\{2a=45mm\}$	14,0	9,4	24,4

$$\sigma_c = \sqrt{J_c \frac{E}{\pi a}} \frac{1}{\phi\left(\frac{a}{W}\right)} = \sqrt{\frac{1}{\sqrt{3}} \left(\frac{\sigma_{bA} h_A^2 + \sigma_{bL} h_L^2}{h_A + h_L} \right) \frac{E}{\pi a}} \frac{1}{\phi\left(\frac{a}{W}\right)} . \qquad (13)$$

Stress Corrosion Crack Growth

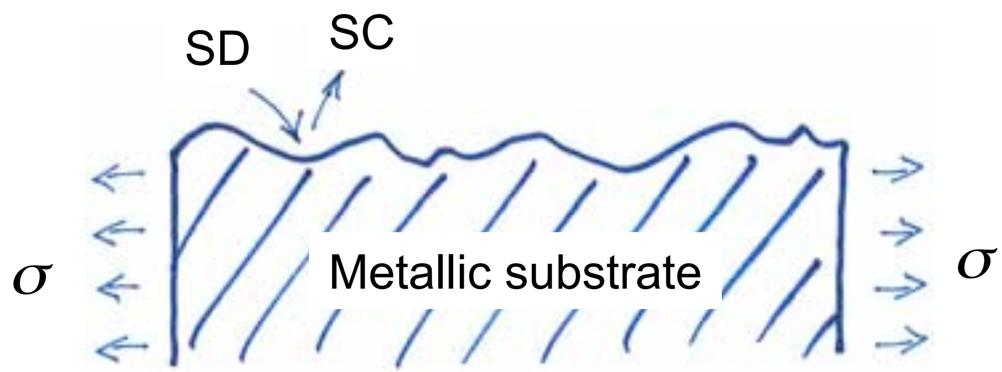
Per Ståhle, Andrey Jivkov and Christina Bjerkén
Malmö University, Sweden

Biocorrosion



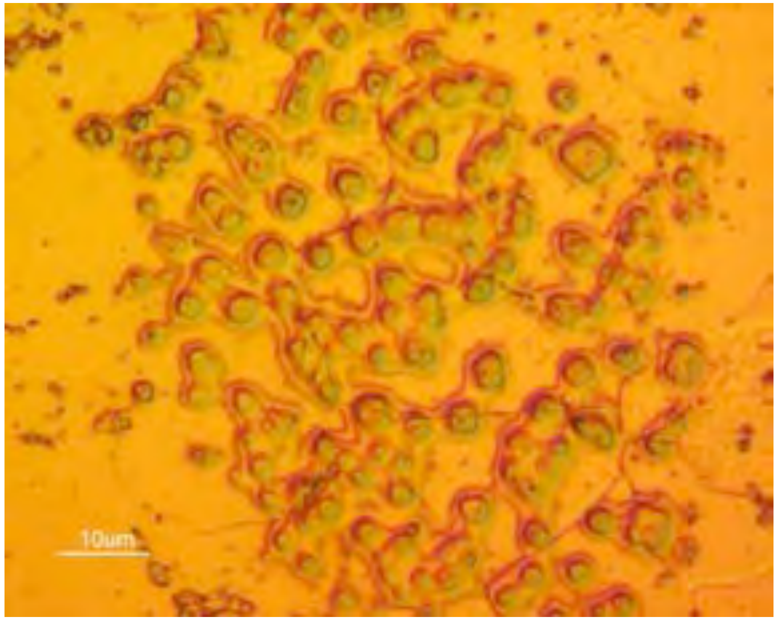
Competing mechanisms

SD - surface diffusion
SC - stress corrosion

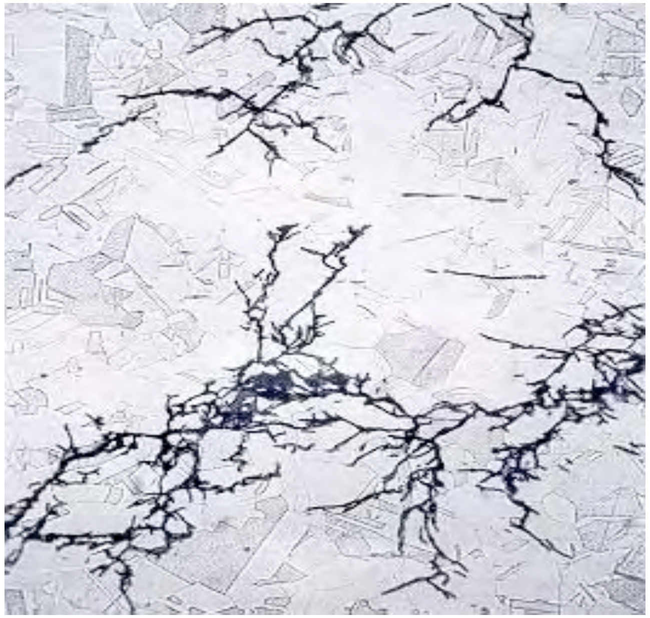


Organisms believed to cause SC

Anaerobic Bacteria:*Desulfovibrio, -maculum, -monas*
Aerobic Bacteria: *Thiobacillus*
Fungi, Algae, Protozoans



Surface with different levels of pitting



Corrosion in stainless steel in the presence of Gallionella Bacillus

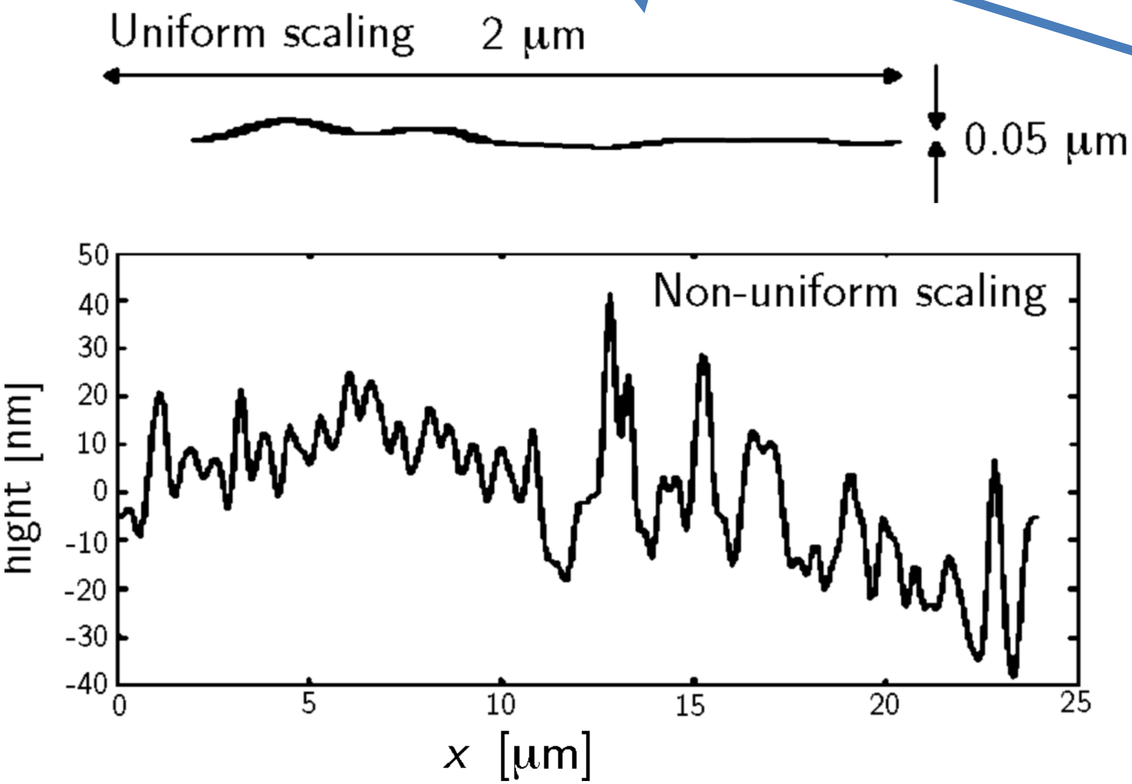
Corroding environment leads to:

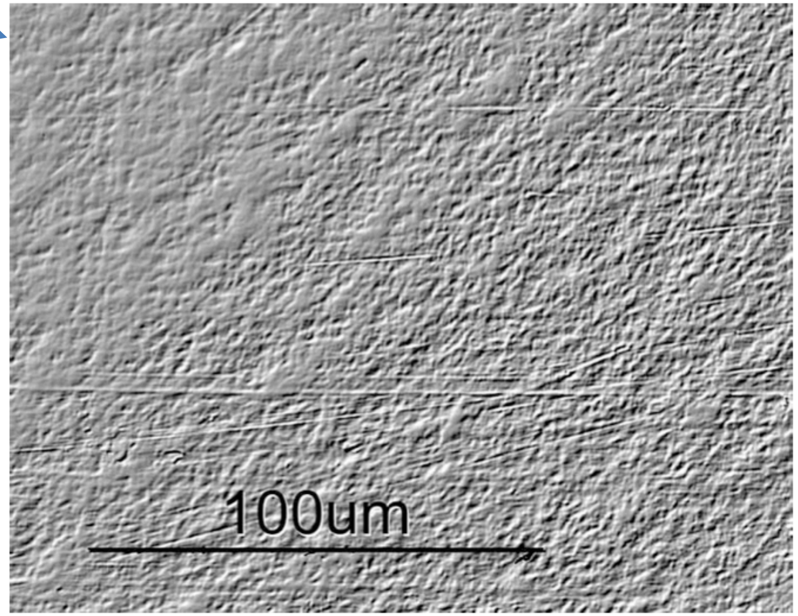
1. Continuous loss of mass
2. Pitting

... and with mechanical stress present

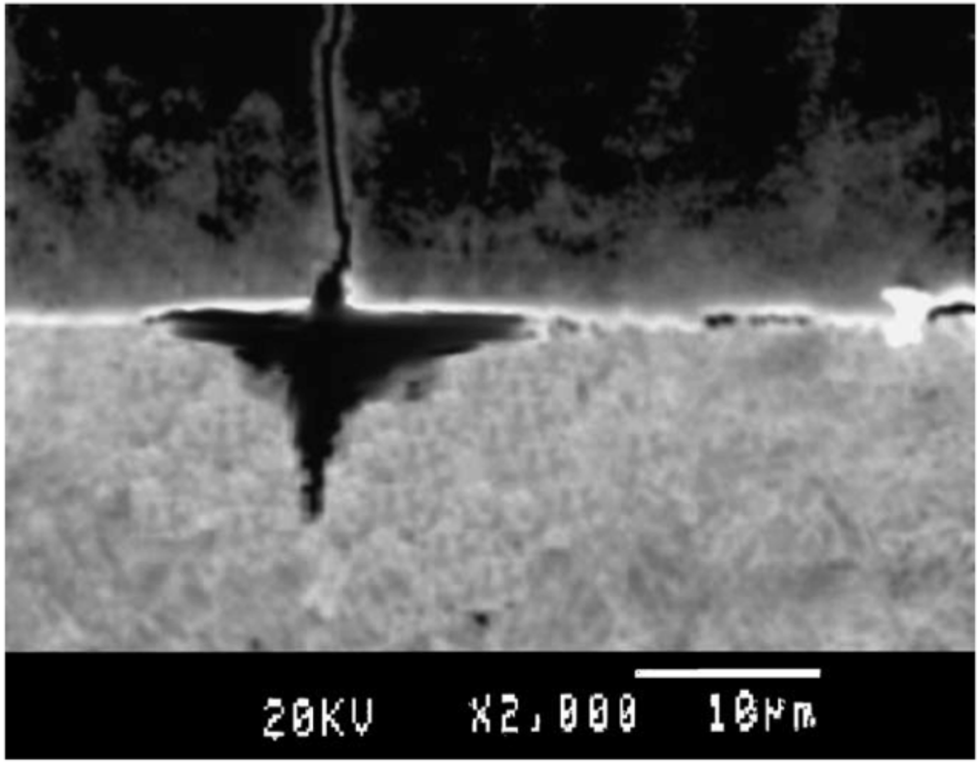
3. Surface roughening

(from Kim *et al*. 2000)

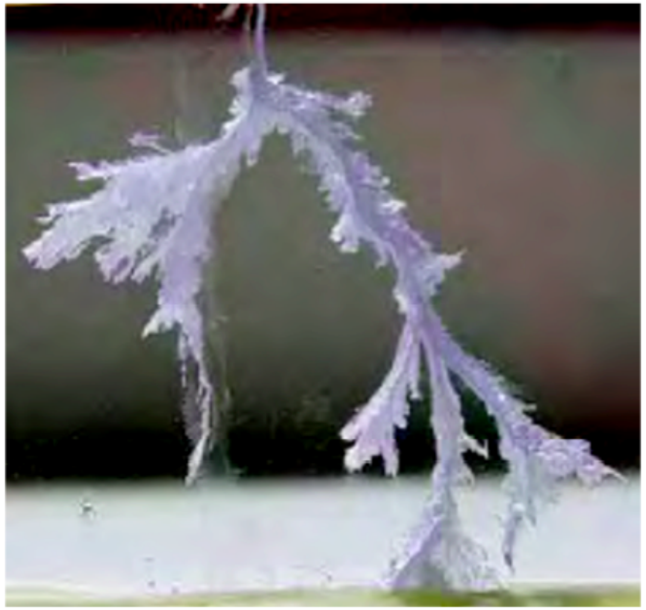




4. Evolving pits
5. Formation of cracks
6. Crack growth
7. Crack branching

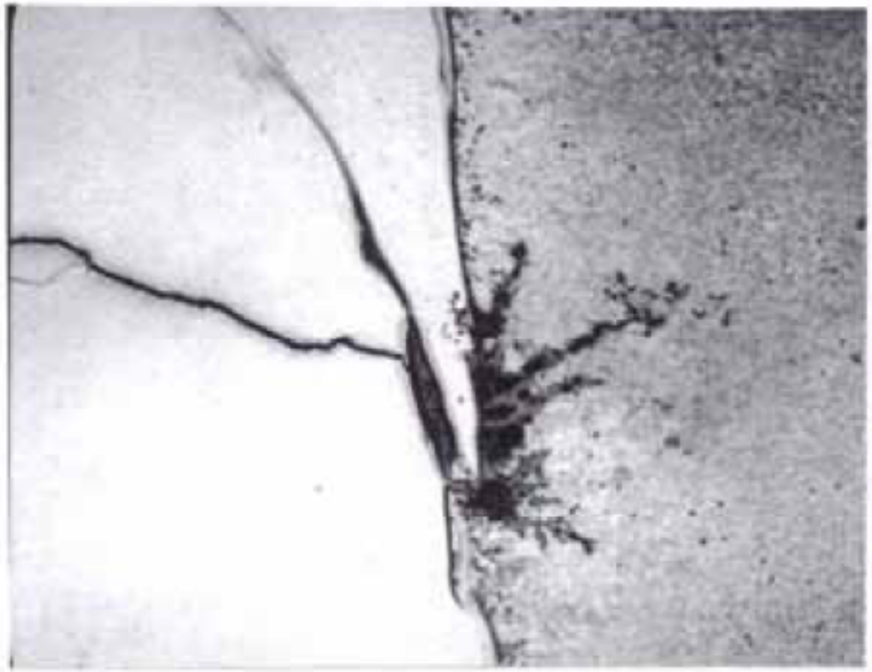


Cr/zone six charge related of land and groove substrate erosion through a micro-crack at the 12:00 bore origin. (Sopok *et al.* 2005)

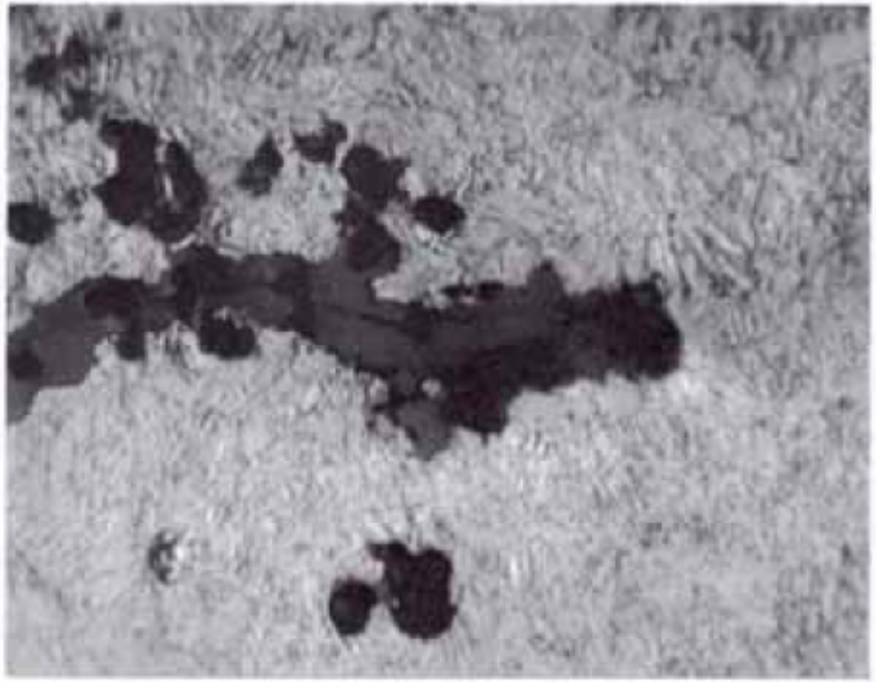


Growing crack in a polycarbonate exposed to acetone (Hejman 2011)

Corrosion Crack crossing a bi-material interface



Corrosion crack penetrating a bimaterial interface between austenitic and pressure vessel steel of type SA533C11.



The tip of one of the crack branches. Crack length 7 mm, notch width 10 µm. *Reproduced with permission from Vattenfall AB.*

Surface Morphology:

Surface Wave Spectrum

Crack Initiation:

Pit, Cusp and Crack

Crack Growth:

Crack Growth, Blunting and Branching

Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993),
Srolovitz (1989), Freund (1995), Kung-Suk (2000)

Chemical potential,

$$\Phi = U_c + U$$

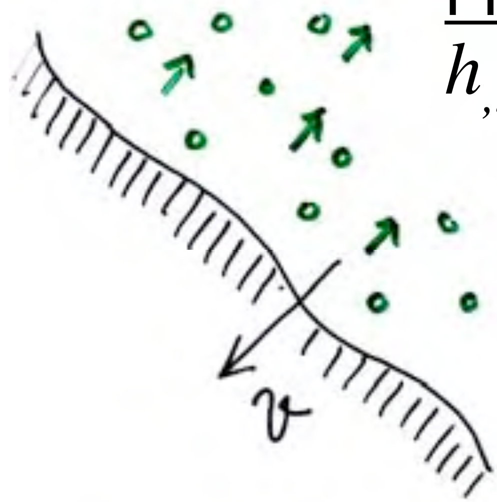
U_c surface energy, U elastic strain energy

Surface diffusion

$$h_{,t} = D\nabla^2\Phi$$

Fluid

$$h_{,t} = k\ \Phi$$



Governing equations:

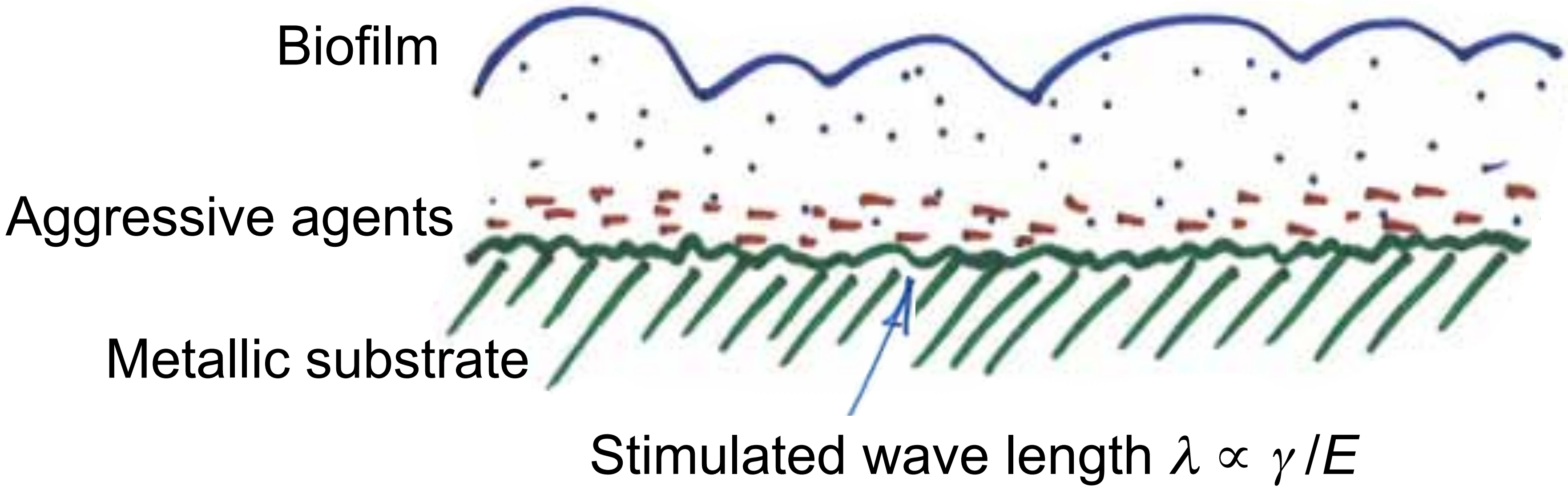
Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left(\gamma \frac{\partial^2 h}{\partial x^2} \text{ - } \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

or surface diffusion

$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left(-\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

Biocorrosion



Competing mechanisms

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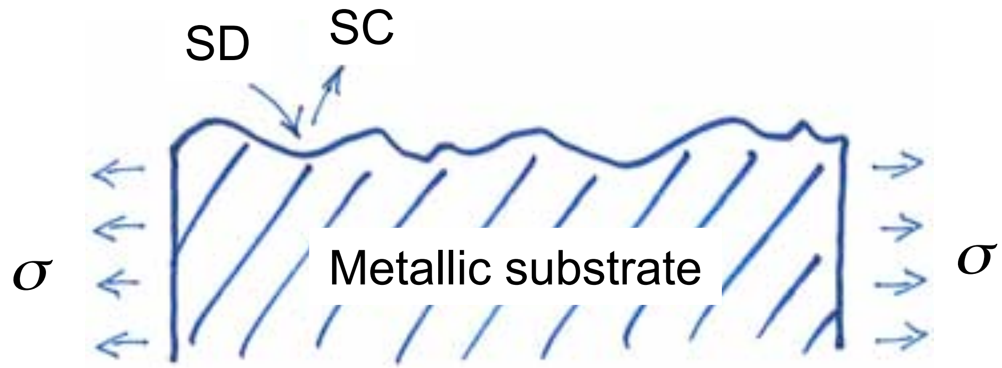
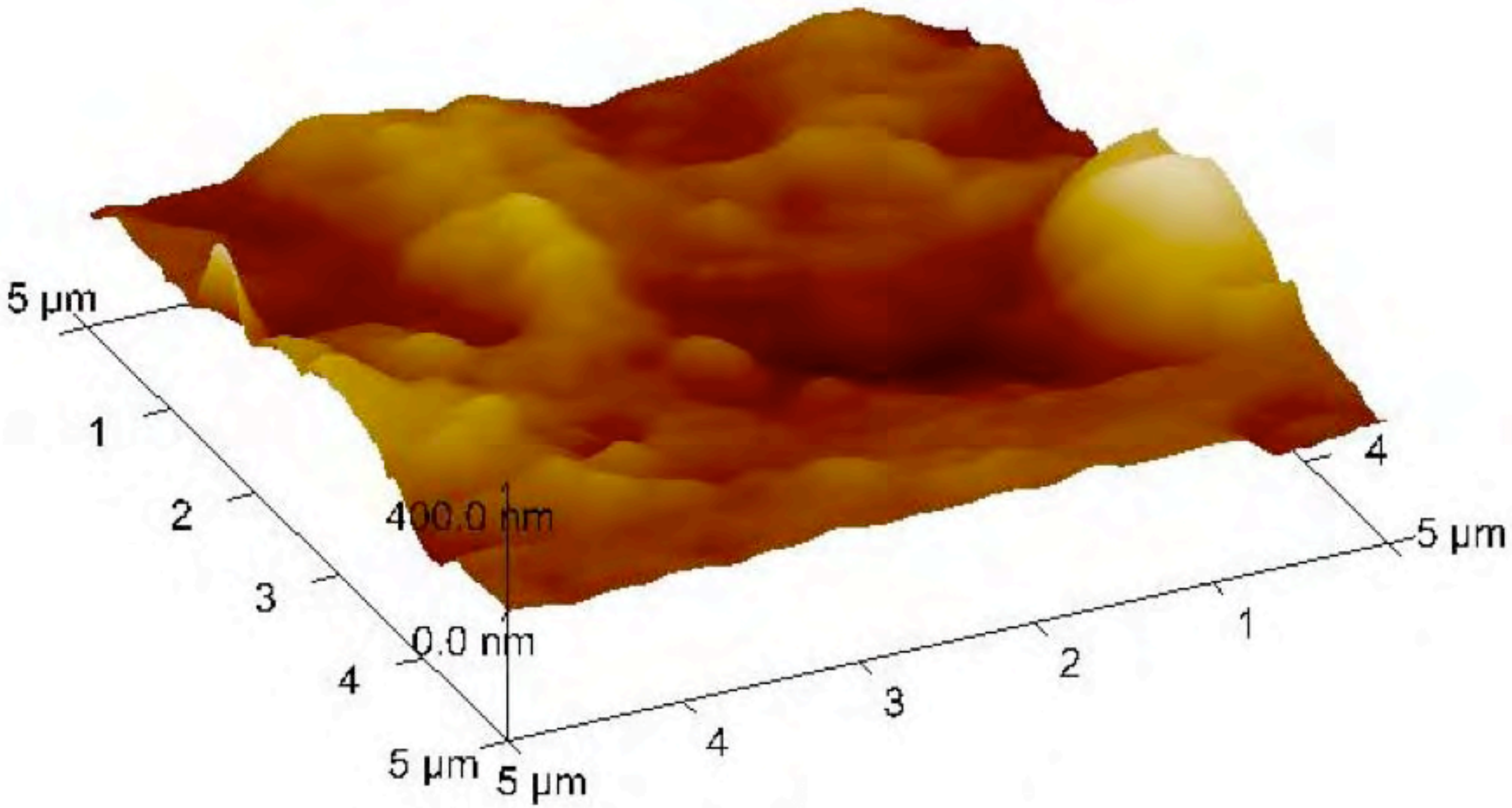
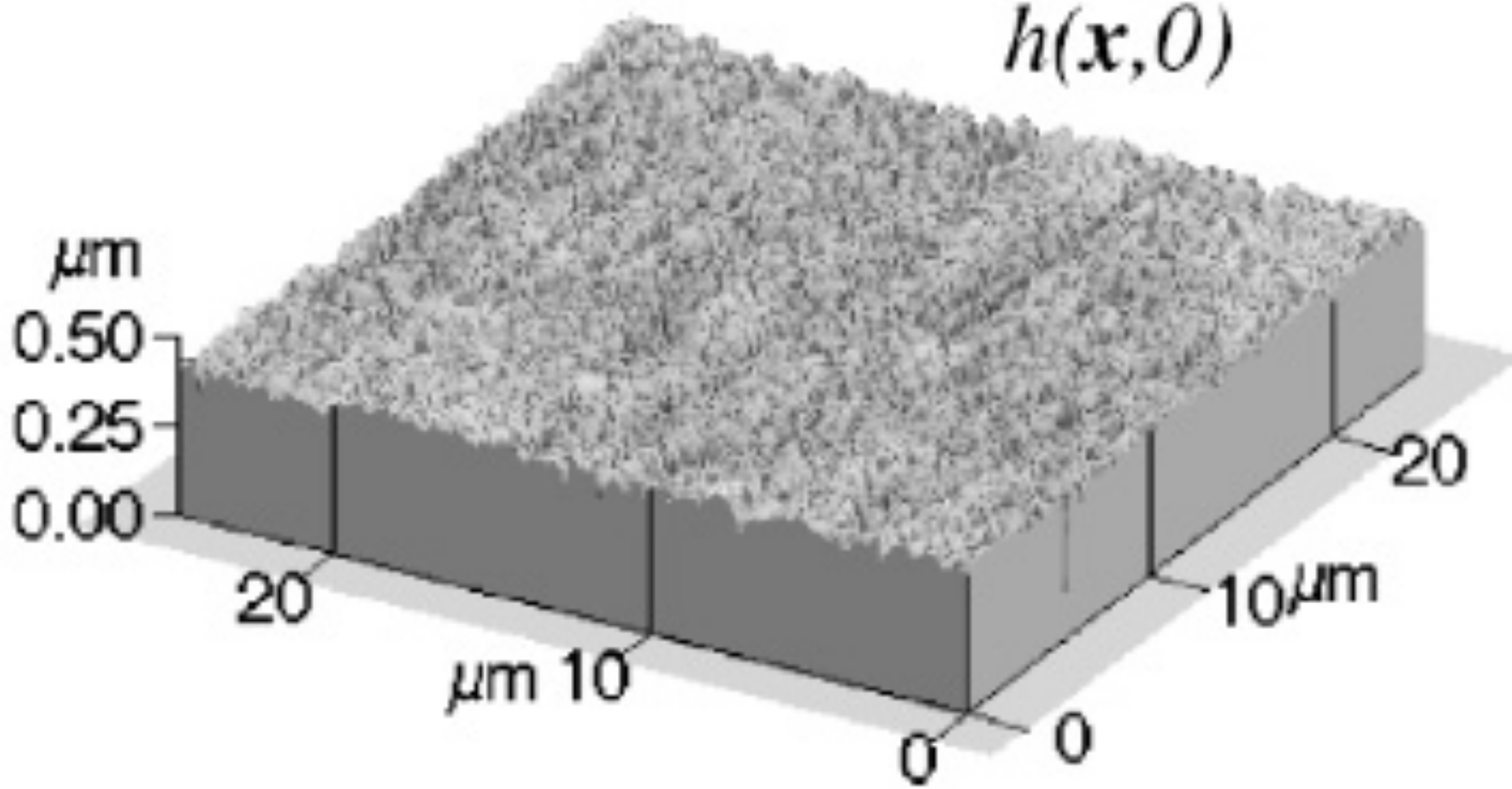
SC - stress corrosion





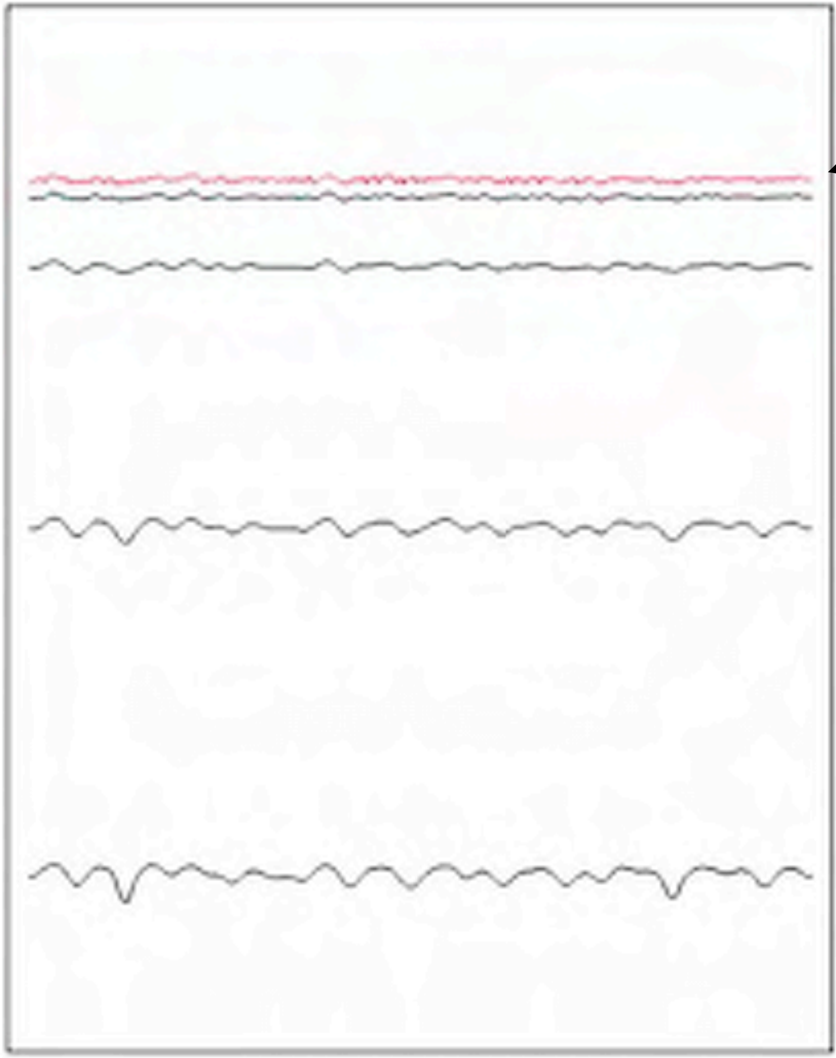
AFM image of corroded EBM surface (Hejman 06)



AFM image of shallow etched aluminium (Kim *et al*. 2000)

