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LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# On the role of process regions at stationary and growing cracks

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av

Per Ståhle

Civilingenjör, Mlm



## ON THE ROLE OF PROCESS REGIONS AT STATIONARY AND GROWING CRACKS

The thesis comprises the following parts, referred to below as papers I-V.

- I. 'On the small crack fracture mechanics' International Journal of Fracture 22 (1983) 203-216.
- II. 'On the formation of necks at crack tips under plane stress conditions' Proceedings of the 3rd International Conference on Numerical Methods in fracture Mechanics, Swansea (1984) 353-362
- III. 'A one step solution technique for elastic-plastic self-similar problems' Report from the Division of Solid Mechanics, TFHF-3015, Lund institute of Technology, Lund, Sweden (1985).
- IV. 'Using elastic-plastic plane stress programs to calculate anti-plane strain' Report from the Division of Solid Mechanics, TFHF-3016, Lund institute of Technology, Lund, Sweden (1985).
- V. 'Process region characteristics and stable crack growth' Report from the Division of Solid Mechanics, TFHF-3019, Lund institute of Technology, Lund, Sweden (1985).

### Introduction

Studies of crack behaviour and fracture are generally performed using continuum analysis. Thus the process region is either not considered or assumed to be point-sized. In the latter case it is often ascribed a physical property, usually the ability to consume energy.

A major step towards the understanding of the fracture process was made as the concept of the autonomy of the near tip field was introduced by Barenblatt [1]. He also introduced a cohesive modulus as a descriptive parameter for material behaviour in the crack tip vicinity. This implied again that the process region was small in some sense. In an elastic surrounding the smallness of the process region must be related to some significant geometrical length, usually the crack length. This introduces the question about what happens in the case of small cracks. Paper I tries to answer

question.

For an elastic-plastic material it is logical to compare the size of process region with the size of the plastic region. One then finds that process region often is big enough that its extension must be considered. such case is necking in thin plates. In this case the process region - necking region - is determinable by continuum mechanics, i.e. the micro-structure does not play the same role as it does for instance at plane strain. The necking phenomena near crack tips is studied in paper II.

At plane strain (in the crack tip vicinity) the decohesion cannot be described by means of continuum mechanics. Micro-structural processes, for instance formation and opening of voids or micro-cracks play an important role. However, simple models of the process region can be designed for use in analysis. The model used here is the Barenblatt line model, but its length is not regarded as small compared to all other significant length dimensions. The discontinuous behaviour is modelled by a relationship between boundary stress and displacement and not between stress and strain as for a continuum.

In order to perform some investigations for the early stages of development of the process region a method was devised by which the load history in finite element computation could be applied in one single step instead of several steps as is normally required. This technique is described in paper III.

Since most elastic-plastic mode I crack problems cannot be solved analytically there could be some doubt about the finite element (FEM) method being used. In paper IV a method is described in which a FEM code developed for in-plane stresses can be modified for solution of anti-plane strain problems. For anti-plane strain there are several analytical solutions available to compare with the problems (mode III problems). This gives an opportunity of indirect testing of the in-plane FEM code. As a by-product paper IV envisages methods for modifying the FEM code for other problems, for instance heat conduction.

Paper V deals with mode I, plane strain. The early stages of development of the process region, its development to instability and steady-state case are investigated. Both large and small cracks are considered.

### Necking in thin plates

Necking is the dominant process at fracture of thin plates. This continuous cross-sectional slip and needs a three-dimensional analysis fully understood. Dugdale [2], however presented a solution for a crack developing necks by using a two-dimensional approach. He assumed that plasticity was confined to a narrow line shaped region the crack tip.

Drucker and Rice [3] later argued that the Dugdale model is only at plane stress and the Tresca flow rule. At plane strain the effective stress exceeds the yield stress in regions off-side the necking region both Tresca and von Mises materials. At plane stress the effective stress is less than the yield stress for both materials in the whole plane outside the necking regions. However, since the plastic flow rule implies that the plastic increments should be normal to the flow surface a Dugdale region in a von Mises material is impossible, at least at uni-axial remote stress. It is possible for the author to show that the Dugdale solution is not consistent with the Tresca yield criterion for very small cracks (unplastic work), since the effective stress is exceeded in regions near the center of the crack. A natural development is therefore to consider the plasticity in the necking region. Numerical solutions for von Mises materials reveal that a diffuse plastic zone develops ahead of the crack tip, see e.g. Swedlow [4]. They, however, could not consider the possibility of necking because of their two-dimensional analysis.

