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Abstract

Crack growth under steady state conditions is analysed by means of the finite element method. The calculations are performed in order to form a basis for a later study of experimental observations of crack growth rates at intergranular stress corrosion cracking, IGSCC, at a larger scale of yielding and different transient loads.

An attempt is made to single out the near tip behaviour. For comparison a study of steady state growth at a larger scale of yielding is performed as well.

In both cases the same near tip behaviour was found.

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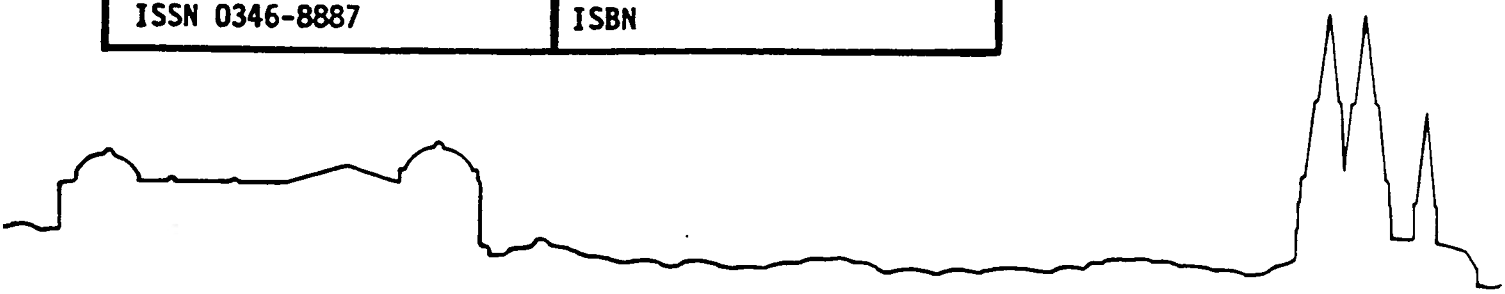

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A comparative study of two steady-state situations at a quasi-statically growing crack in a stainless steel.

This report is the first part of a finite element (FEM) investigation of CERT-testing and fracture mechanical for corrosion-fatigue and IGSCC conditions.

The investigation aims at an analysis of crack growth considering the geometry of the specimen and the specific distribution of applied loads. The analysis should also reveal the details of the stress and strain distribution in the vicinity of the crack tip. This, however, can not be done in one single computation even with very effective computors, or at least not at a reasonable computor cost.

A quite recently reported crack tip analysis [1] for a related situation including only the crack tip surrounding shows a comparatively course picture of the immediate crack tip vicinity even though super computor facilities have been taken advantage of.

Due to this the existance of an asymptotic solution is assured in a very detailed and rather time consuming numerical analysis. The asymptotic solution is compared with analytical solutions [2] and [3]. In [2] a stress function is approximated with a Taylor expansion. The possibility of a secondary plastic zone in the wake, trailing the plastic zone at the crack tip is not considered.

In the present study calculations have been performed for two different remotely applied stress distributions. The same asymptotical behaviour for the crack surface displacement has been extracted from the respective near tip solutions. The result of the present analysis is planned to be matched to solutions for global problems for the different test specimens.

Model

A plane body is assumed to have a straight through-crack. The modulus of elasticity E=180 000 MPa and poisson's ratio ν=0.3. Plastic flow is assumed to be governed by von Mises yield criterion and its associated flow rate. The material is assumed to be linearly hardening with respect to the effective plastic strain ε_e^p. The hardening rate - measured at a tensil test - $H = \frac{d\sigma_e}{d\varepsilon_e^p} = 1800$ MPa (see fig. 1).

The crack is assumed to have extended under stationary plane strain conditions in the crack tip surrounding.

The remotely applied load is a uniaxial stress field directed normal to the crack plane. The influence of inertia is neglected.