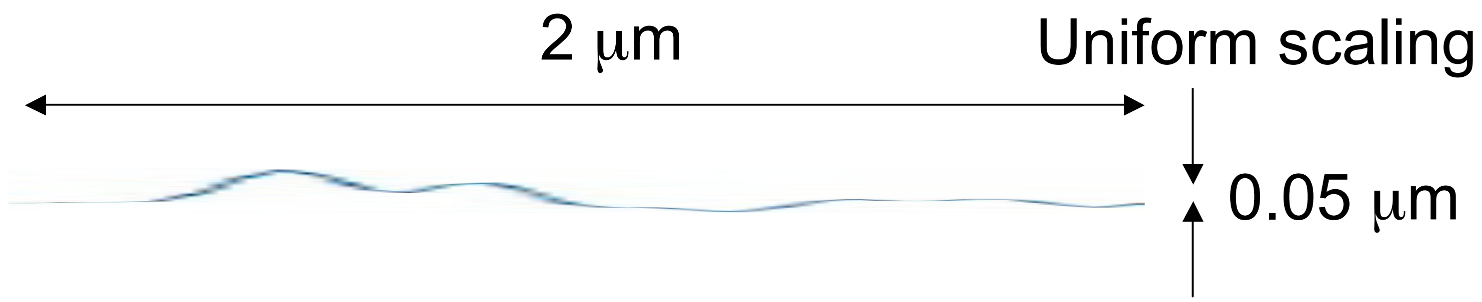
FEM calculation of an evolving surface

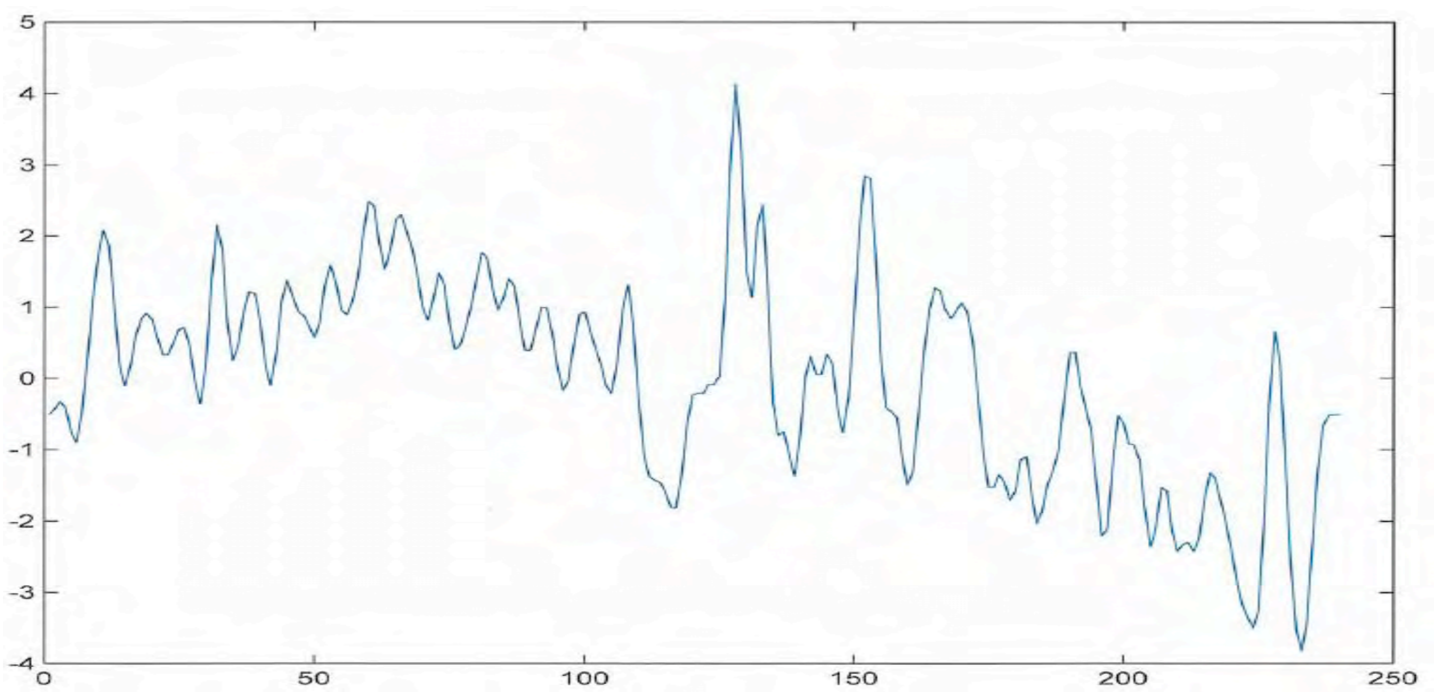
Increasing time and depth



Original surface

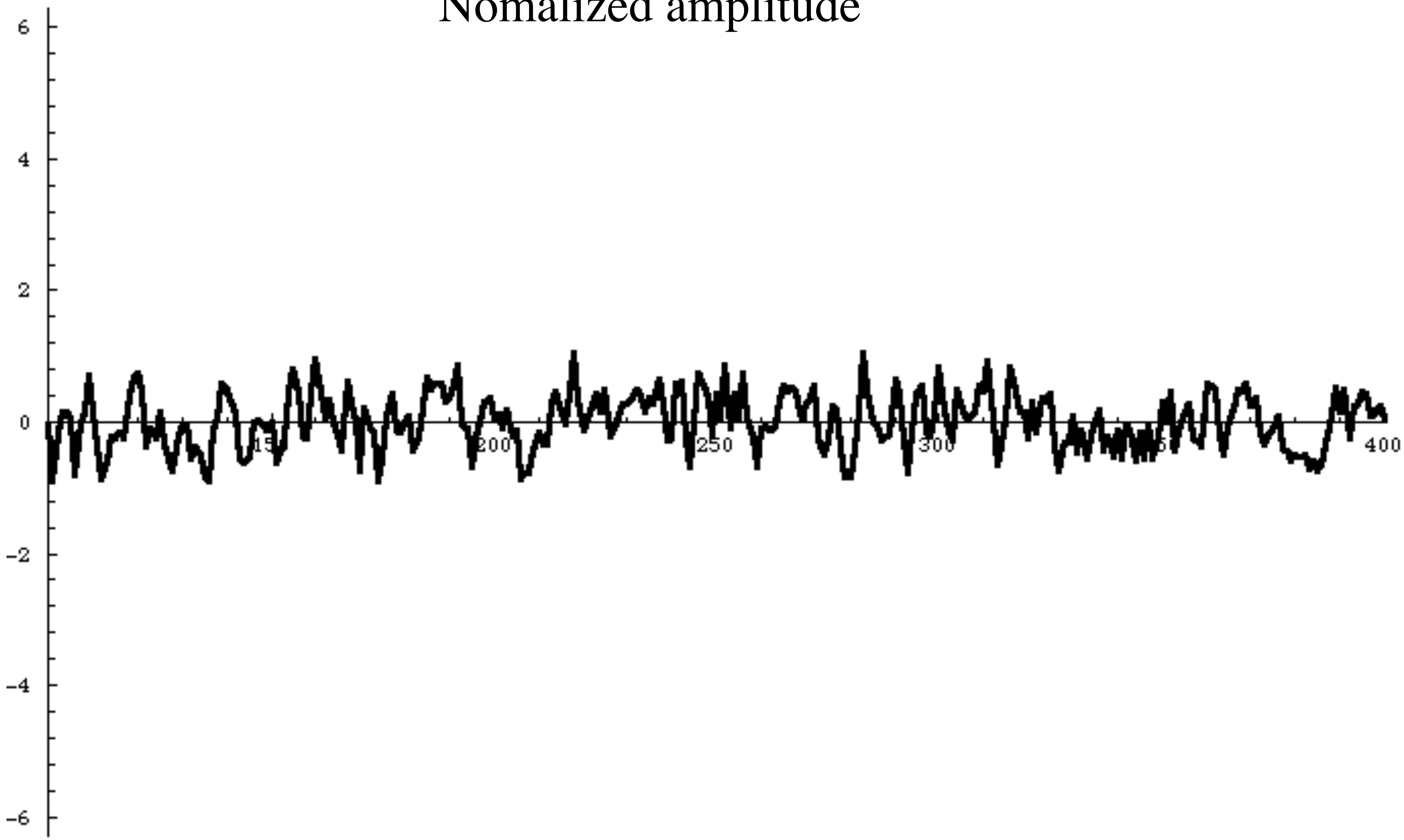
Surface Roughness

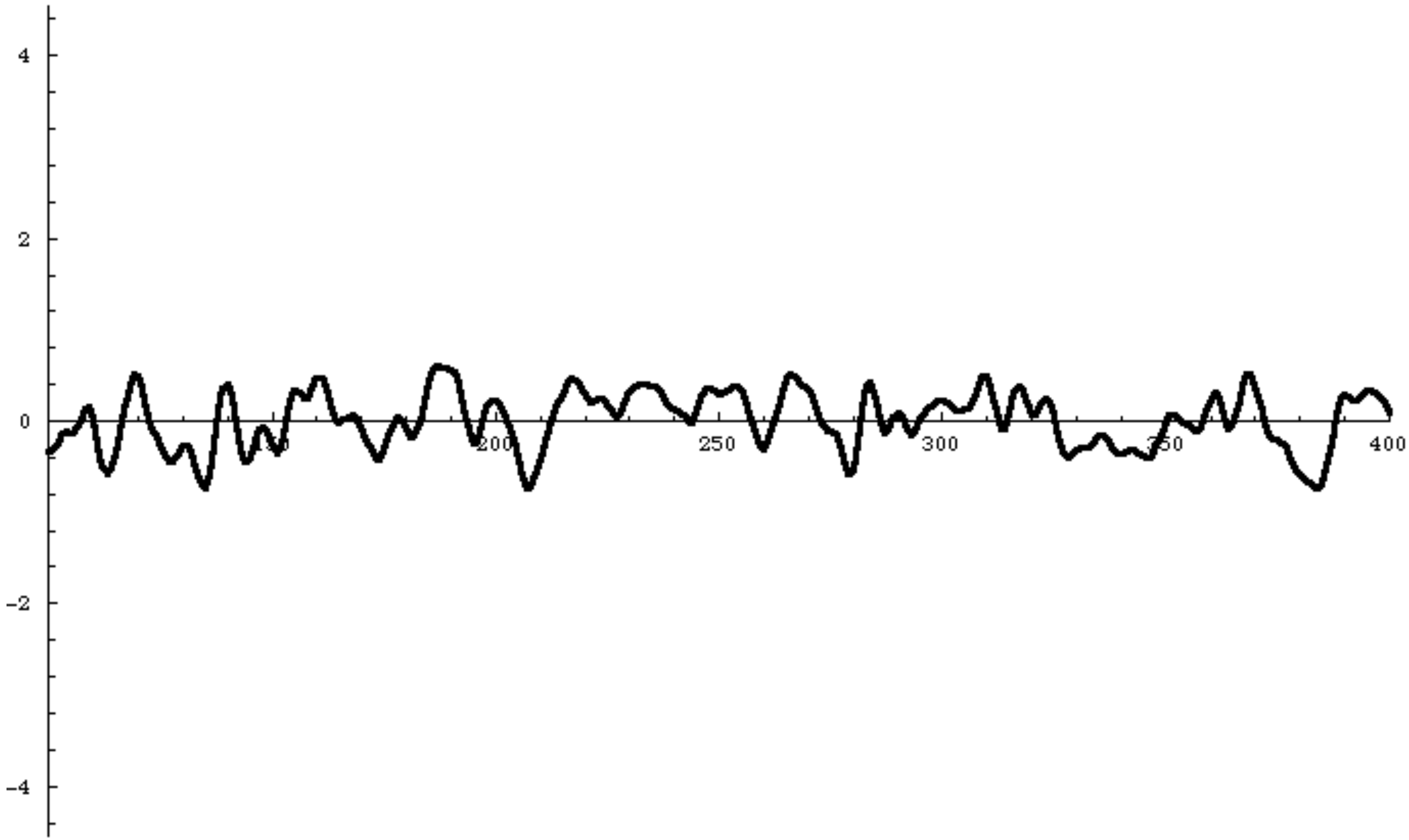




From Kim Kung Suk

Nomalized amplitude





4
2
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200
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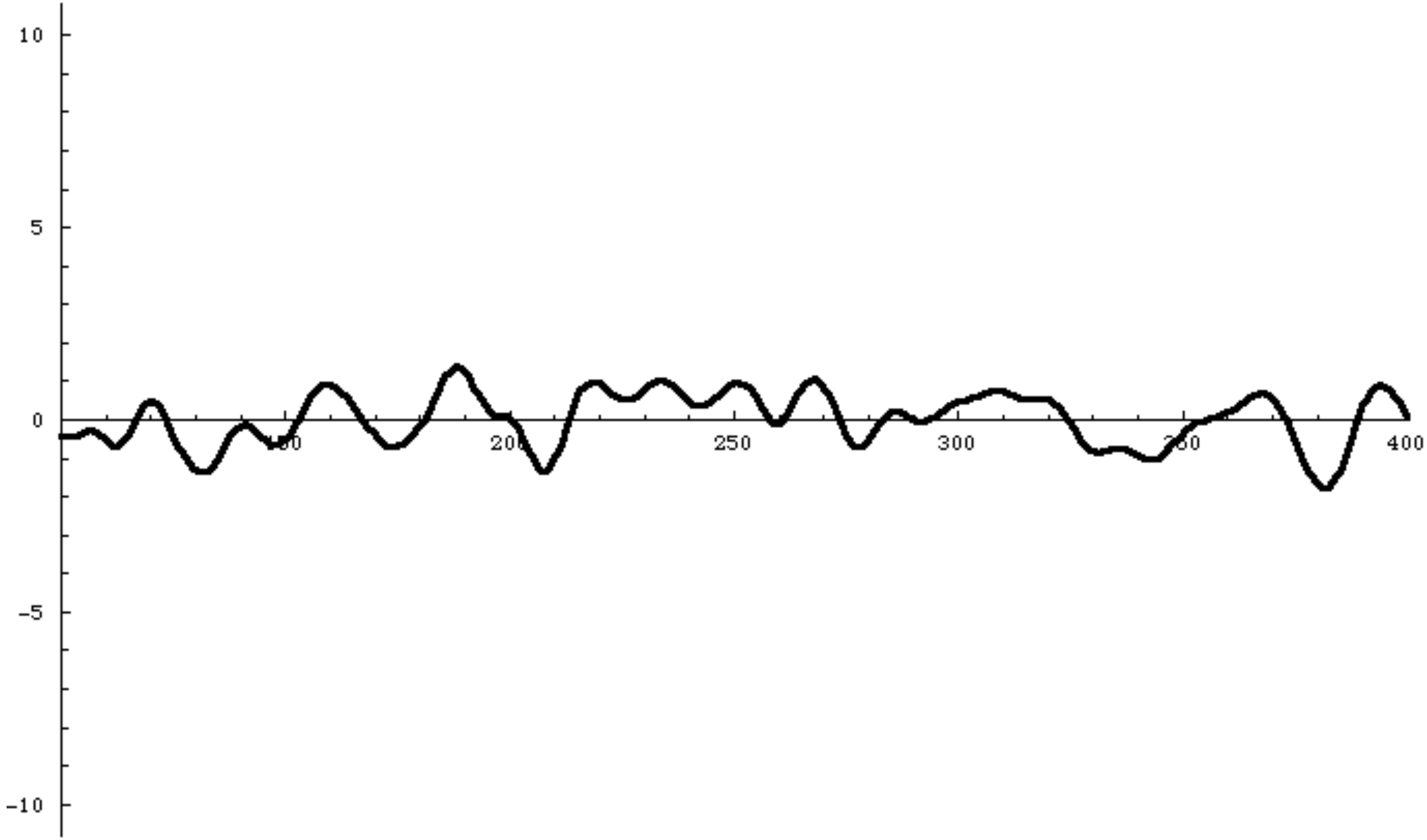
400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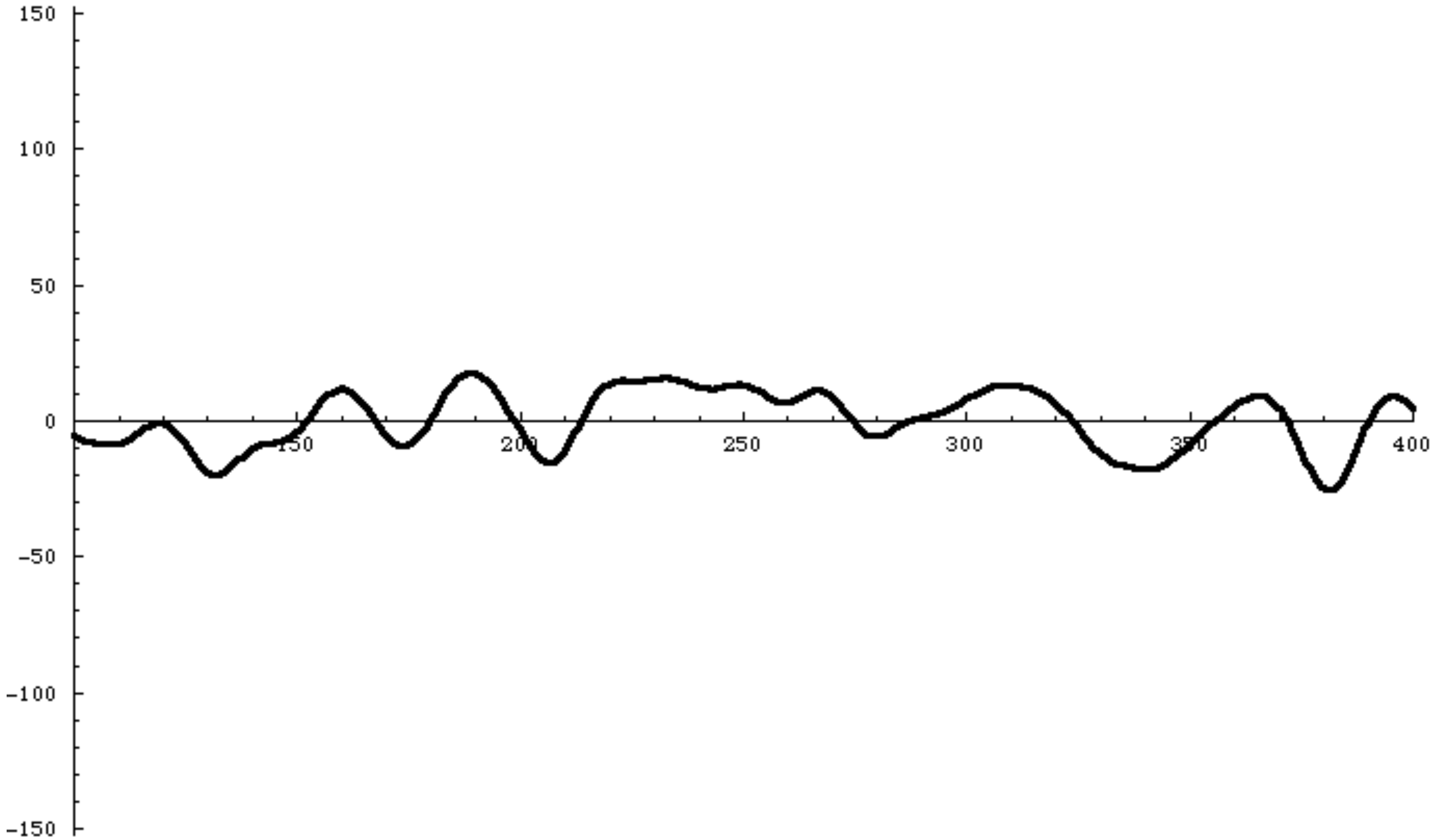
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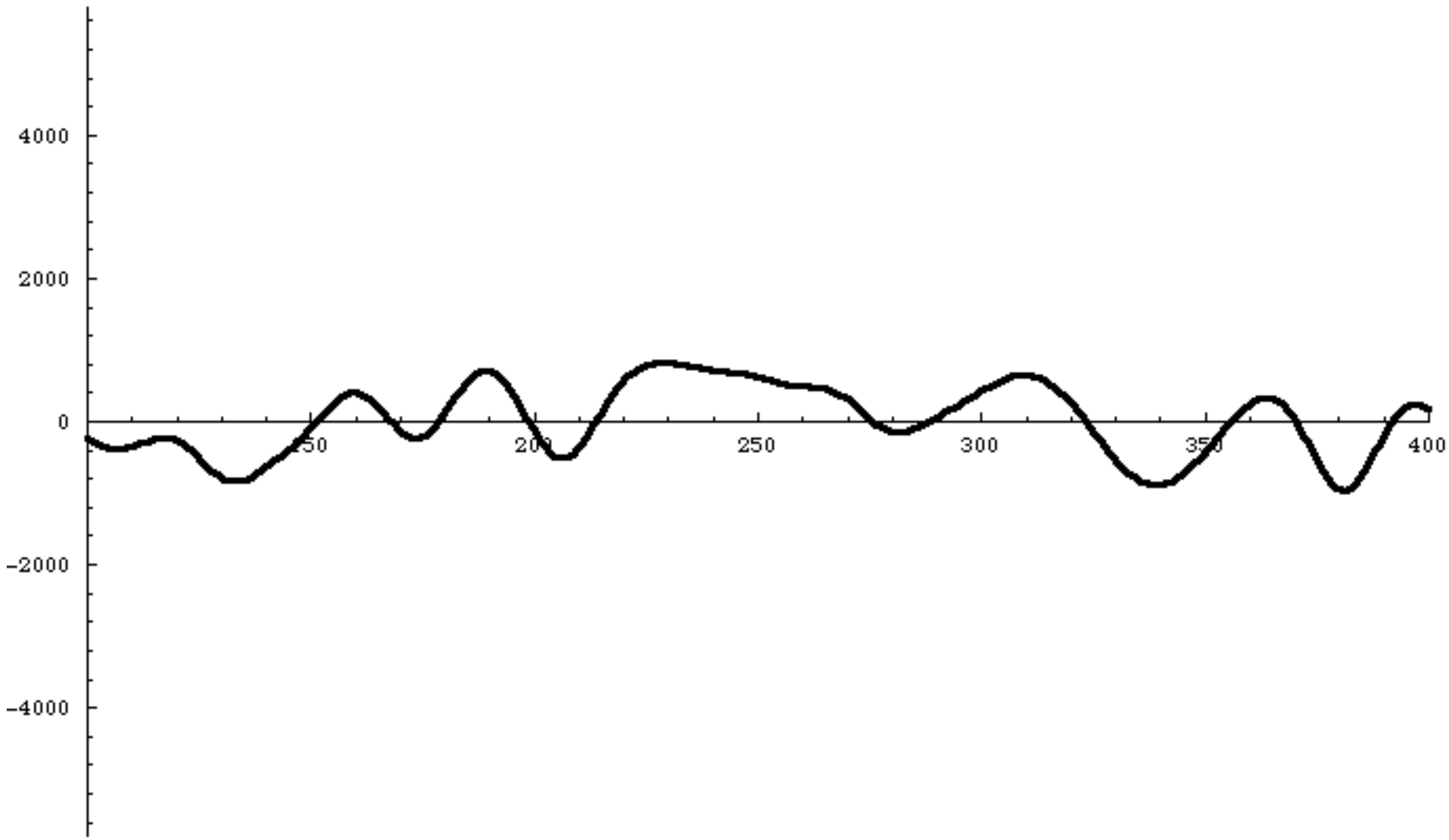
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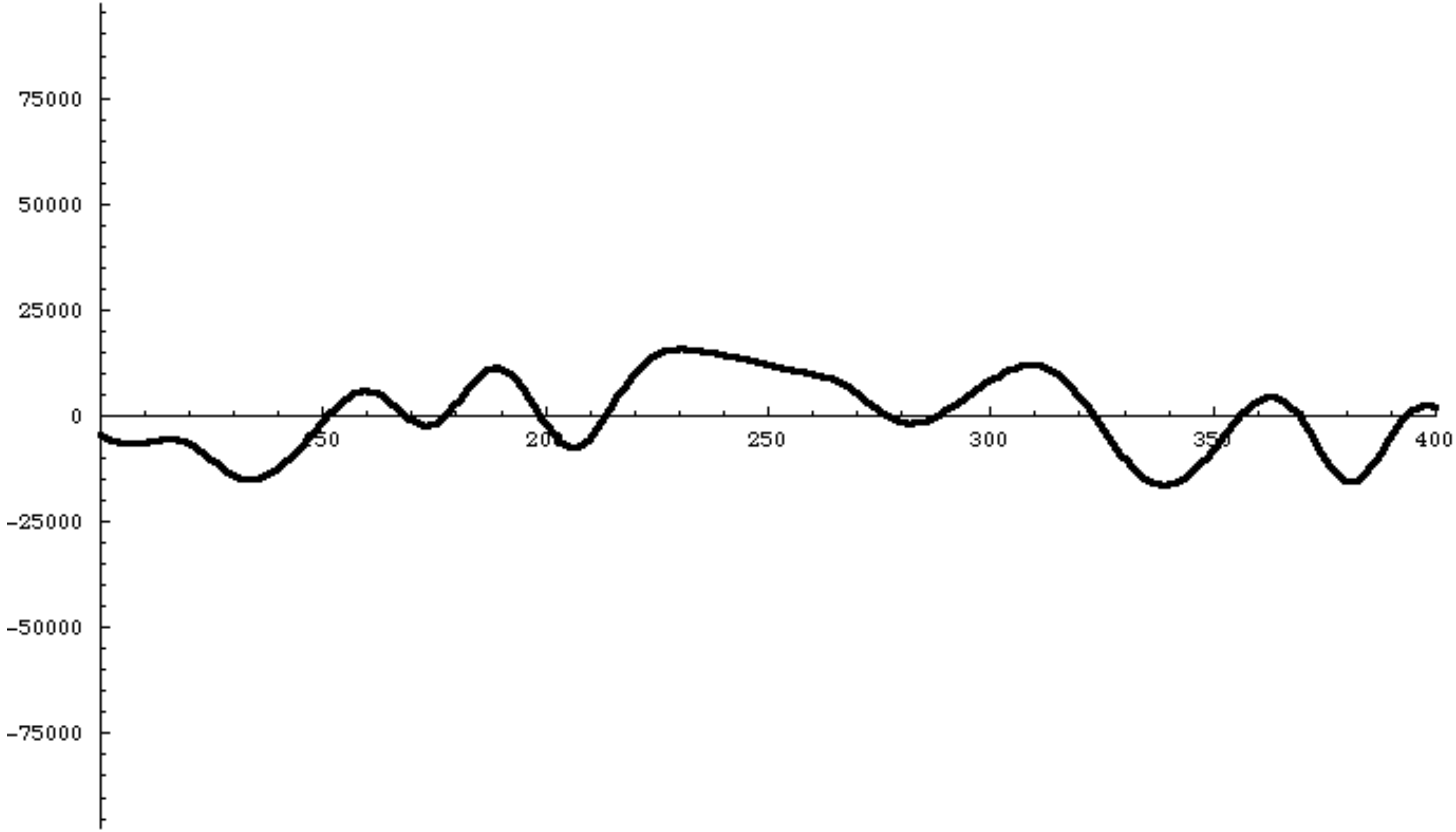
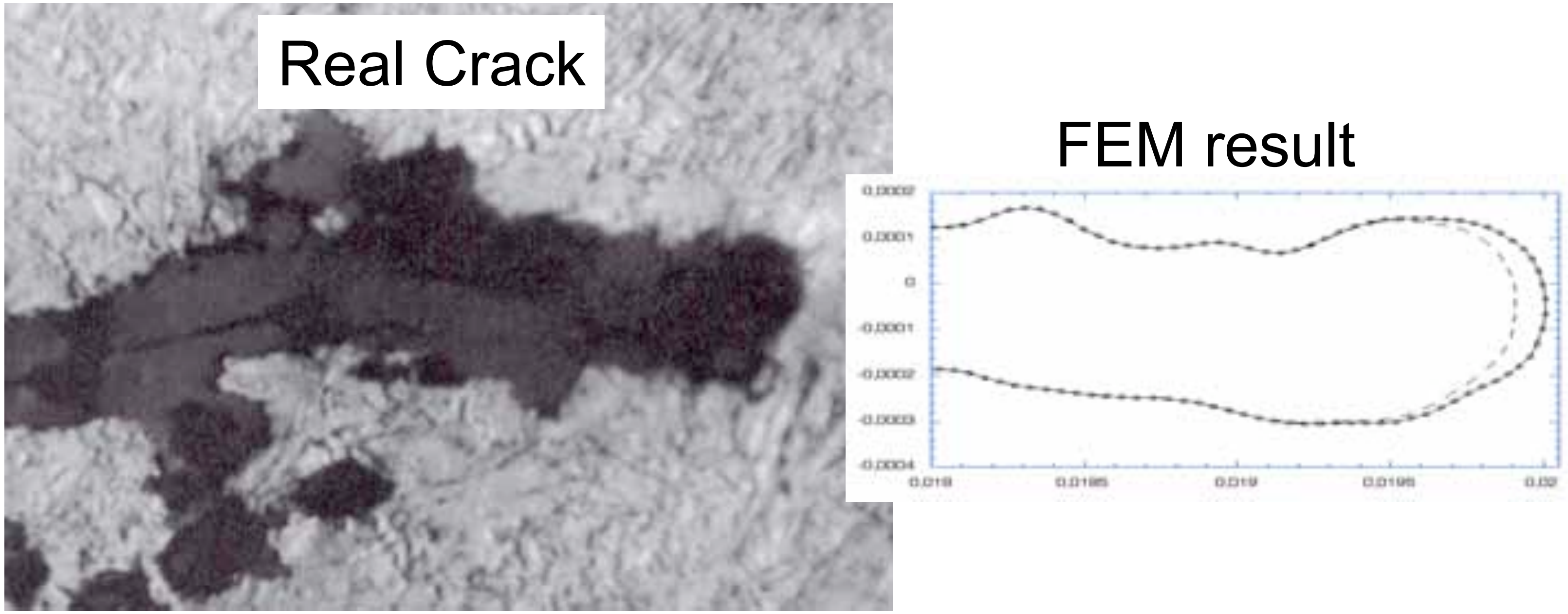
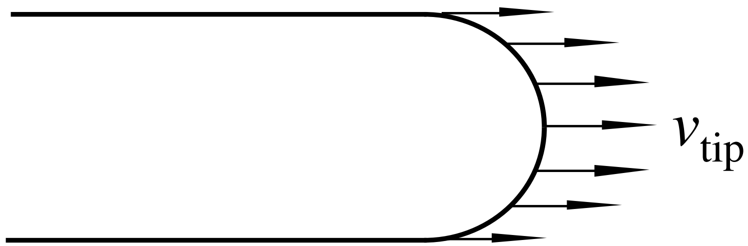
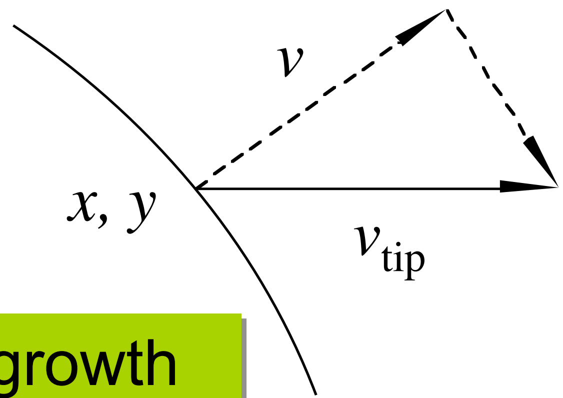


Steady State Crack-tip Shape

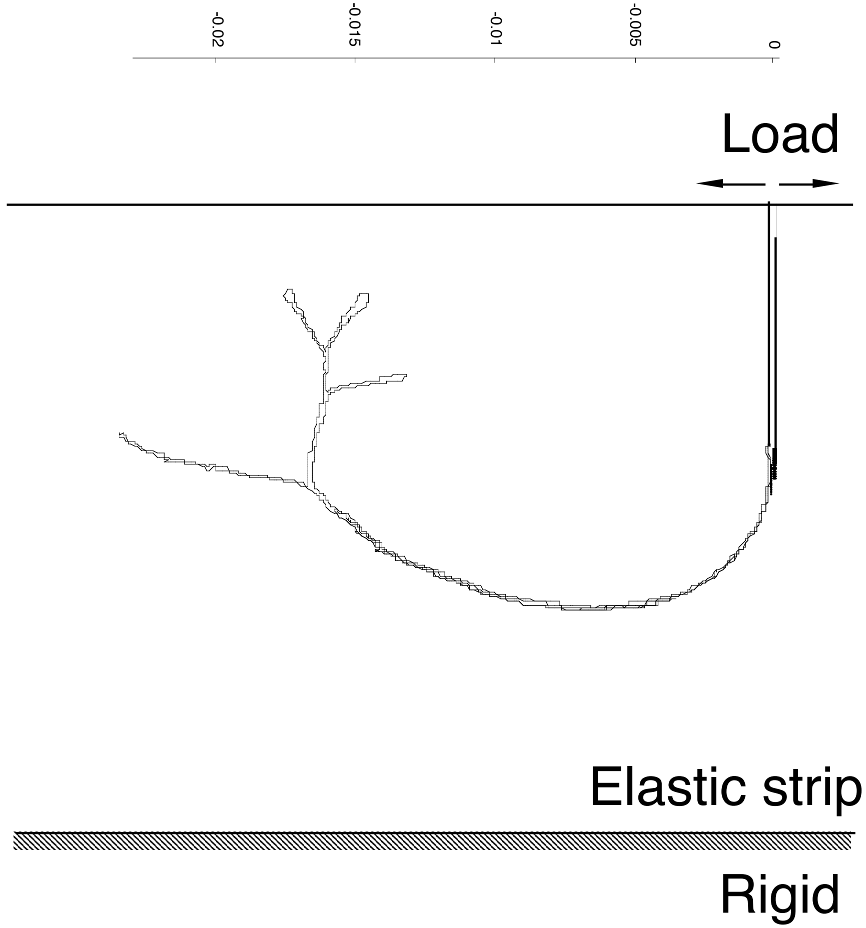


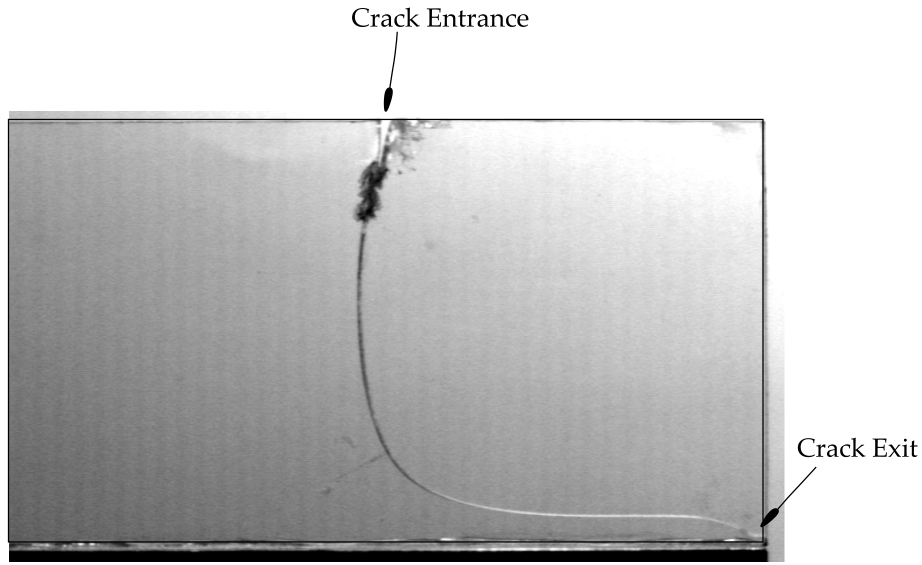




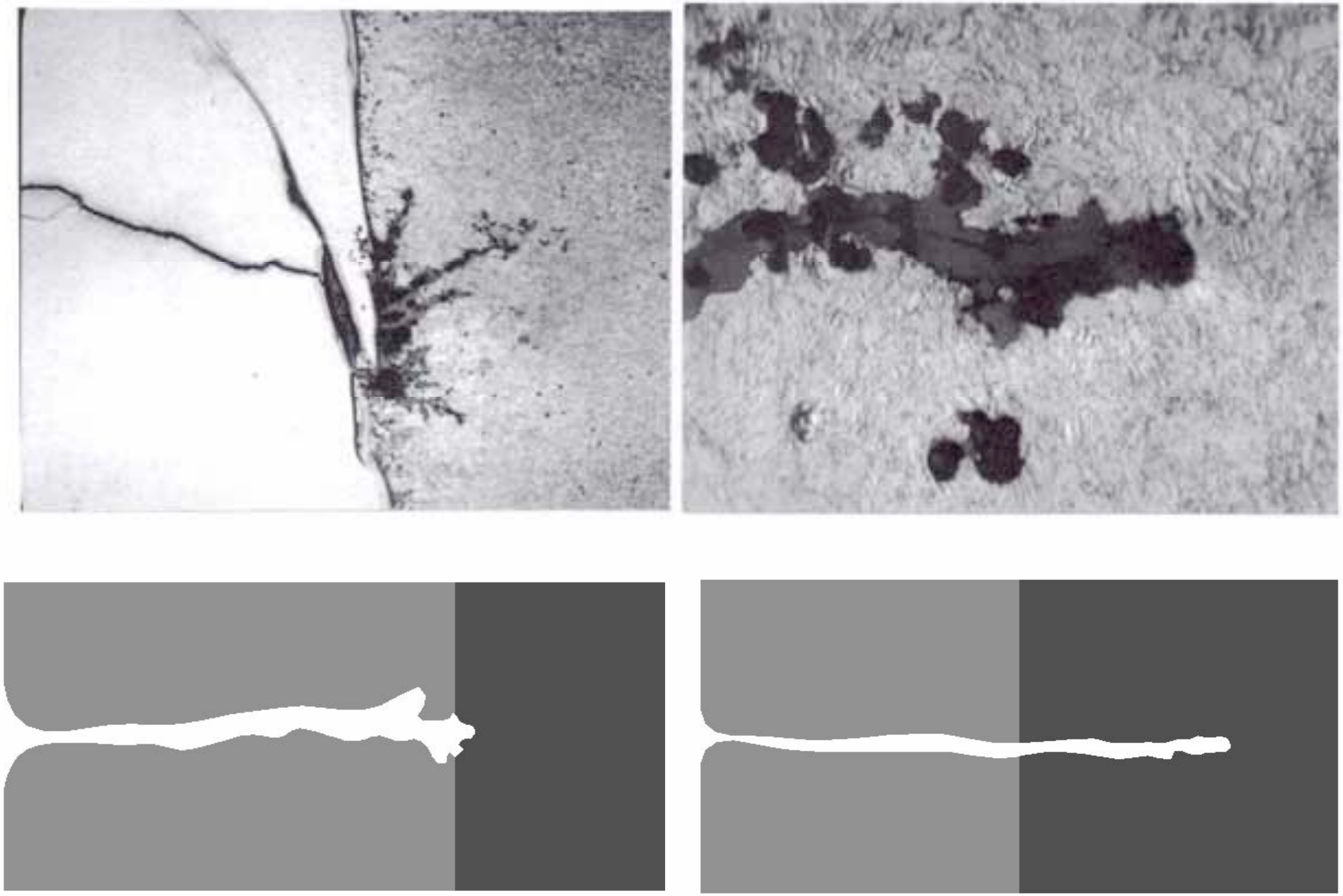
Constant Component in the growth direction: $v = (x/\rho)\ v_{tip}$

Crack seem to follow a mode I path

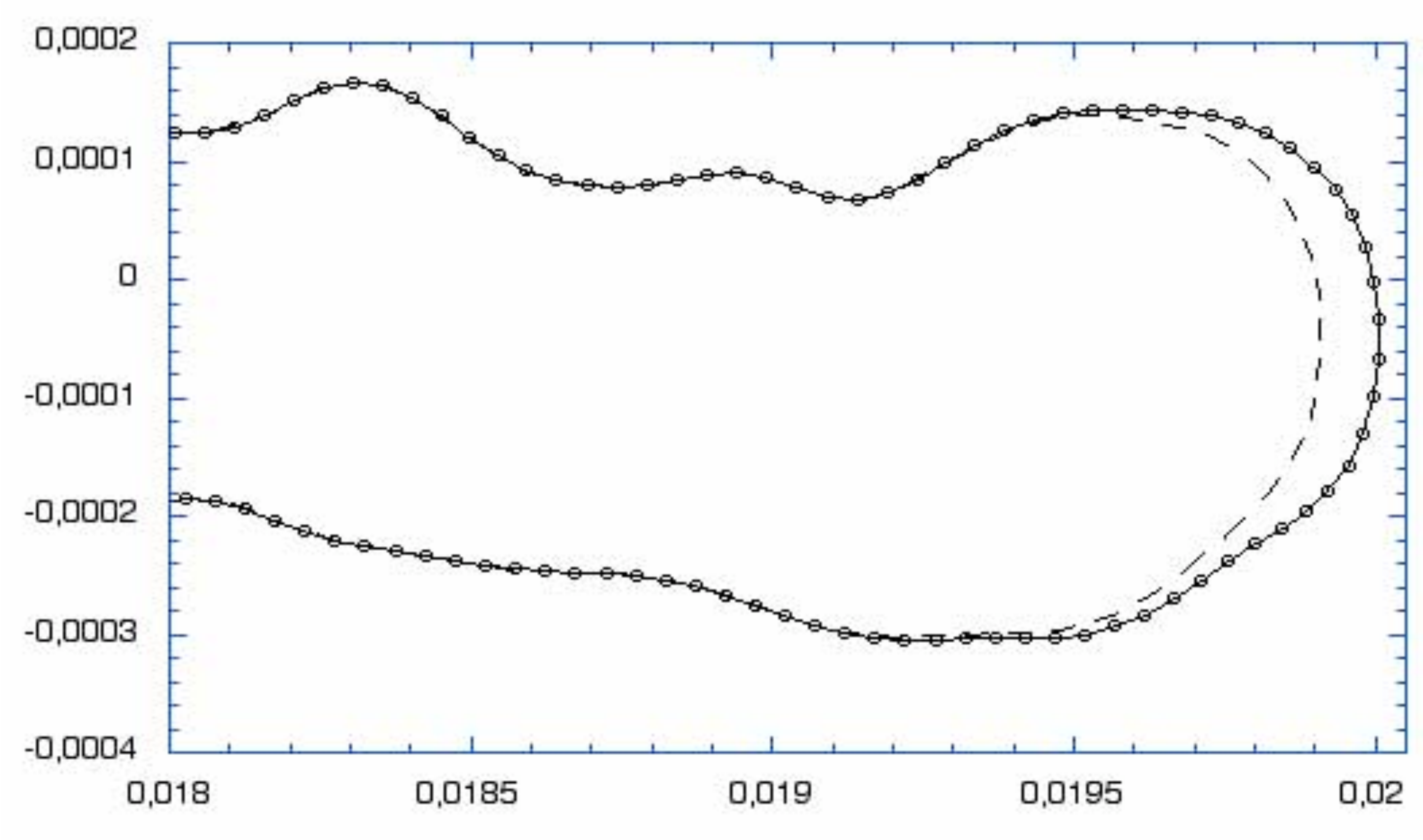




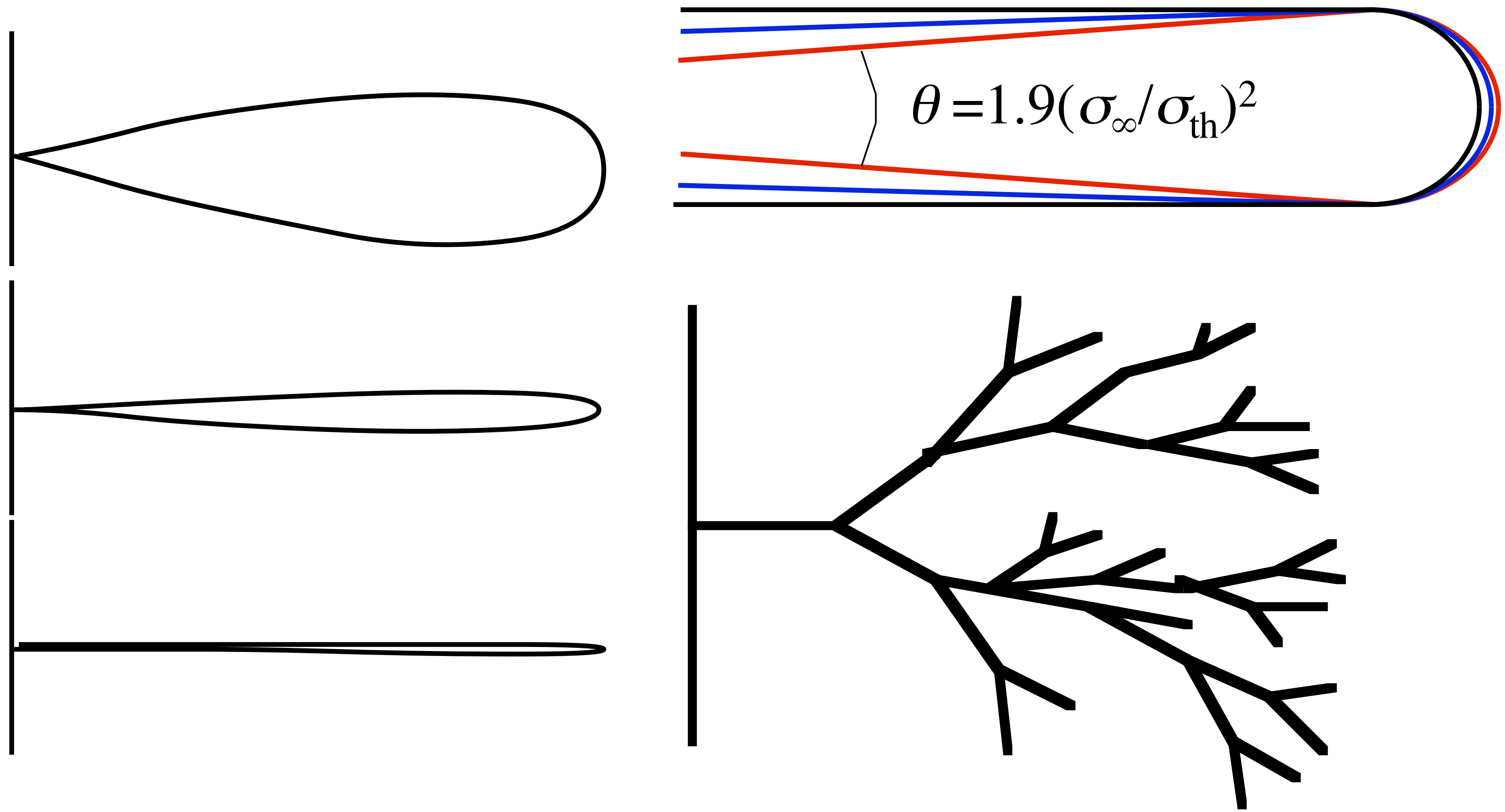
From Cladding to Grey Material



Steady-state Advancing Crack Tip



Missing lengthparameter => Selfsimilar growth



2. Cusp solutions

Spencer and Meiron(94), Chiu and Gao (95),
Yang Xiang and Weinan (02)

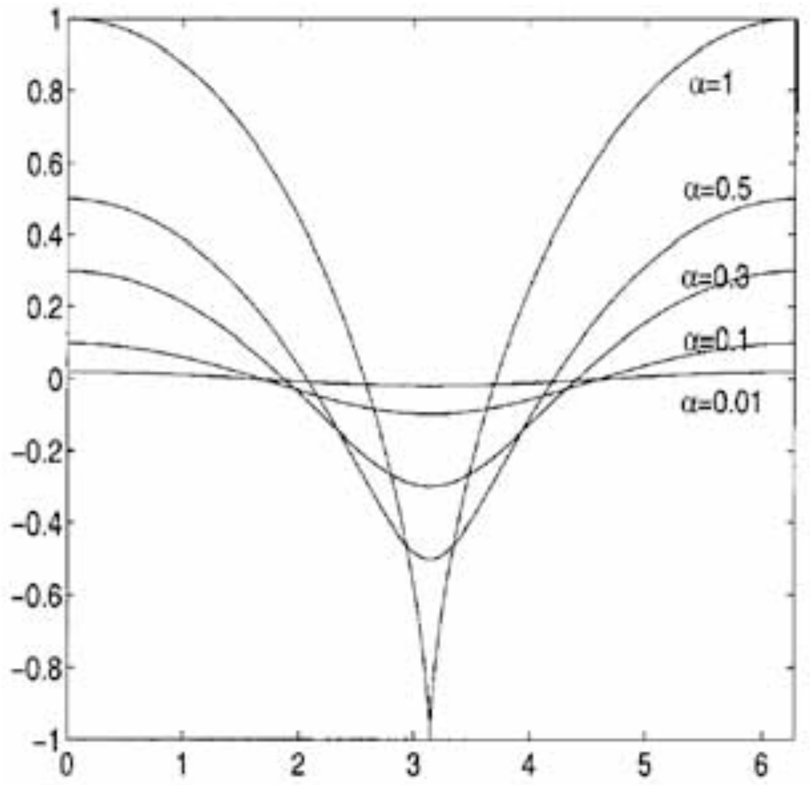
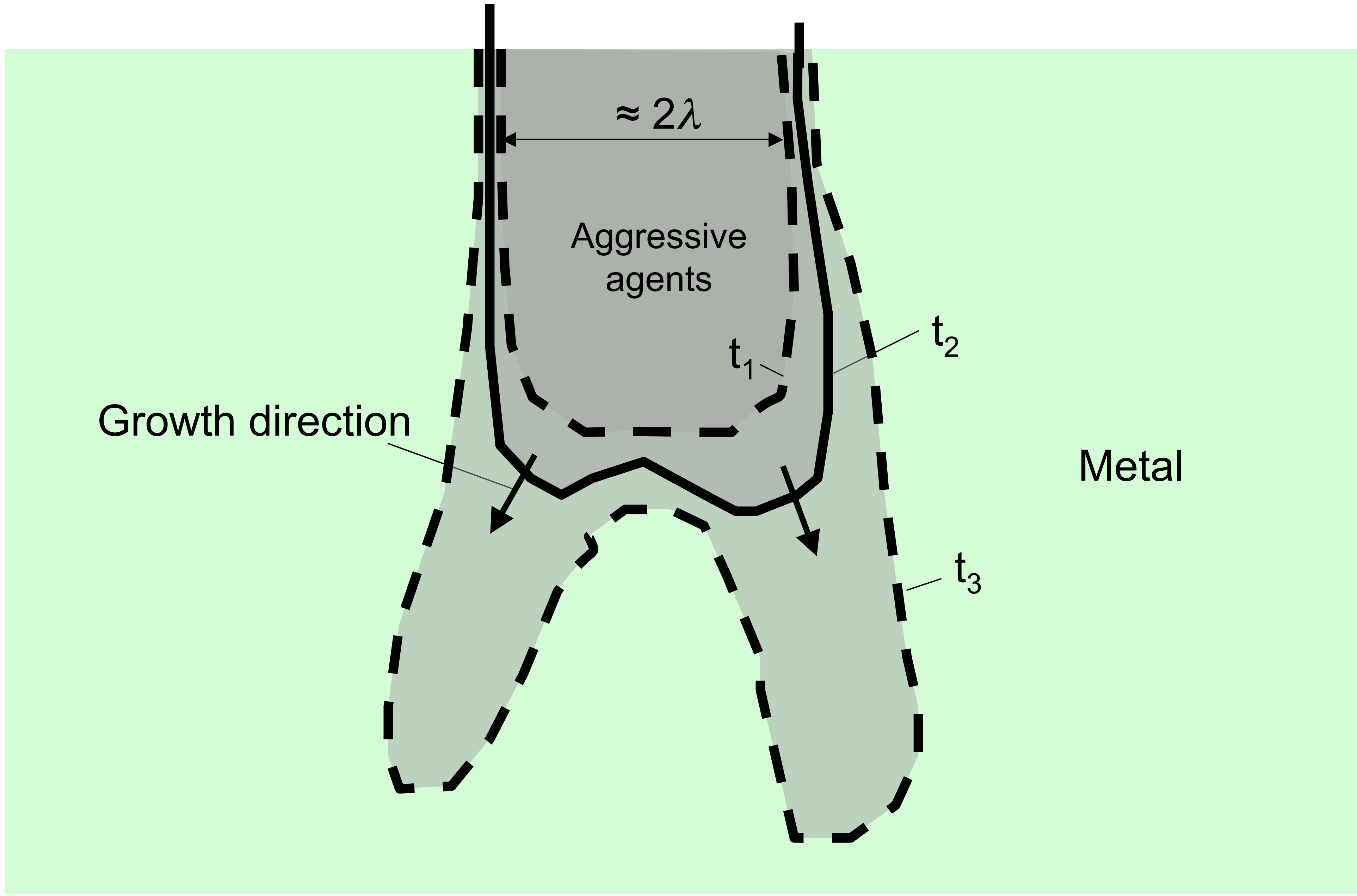


FIG. 1. Cycloid surfaces.

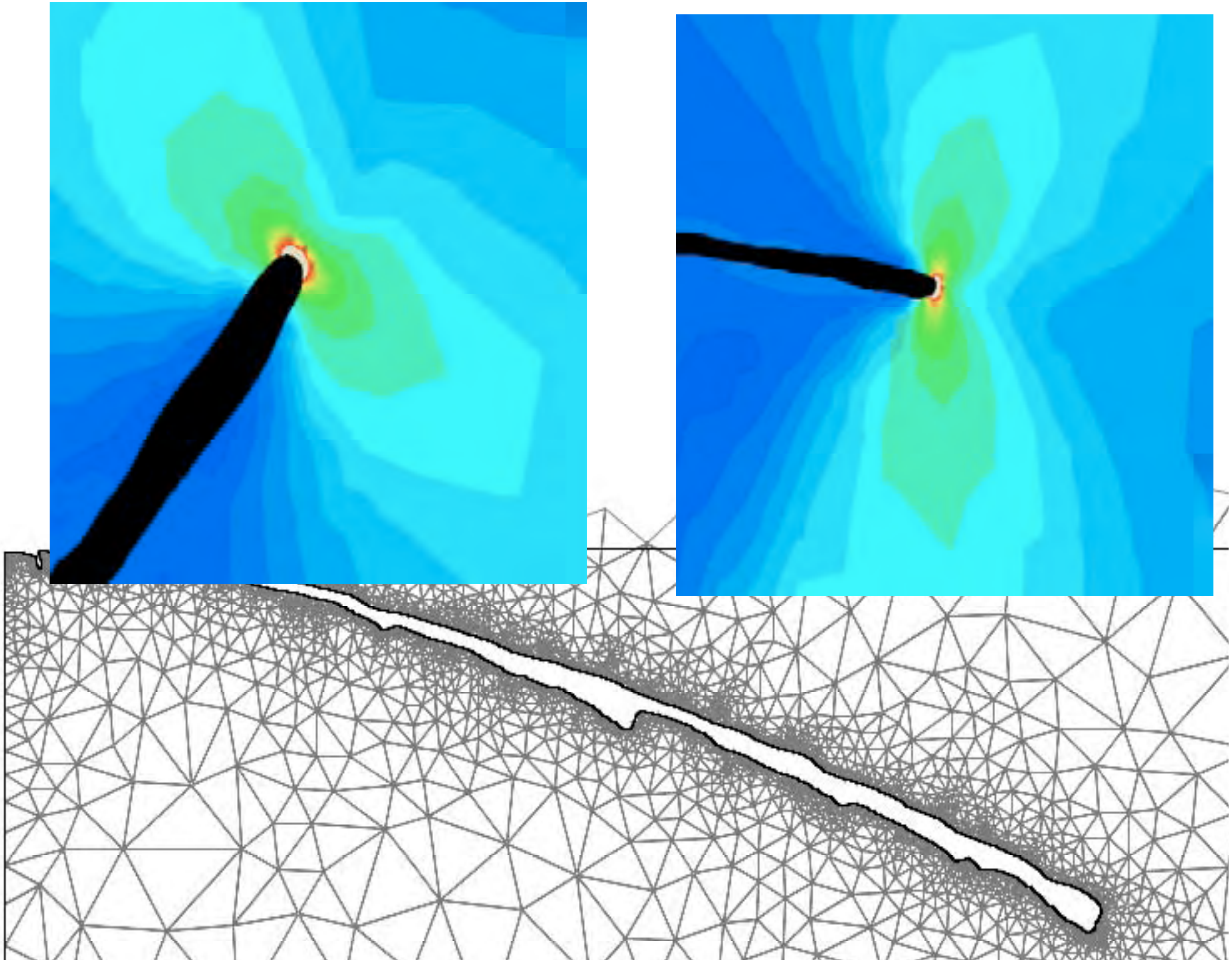
FEM stress corrosion

	$\varepsilon_f/\varepsilon_\infty = 0$	0.3	0.5	0.75	1	1.2
$\sigma_x/\sigma_y = 0$						
0.5						
0.7						
0.9						