In paper II, a finite element analysis, the developing necking region was modelled by means of a slowly progressing node release. Both Tresca and von Mises materials were investigated. It was possible to show that the conditions for necking are fulfilled for both Tresca and von Mises materials. For both materials the necking region was generally embedded in a plastic region. For a certain ratio of hardening modulus versus yield stress the Tresca material plasticity was absent outside the necking region. In some cases a diffuse plastic region appeared together with a necking region whereas for the non-hardening von Mises material, a diffuse plastic region resulted together with a vanishing necking region. The results compare well with those obtained by Hutchinson [5], who did not consider the possibility of necking.

### The role of the process region in an elastic surrounding

For a pilot study of the role of the process region (paper I) linear elasticity was assumed throughout together with a line shaped process region. Somewhat different models of the cohesive properties of this region were used with rather small differences in results. On the other hand the results obtained with these reasonably realistic process region models (i.e. finite cohesive strength) turned out to be very different from results obtained by assuming a point-sized process region, especially for small cracks. Even for cracks with the minimum length allowed by the ASTM-convention for linear fracture mechanics the difference was substantial - about 5 per cent as regards the critical load. Fig. 1 shows the relative difference in critical load as predicted by large crack fracture mechanics (the so called linear fracture mechanics) and as found by use of one of the process region models in the study.

The procedure used allows treatment of arbitrarily small cracks, even in the infinitesimal limit.

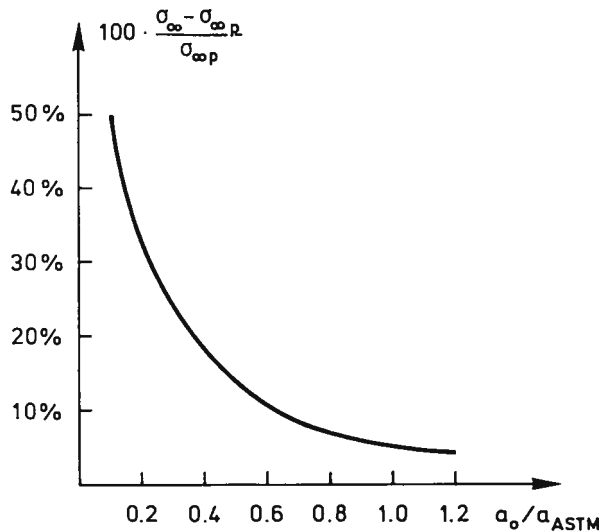


Figure 1. The relative difference in maximum remote stress  $\sigma_\infty$  as predicted by large crack fracture mechanics and a process region model.  $a_{ASTM}$  is minimum crack half-length allowed by the ASTM-convention.

### FEM-treatment of self-similar elastic-plastic problems

In elastic-plastic cases at plane strain finite element computation generally very laborious because the load has to be increased in increments. This also has a negative influence on the numerical accuracy. However, in a few cases the load can be applied in one single step. These cases are the steady-state and cases of self-similarity. The early stages and the development of the process region are characterized by self-similarity. This implies that the same solution holds, apart from a length scale factor, for all different stages. Of course, at the development of the process region this is only approximately true and only in a sufficiently close neighborhood of the crack tip.

A characteristic feature of self-similarity is that the whole history of development is embedded at each instant in the current stress-strain history. This fact is taken advantage of in paper III which contains the description of a method whereby the load can be applied in one single step. The method is used in paper V.

### On the control of the accuracy of the FEM code

Very few analytical solutions exist for mode I and II crack problems, and only a few such solutions are available for mode III. Therefore control of the accuracy of the FEM code is difficult. In existing FEM codes for elastic-plastic problems specializations to in-plane stress and strain are common but seldom to the anti-plane strain case. However, a way was detected to use the FEM code for plane stress in anti-plane strain computations without any changes in the program, but simply by making certain specializations, translations and interpretations of variables and parameters. The procedure is described in paper IV. An indirect control of the FEM-code used was made by treating the Hult-McClintock [6] problem. The accuracy turned out to be very good. Displacements on the crack surfaces were found to coincide with the analytical solution with a maximum error of 0.2%.

As a by-product a way was found to adopt a plane stress FEM code for the solutions of problems in general which are governed by the Poisson equation. As an illustration a problem of stationary heat conduction was treated. The maximum error of less than 0.2% in temperatures along a certain line was resulted.

## The role of the process region in an elastic-plastic surrounding

Paper V deals with the development of the process region, stable growth and onset of global instability for both long and short cracks. Investigation is carried out by means of finite element methods.