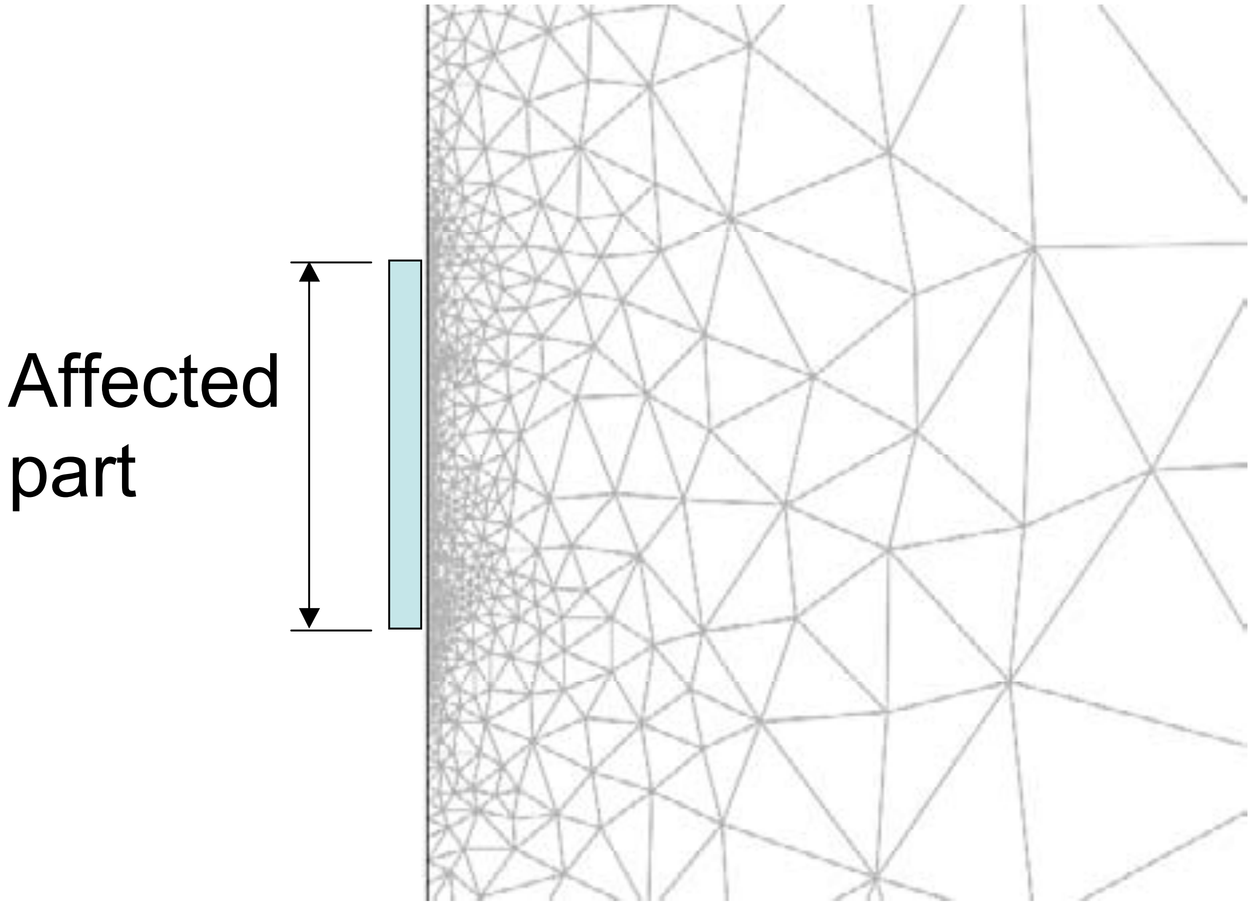
Branching



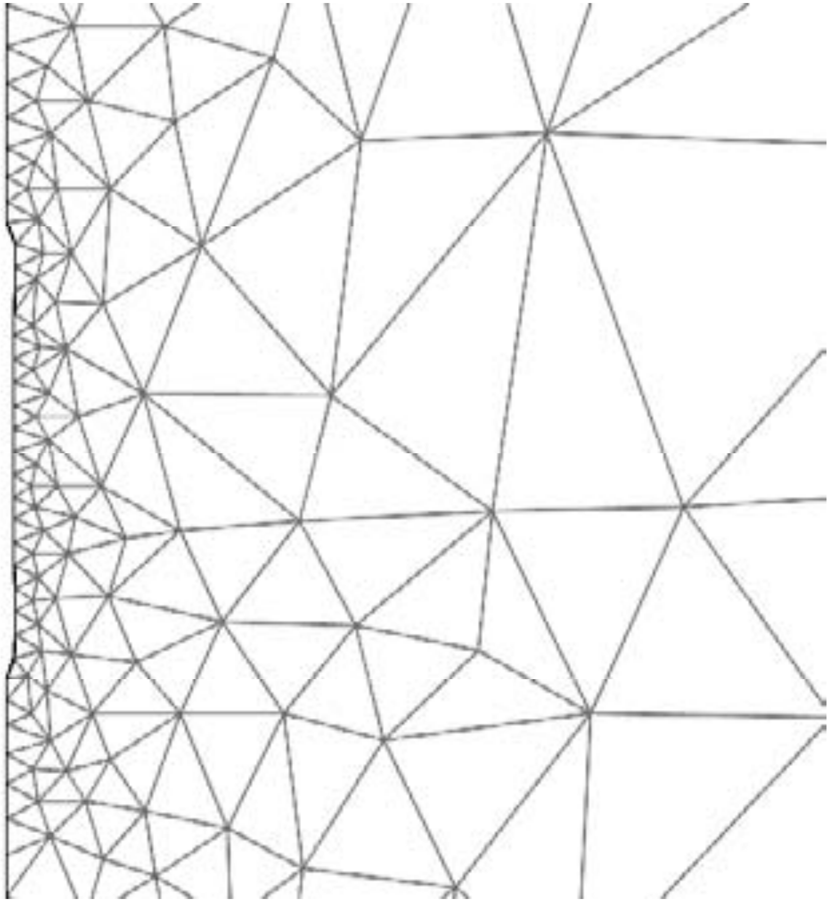
Crack-tip Load



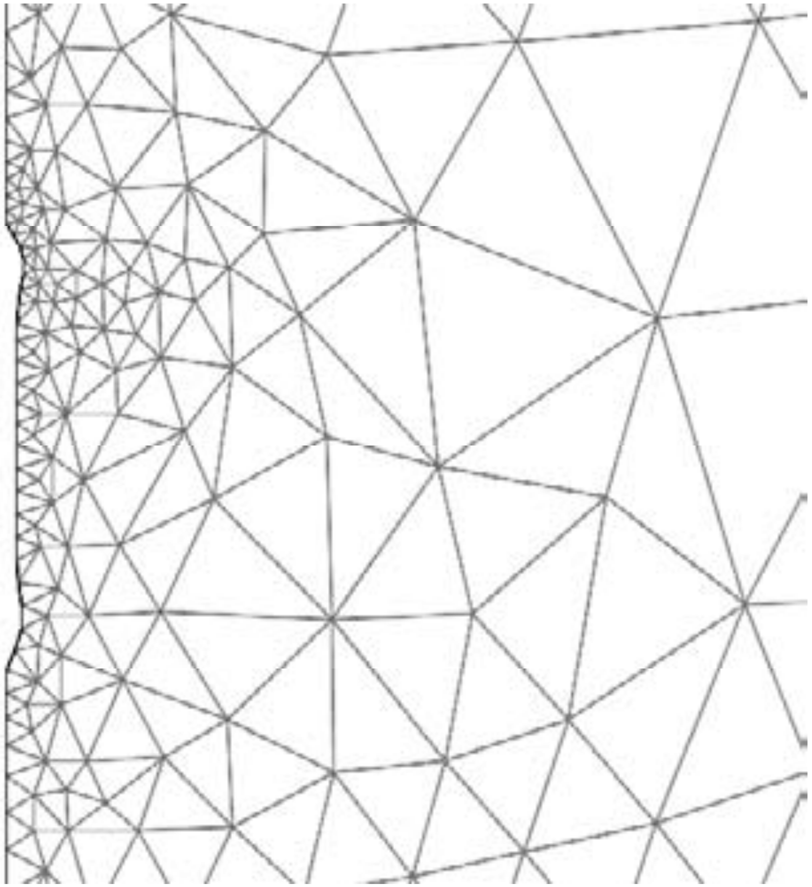
Corroding Surface



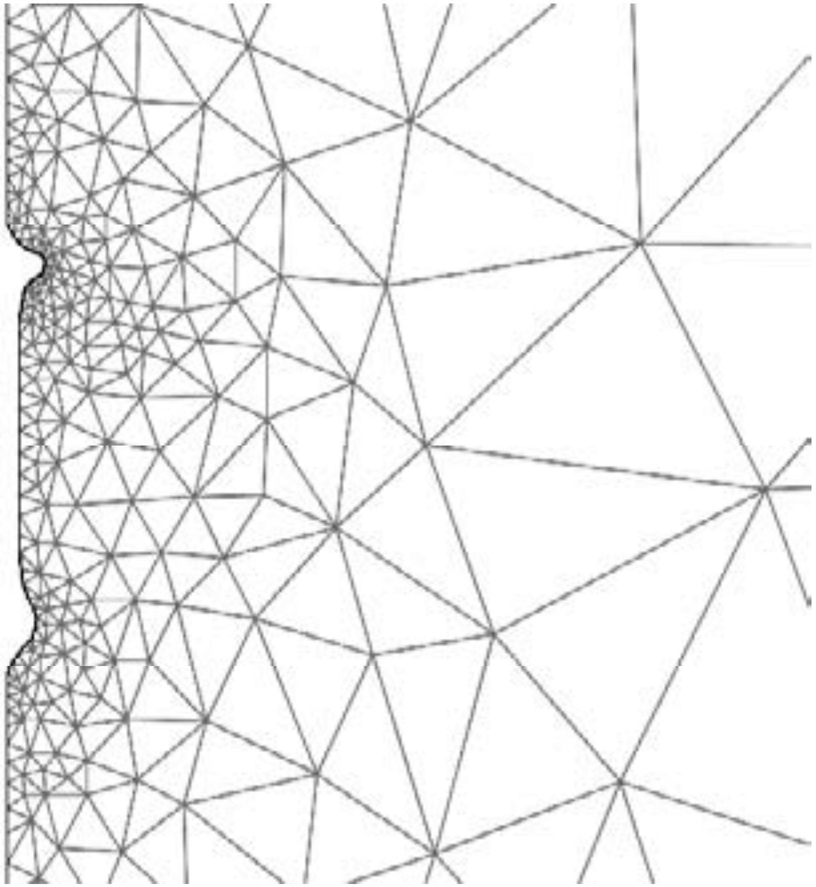
Corroding Surface



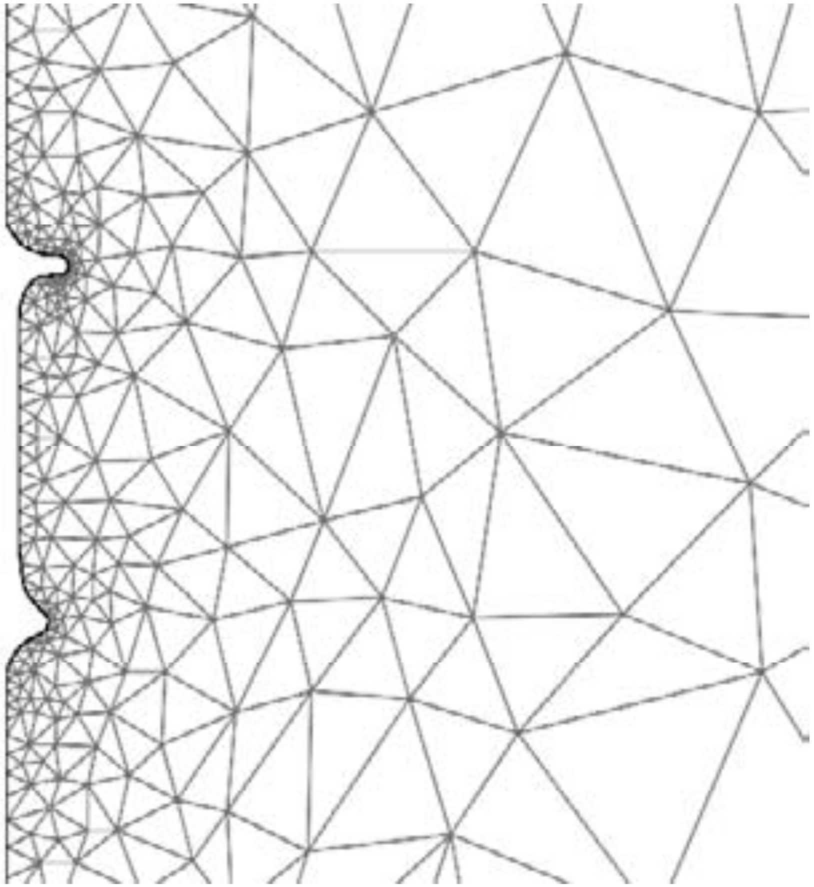
Corroding Surface



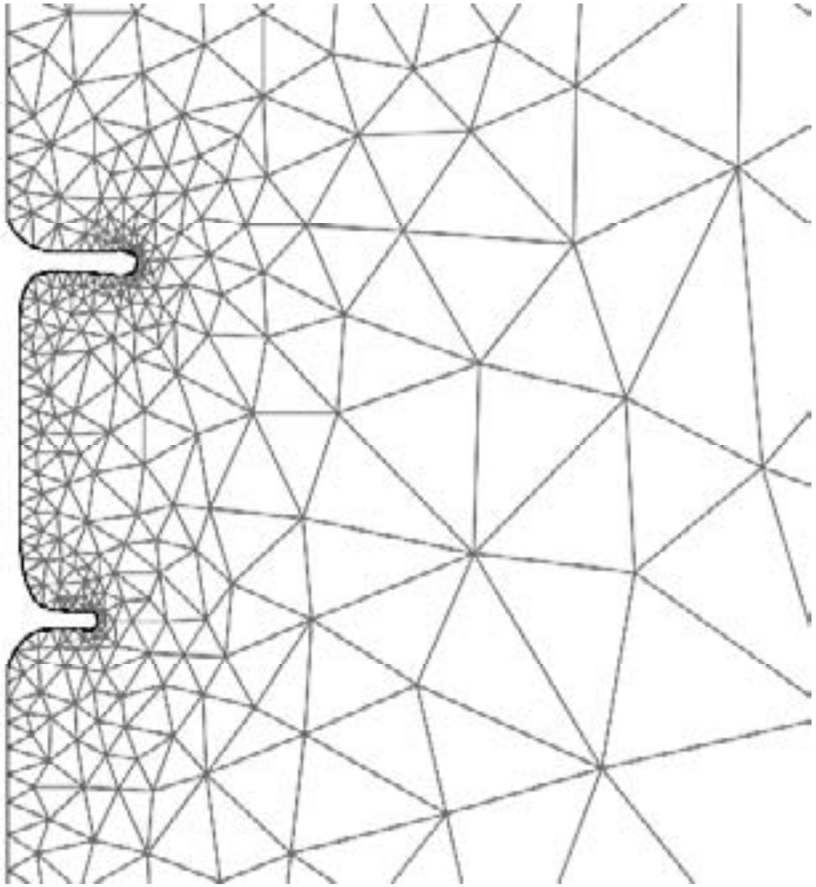
Corroding Surface



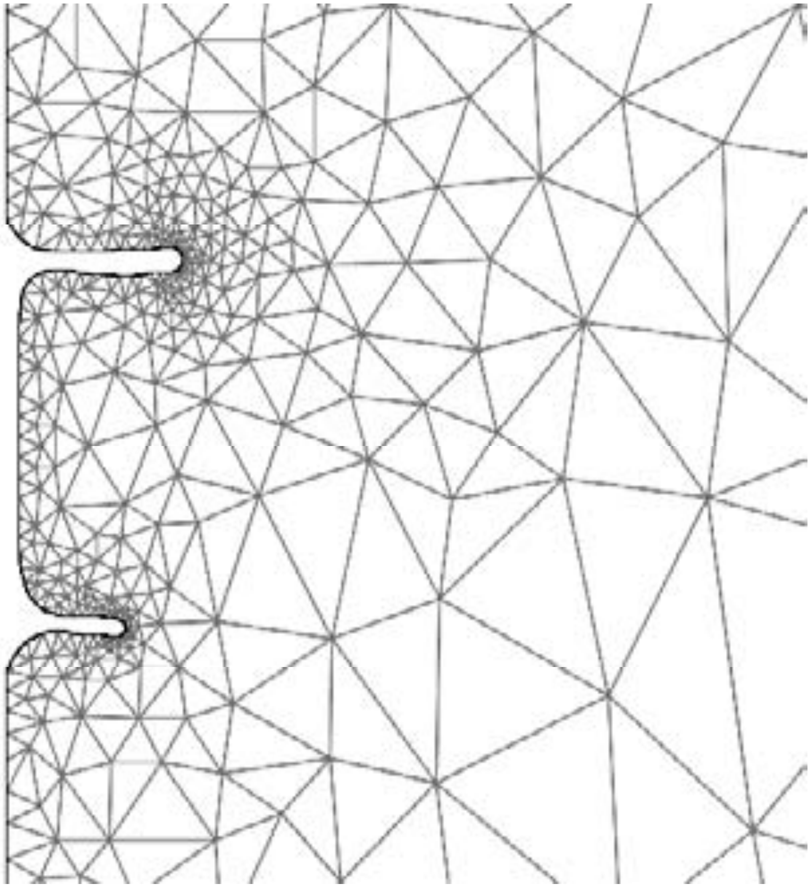
Corroding Surface



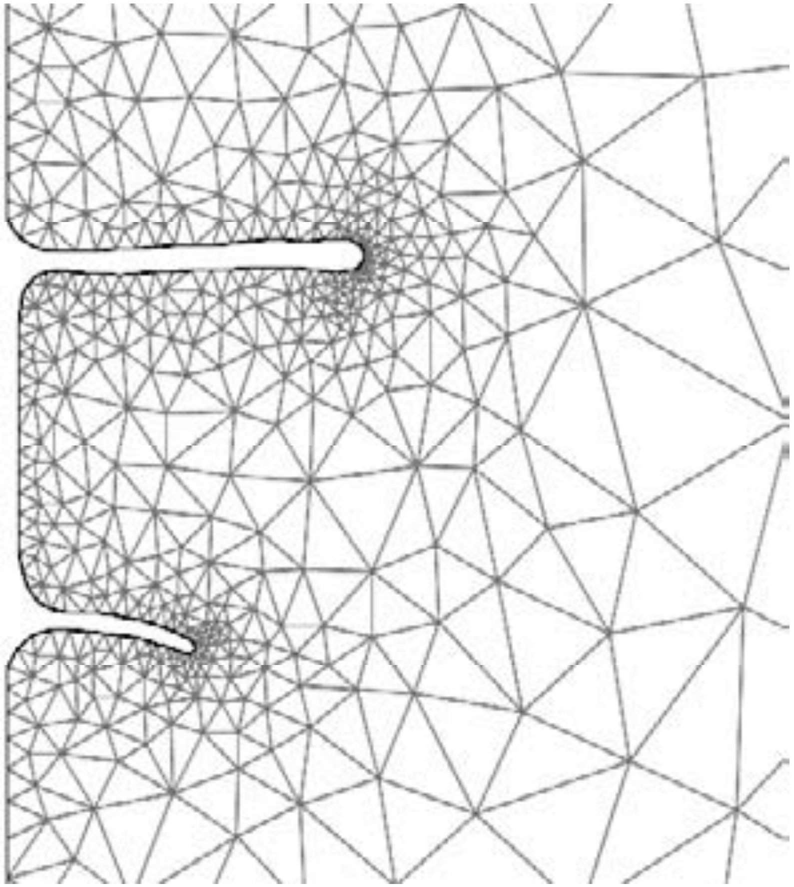
Corroding Surface



Corroding Surface

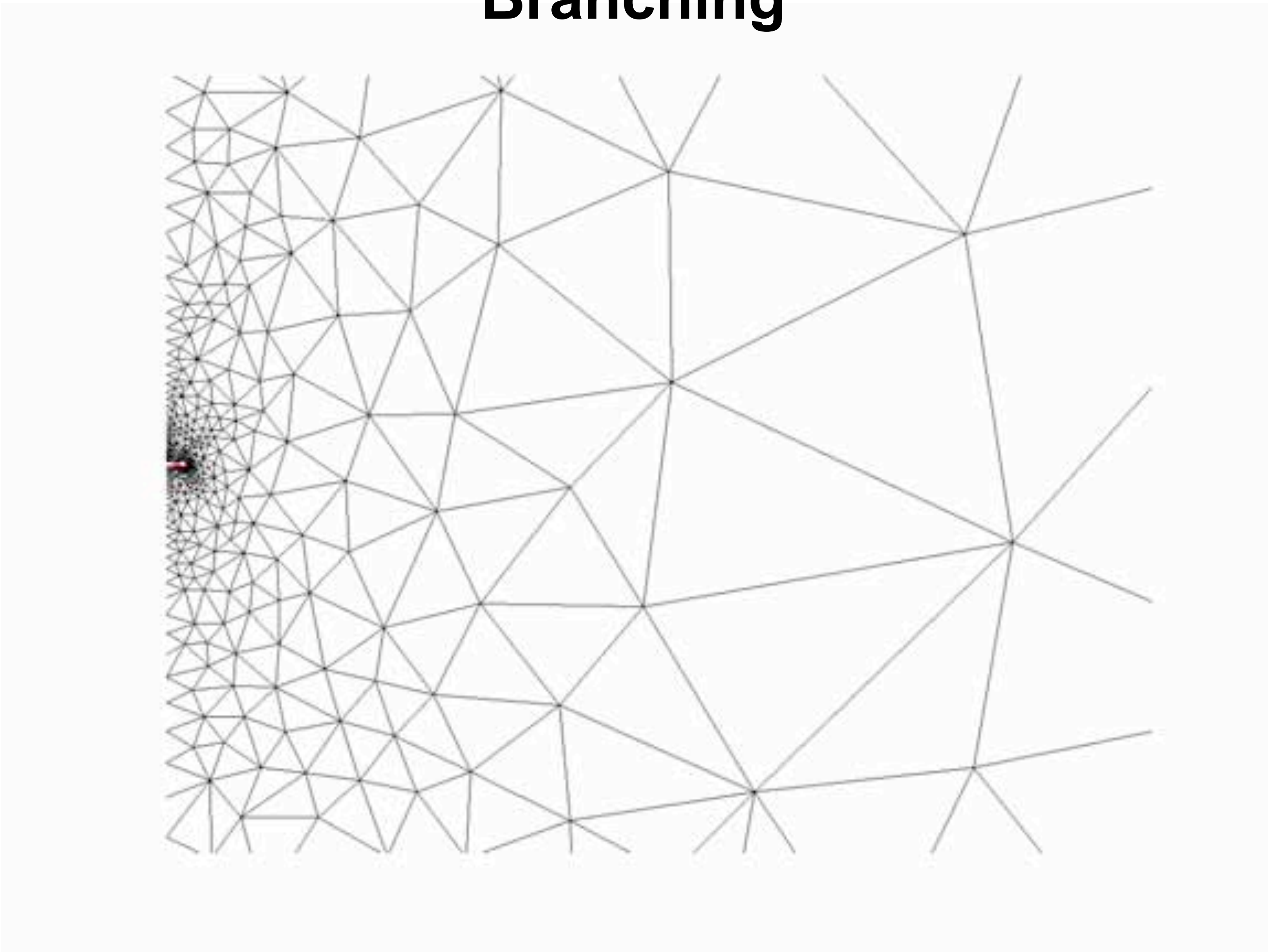


Corroding Surface



Branching

Branching



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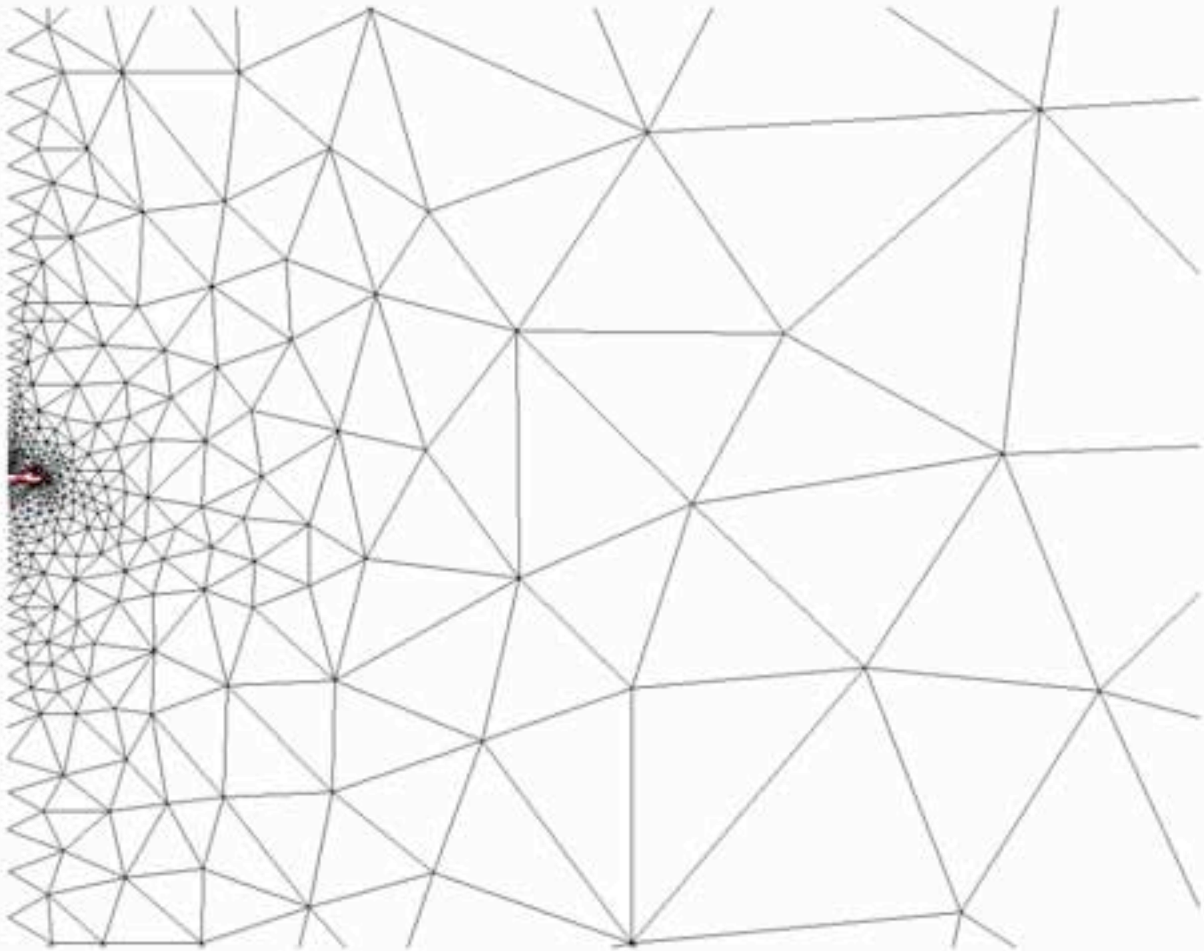

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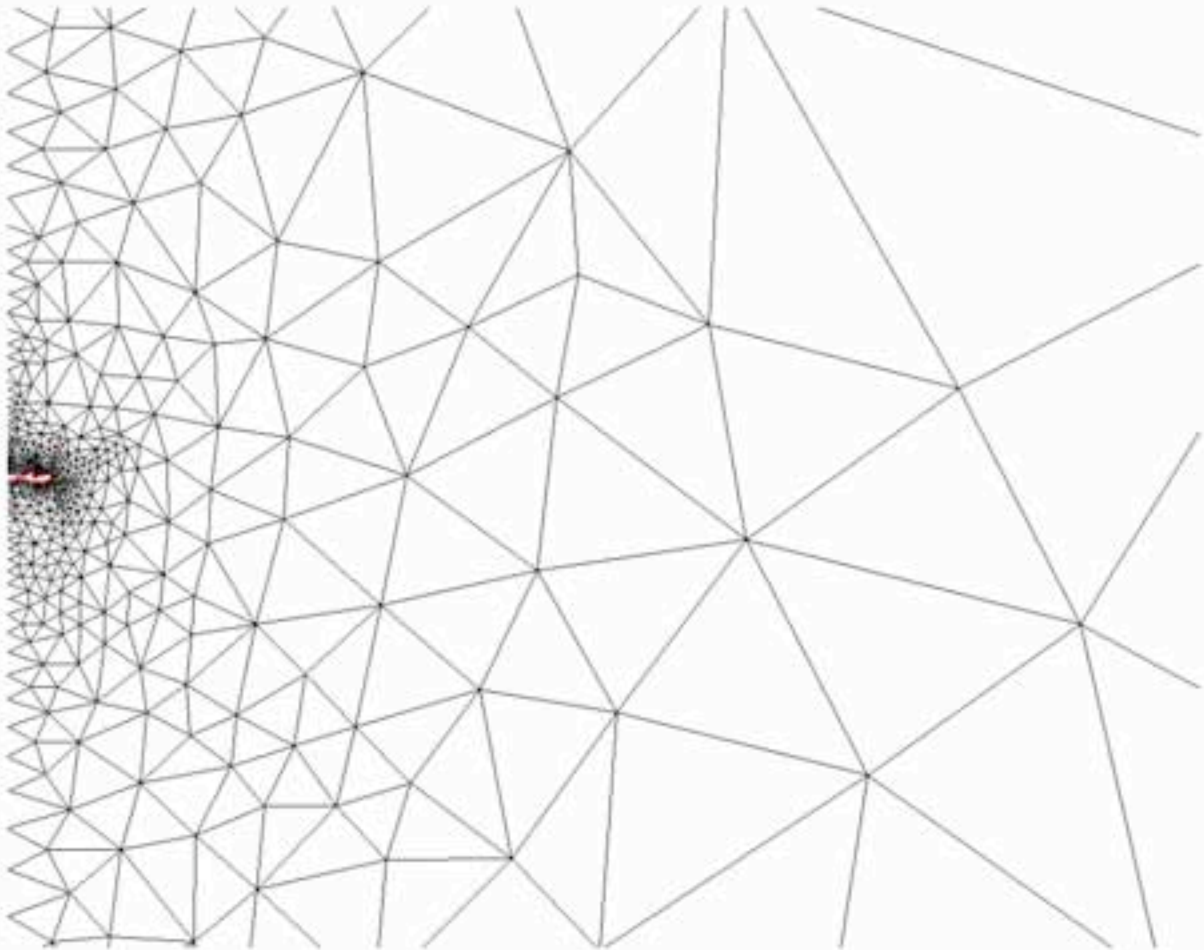

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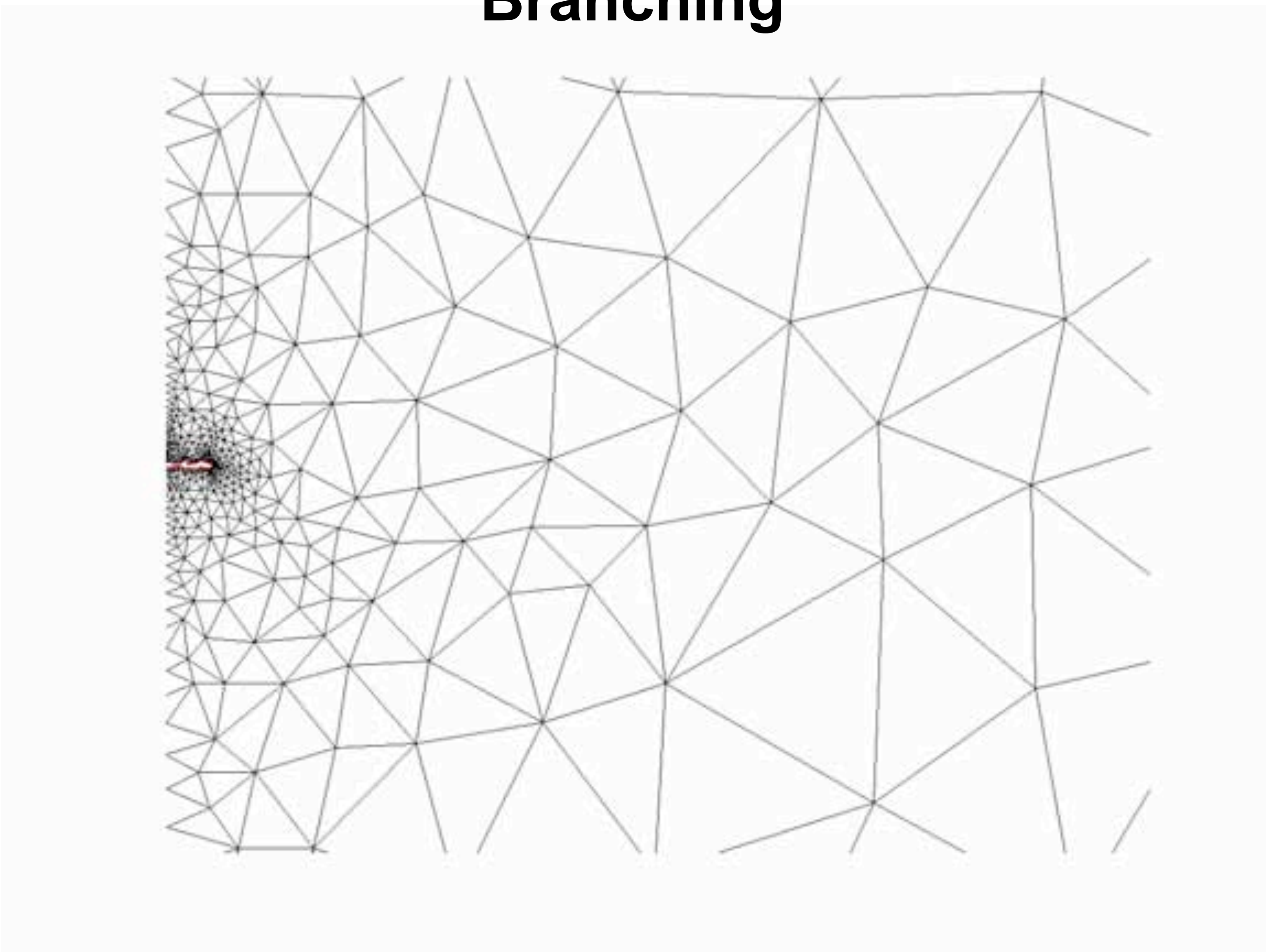

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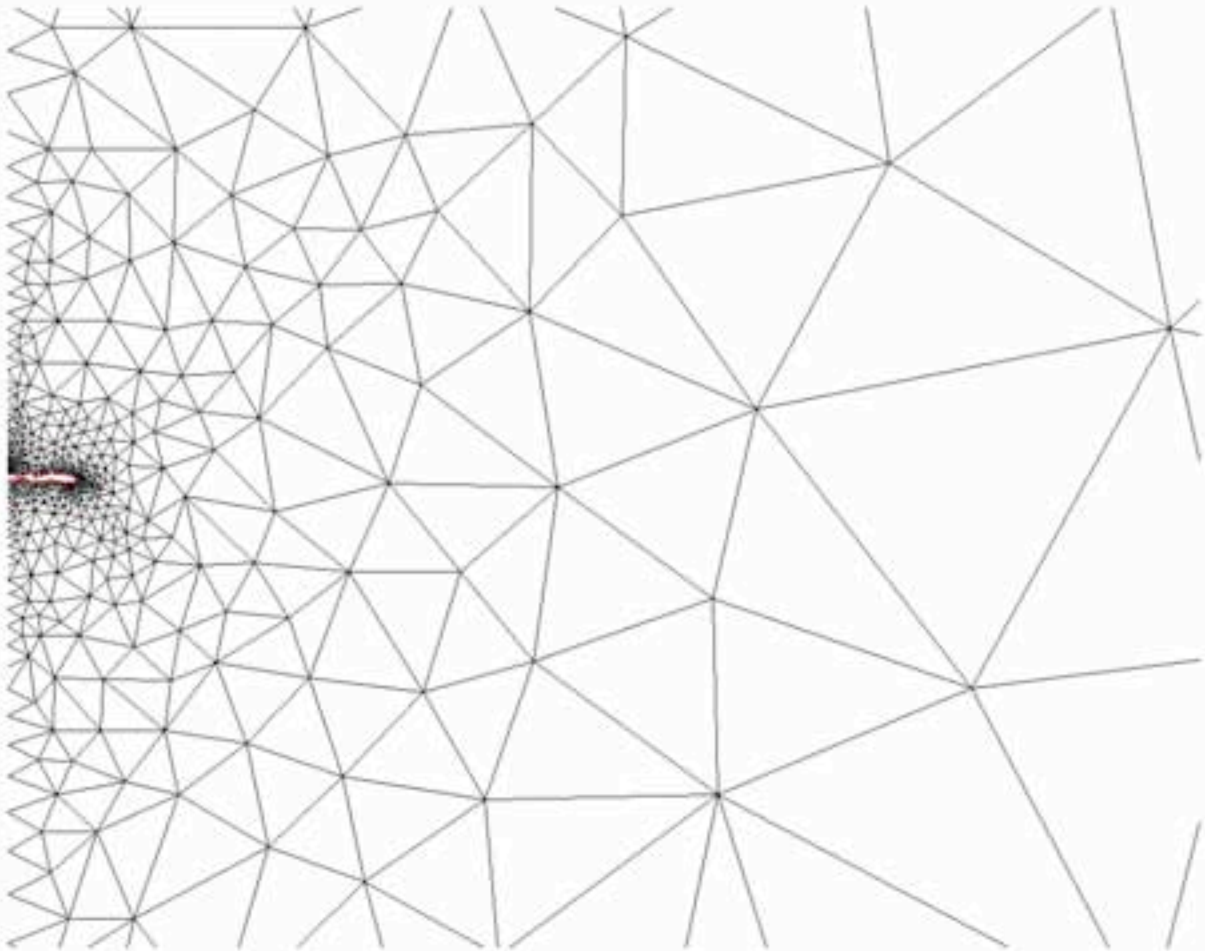

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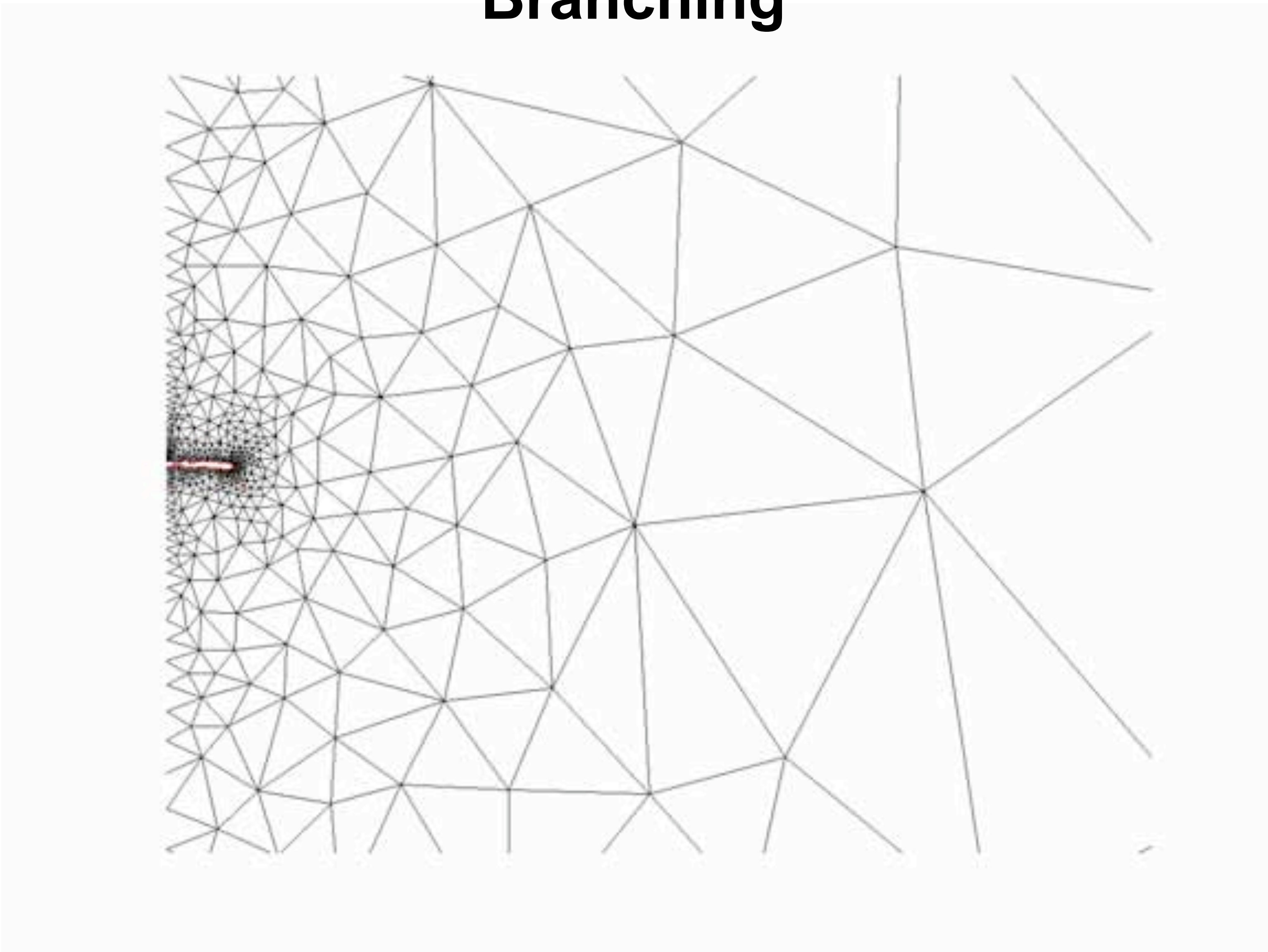

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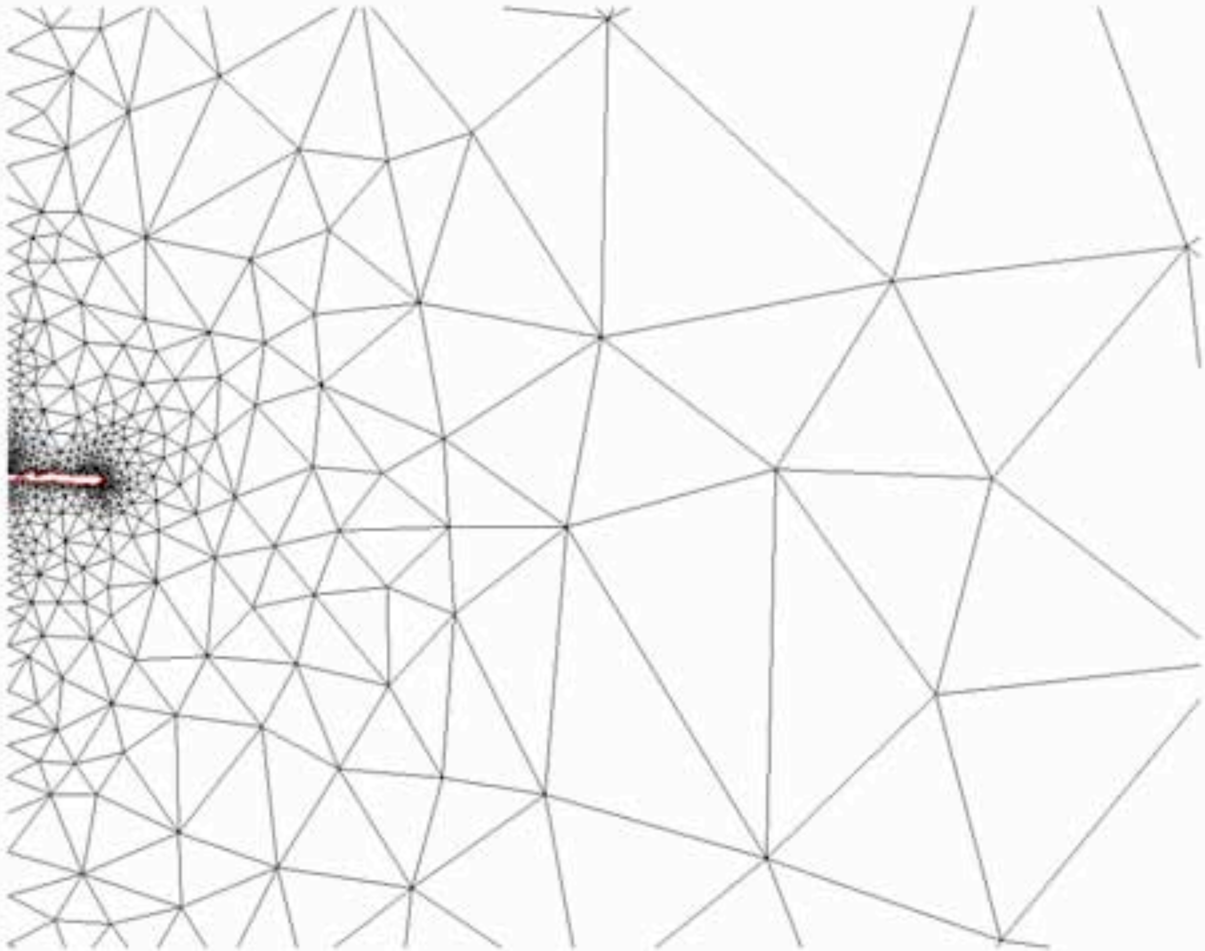

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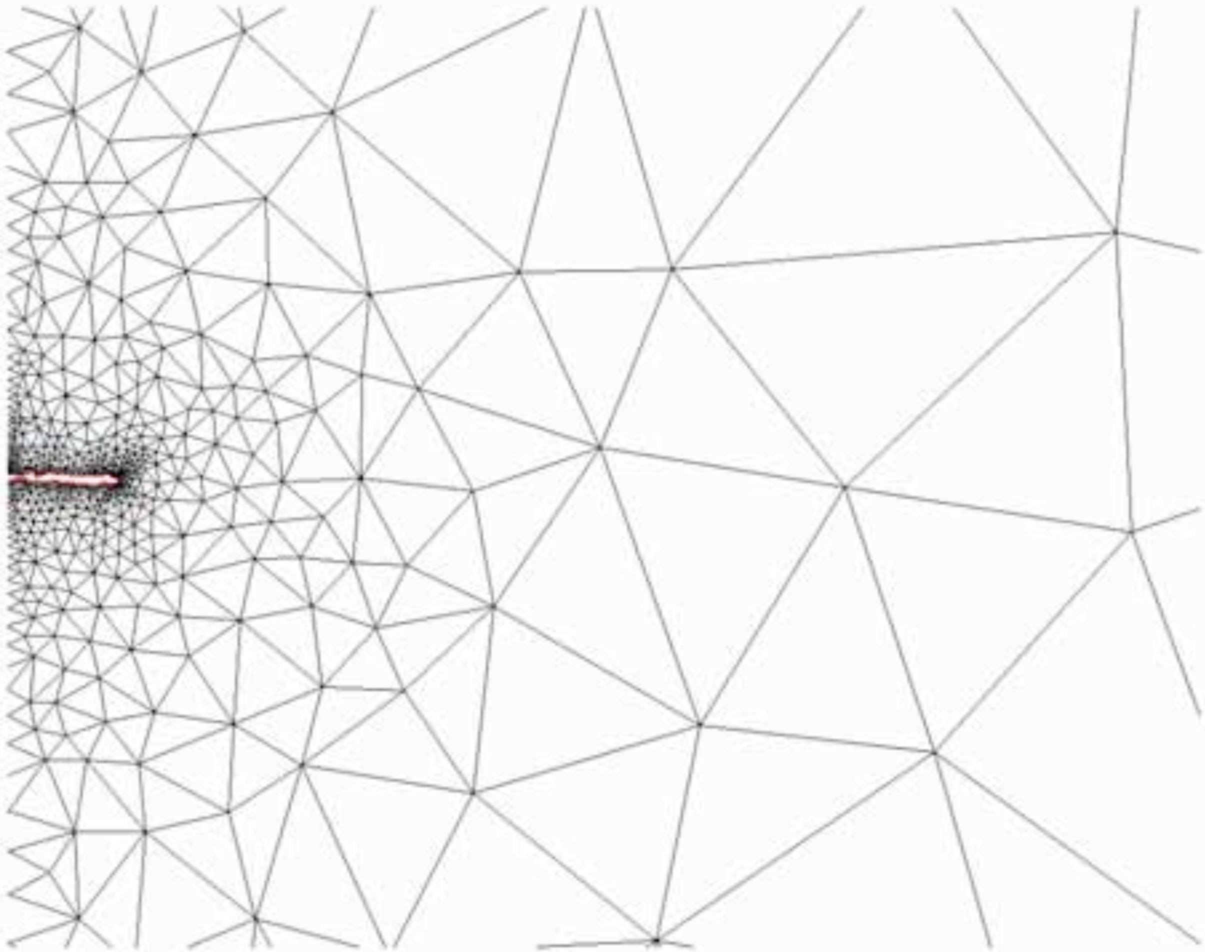

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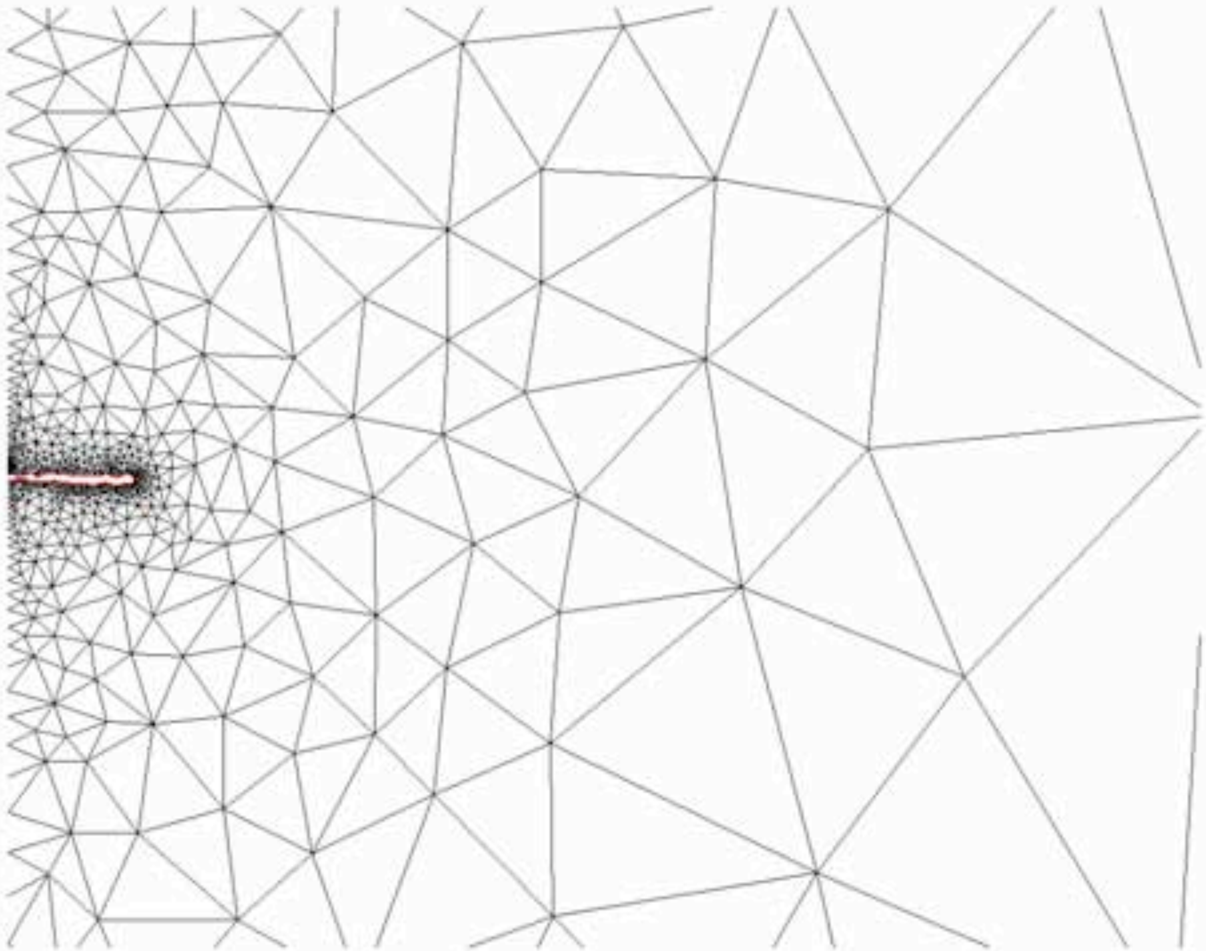

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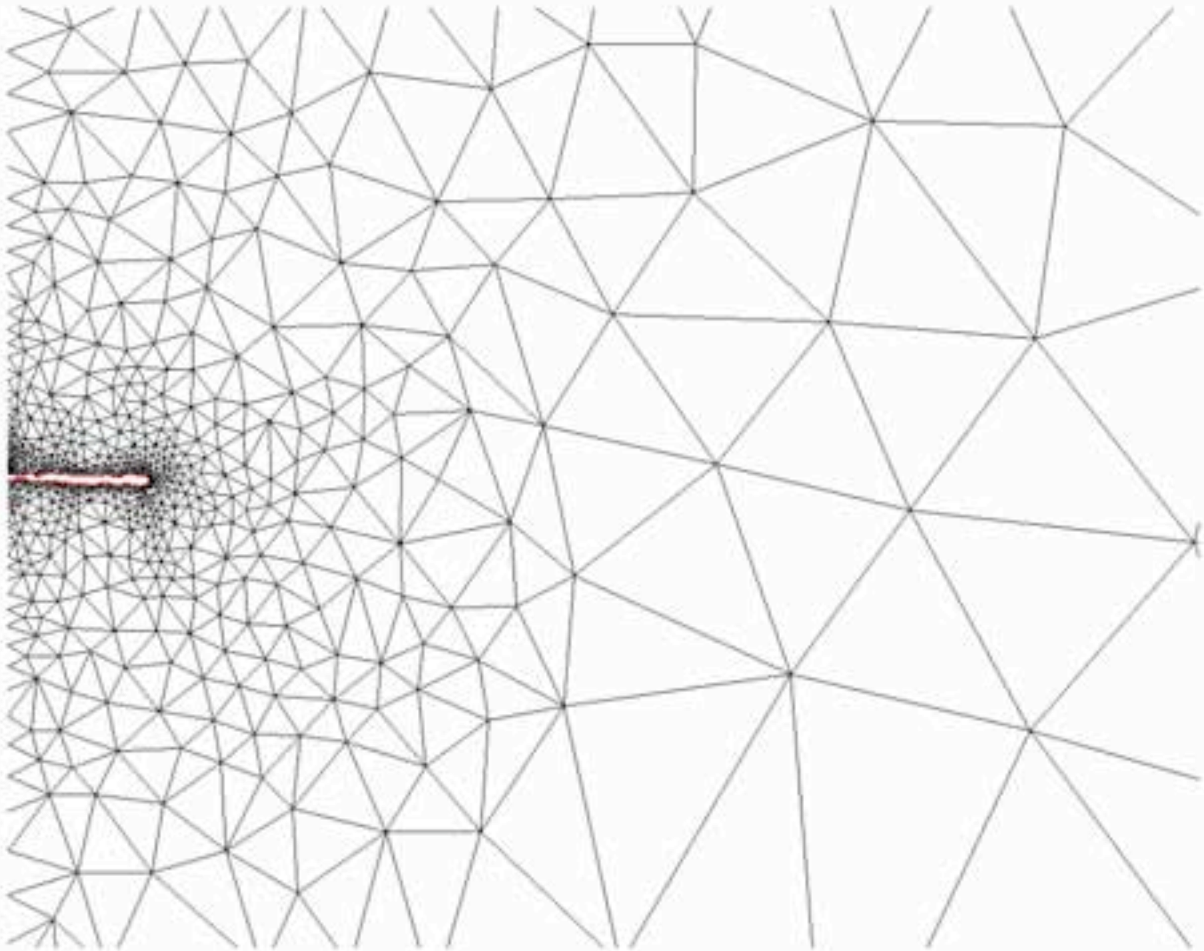

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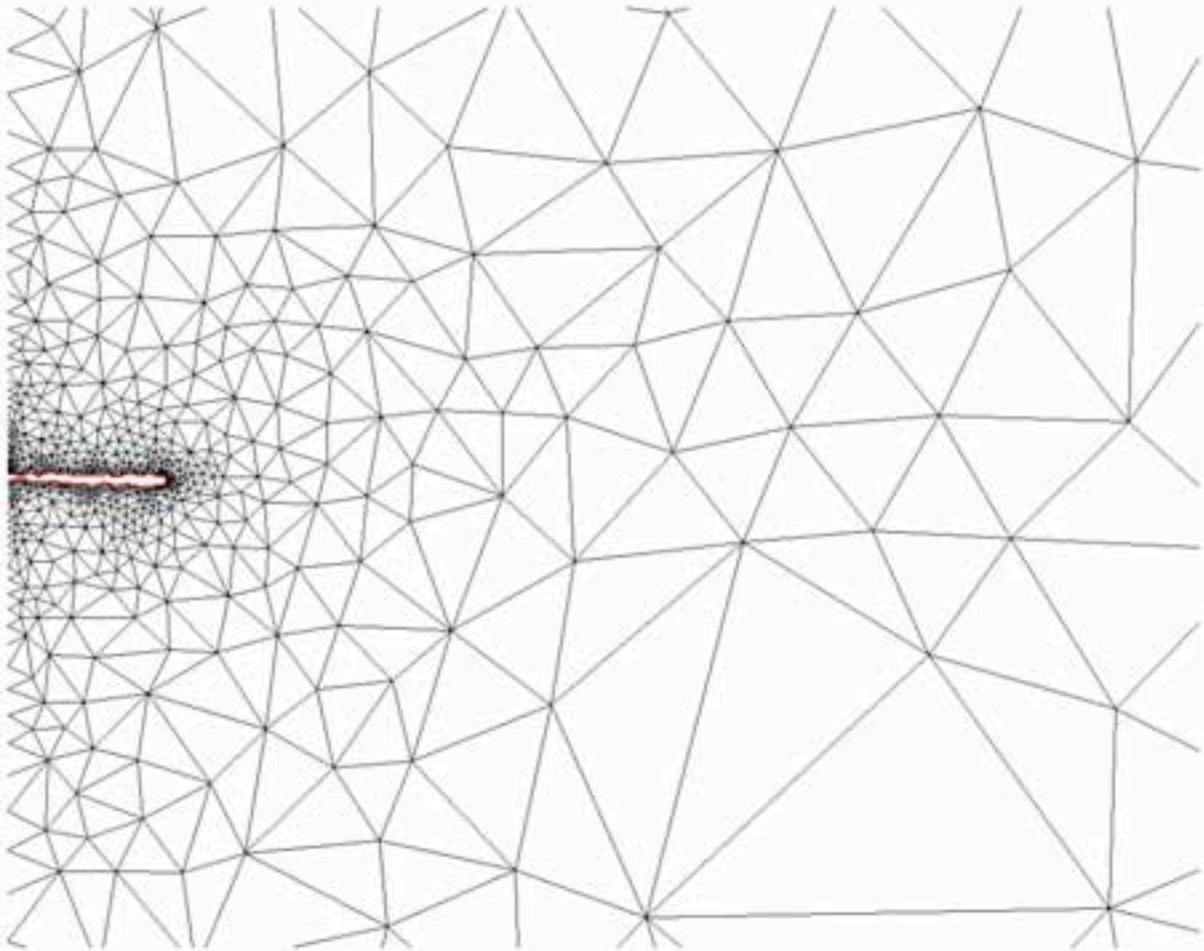

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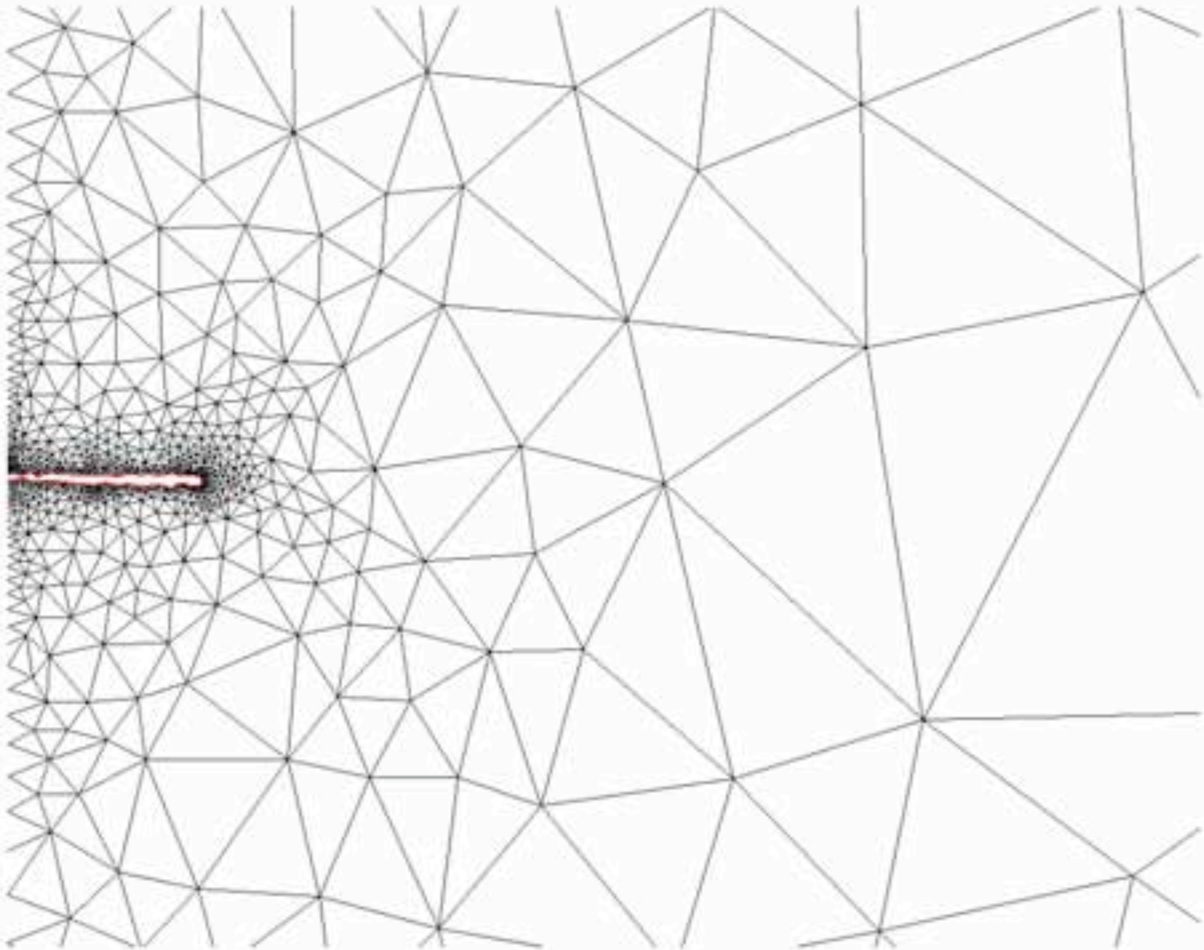

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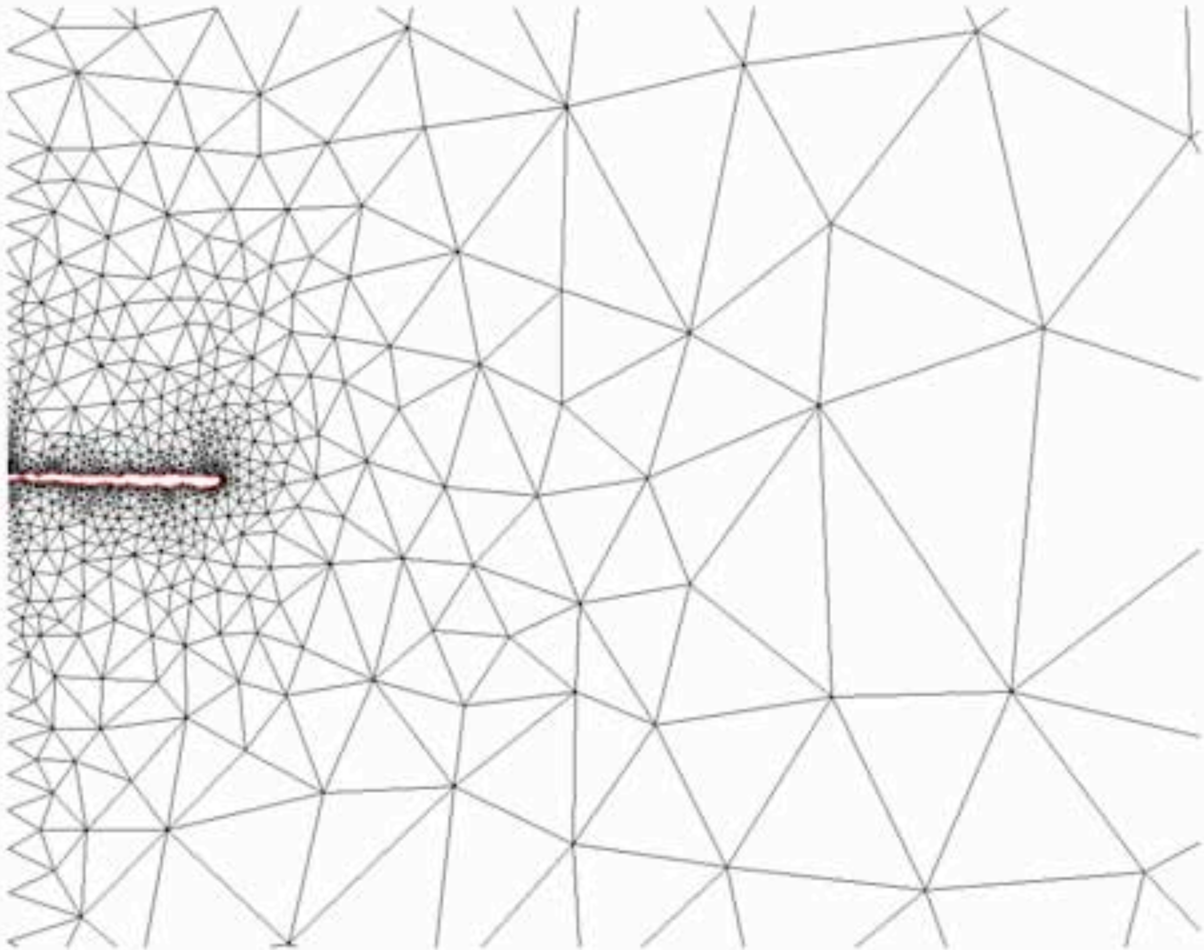

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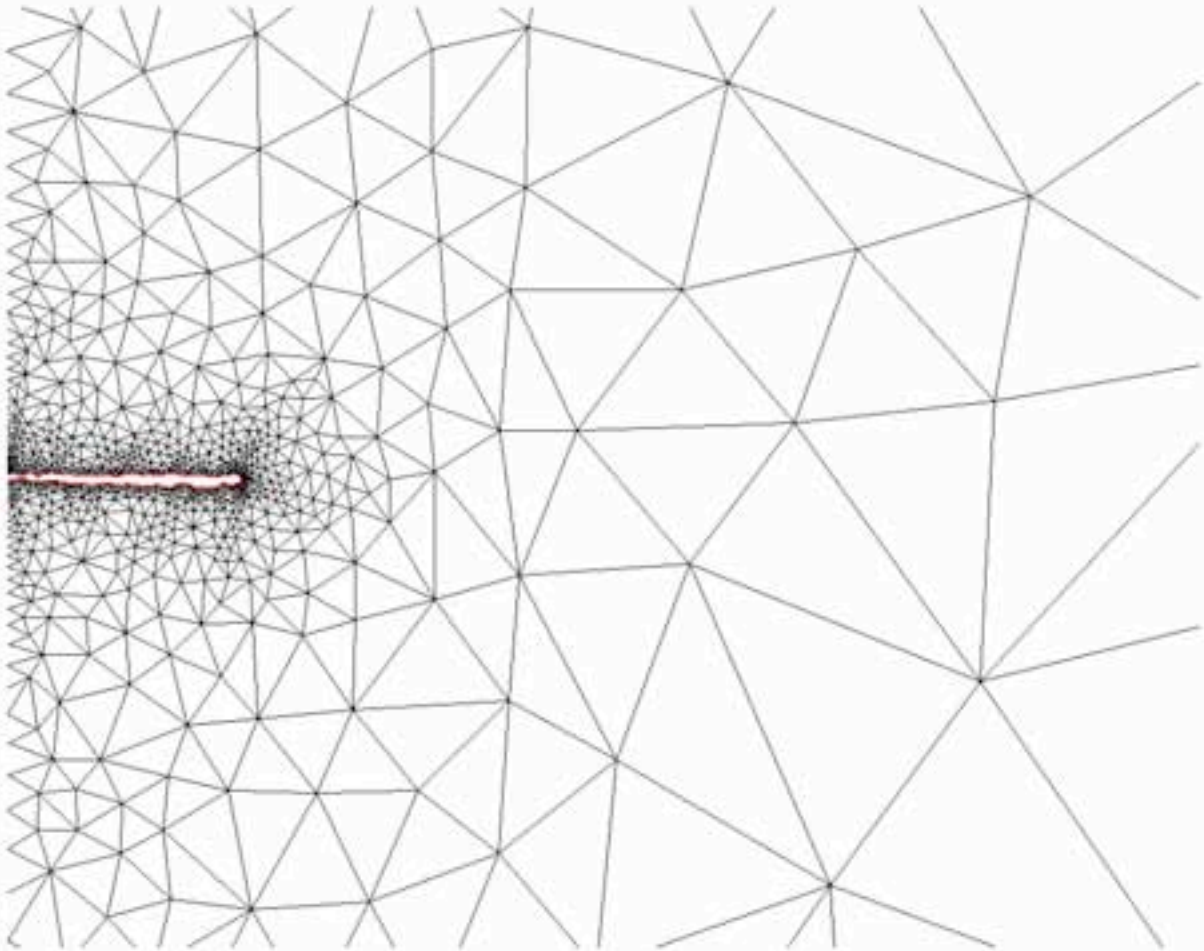

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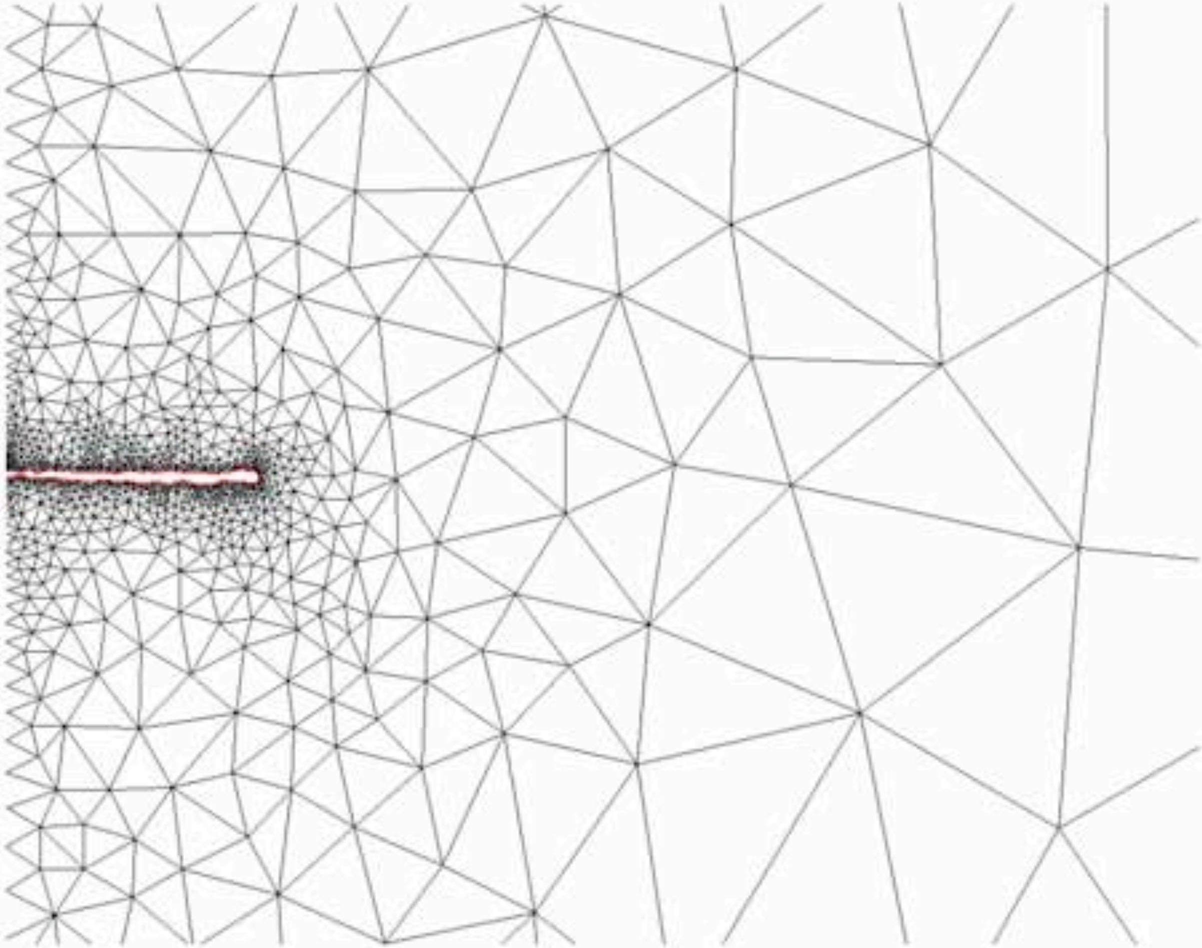

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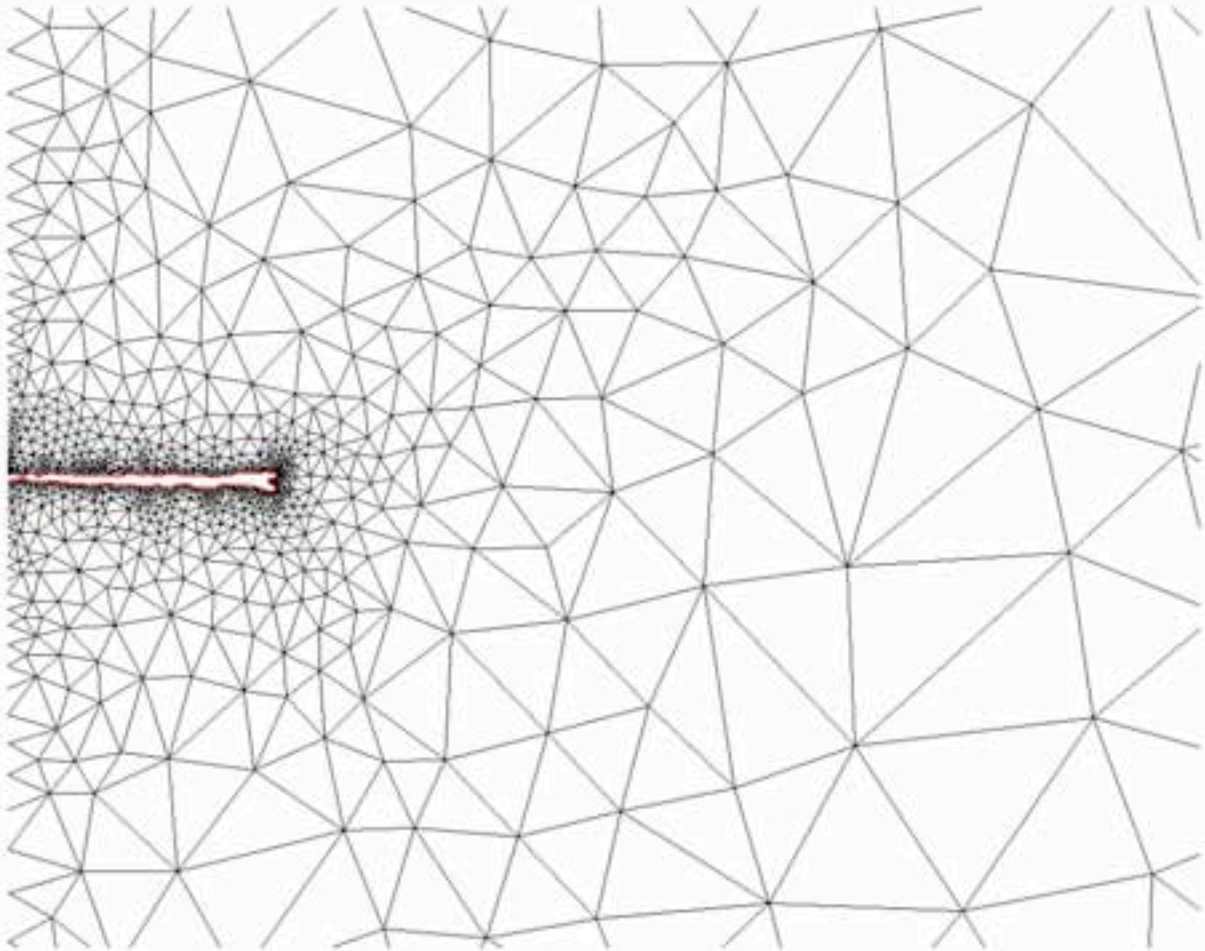

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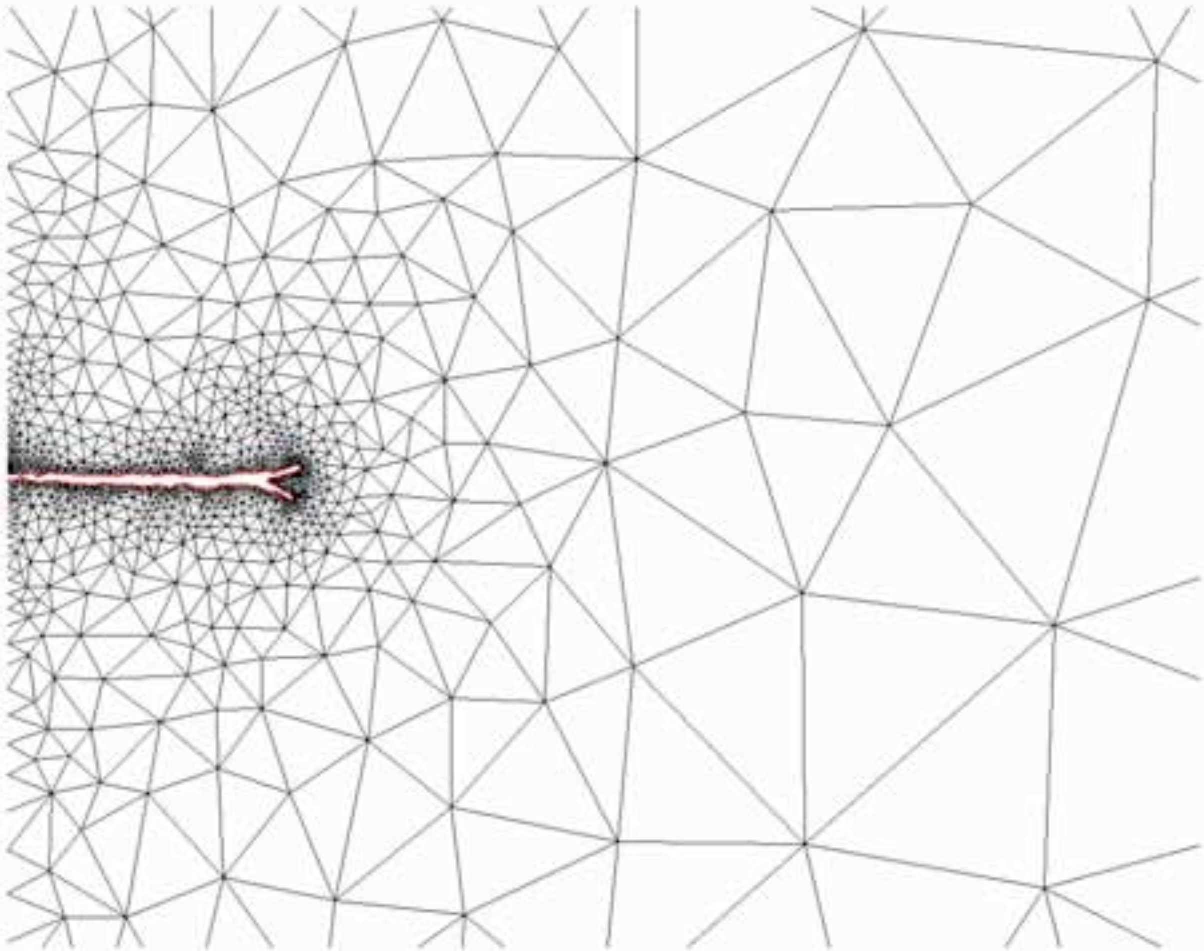

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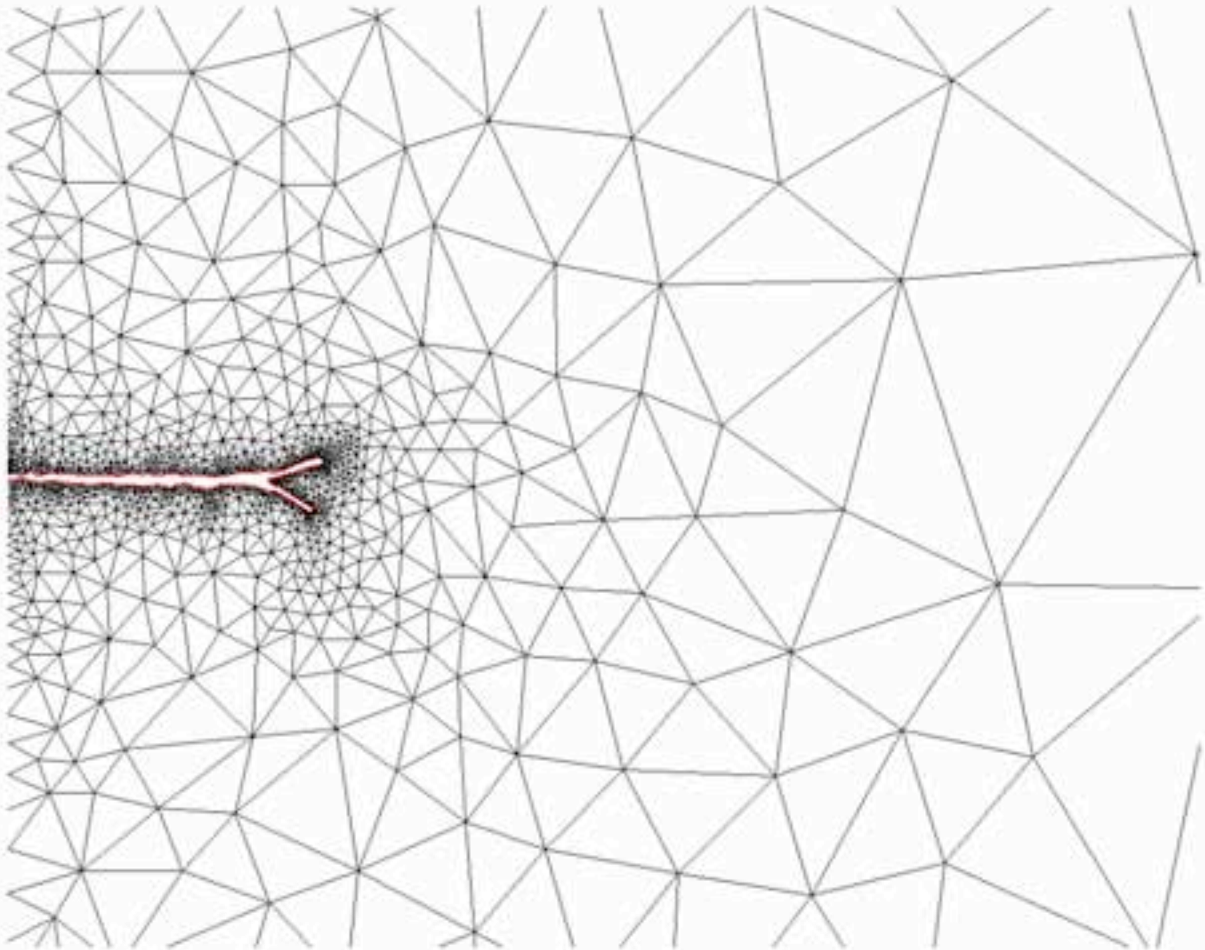

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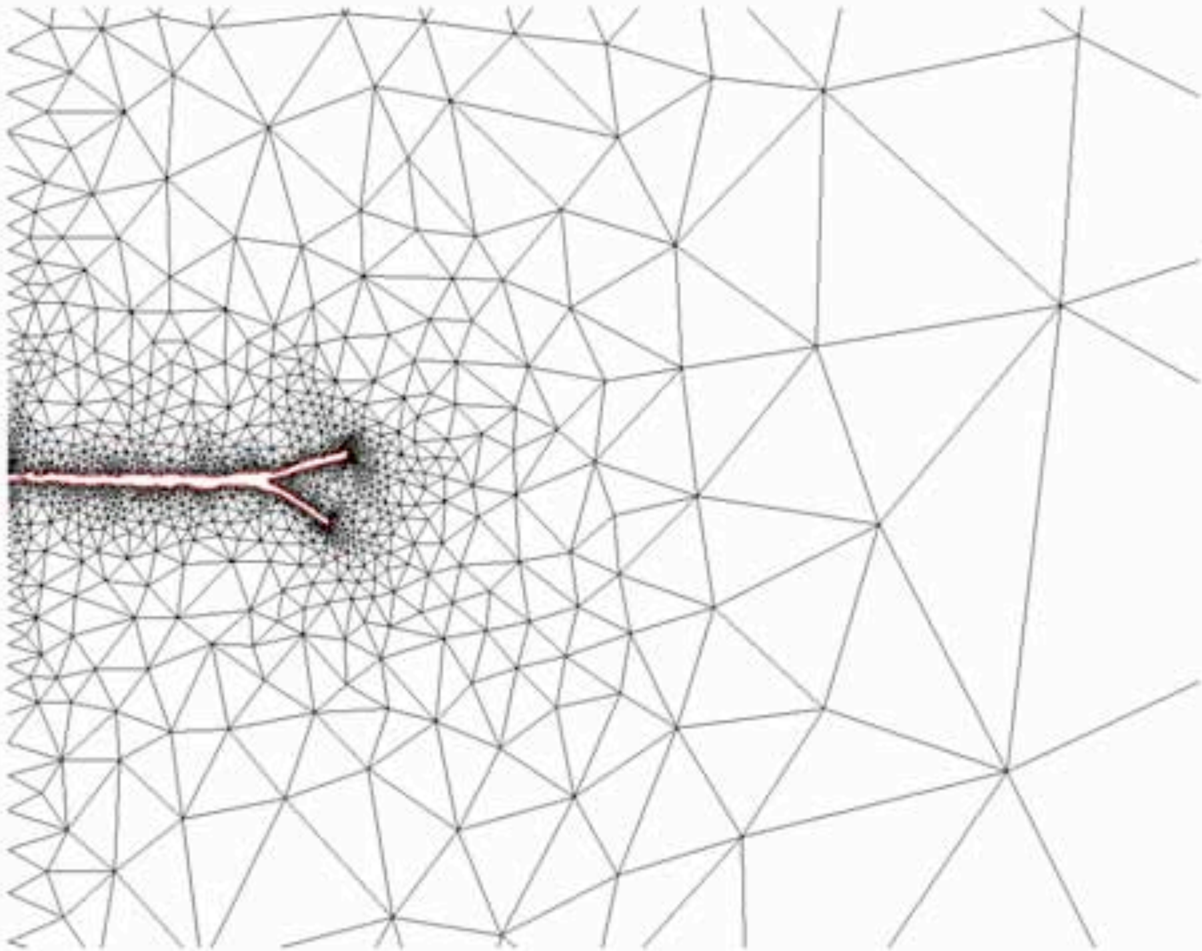

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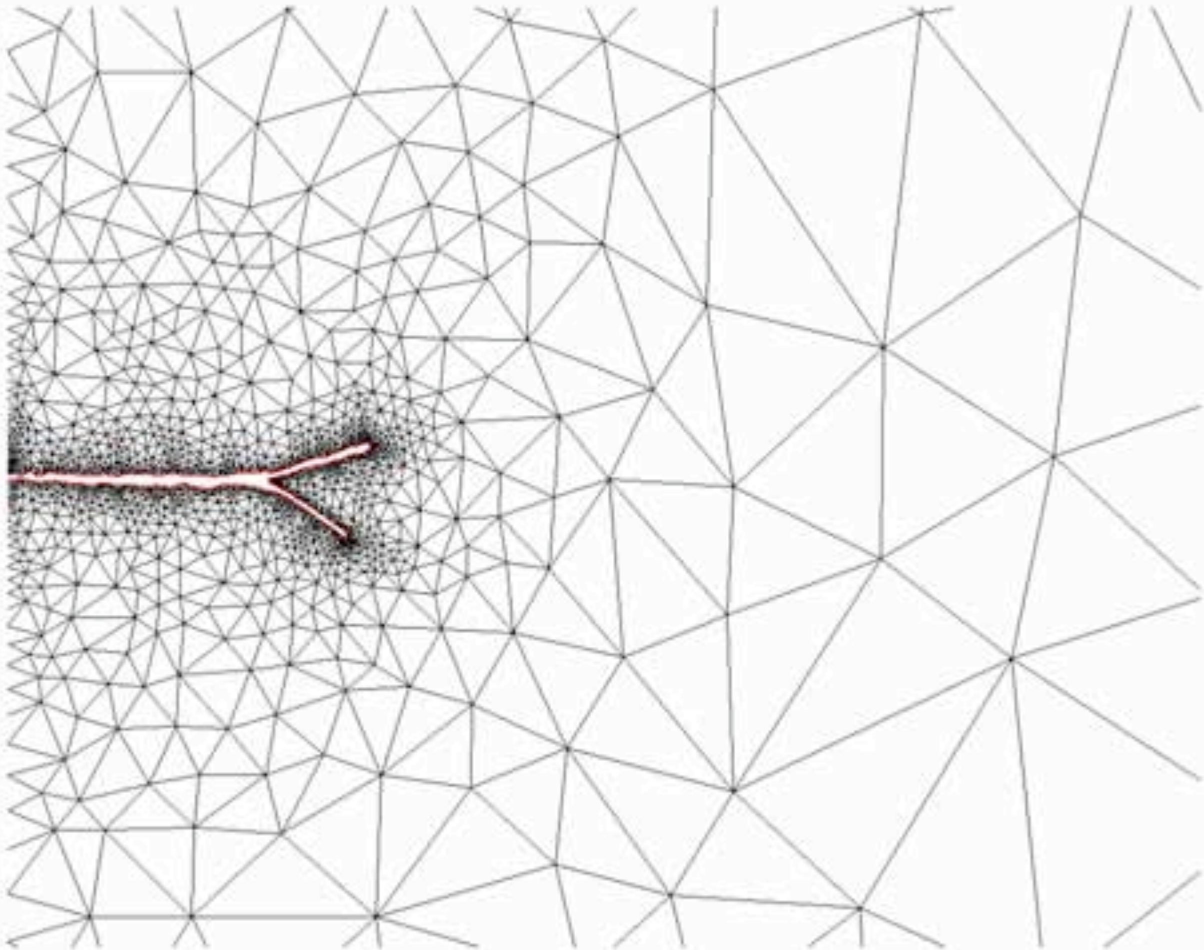

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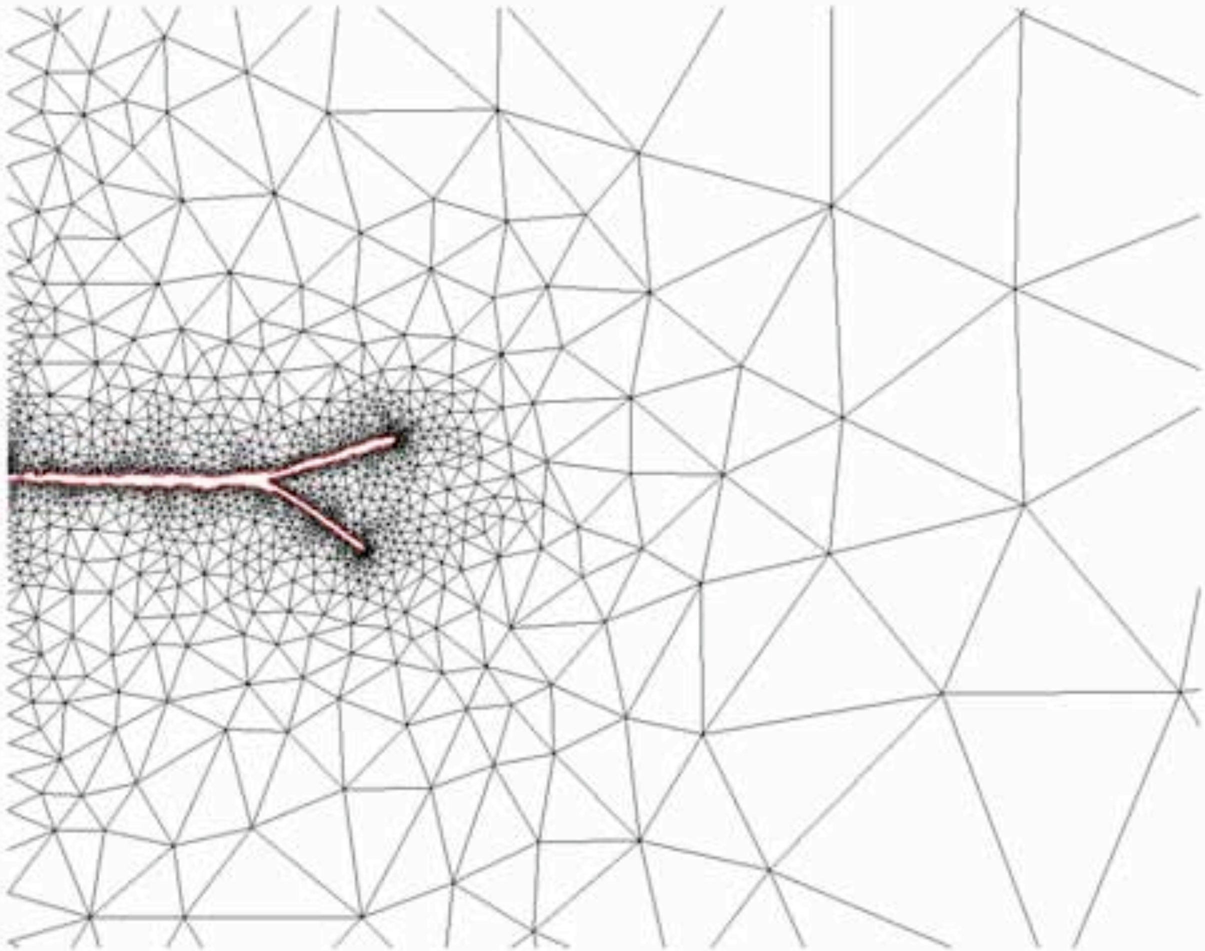

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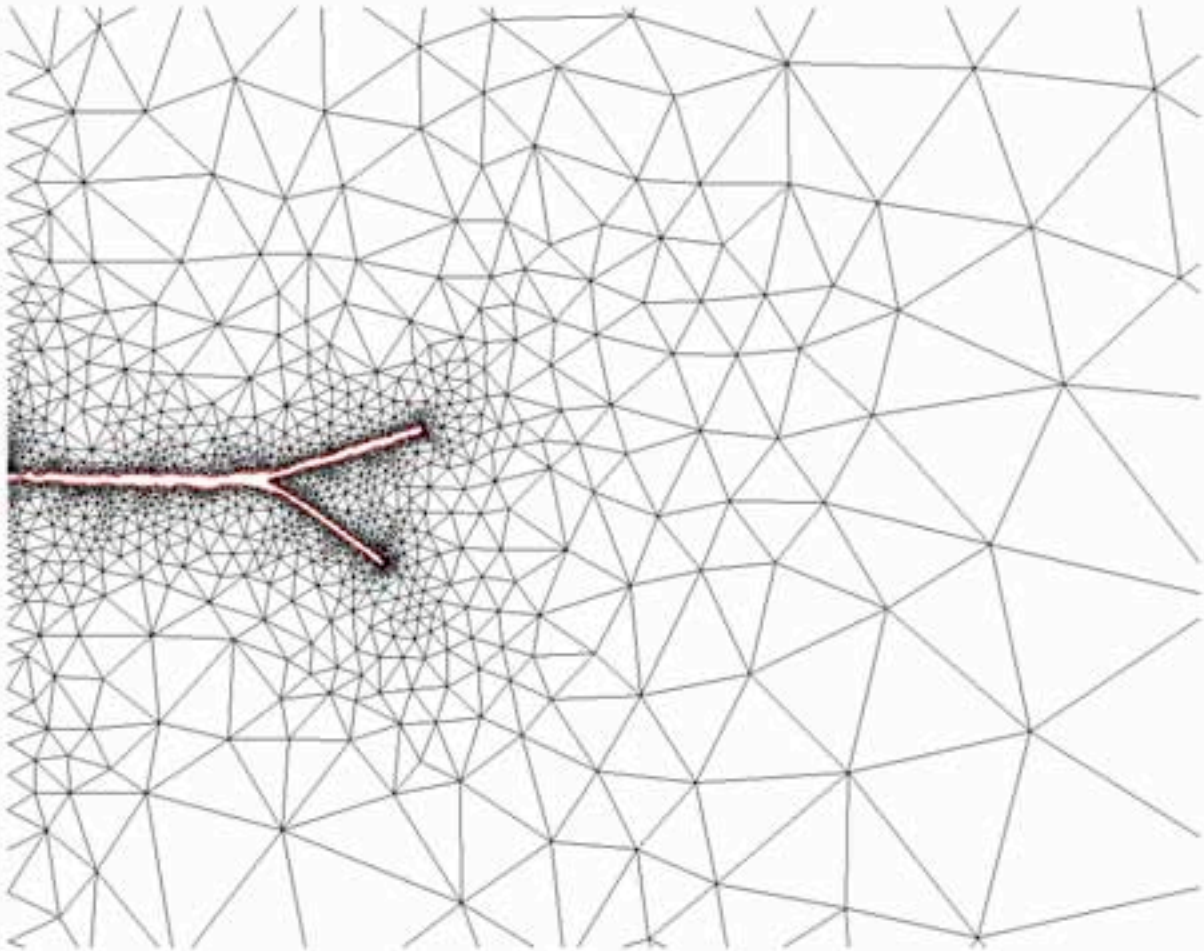