Four different types of materials were studied. After some investigation the influence of the strain hardening turned out to be fairly small in a range of practical interest. Therefore - and also due to some circumstances - the number of material types was reduced to two, different as regards the cohesive strength of the process region.

For one material the cohesive strength was put to  $\sigma_D = 3\sigma_Y$  where  $\sigma_Y$  is the yield stress. This corresponds approximately to the maximum possible cohesive strength in the plastic region when strain hardening is very small. For the other material the cohesive strength  $\sigma_D = 2.5\sigma_Y$  was chosen.

The difference in cohesive strengths may appear to be small, but it turned out to imply quite remarkable differences on the shape of the plastic zone, on the process region size, on the amount of stable crack growth at the instability (fracture) load. The plastic region size was comparatively smaller for the material with the lower cohesive strength. The process region was larger than the forward extension of the plastic zone for this material but it was not so pronounced for the material with the higher cohesive strength. Figs. 2-3 show the plastic and process regions for the two materials.

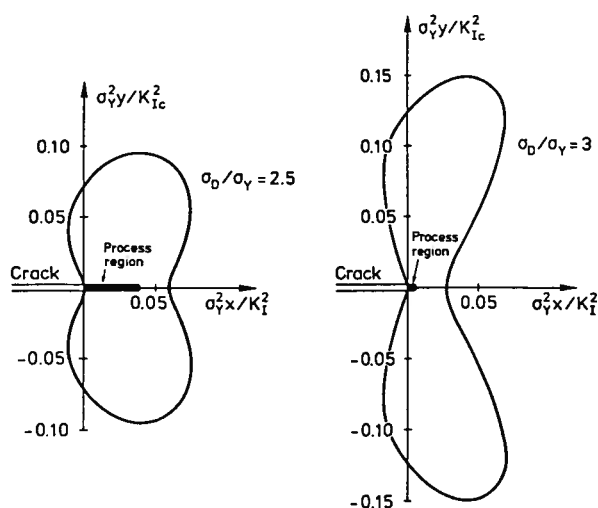


Figure 2. Plastic and process regions at the early stages of the development of the process region.

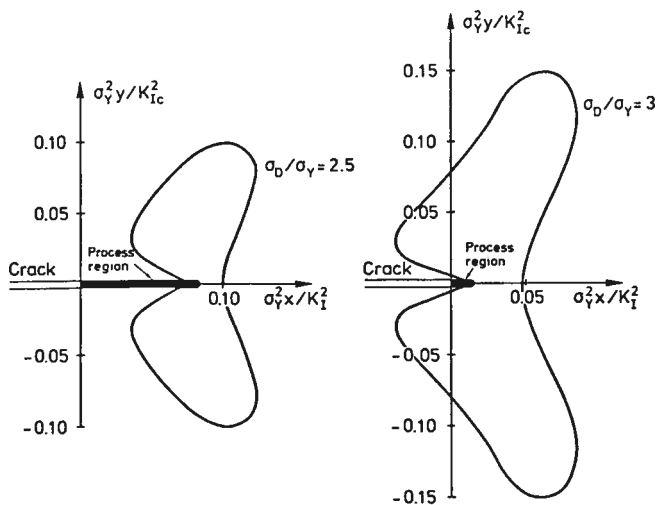


Figure 3. Plastic and process regions at steady-state.

at the early stages of the development of the process region and at state conditions.

The energy consumed in the process region per unit of crack growth is remarkably large. Thus it amounts to about 80% of the total dissipated for the material with the lower cohesive strength and to more than half the energy for the material with the higher cohesive strength. The large extension of the process region and its dominating role in the dissipation implies that analyses of stable crack growth carried out under the assumption of a vanishing small process region should be regarded with caution. The existence of a HRR-singularity at stationary cracks and the existence of a  $\ln(r)$ -singularity at moving cracks should therefore not be taken for granted. This has been pointed out earlier by Broberg [7].

The influence of the cohesive strength on the fracture toughness seems to be more pronounced than a proportional one. The progress of crack growth seems to be due to the influence of the process region characteristics of the plastic region. Thus, see Fig. 3, low cohesive strength seems to cause 'embrittlement', since obviously the development of the plastic region is impeded.

Small cracks are studied as well as large cracks. The amount of crack growth decreases, as expected, with decreasing length of the crack. Probably, for very small cracks the stable phase disappears completely as found in the pilot study, paper I. However, cracks small enough to induce such a result were not studied.

### Acknowledgements

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