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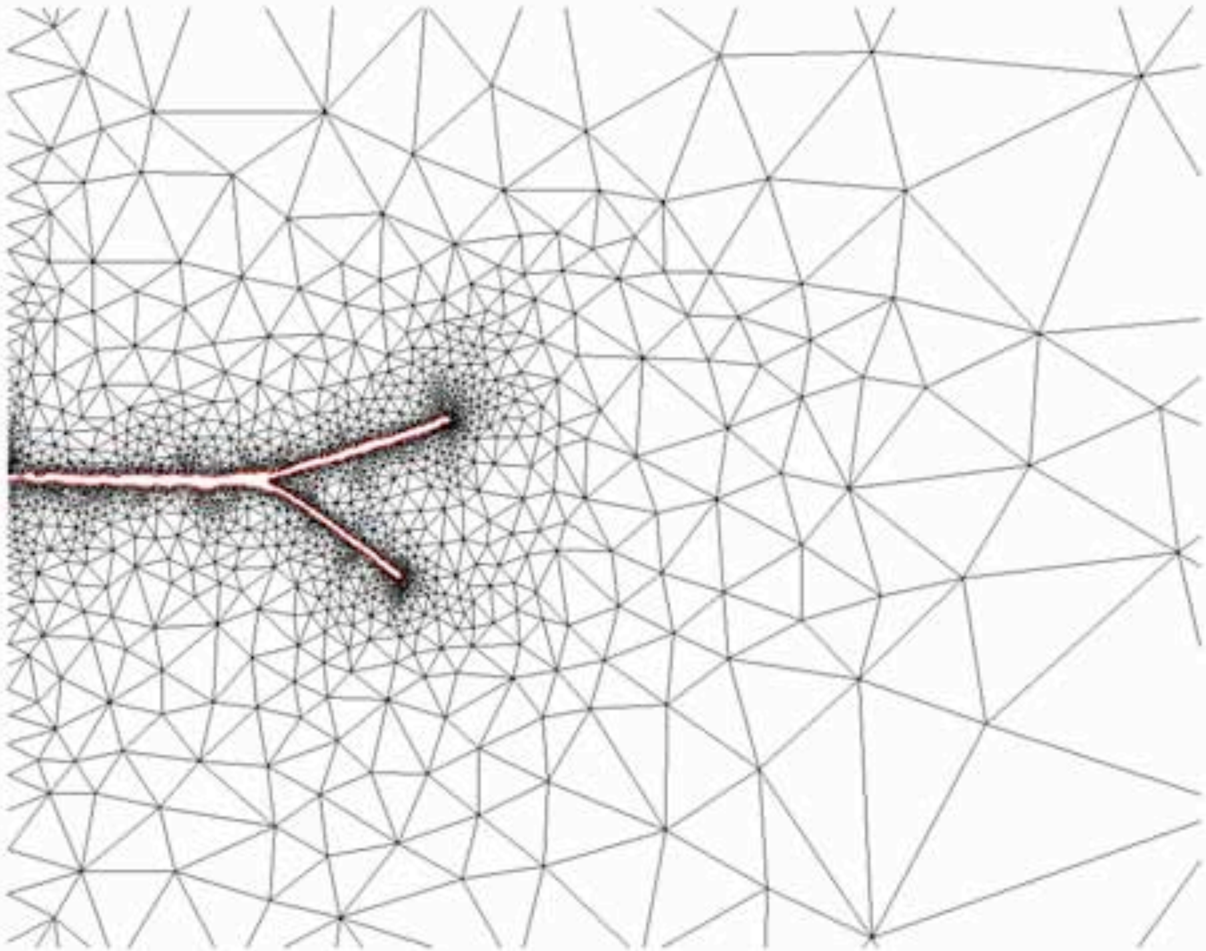

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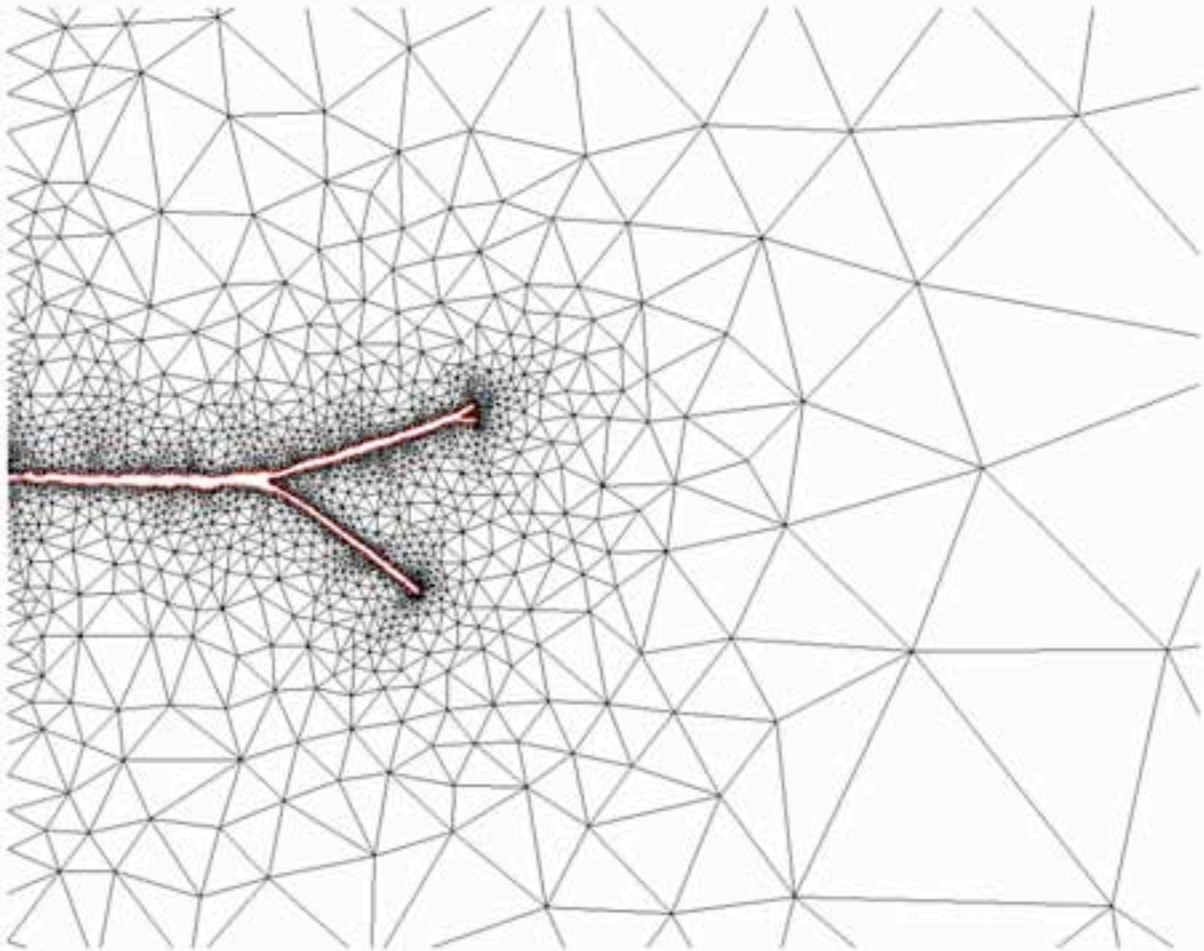

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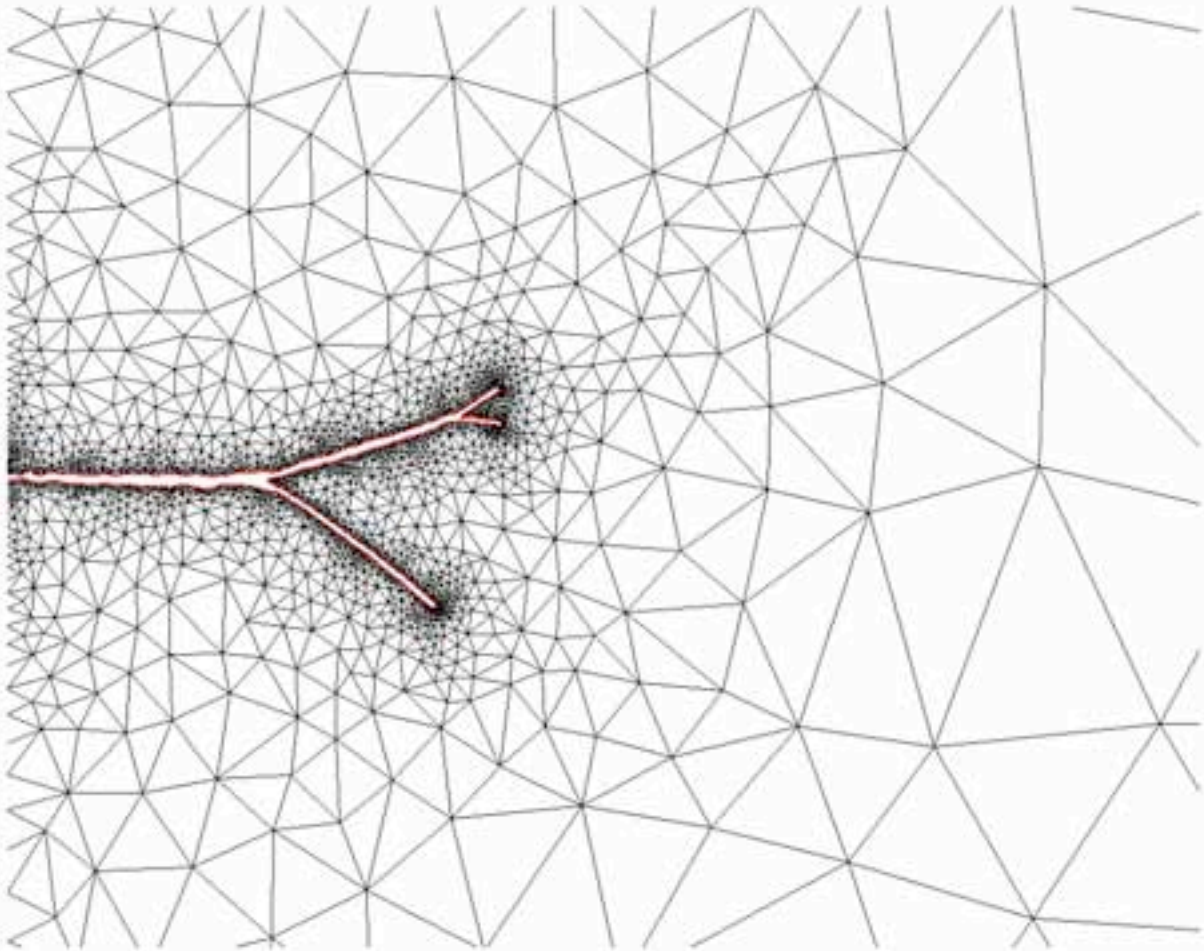

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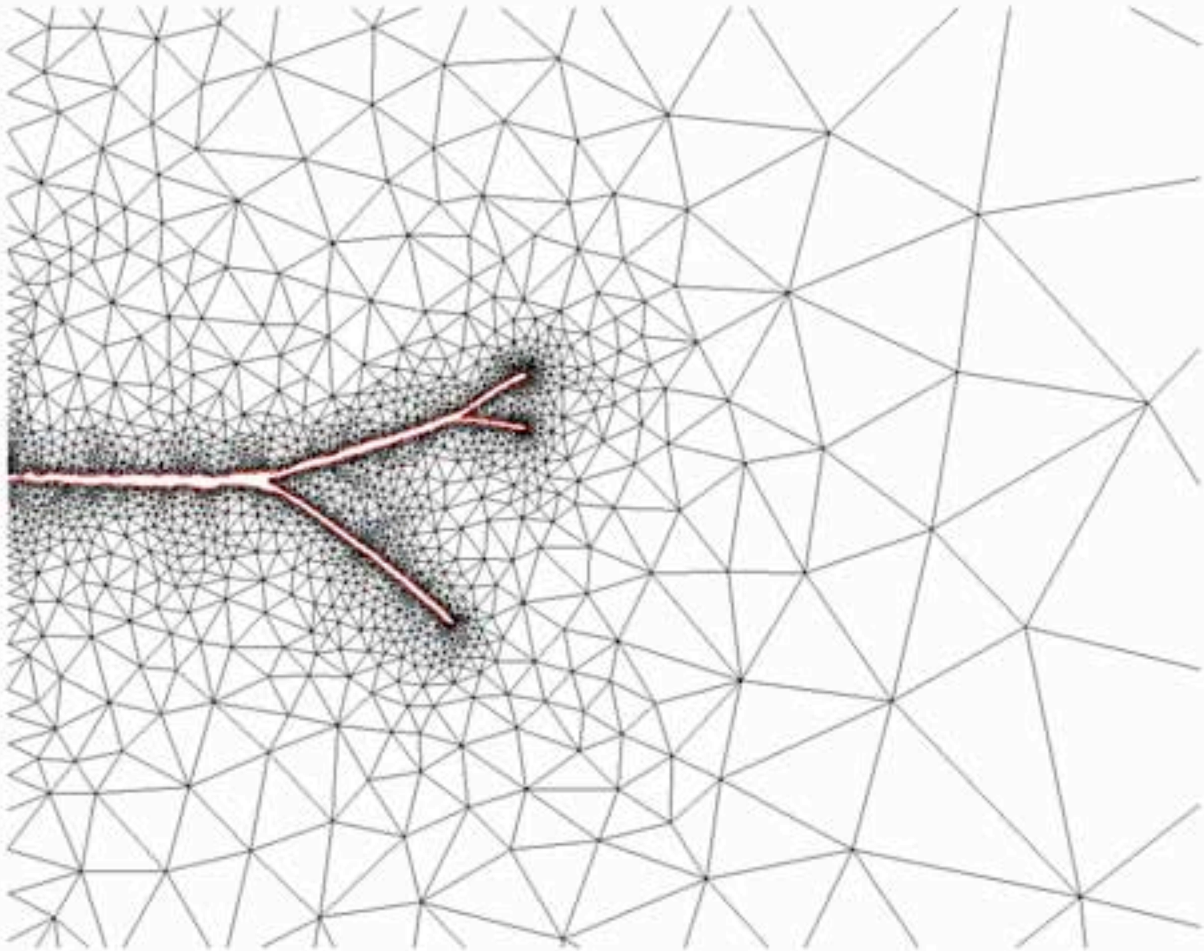

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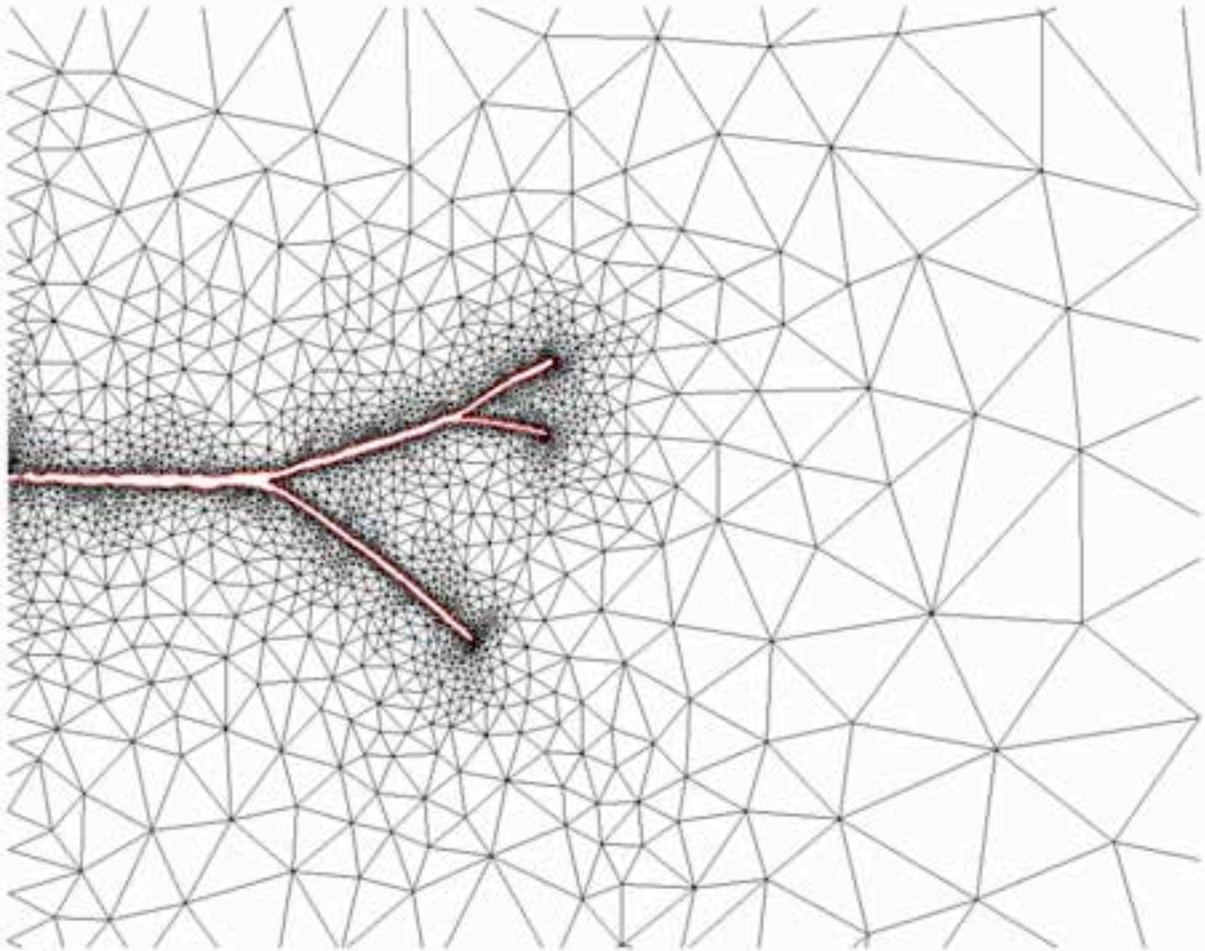

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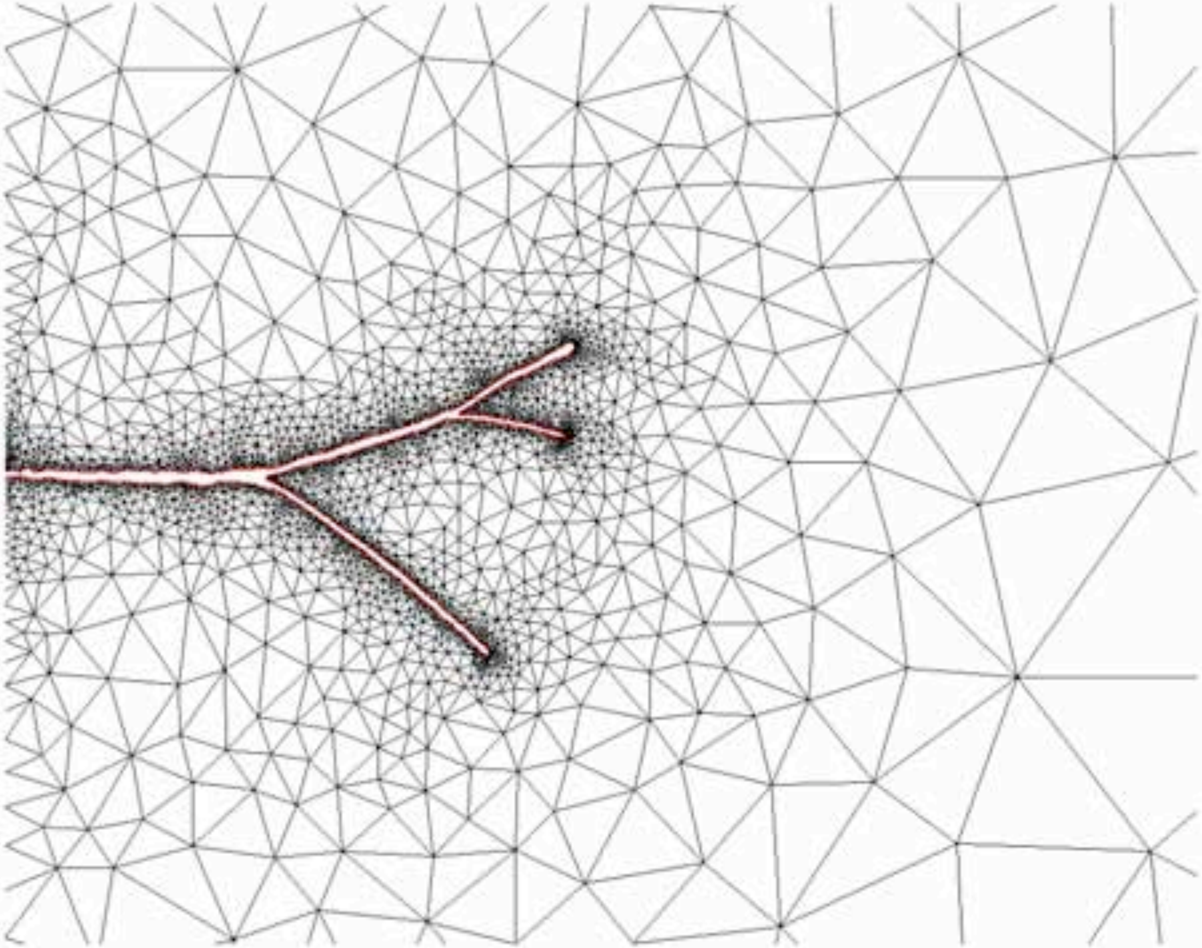

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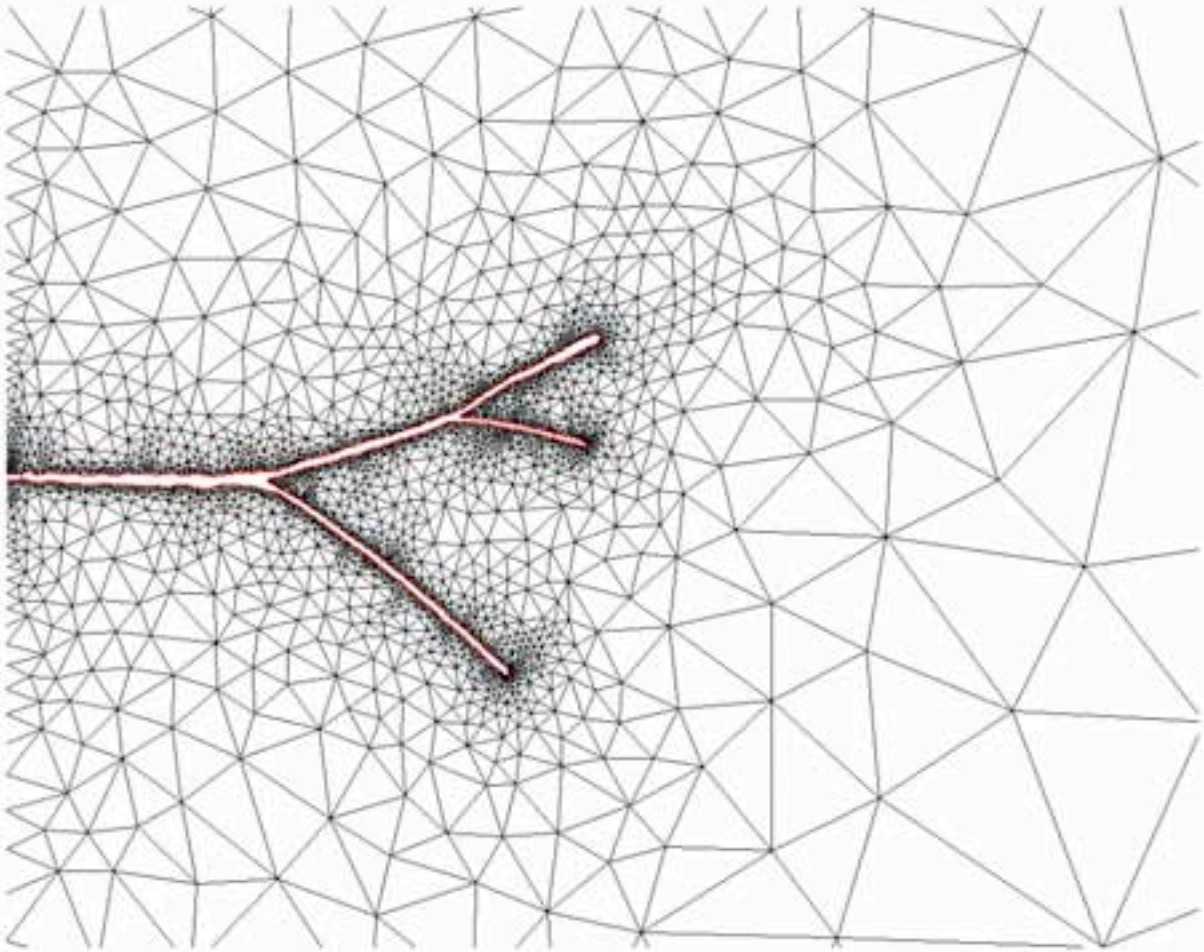

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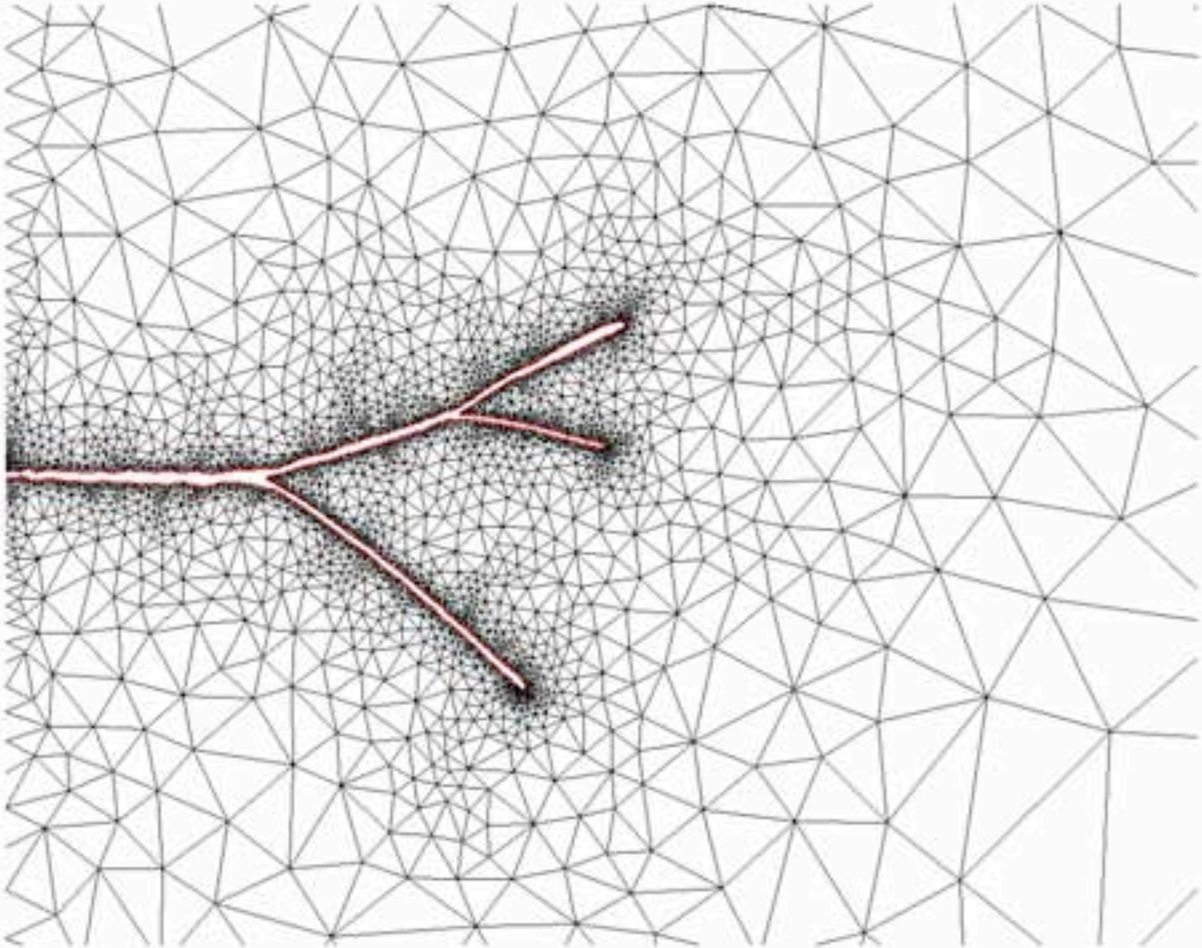

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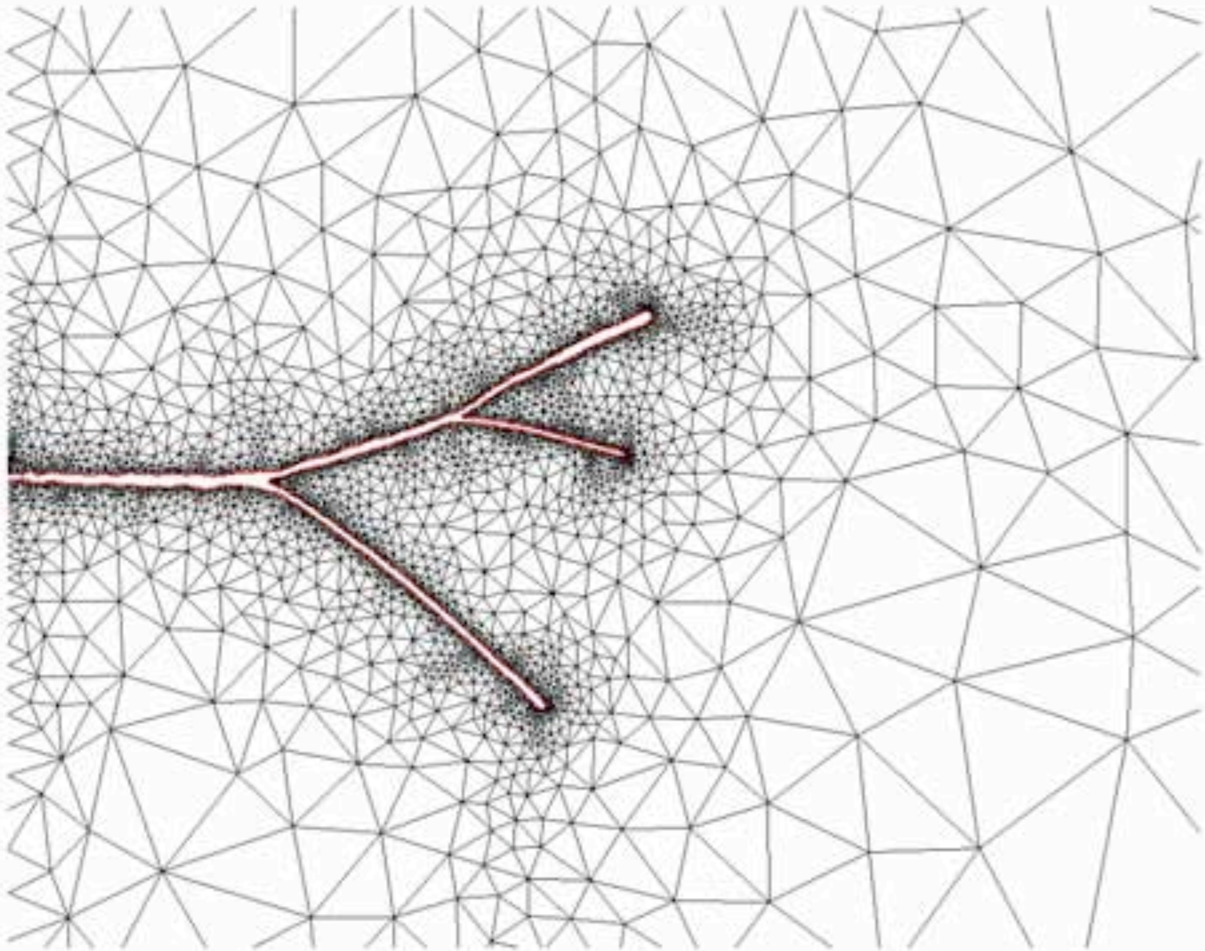

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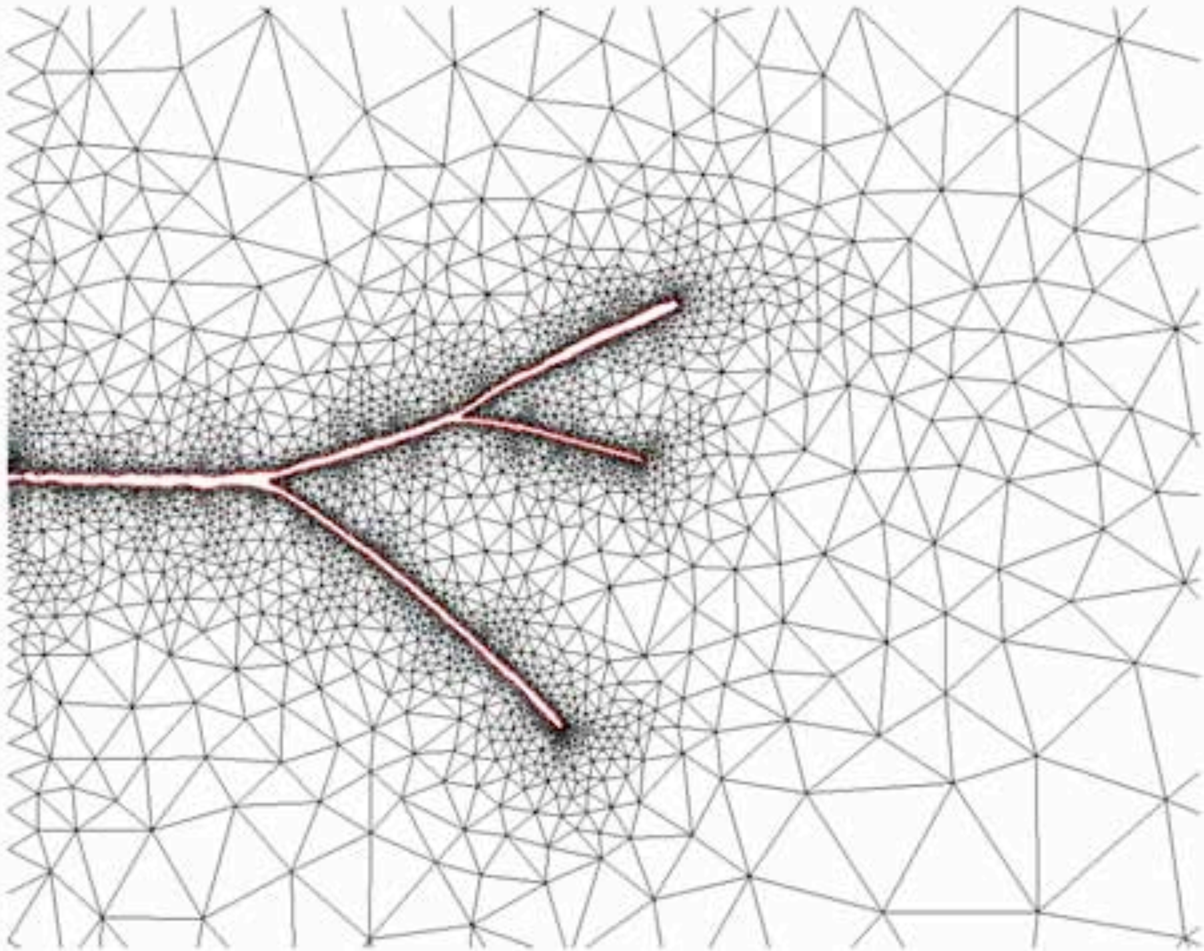

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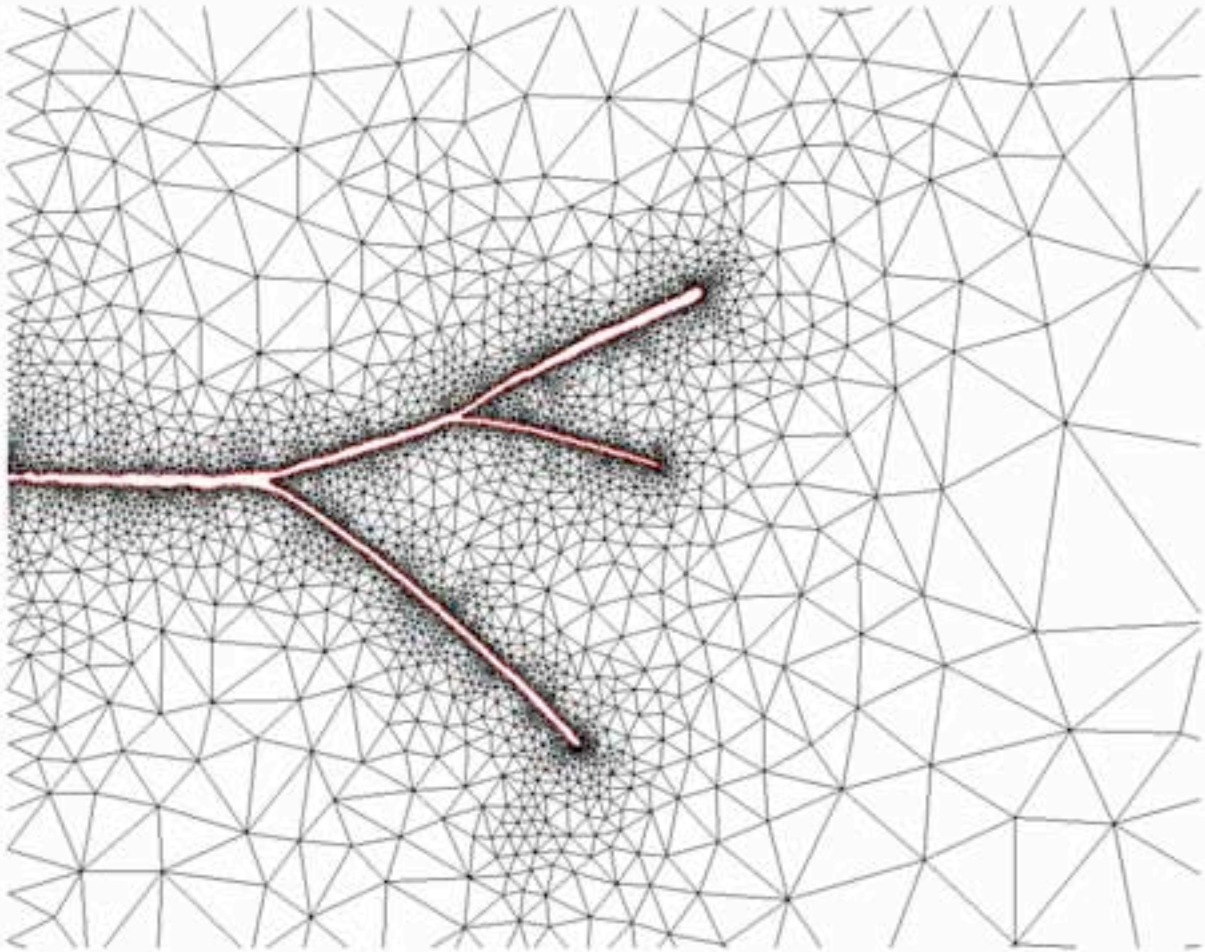

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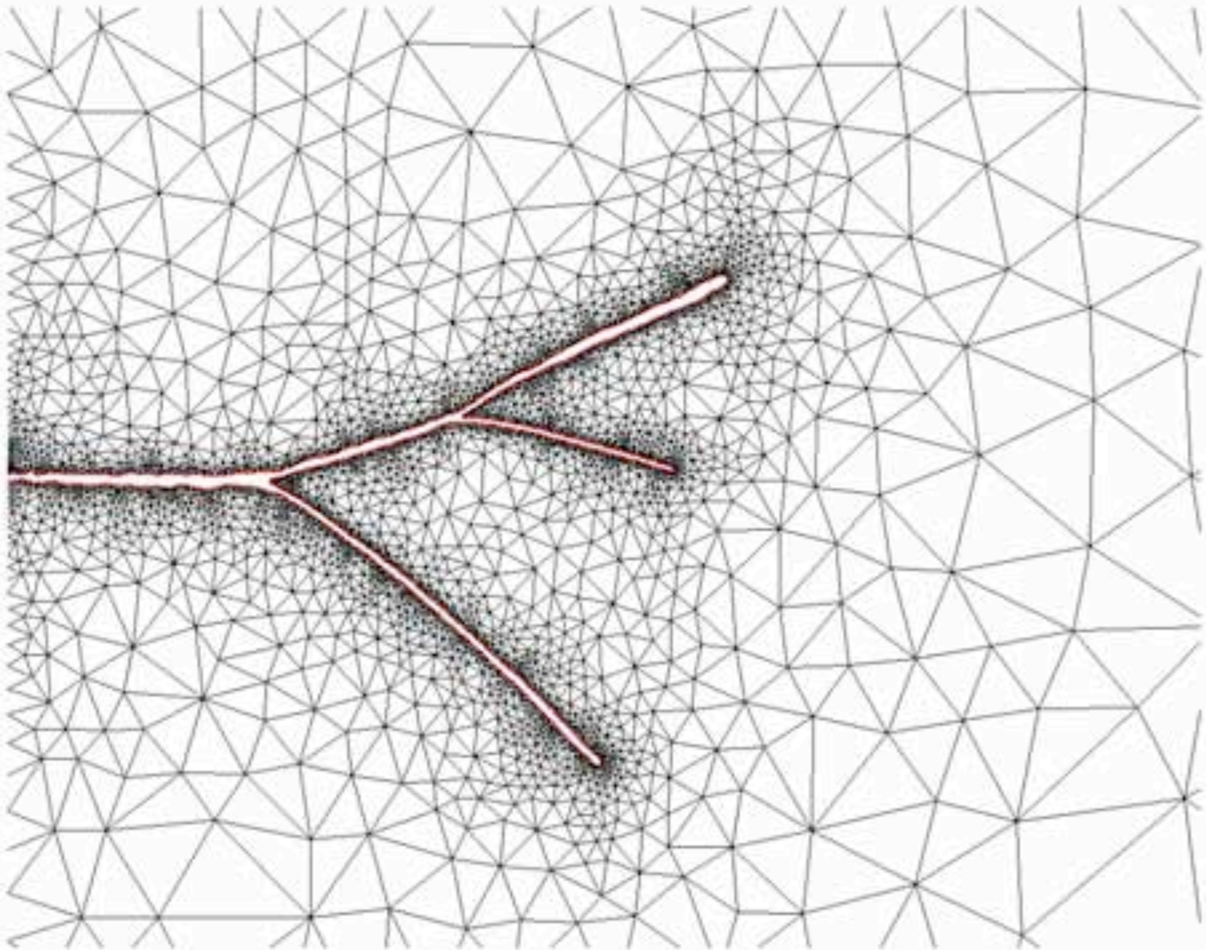

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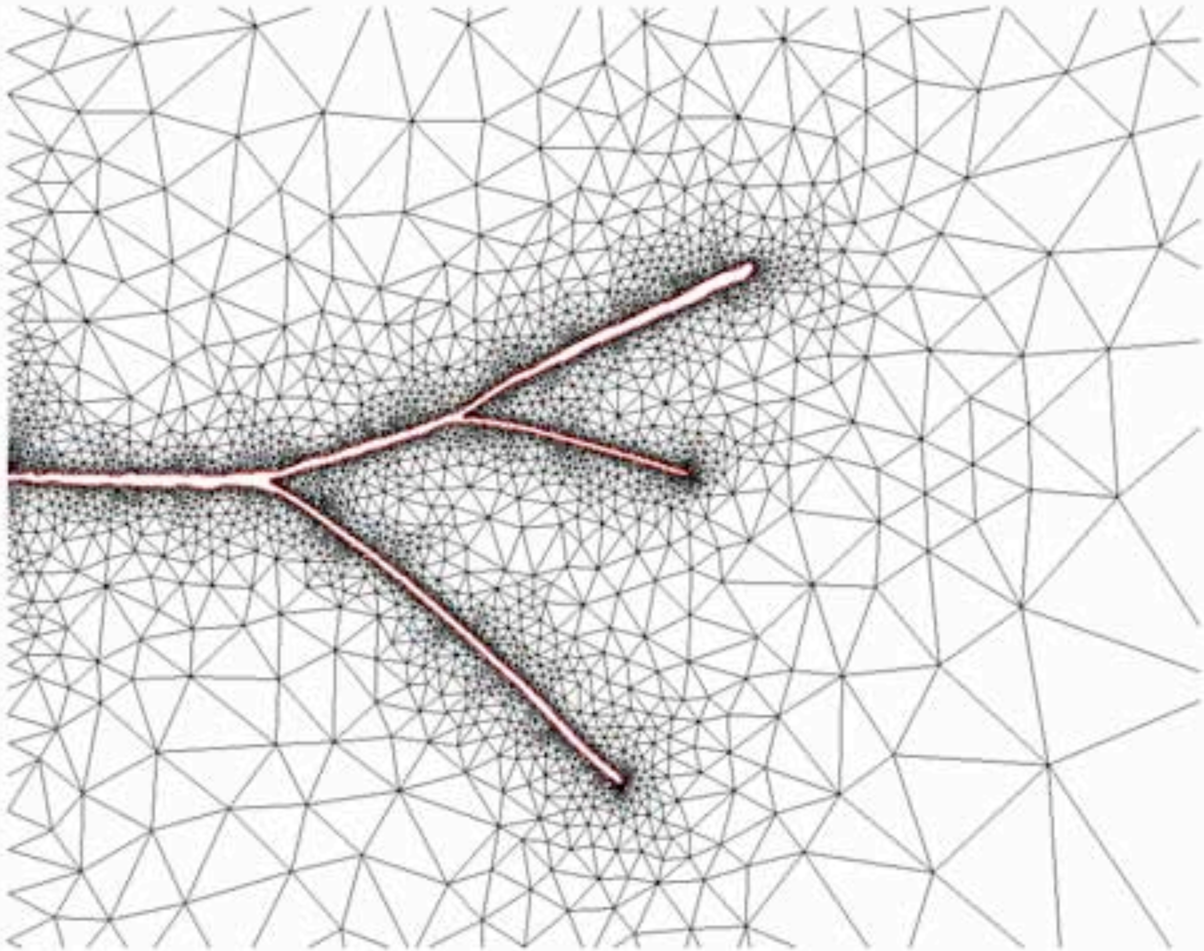

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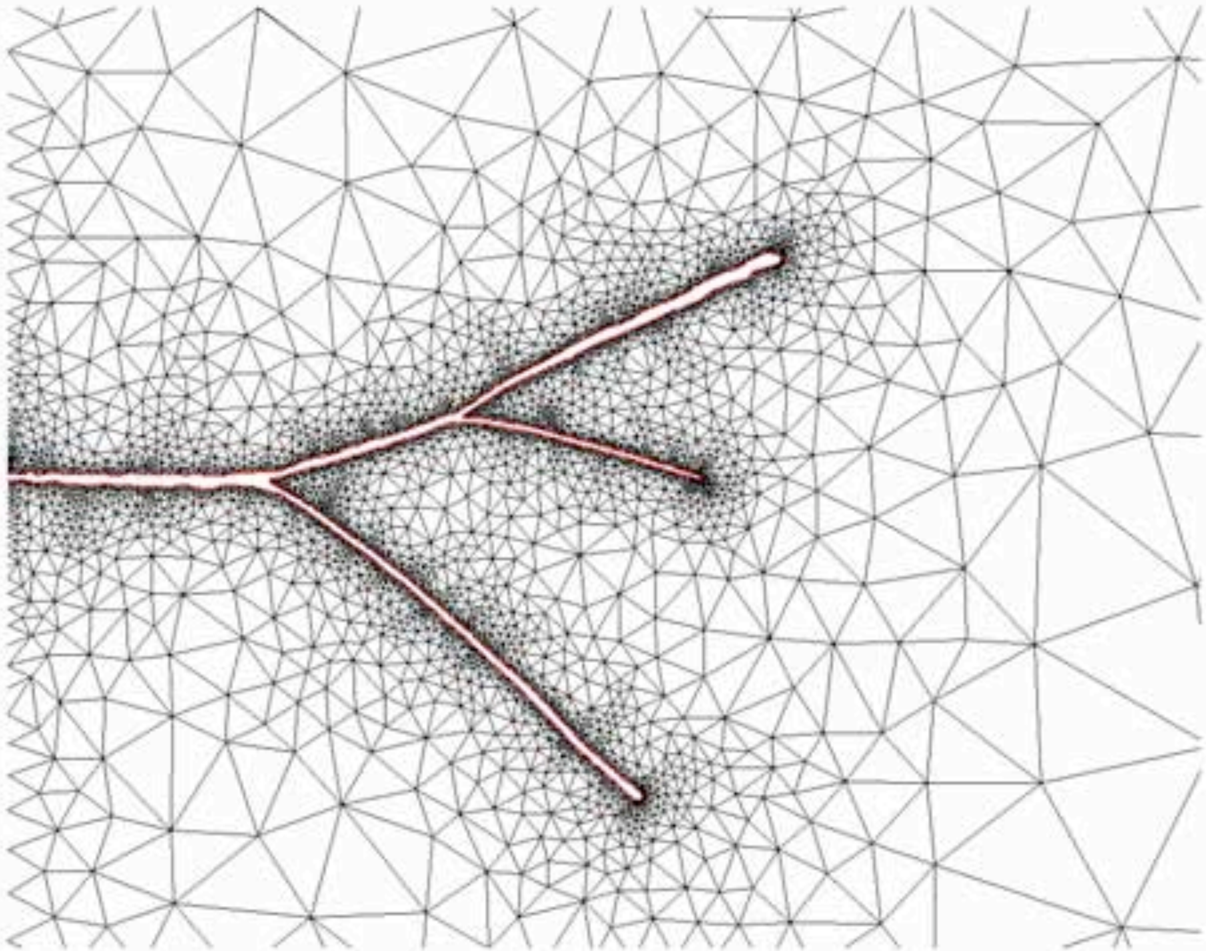

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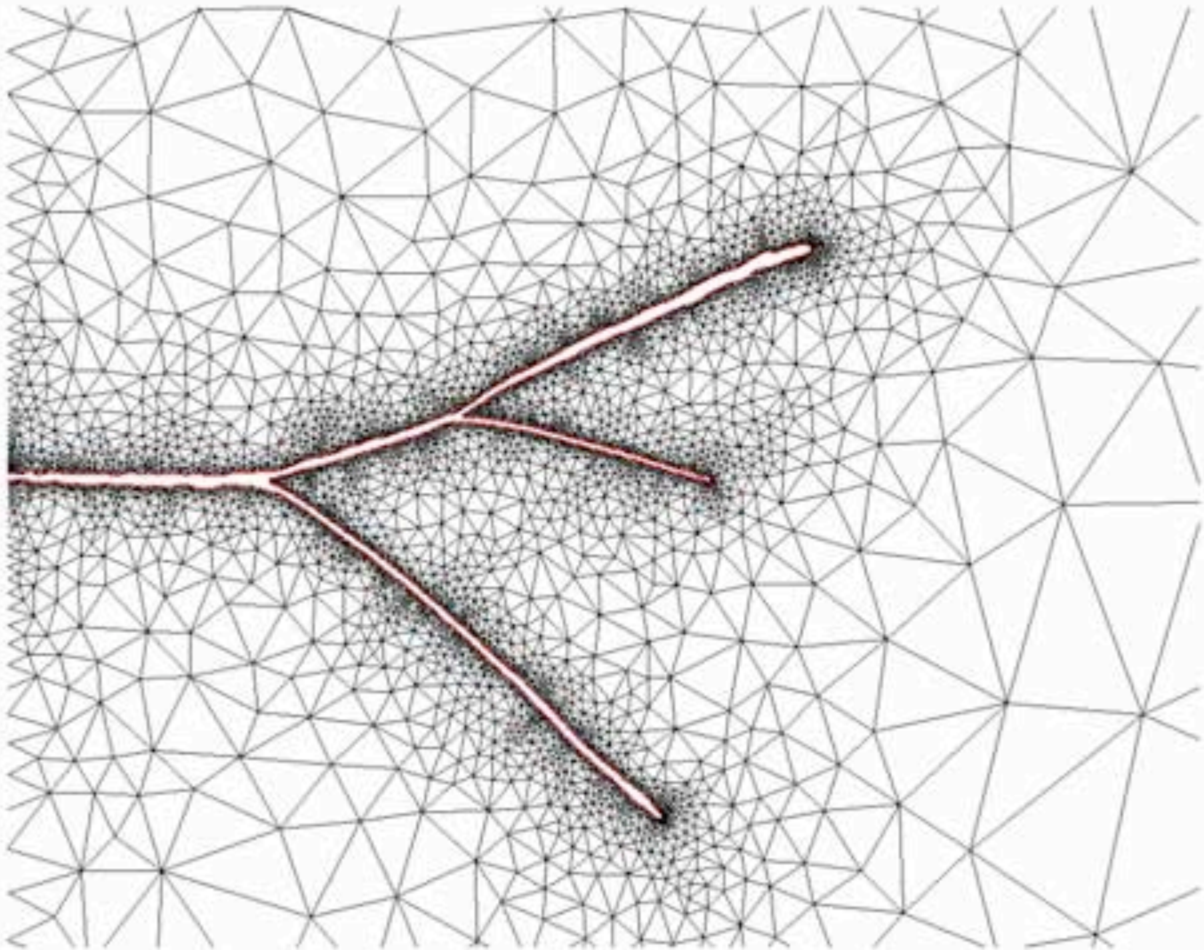

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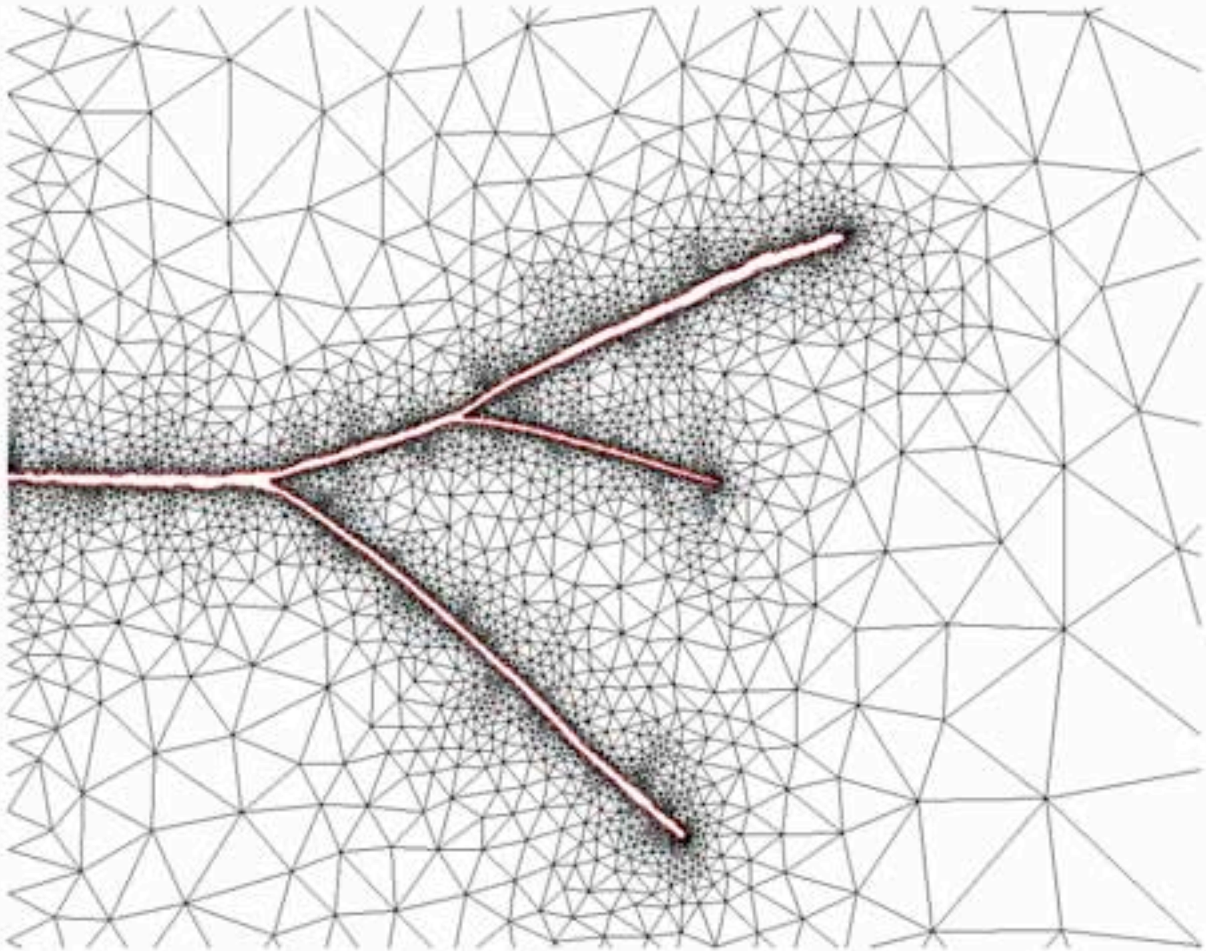

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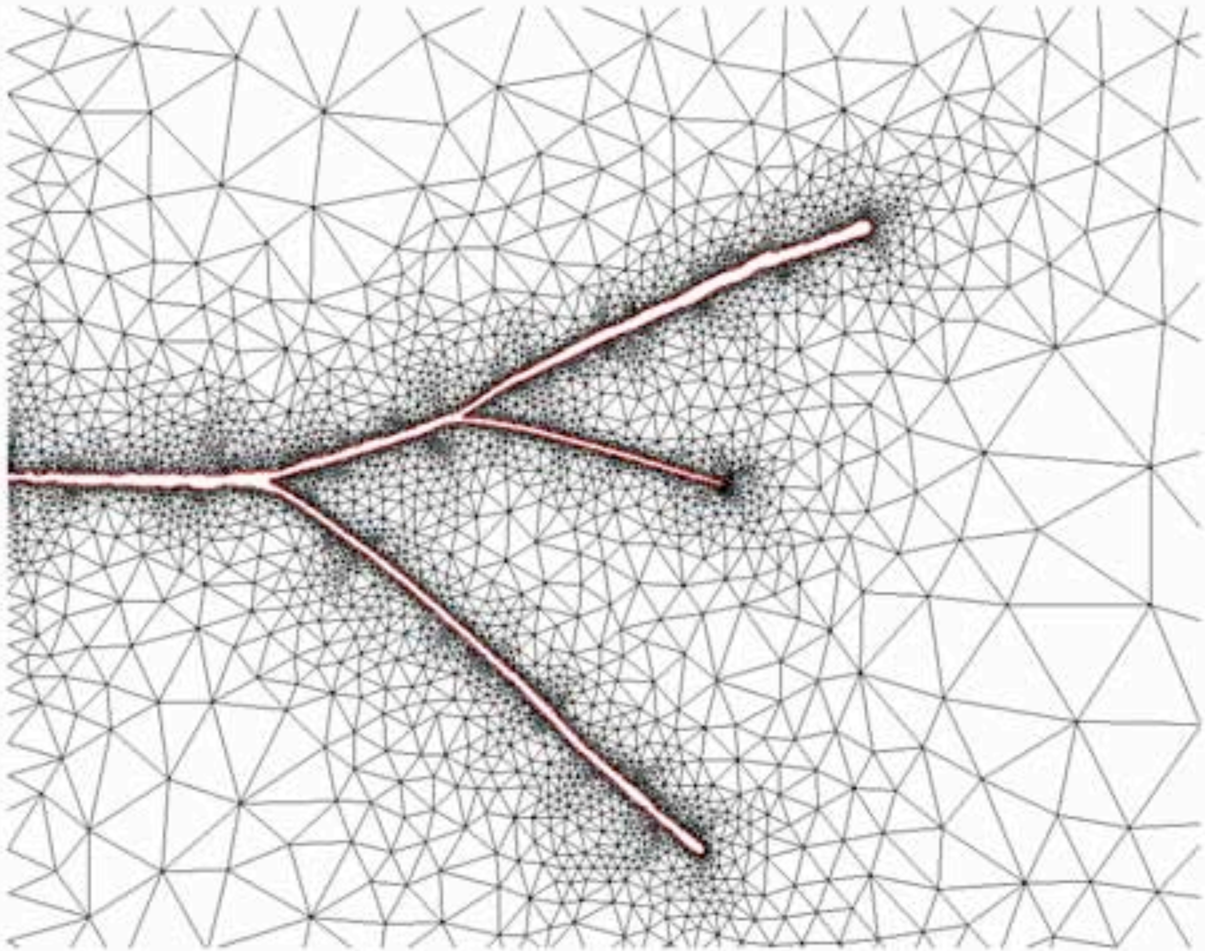

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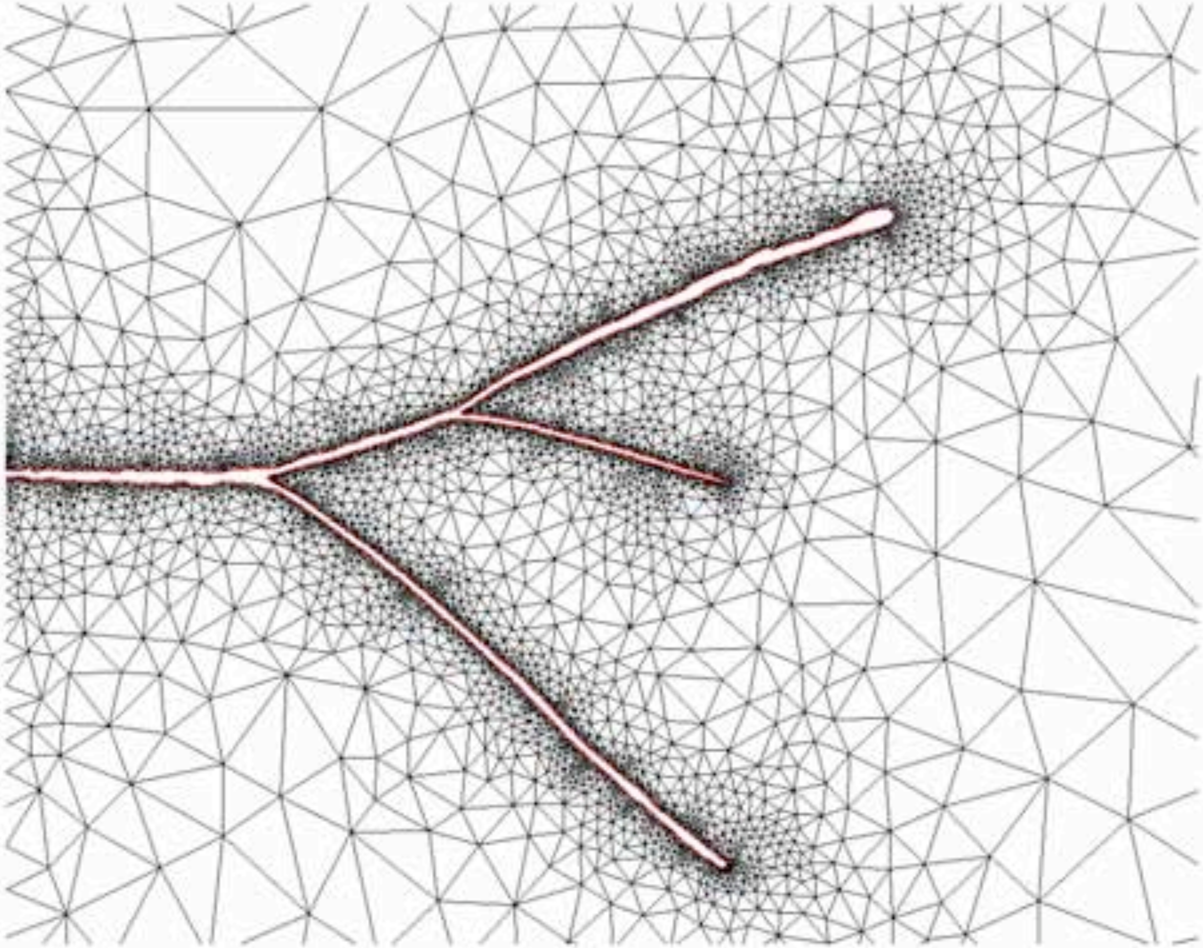

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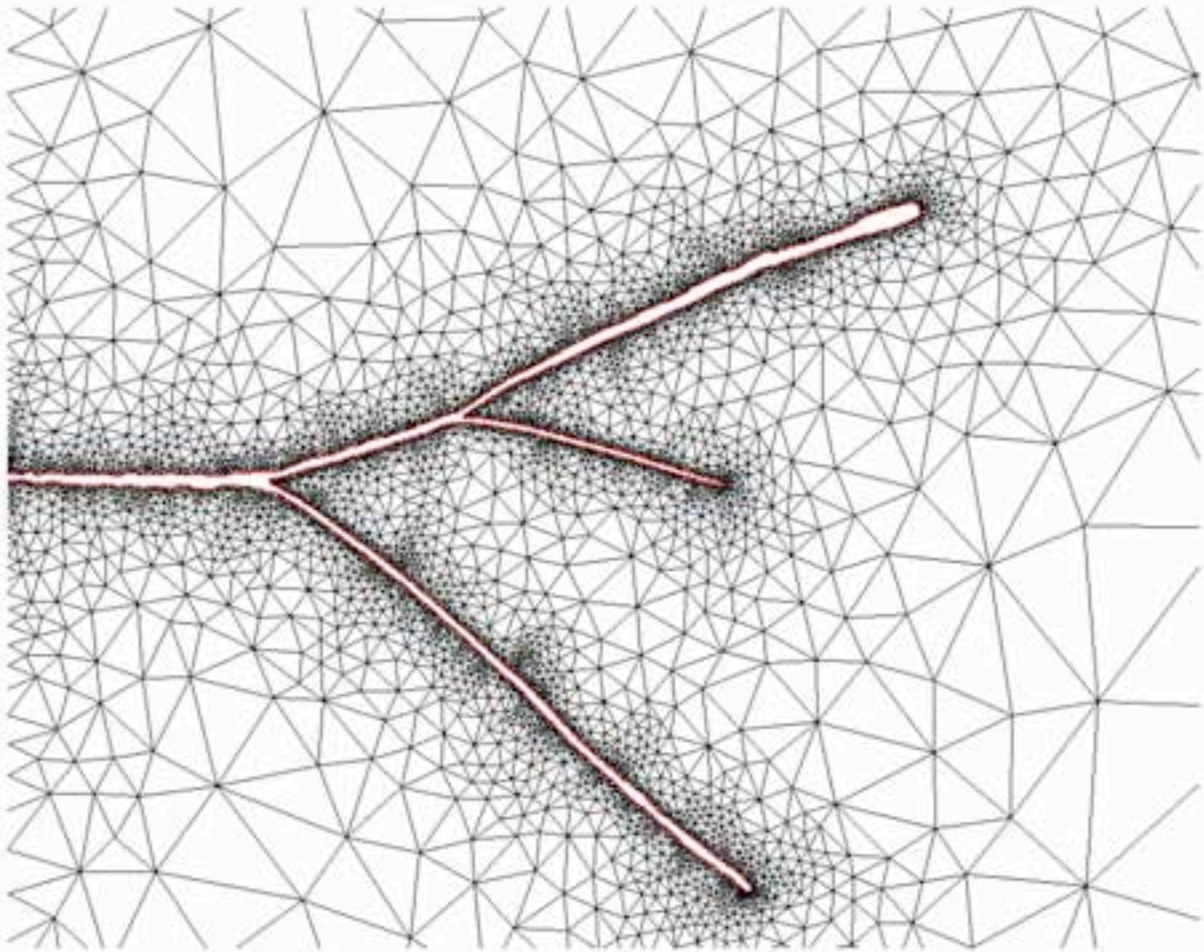

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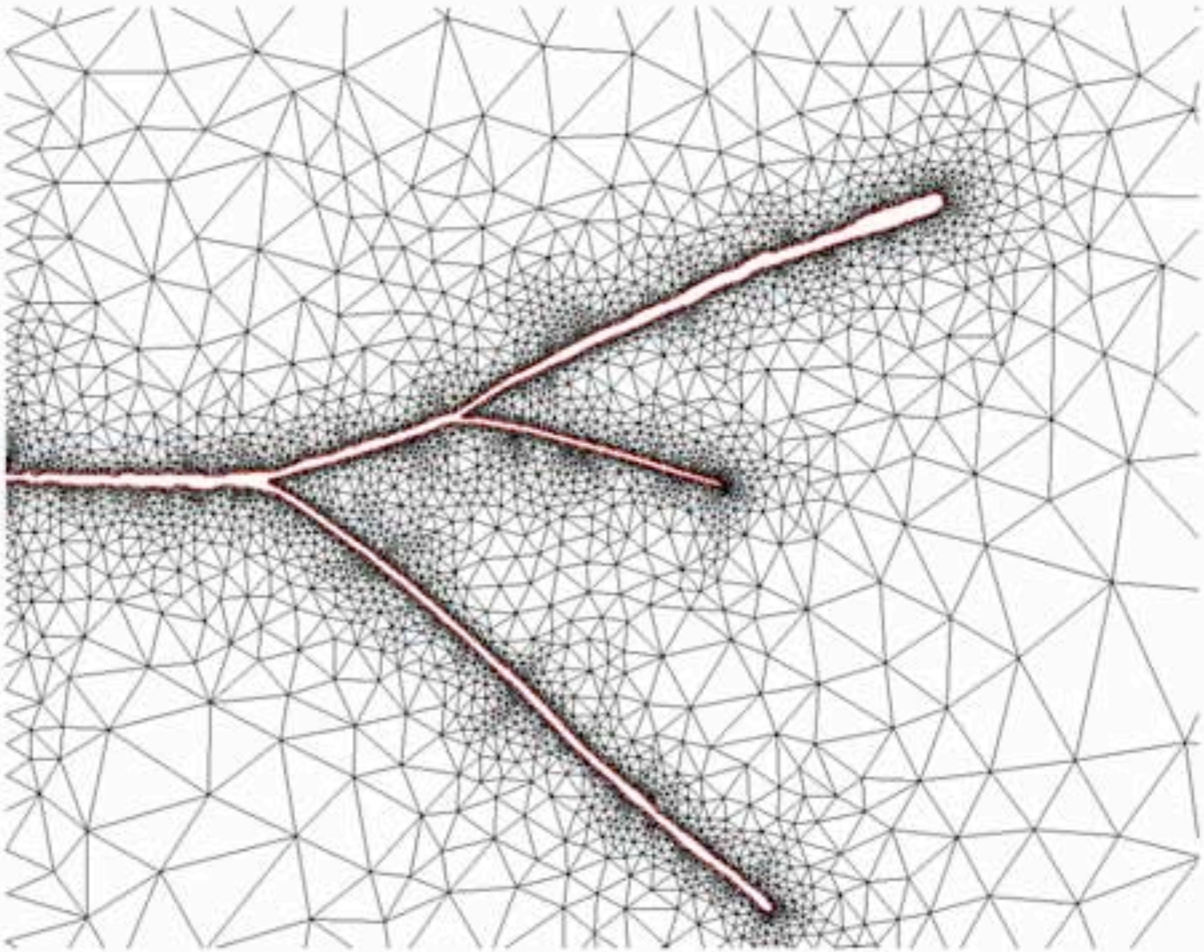

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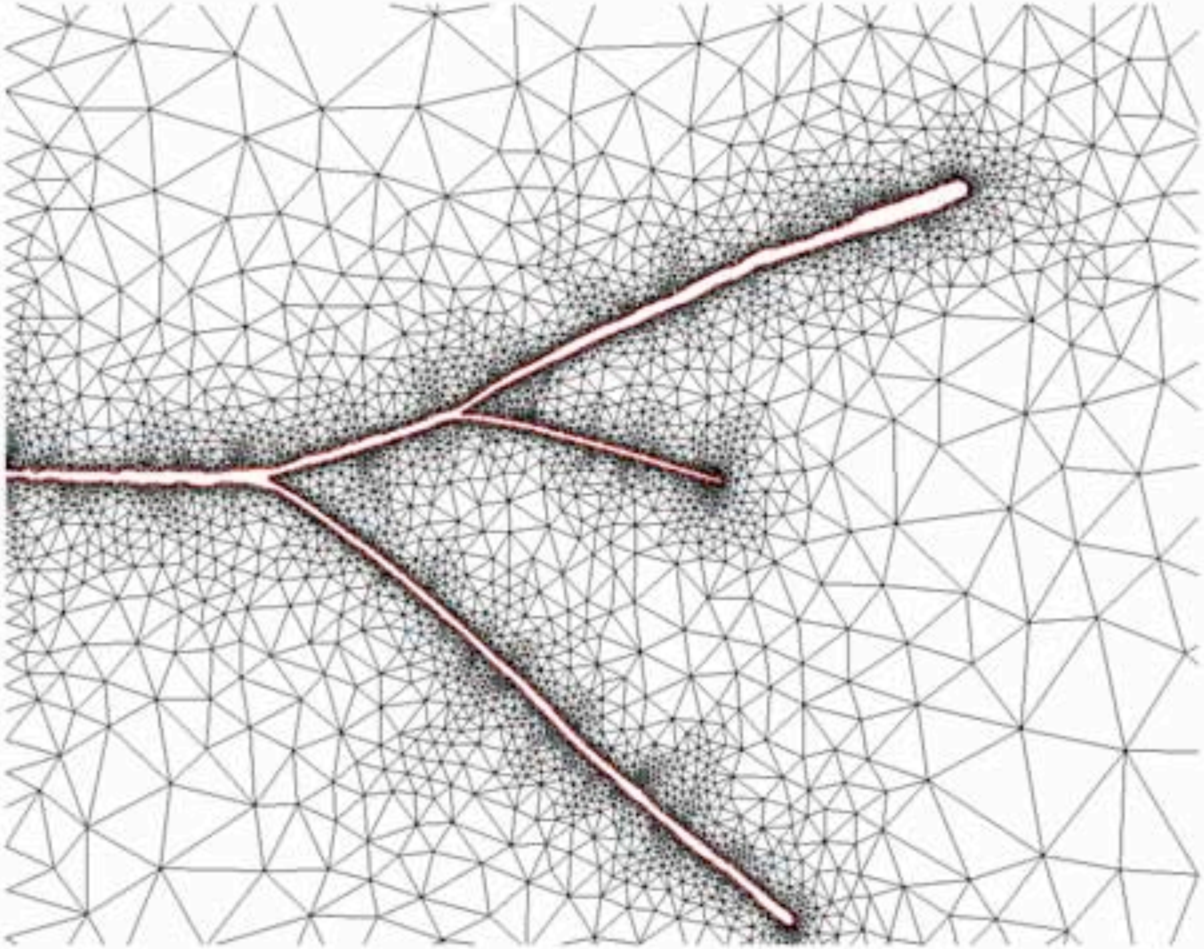

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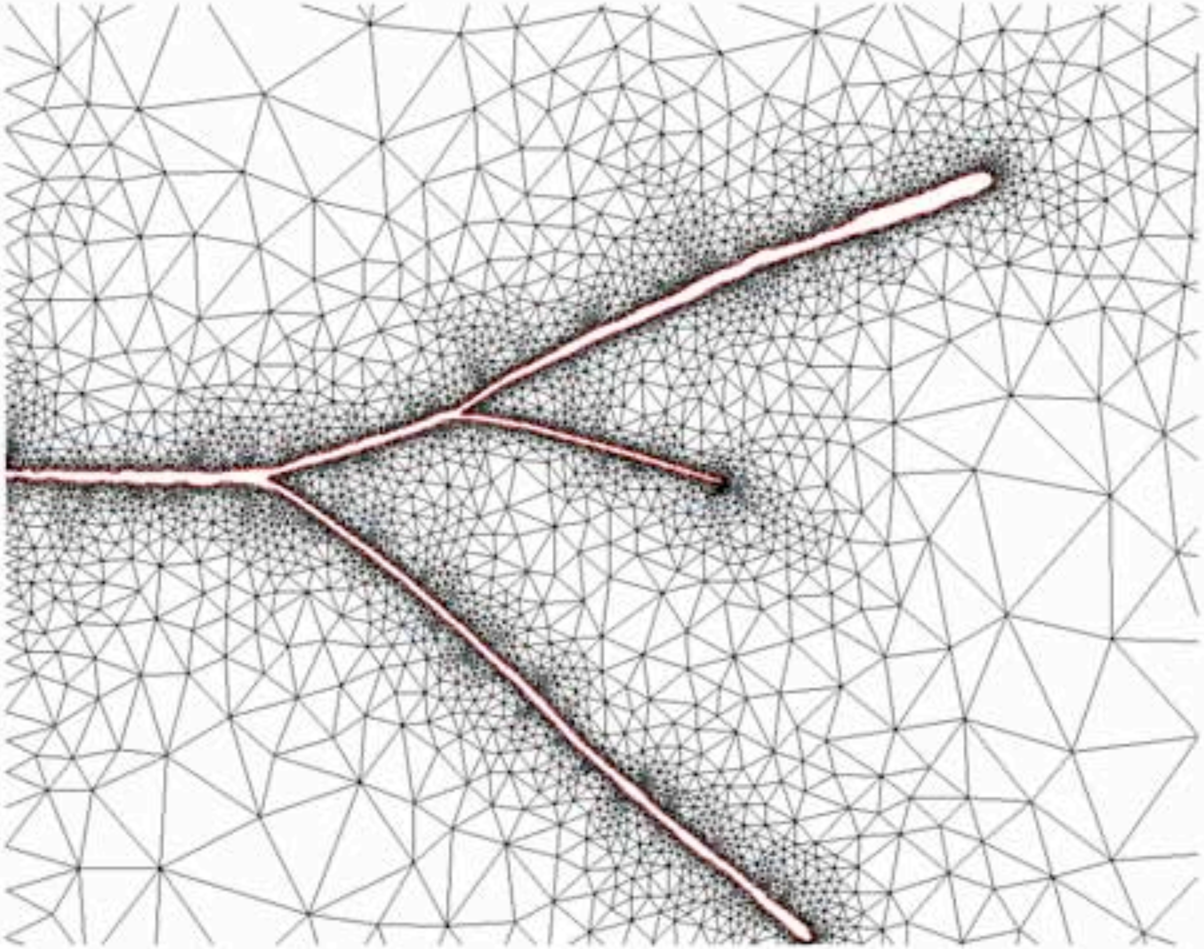

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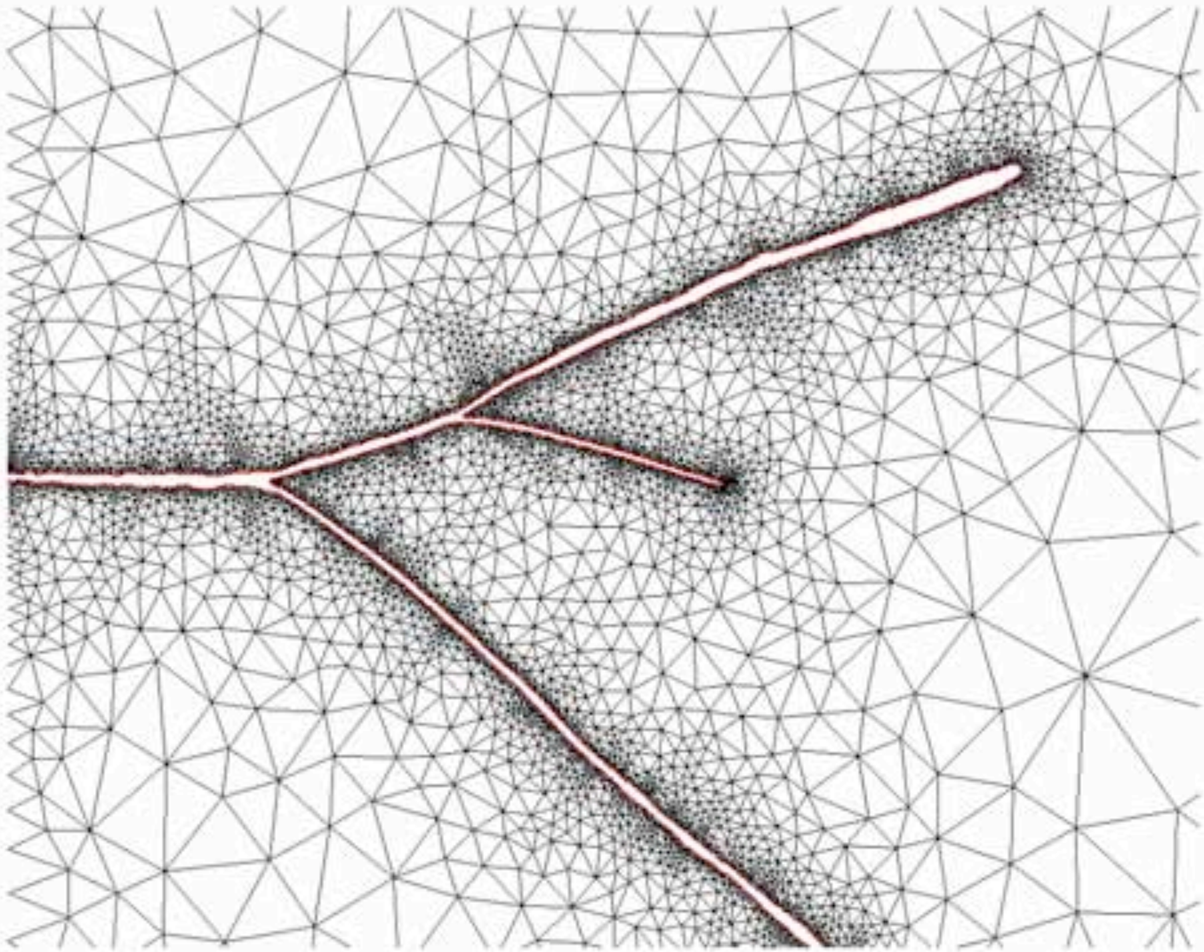

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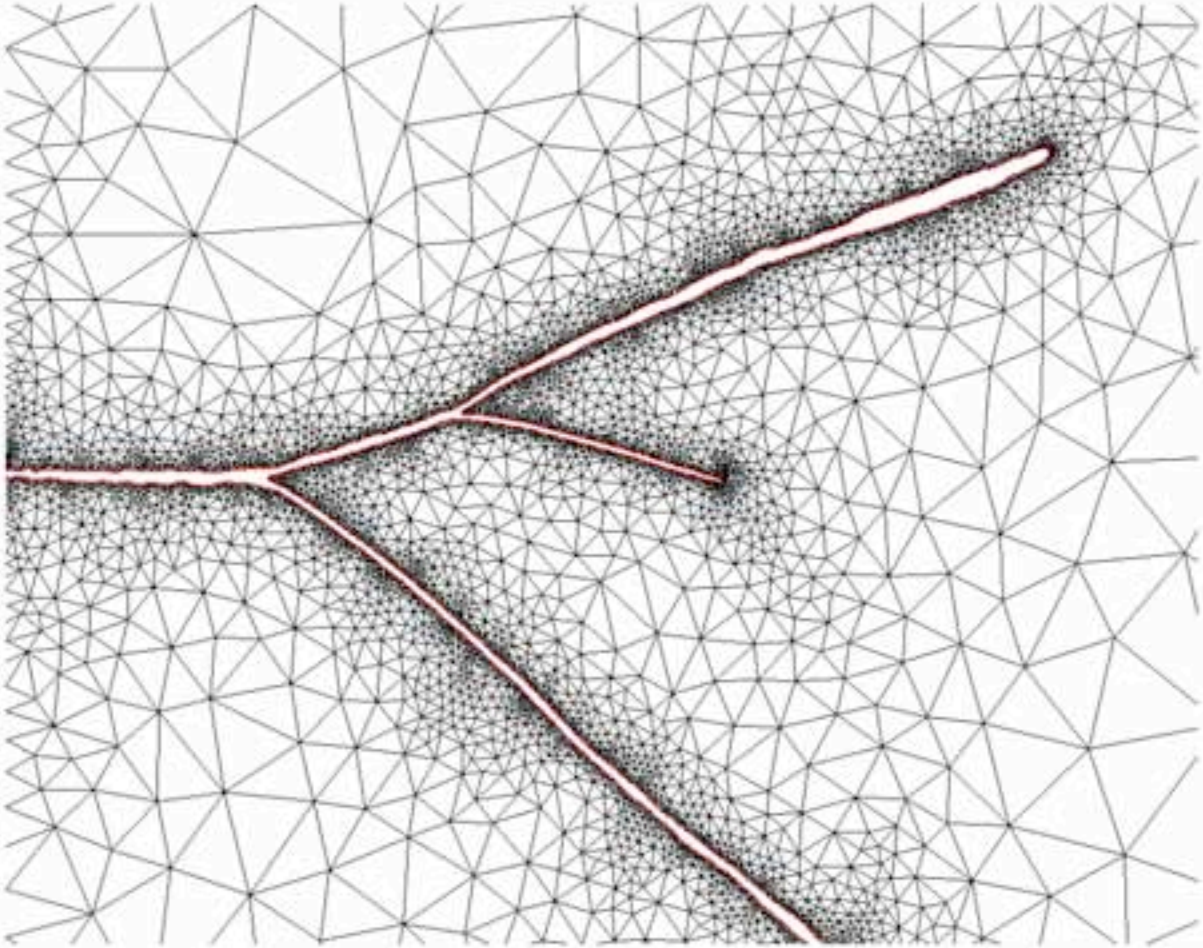

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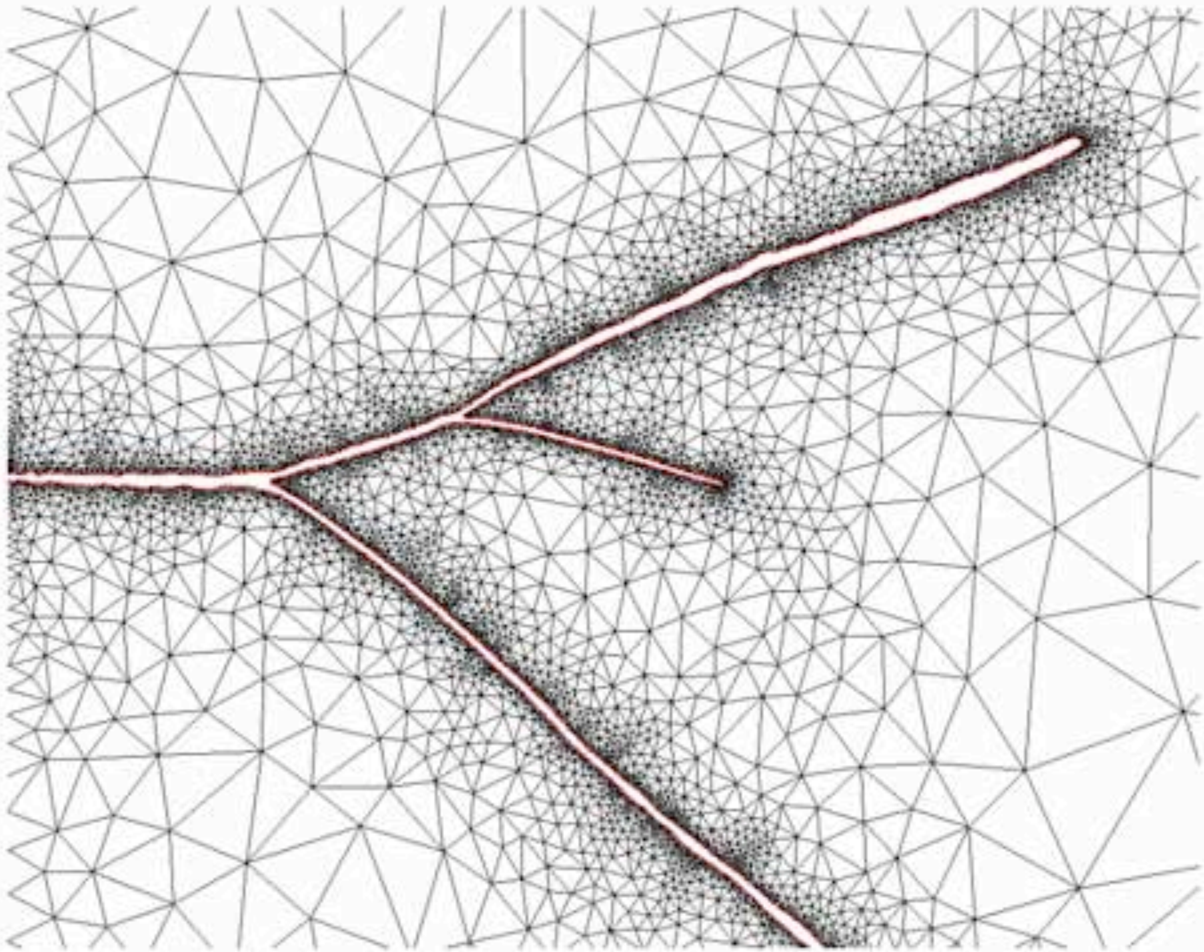

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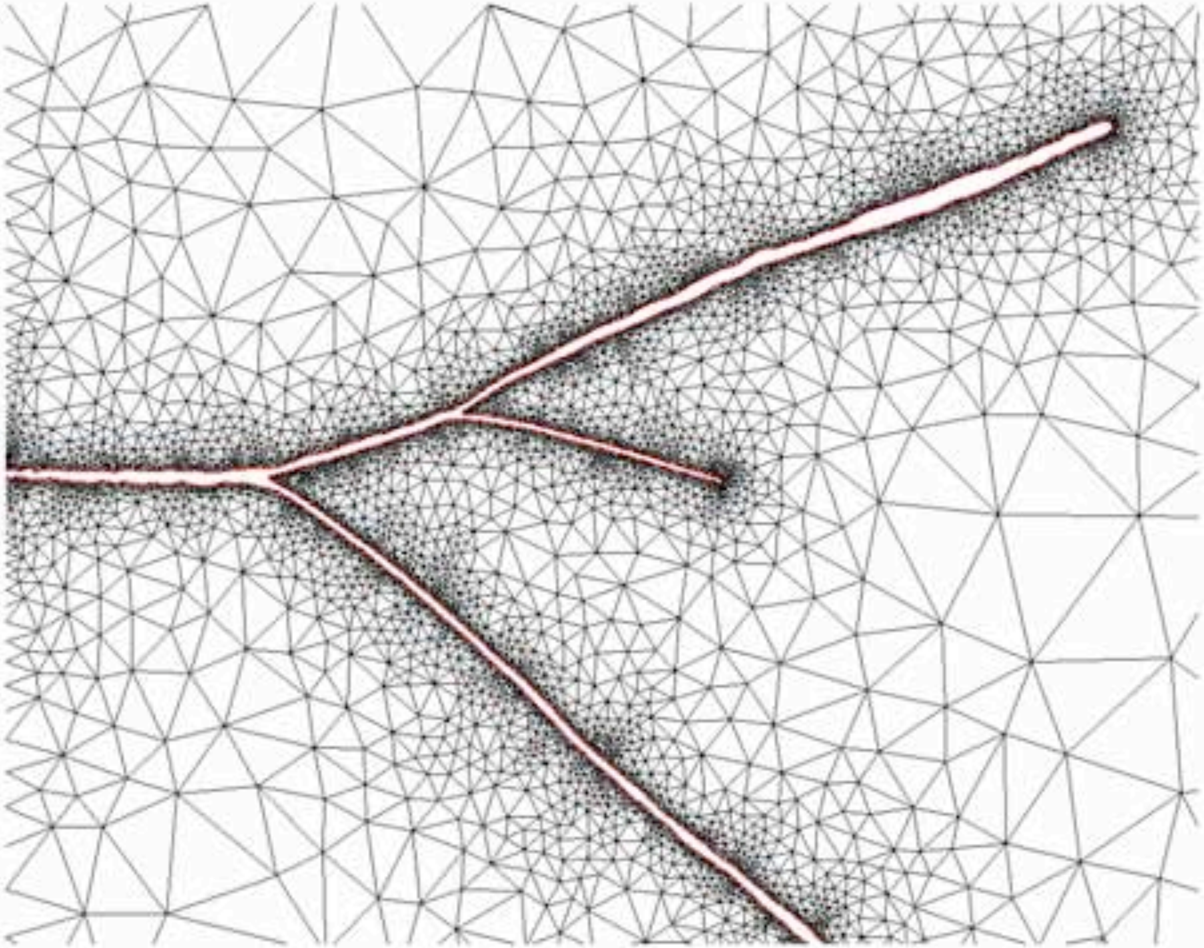

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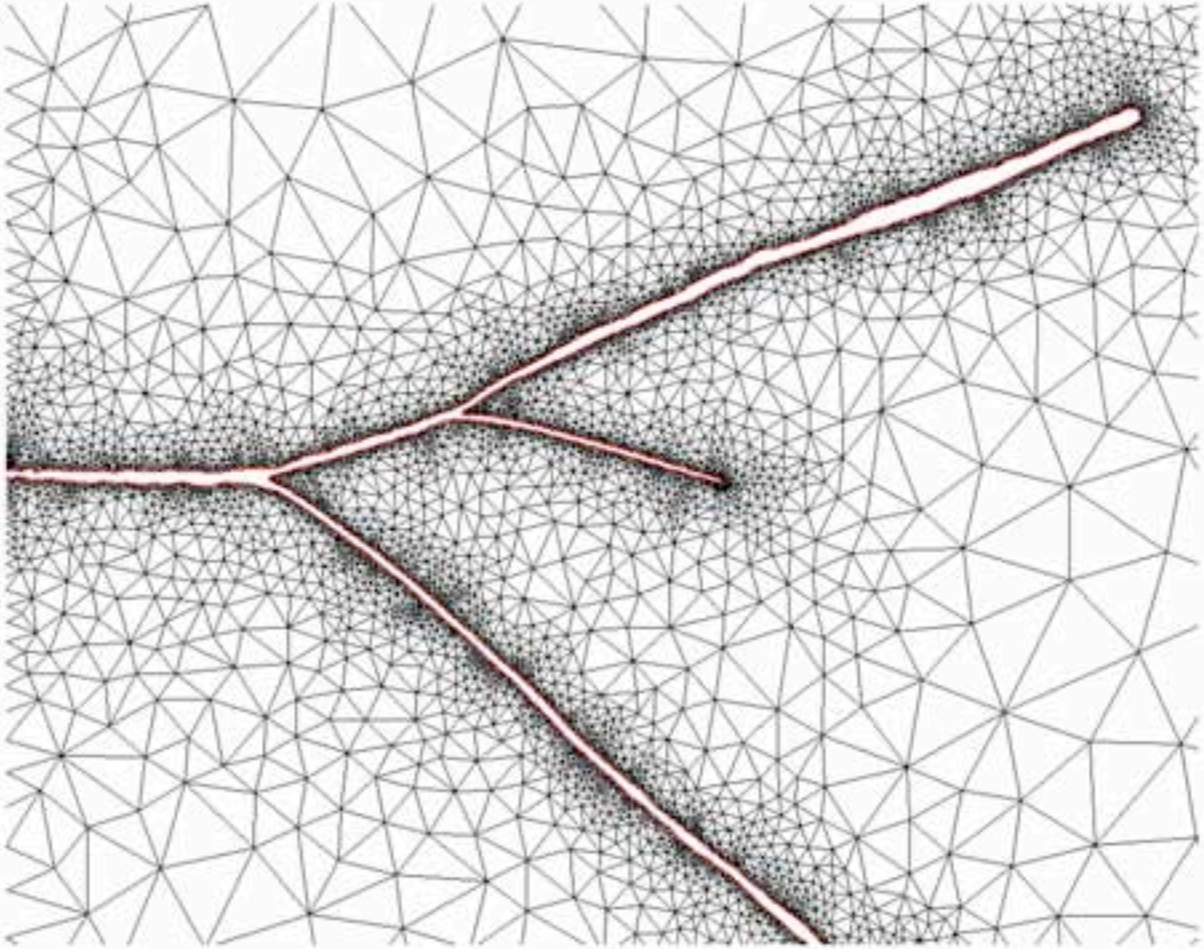
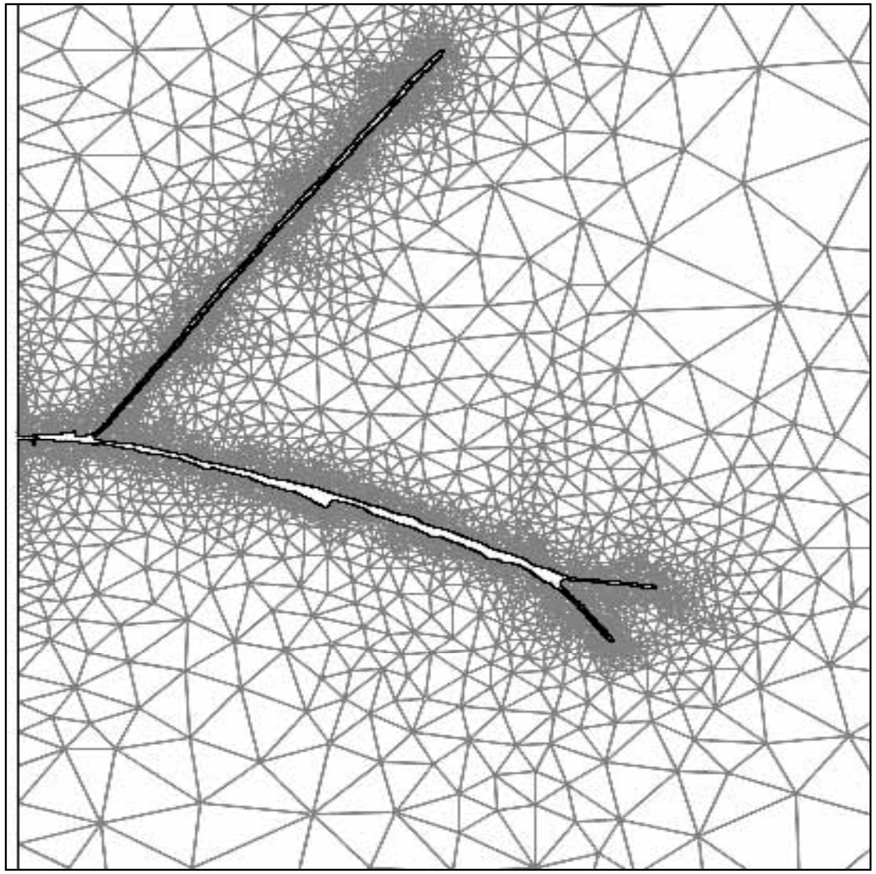
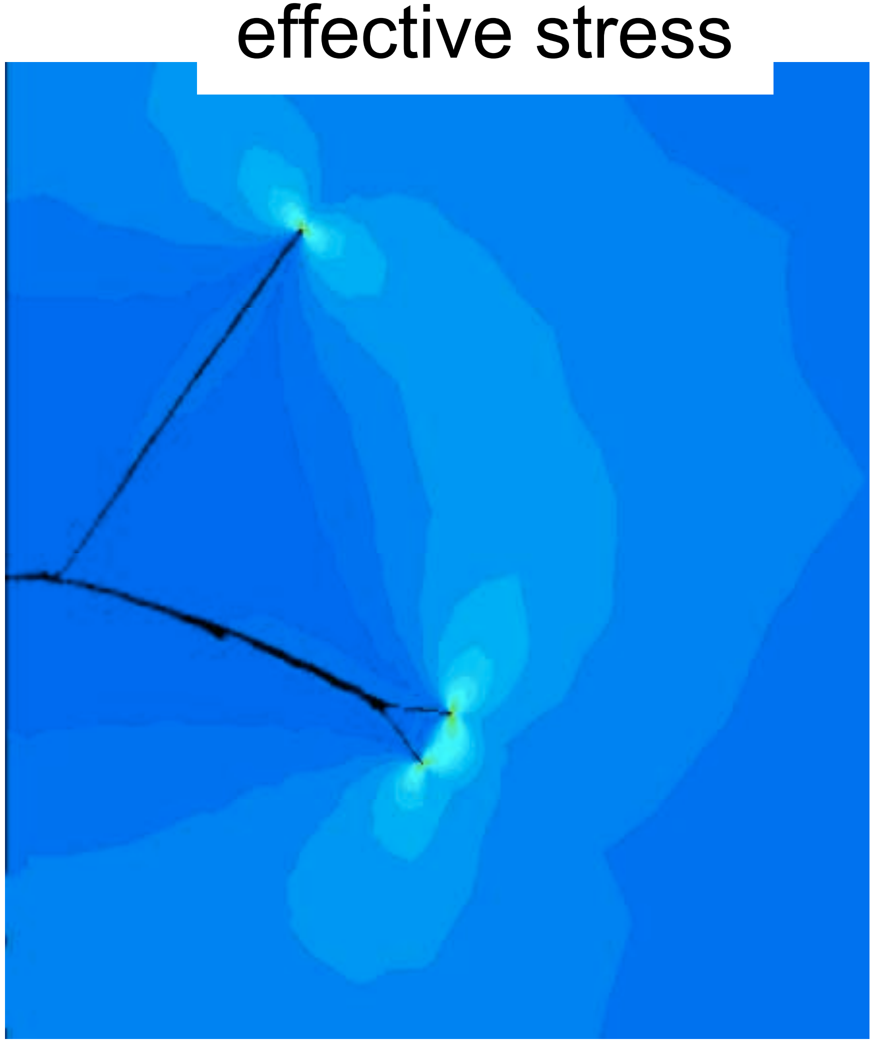


Crack-tip load

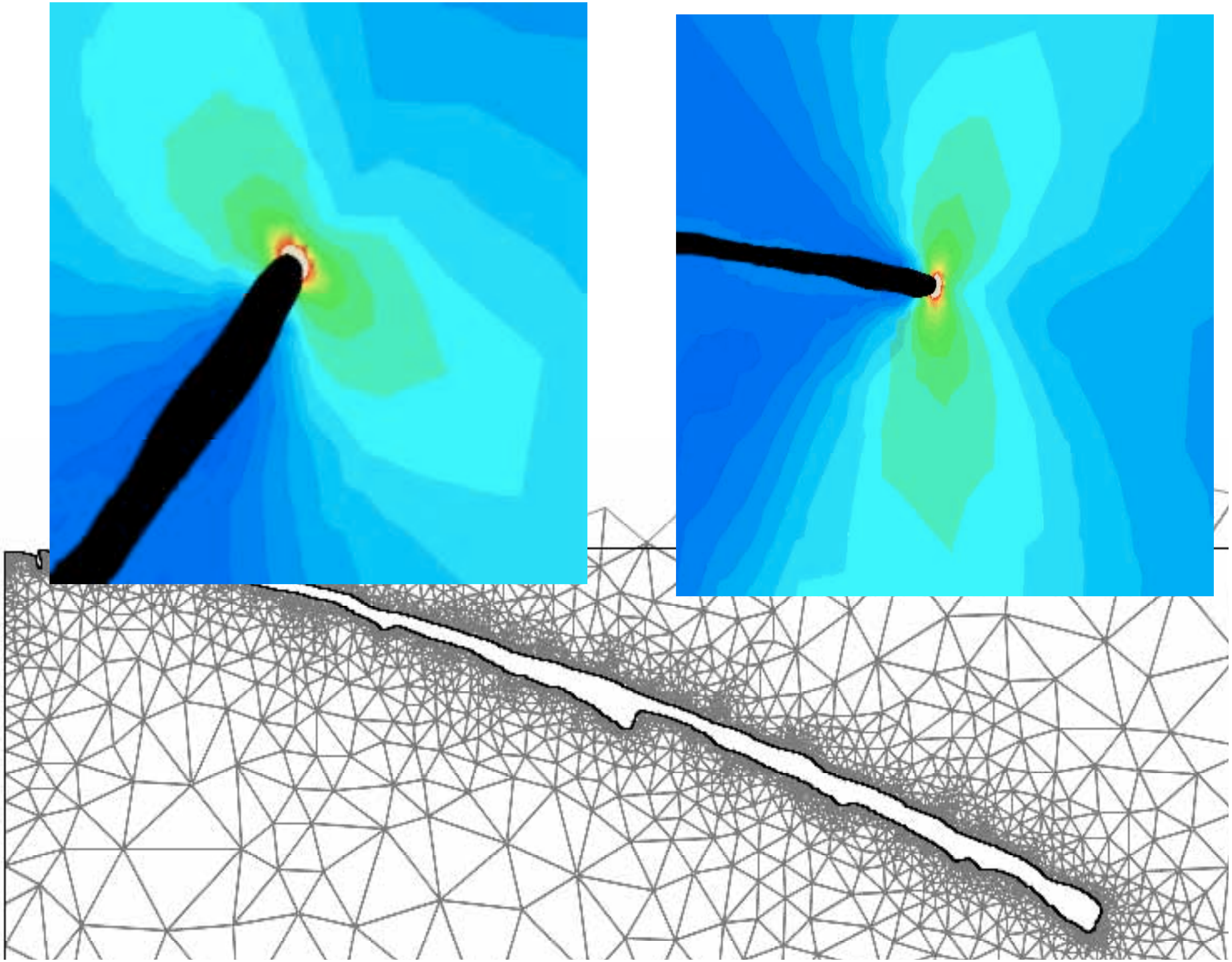




Before branching
$K_I = K_I', \rho = \rho'$

After branching
$K_I \approx 0.7K_I', \rho = \rho'/2$

Crack-tip Load



Phase Field Modelling of Stress Corrosion

Per Ståhle, Eskil Hansen

LU, Lund Sweden

Contributions to the free energy (Ginzburg & Landau, 1950)

$$\mathcal{F} = \mathcal{F}_{el} + \mathcal{F}_{ch} + \mathcal{F}_{gr}$$

Volume totals:

Elastic energy $$\mathcal{F}_{el} = \int \frac{G(\psi)}{2} (\nabla w)^2 dV$$

Chemical energy $$\mathcal{F}_{ch} = \int U(\psi)\, dV$$

Gradient energy $$\mathcal{F}_{gr} = \int \frac{g_b}{2} (\nabla \psi)^2 dV$$

Antiplane deformation => Two free variables

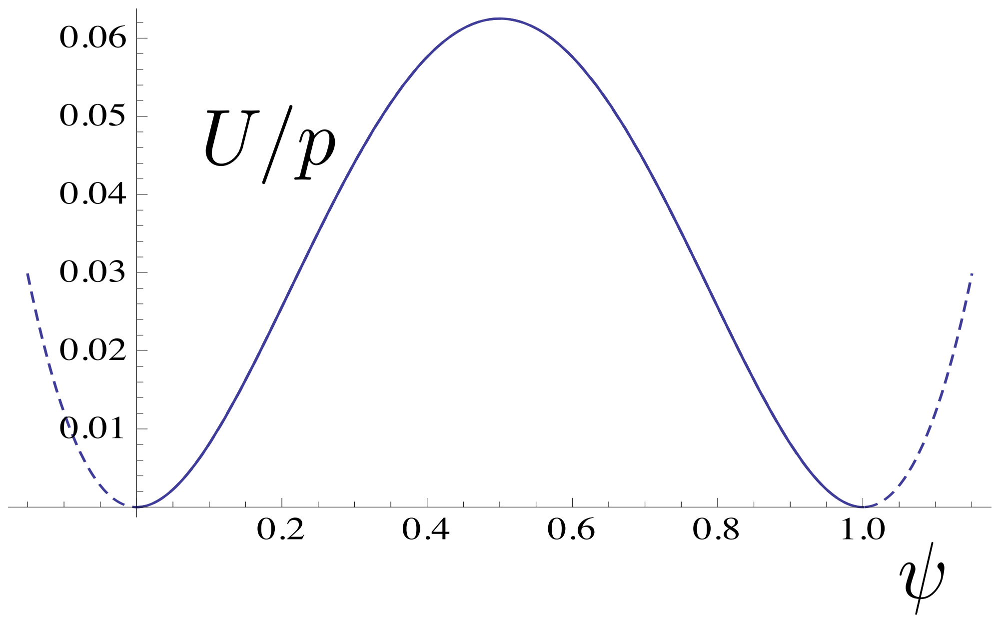
Displacements w and phase (density) ψ

$$\frac{\partial \psi}{\partial t} = -L_\psi \frac{\delta \mathcal{F}}{\delta \psi} \quad , \qquad \frac{\partial w}{\partial t} = -L_w \frac{\delta \mathcal{F}}{\delta w}$$

Ginzburg, Landau (50)

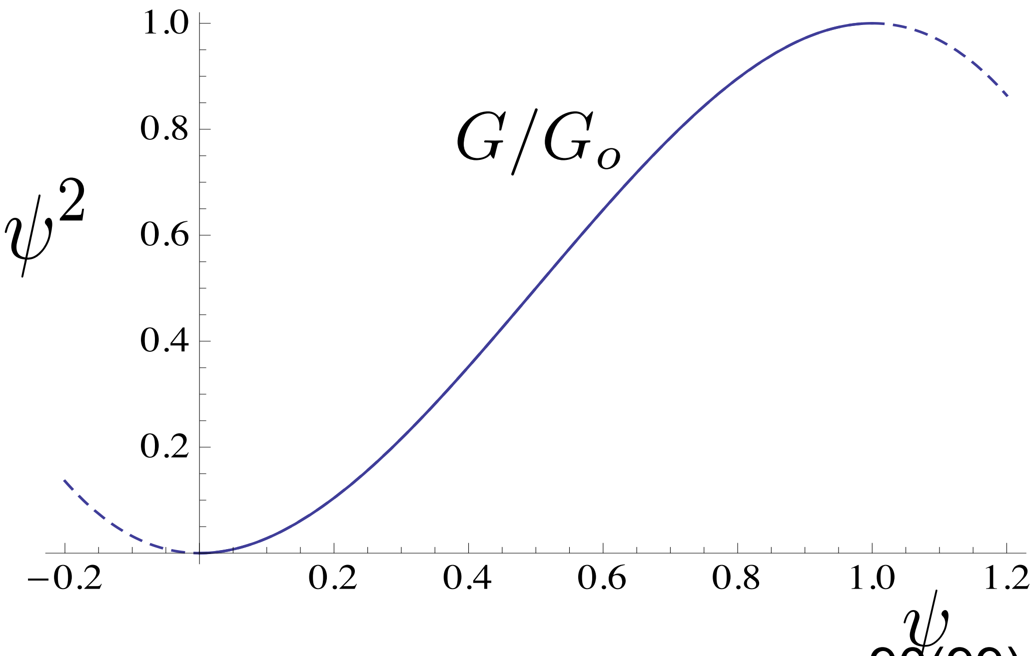
Double-well chemical potential

$$U(\psi) = p\,\psi^2(1-\psi)^2$$



Shear modulus

$$G(\psi) = G_o(\psi)(-2\psi+3)\psi^2$$



Evolution of the phase

$$\frac{\partial\psi}{\partial t} = -L_\psi[\frac{1}{2}G'(\psi)(\nabla w)^2 + p\psi(\psi^2 - 1) - g_b\triangle\psi]$$

Evolution of the displacements

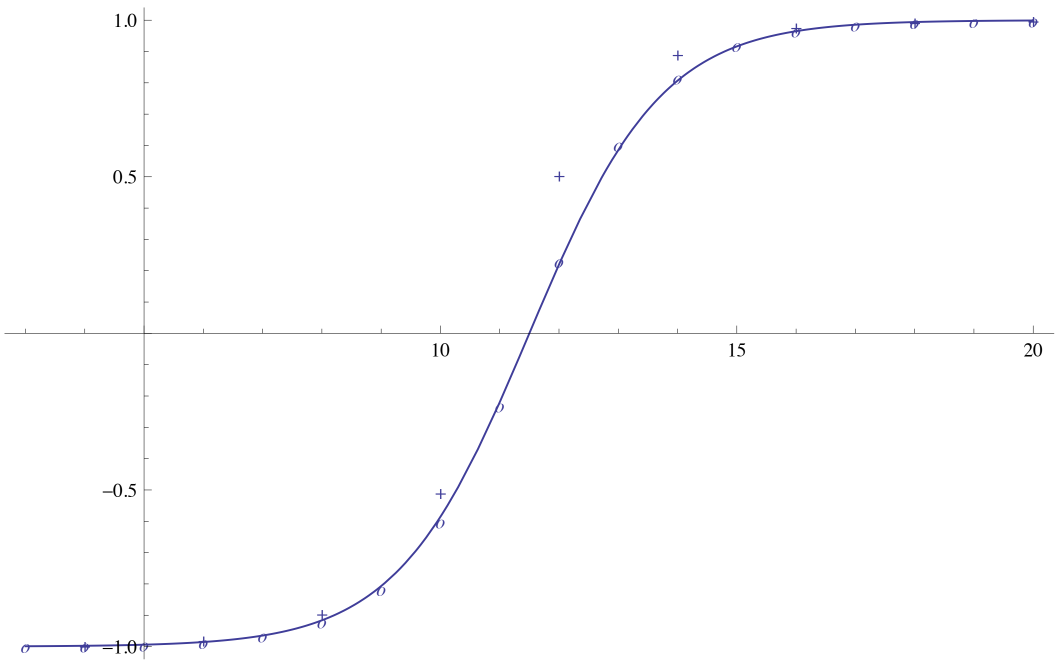
$$\frac{\partial w}{\partial t} = L_w\nabla\cdot[G(\psi)\nabla w]$$

At equilibrium: $\nabla\cdot[G(\psi)\nabla w] = 0$

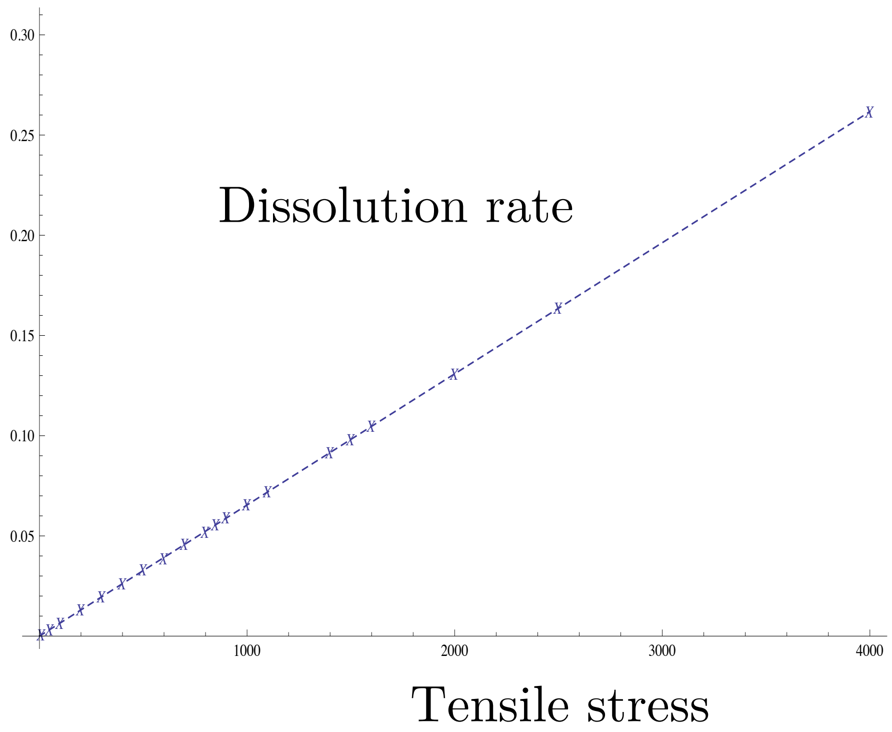
Steady state solution

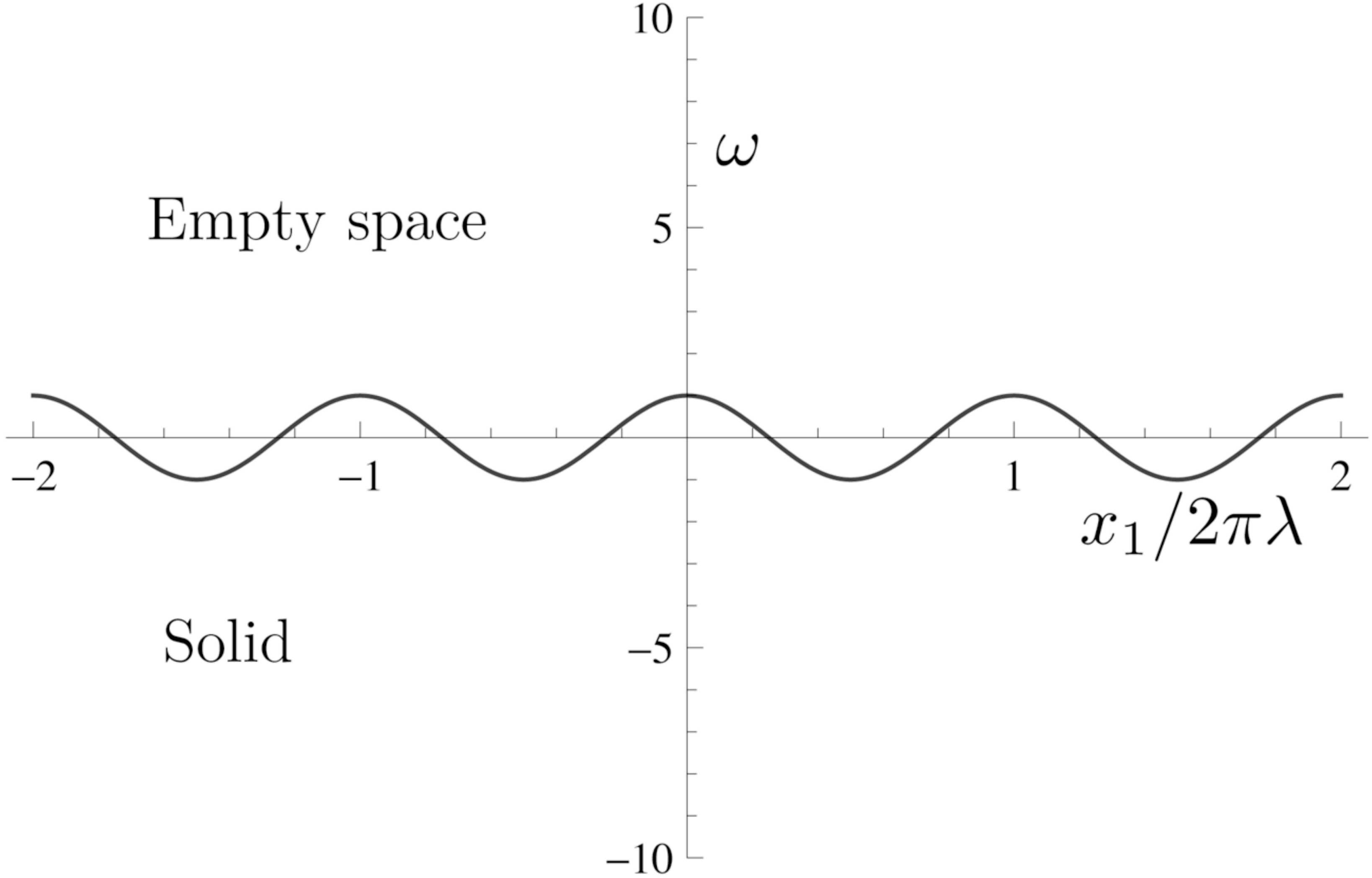
$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$

Transition Rate vs. Distance



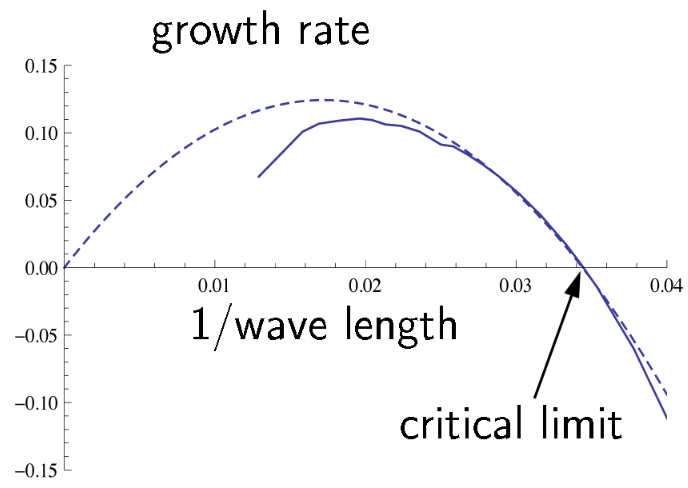
Dissolution Rate vs. Tensile Stress

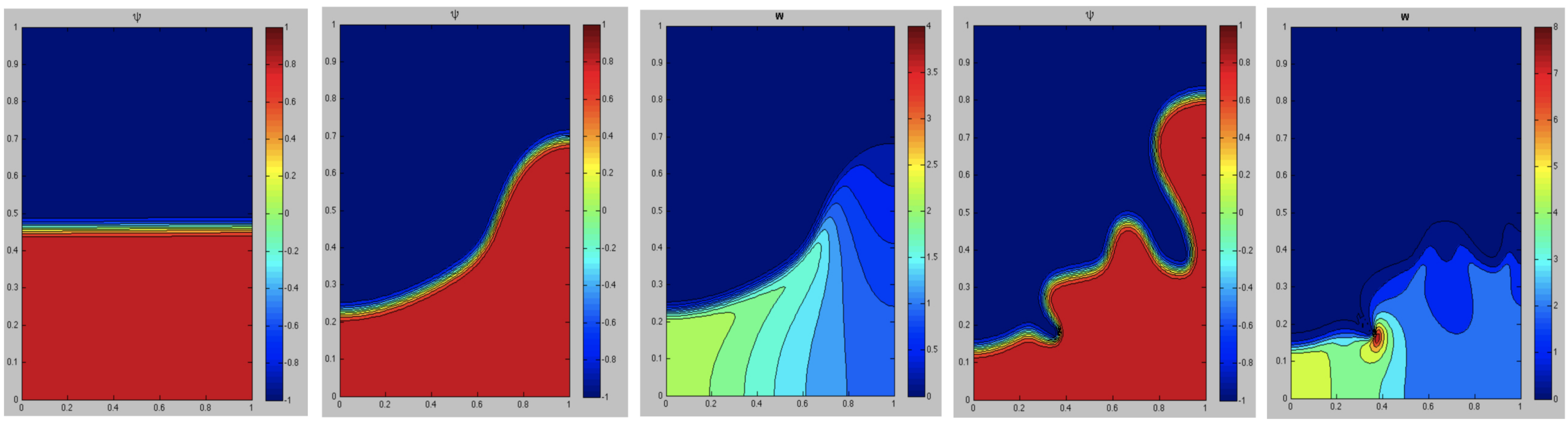



10
ω
Empty space
5
–2
–1
1
2
$x_1/2\pi\lambda$
Solid
–5
–10

$$\left(\frac{\mathrm{d}}{\mathrm{d}x_2} - 2\beta\right)(f' + f^2 - 1) = 0$$

$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$

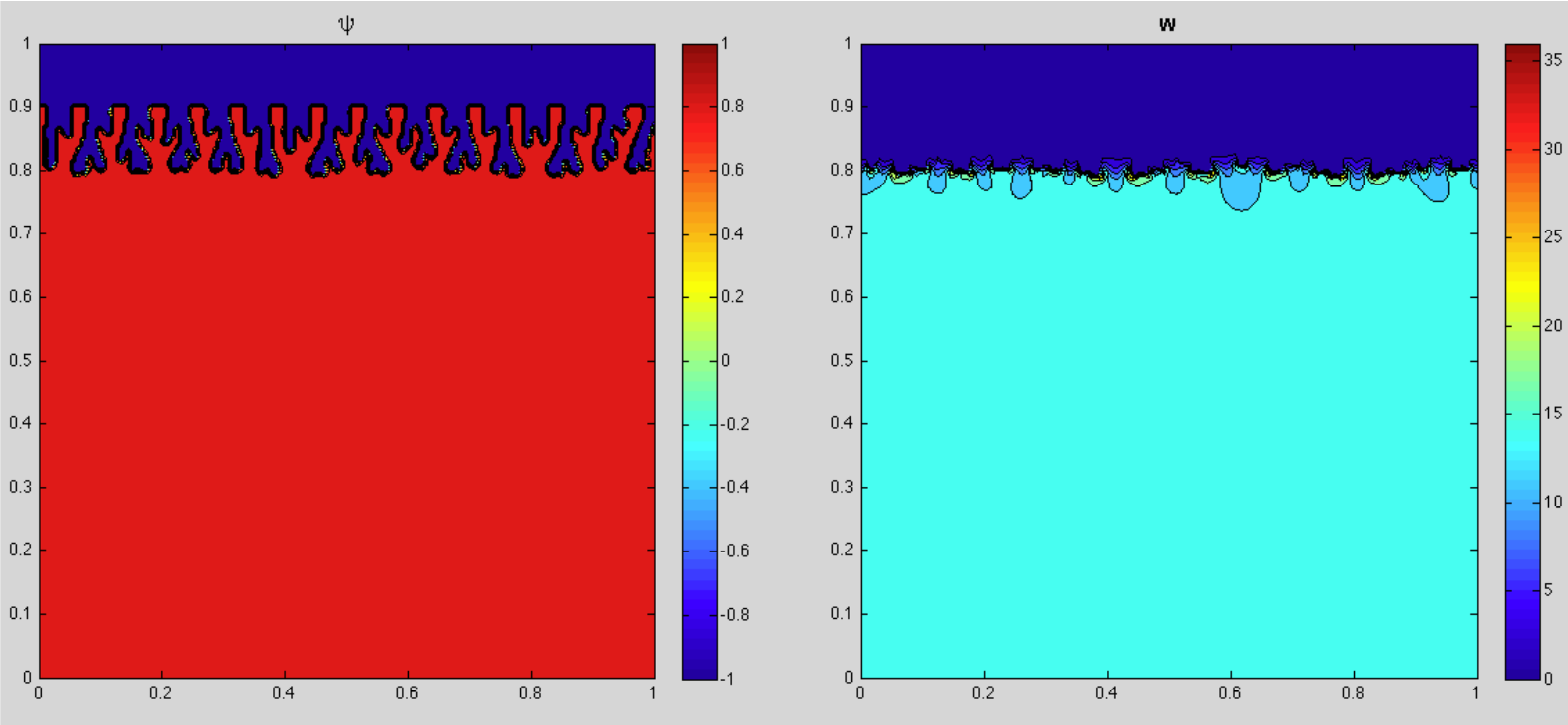




Red is remaining material

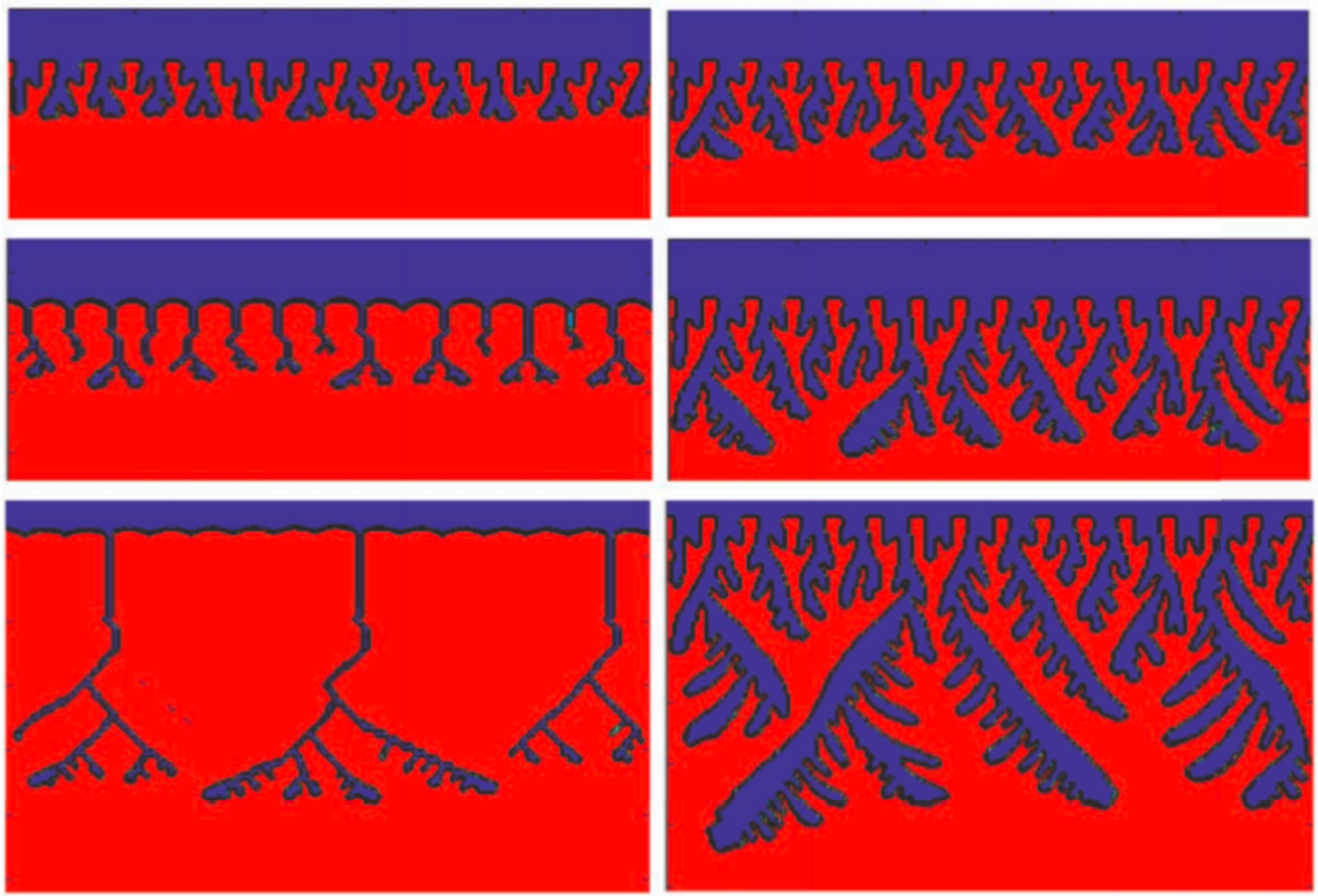
Effective Stress

Effective Stress



Red is remaining material

Effective stress



Without general corrosion with general corrosion

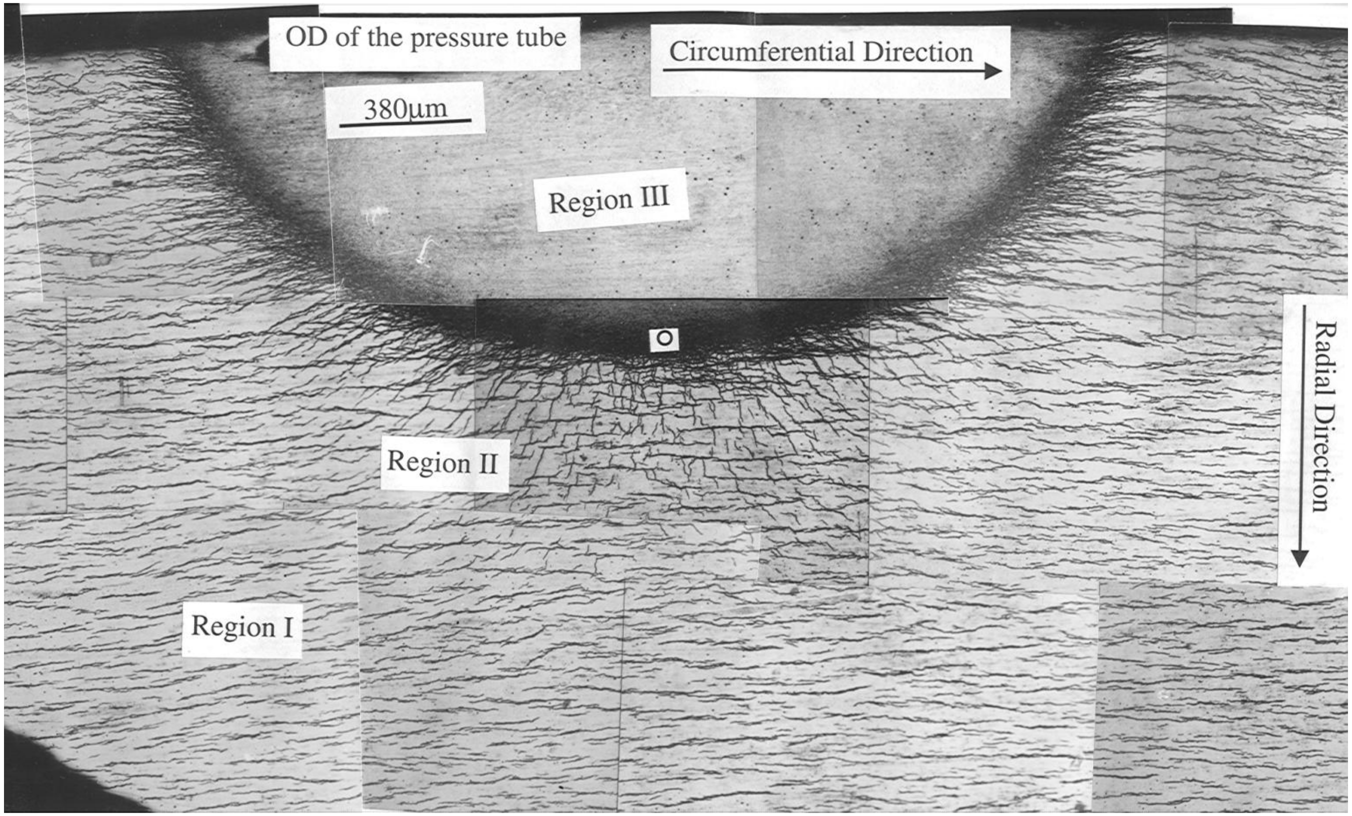
Formation and growth of hydrides

Per Ståhle

Solid Mechanics,
Lund University, Sweden

Collaborators:
Wurigul Reheman, Ram N Singh,
Martin Fisk, Srikumar Banerjee,
Ali Massih, Christina Bjerkén

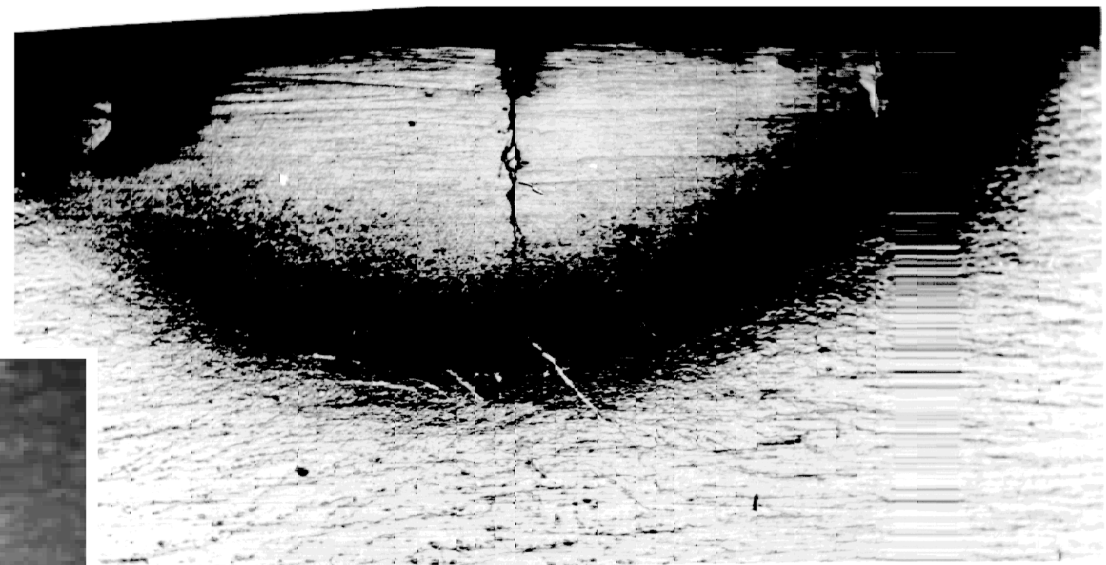
Hydride Blister section

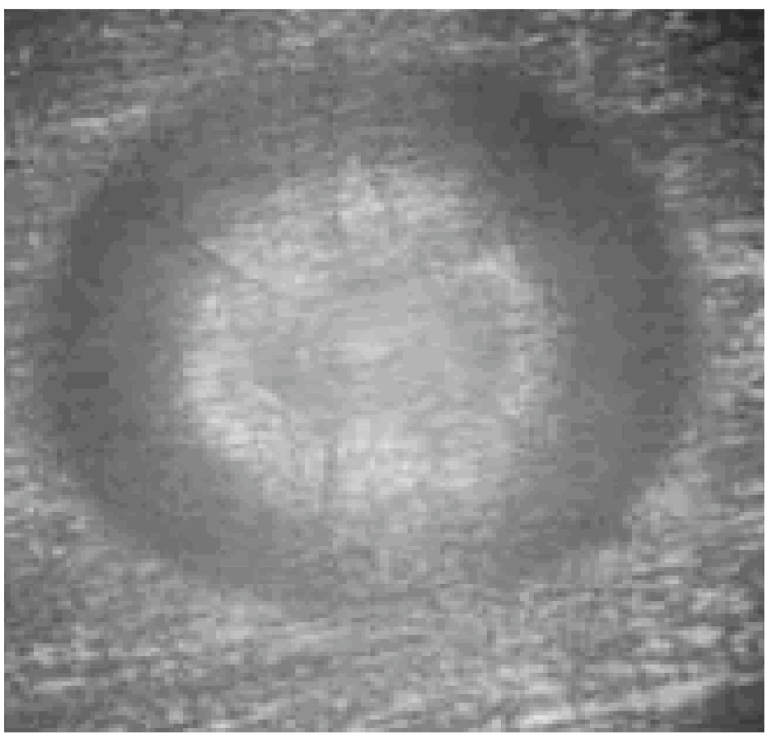


Optical micrograph of hydride blister section, grown in Zr-2.5wt.% Nb pressure tube material. Three regions - Region I - matrix & circumferential hydrides, region II - matrix containing both radial and circumferential hydrides and region III - mainly of δ-hydride.

The basis of radial hydride formation is the stress field of blister in the matrix surrounding it.

a hydride blister grown in Zr–2.5wt%Nb pressure tube alloy (Singh *et al*., 2001)

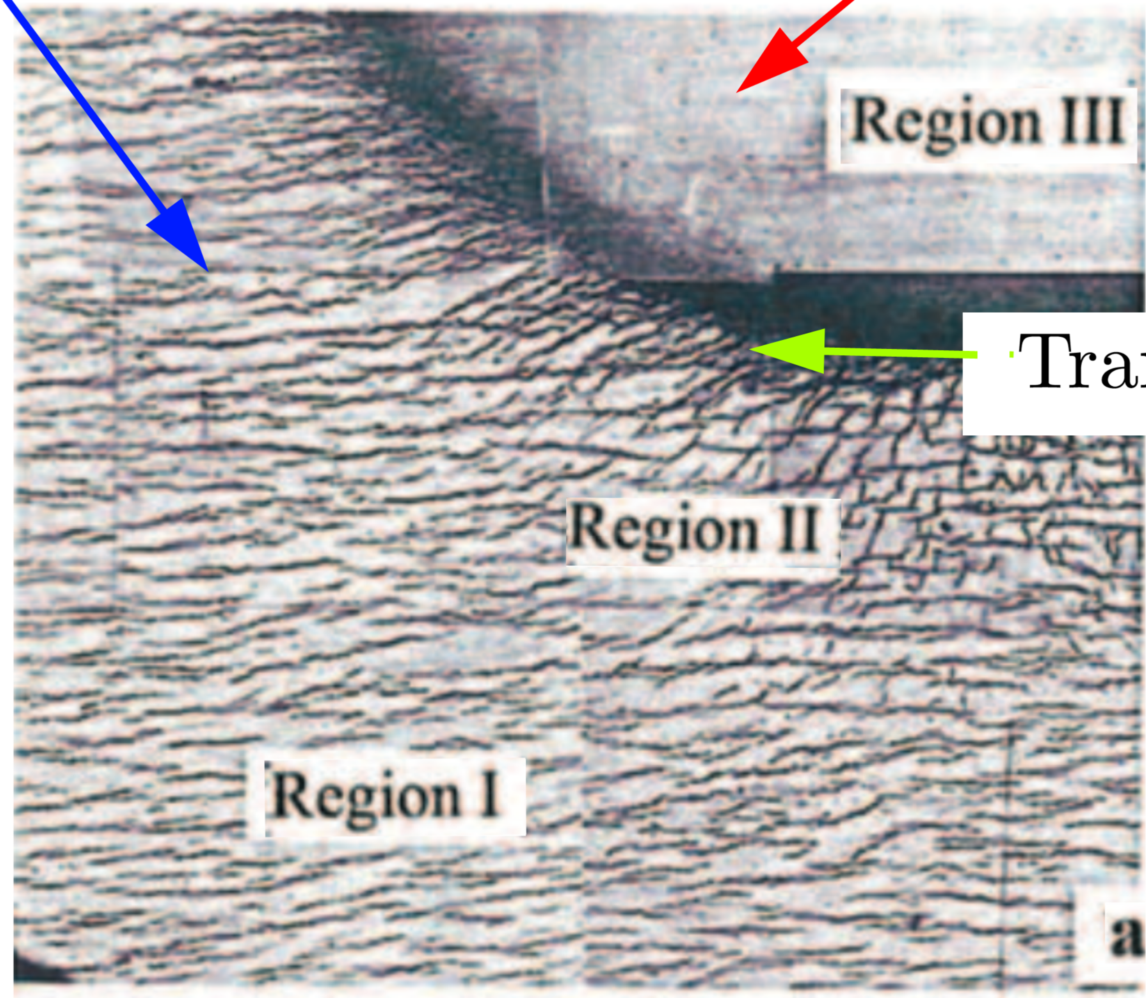




The Phase Field

$\psi = 0 \rightarrow$ Zr

$\psi = 1 \rightarrow$ Zr_nH



Transient region

$0 < \psi < 1$

Contributions to the free energy

$$\mathcal{F} = \mathcal{F}_{el} + \mathcal{F}_{ch} + \mathcal{F}_{gr}$$

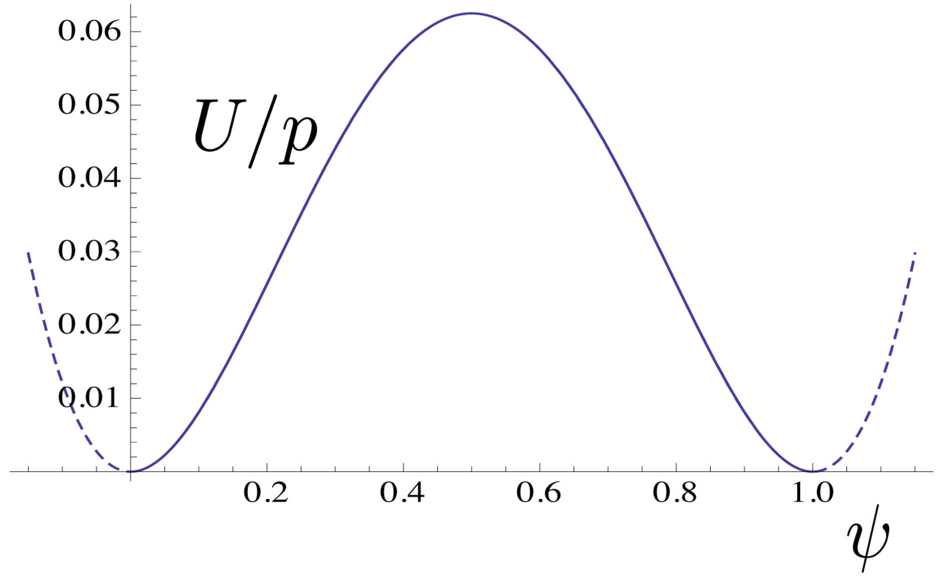
Elastic energy $$\mathcal{F}_{el} = \int \sigma_{ij} \mathrm{d}\epsilon_{ij}$$

Chemical energy $$\mathcal{F}_{ch} = U(\psi)$$

Gradient energy $$\mathcal{F}_{gr} = \frac{g_r}{2} \left(\psi_{,i}\right)^2$$

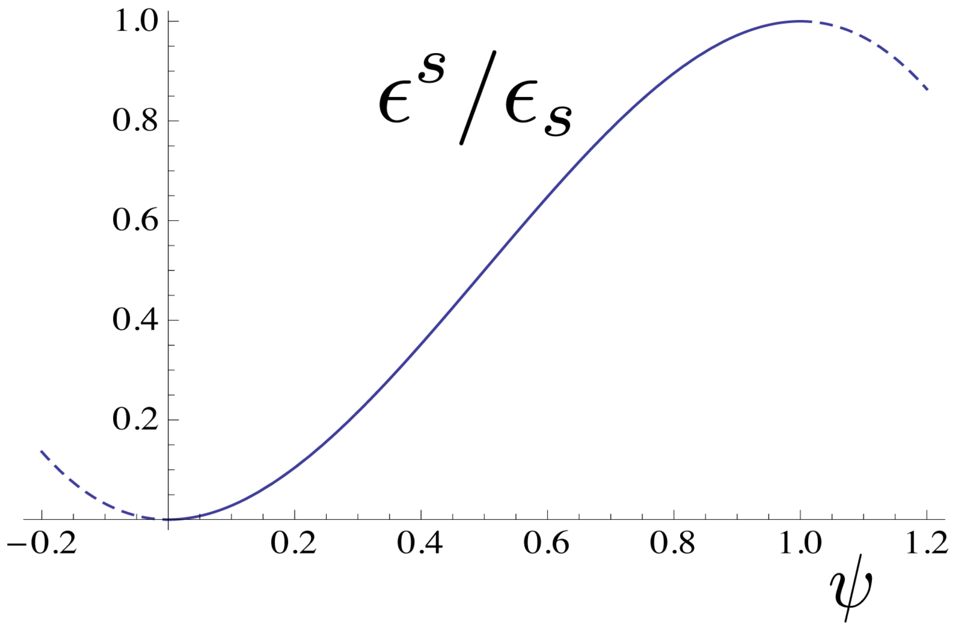
Double-well chemical potential

$$U(\psi) = p\,\psi^2(1-\psi)^2$$



Expansion

$$\epsilon^s(\psi) = \epsilon_s(3-2\psi)\psi^2$$



Plane cases. Unknown: ψ, u_1, u_2

Phase: $$\frac{\partial \psi}{\partial t} = -L_\psi \left(\frac{\partial \mathcal{F}}{\partial \psi} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla \psi)} \right)$$

Displ.: $$\frac{\partial u_i}{\partial t} = -L_{u_i} \left(\frac{\partial \mathcal{F}}{\partial u_i} - \nabla \frac{\partial \mathcal{F}}{\partial (\nabla u_i)} \right)$$

Heat transfer with heat generation

$$\psi_{,ii} - \frac{\partial \psi}{\partial \tilde{t}} = \left\{ 3\epsilon_{ii}^{el} \tilde{\epsilon}_s + 2(1 - 2\psi) \right\} (1 - \psi)\psi$$

Mechanical equilibrium with thermal expansion

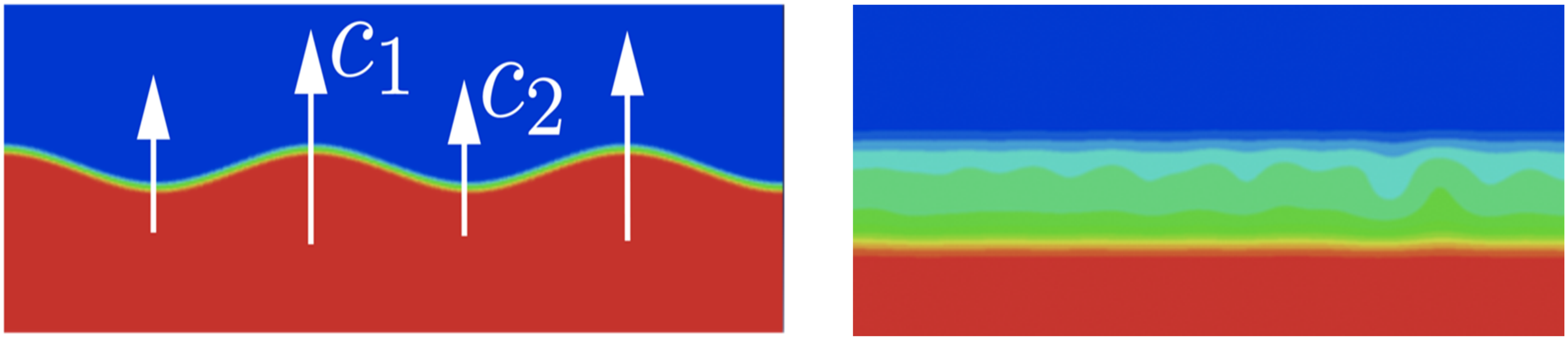
$$\tilde{u}_{i,jj} + \frac{1}{1 - 2\nu} \tilde{u}_{j,ij} - \left(\tilde{\epsilon}_s \right)_{,j} = 0$$

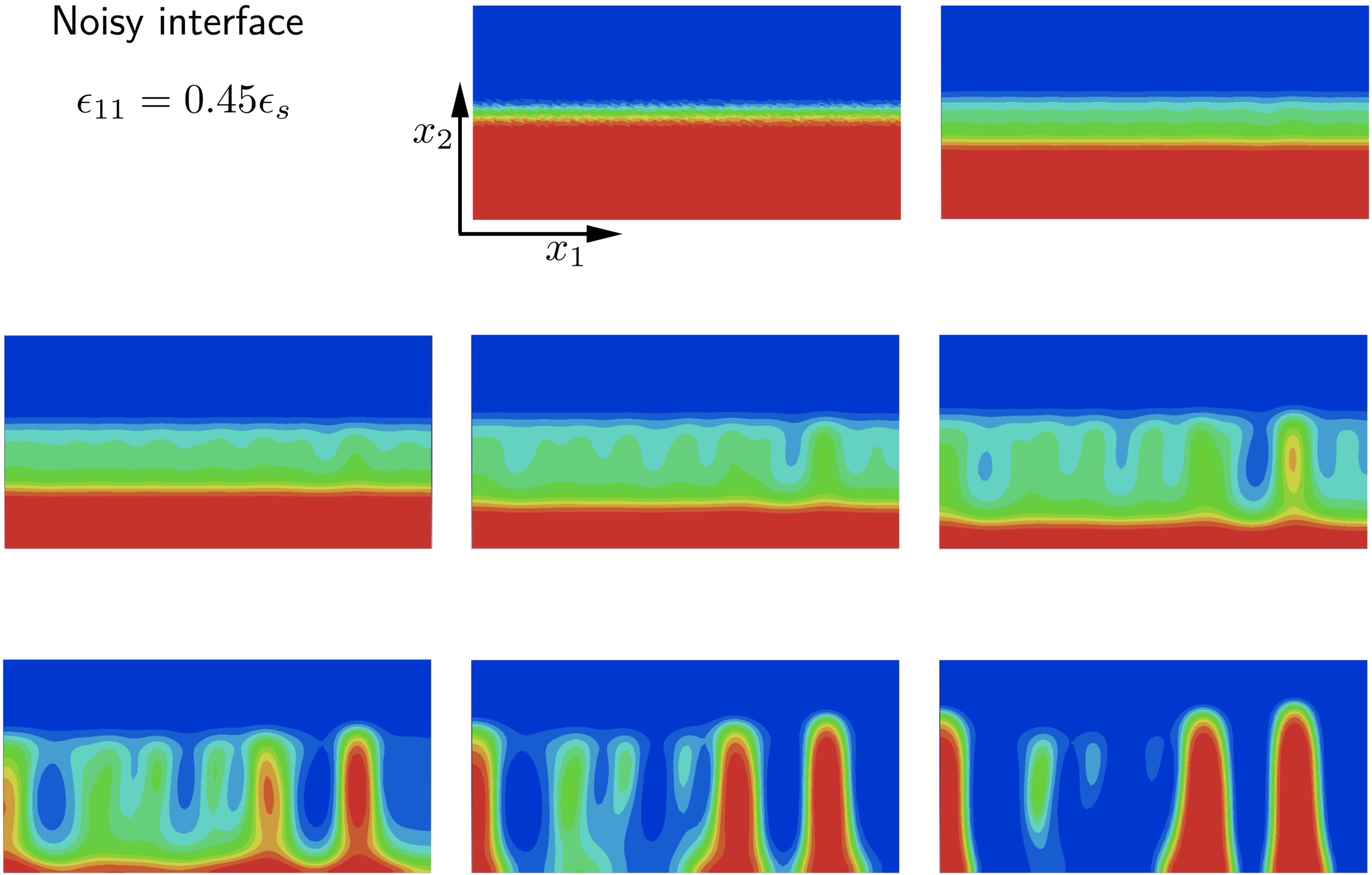
In analogy with a fully coupled thermal-stress

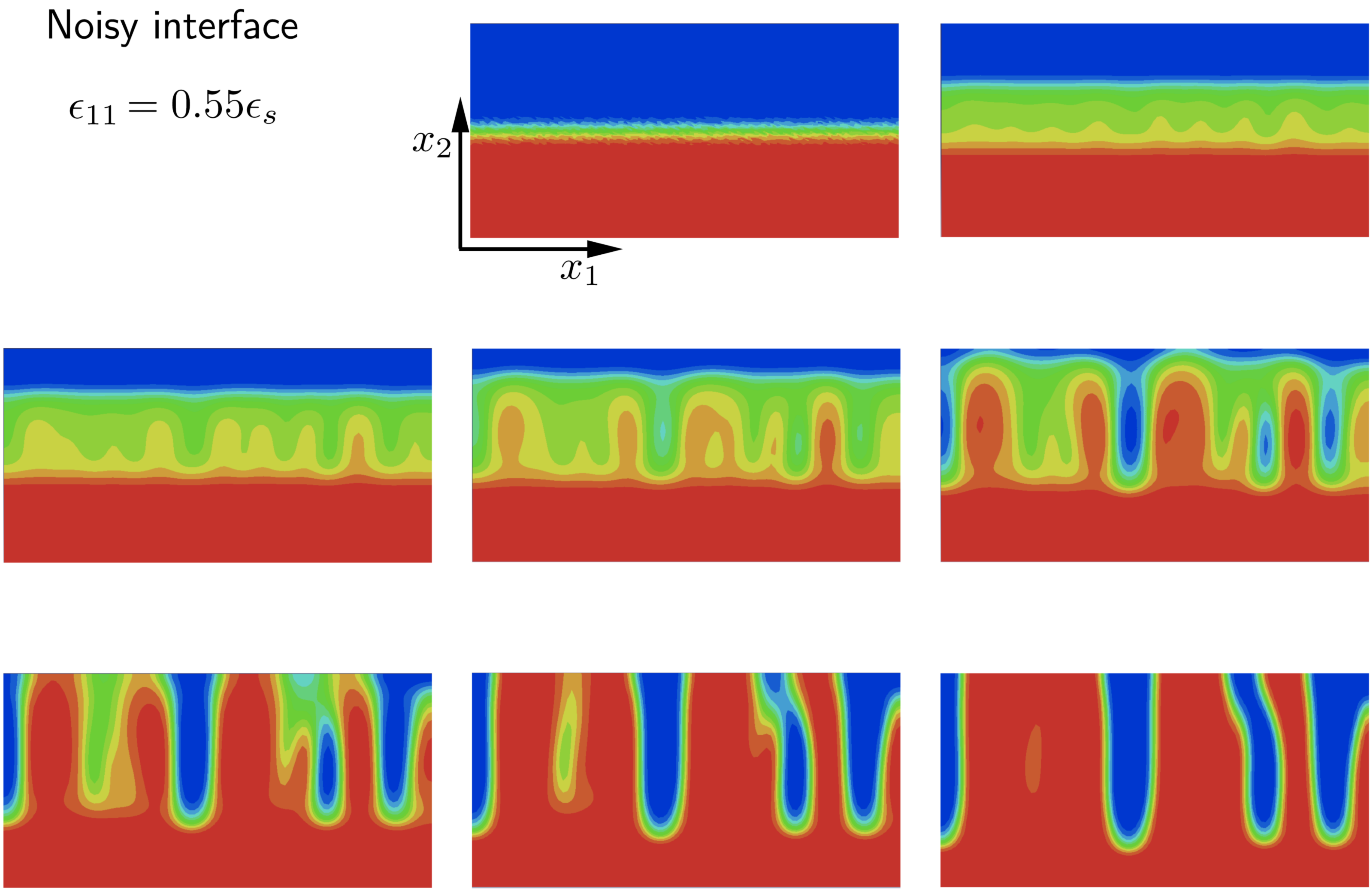
Surface energy $\gamma = \sqrt{2pg_b}$ [F/L]

Strain energy density $W = \sigma_{ii}\epsilon_s$ [F/ L²]

Length parameter γ / W [L]



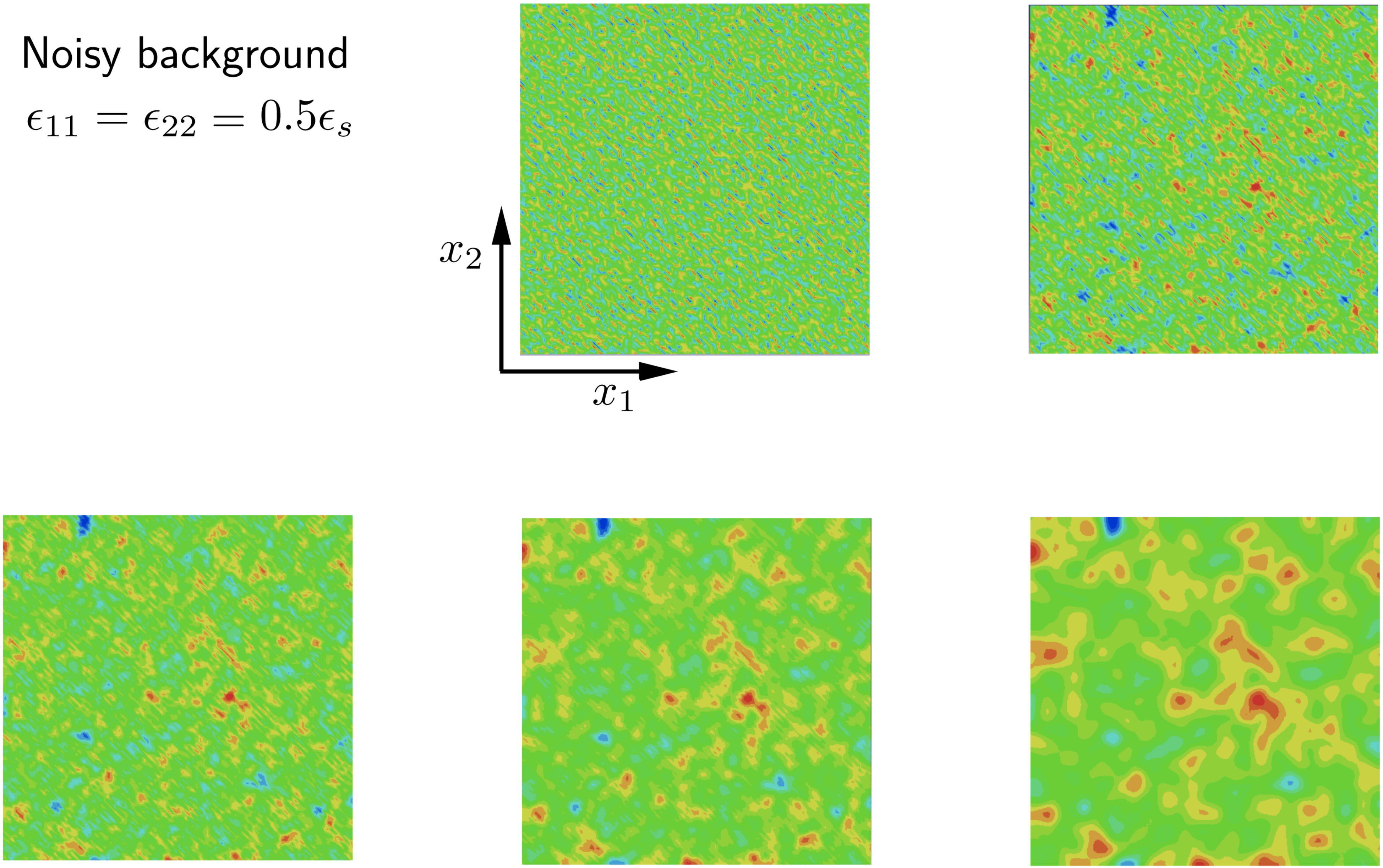

Noisy interface
$\epsilon_{11} = 0.45\epsilon_s$
x_2
x_1

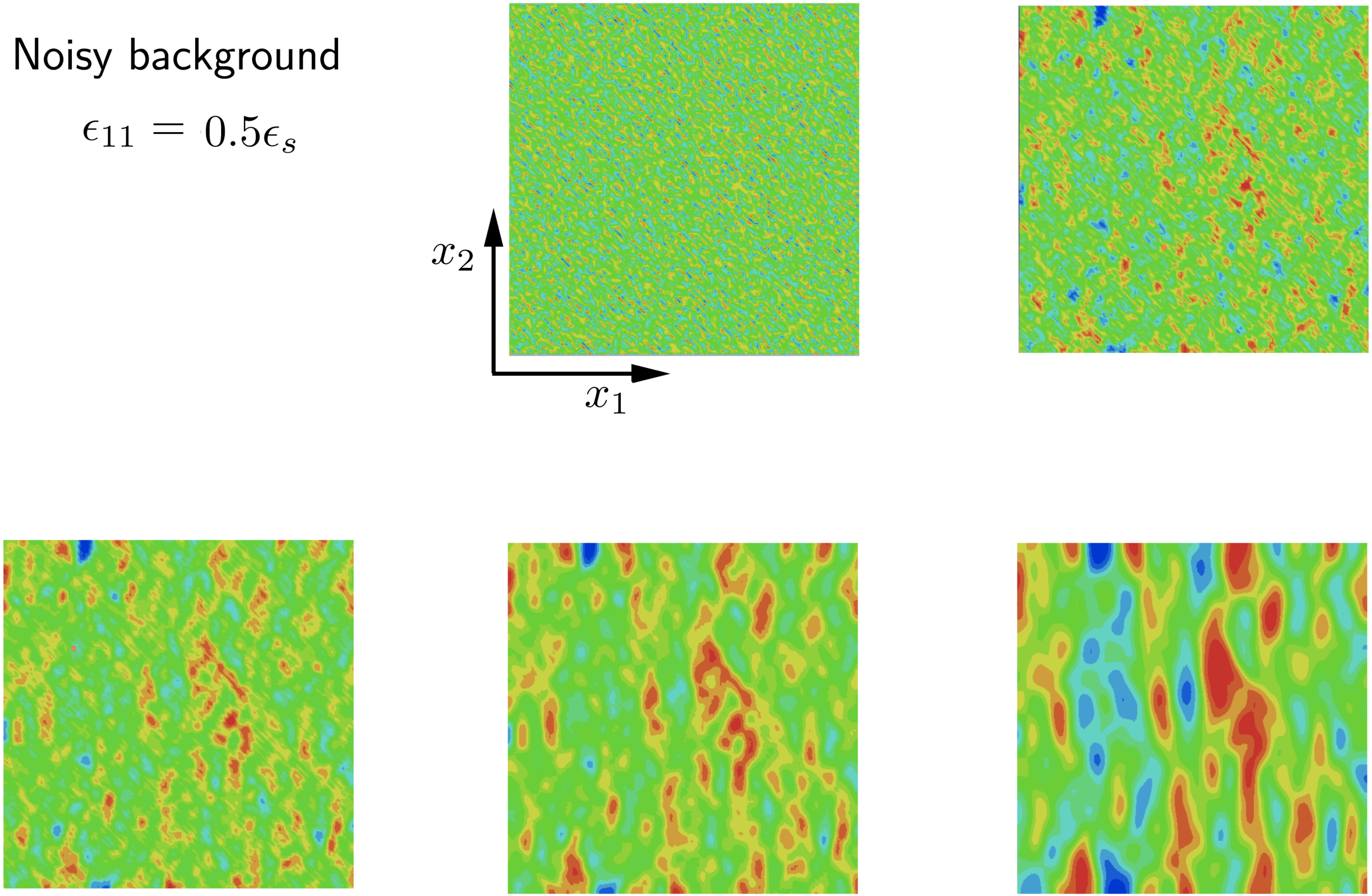


Noisy interface
$\epsilon_{11} = 0.55\epsilon_s$
x_2
x_1

Noisy background

$\epsilon_{11} = \epsilon_{22} = 0.5\epsilon_s$



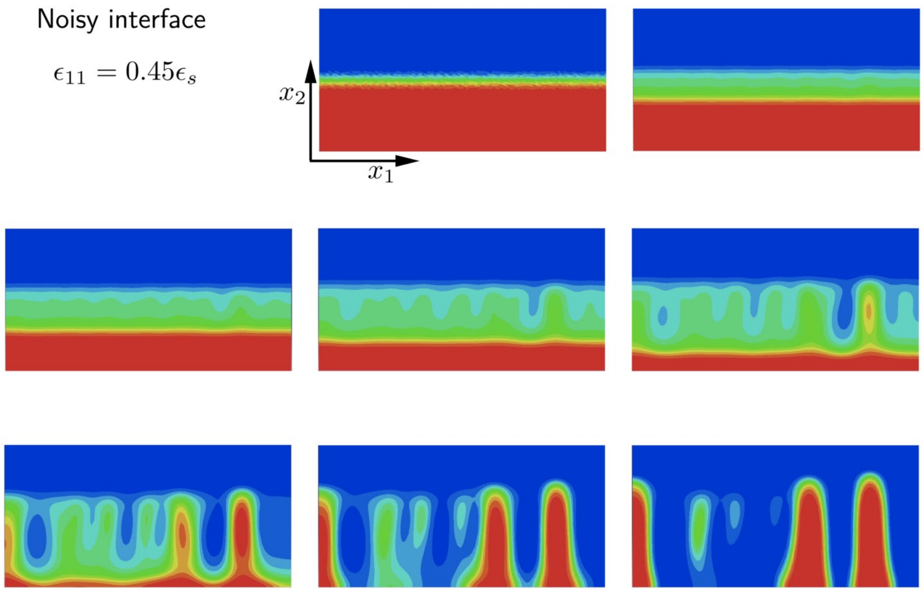

Noisy background
$\epsilon_{11} = 0.5\epsilon_s$
x_2
x_1

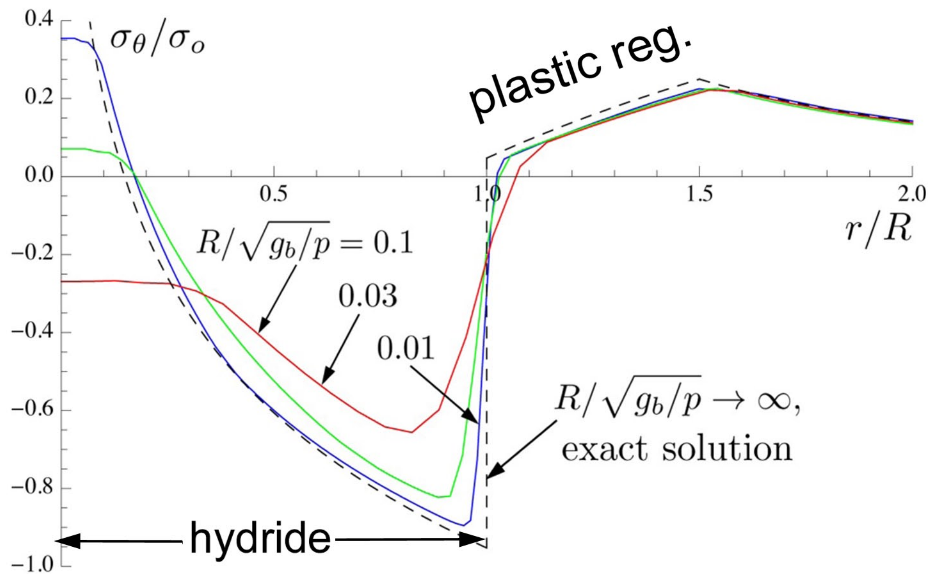
Evolution of the phase

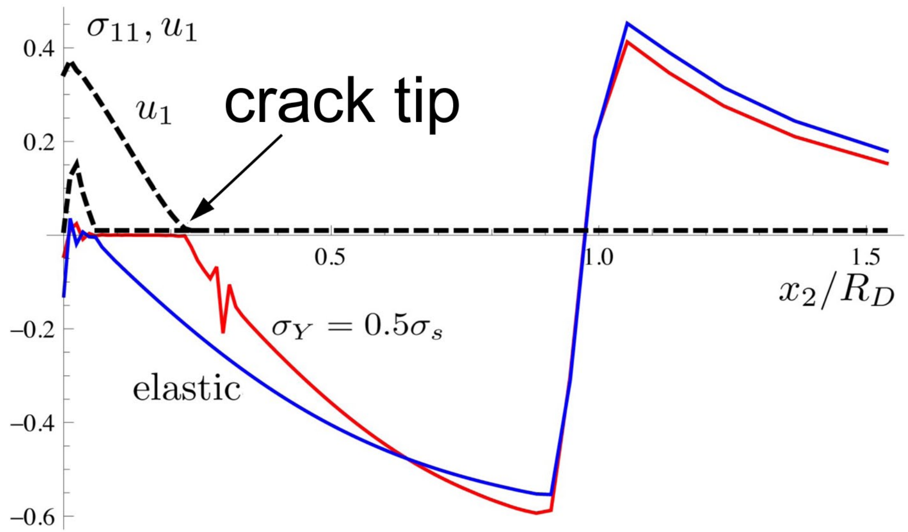
$$\frac{\partial \psi}{\partial t} = -L_\psi \left(\{3\sigma_{ii}\epsilon_s + 2p(1-2\psi)\} (1-\psi)\psi - g_b \psi_{,ii} \right)$$

Evolution of the displacements

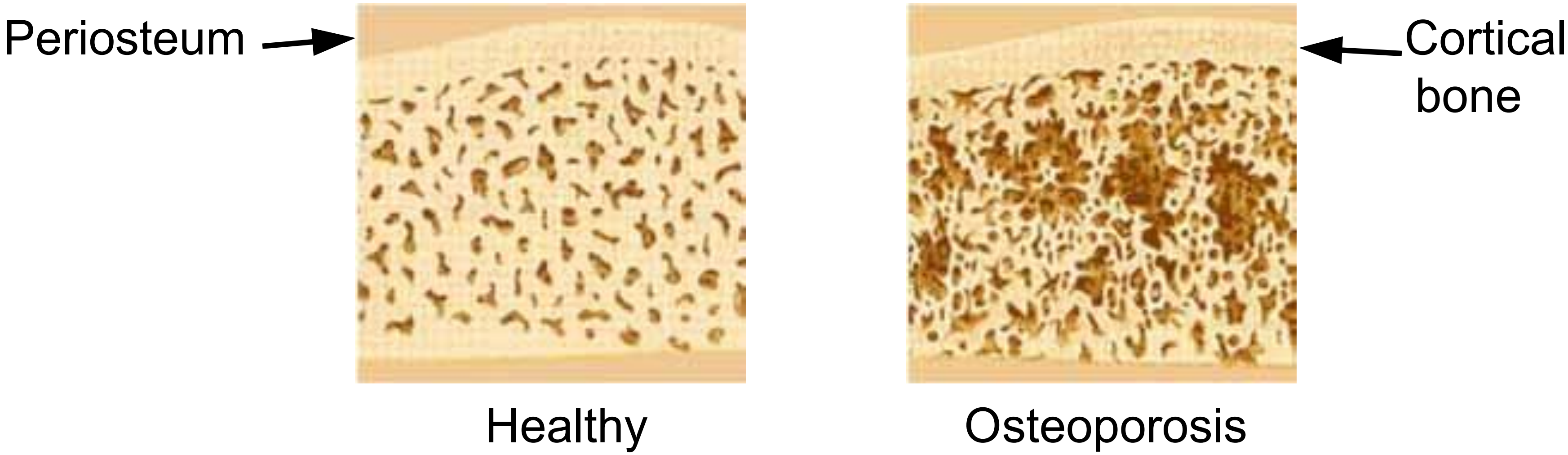
$$\frac{\partial u_i}{\partial t} = -L_u(\mu u_{i,jj} + (\mu+\lambda)u_{j,ij} - (2\mu + 3\lambda)\epsilon^s_{,i})$$







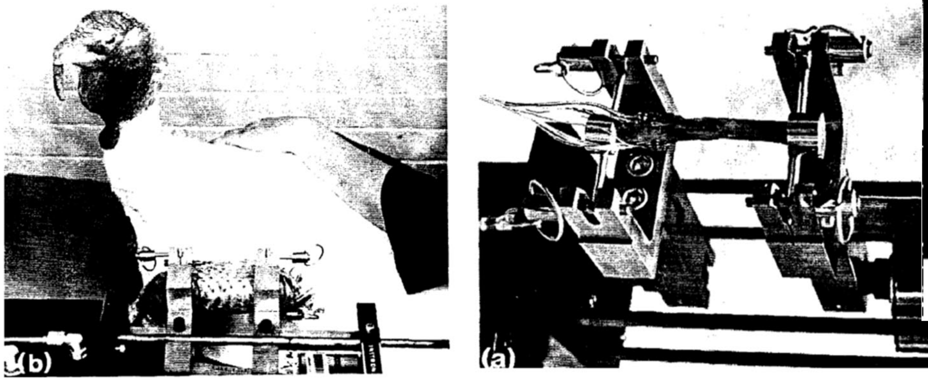
Osteoporosis



100.000 fractures related to osteoporosis each year in Sweden.

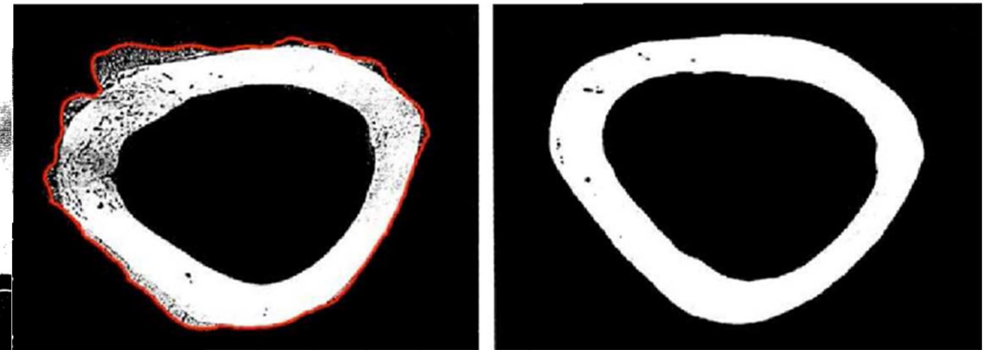
(Sweden has a population of 9 million people)

Observations of exercise stimulated bone growth



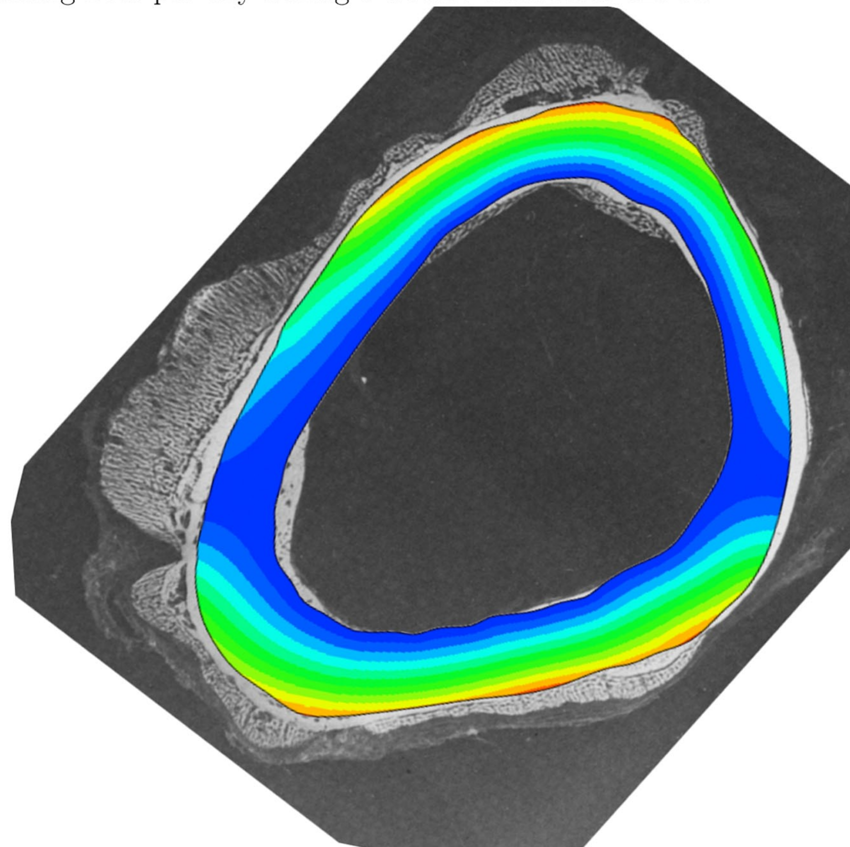
(Lanyon and Rubin, 84)

Experiments performed on the left wing, while the right wing was used as reference for each turkey.



Left wing, dynamic load *Right wing, no load*

The dynamic load was applied at a frequency of 1 Hz during 100s per day during 8 weeks. Max force 525 N.



The flow $\boldsymbol{J}$, and the concentration $\boldsymbol{c}$,

$$\boldsymbol{J} = -\boldsymbol{D}\nabla c + \boldsymbol{B}\nabla\sigma_h$$

$$\nabla\boldsymbol{J} = -\dot{c}$$

Hooke stress tensor

$$\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\delta_{ij} - \beta\delta_{ij}c$$

=> $-\nabla\boldsymbol{D}\nabla c + \nabla\boldsymbol{B}\nabla\sigma_h = -\dot{c}$

Landau phase field model

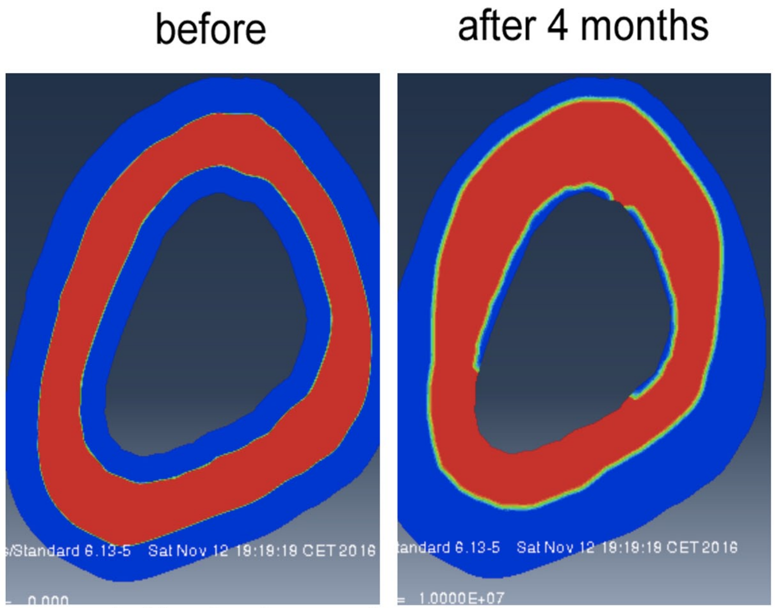
phase field variable $-1 \leq \psi \leq 1$

Young's modulus $E(\psi) = -\frac{1}{4}(\psi^3 - 3\psi - 2)E_b$

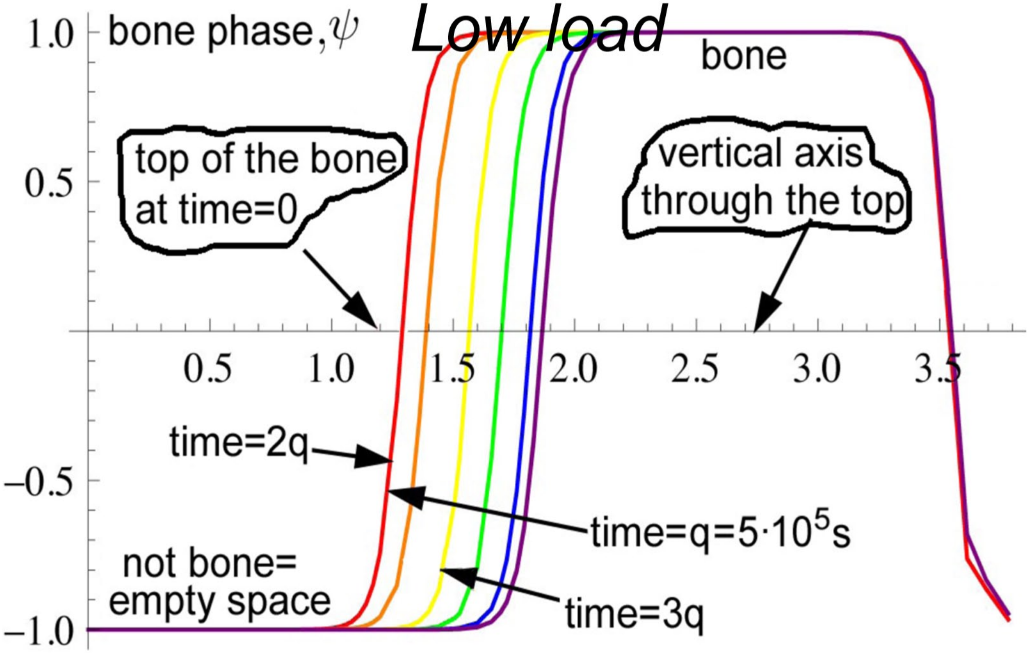
Gibbs free energy $\mathcal{F} = \int_V (F_{ch} + F_{gr} + F_{el})\mathrm{d}V$

Governing PDE

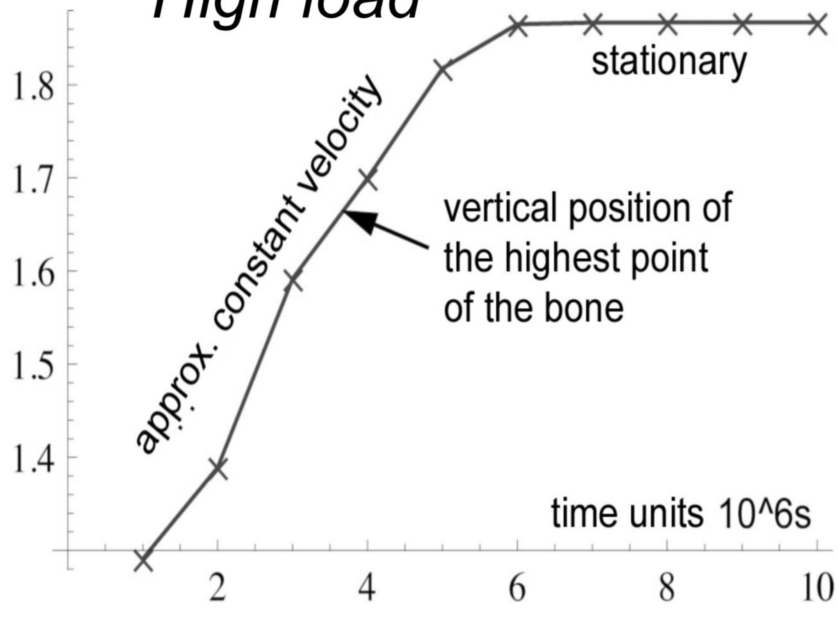
$$\frac{\partial \psi}{\partial t} = L_\psi \Big(p(1-\psi^2)\psi + g_b \psi_{,ii} + \\ + \frac{1}{8}\epsilon_{ij}\sigma_{ij}(1-\psi^2)E_b/E_o \Big)$$



Low load



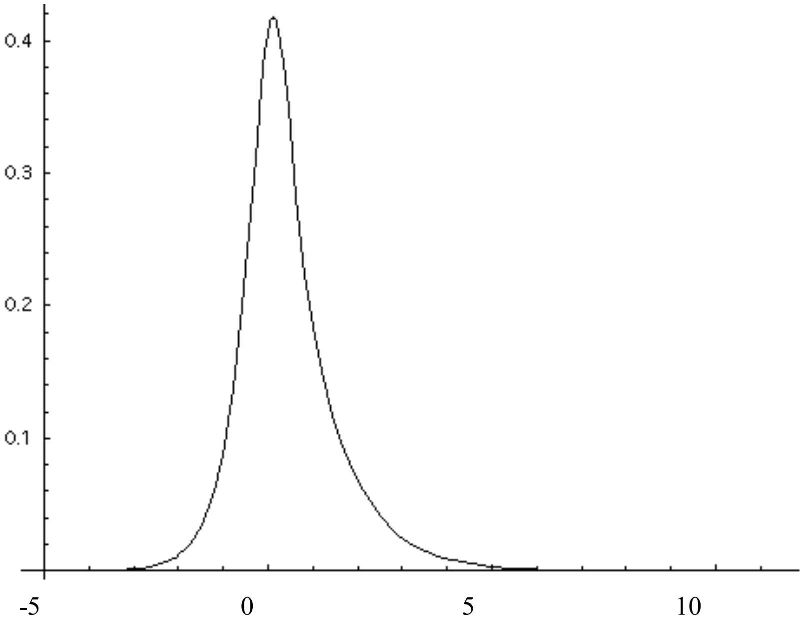
High load



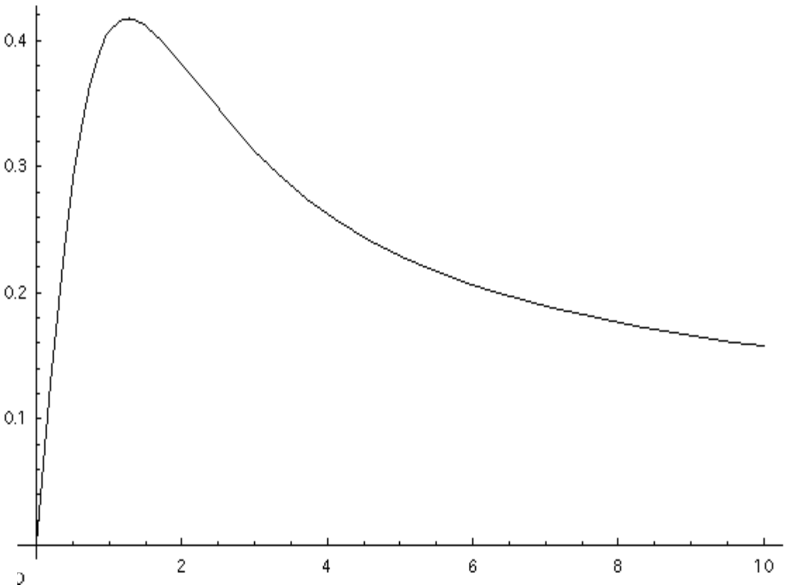
Amplitude, A, as a function of load frequency ω

$$C = C(x, t) = A\, f(x)\, \sin(\omega\, t), \quad f(L) = 1$$

$ADW/(BML)$



$\log(L^2\omega/D)$



$L^2\omega/D$

Concentration per unit of area