A coordinate system is introduced with x=0 and y=0 at the right crack tip. The crack occupies the region $-2a<x<0$, $y=0$.

Invoking the assumptions of small scale yielding and due to the symmetry of the problem only the upper half of a surrounding of the crack tip at x=0, y=0, say $-C<x<C$, $0<y<C$ need to be analysed. Here C is considered to be much larger than the linear extent of the plastic zone.

A polar coordinate system is introduced with $r=(x^2+y^2)^{1/2}$ and $\theta=\tan^{-1}(y/x)$. The boundary conditions for the small scale yielding case (case 1) are

$$\sigma_x = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]$$

on $x=c,\ 0<y<c$

$$\tau_{xy} = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]$$

on $-c<x<c,\ y=c$

$$\tau_{xy} = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

$$\frac{\partial u}{\partial x} = \frac{\partial u_e}{\partial x}$$

on $x=-c$, $0<y<c$

$$\frac{\partial v}{\partial x} = \frac{\partial v_e}{\partial x}$$

u_e and v_e are displacements given by a linearly elastic solution for a remote stress field of the form $\sigma_{ij}^{K_I} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$ where f_{ij} are known functions.

For $y>y_p$, where y_p is the largest extension in the y direction of the plastic zone the last condition reduces to

$$\sigma_x = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]$$

on $x=-c,\ y_p<y<c$

$$\tau_{xy} = \frac{k_I}{\sqrt{2\pi r}} \left[\cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

Further one concludes for symmetry reasons

$$v = 0$$

om $0<x<c,\ y=0$

$$\tau_{xy} = 0$$

and finally the traction free crack surface requires

$$\sigma_y = 0, \quad \tau_{xy} = 0 \qquad \text{on } -c<x<0,\ y=0$$

For case 2 the centre of the small scale yielding field was transfered to a point $x=(1/10)(k_I/\sigma_Y)^2=C/10$, $y=0$ straight ahead of the crack tip. Thus $r=(x^2+y^2)^{1/2}$ is replaced with $r^*=[(x-c/10)^2+y^2]^{1/2}$ and $\theta=\tan^{-1}(y/x)$ with $\theta^*=\tan^{-1}[y/(x-c/10)]$ in the above boundary conditions.

The region $-c<x<c$, $0<y<c$ is covered by 882 8-node isoparametric elements with 2 by 2 integration points (see fig. 2). The size of the elements at the crack tip is c/500.

The code [4] is used for the calculations. The procedure starts at the linearly elastic solution. Plastic strains manifest themselves through unequilibrated nodal forces Ψ_i. These are used to correct the displacement field. After repeated calculations of Ψ_i and subsequent corrections the displacement field is finally brought to equilibrium. The full details of the algorithm are described in [5] and modifications due to the present problem in [4].

Calculations and Results

A total of about 20 CPU-hours was used for each case on a Vax 11/780. Solutions were found for decreasing values of H, starting with a very large value, typically 10E. When a stable solution was found H was decreased about 10%. This was repeated until a solution for H=E/100 was obtained. In all 31 steps were needed.

In order to develop an asymptotic solution within the limitations of the finite element approximation, loads could not be less than $K_I=1.25\sigma_Y C^{1/2}$. At larger loads the stress distribution in the remote parts of the plastic zone deviated substantially from the small scale yielding solution (for case 1) and thus the load could not be increased any further.

Case 1

The shape of the plastic zone for case 1 extended to about 1/4 of the linear extension of the mesh C (see fig. 3). Size and shape are in accordance with the result of [6] who investigated a perfectly - plastic material. [6] also reports very similar shapes and sizes for fairly low hardening materials.

The largest extention y_p in the y-direction of the plastic zone was found to be $0.158\ (K_I/\sigma_Y)^2$. The largest extension in the x-direction was $0.119\ (K_I/\sigma_Y)^2$. This should be compared with the extension straight ahead of the crack tip $x_p=0.063\ (K_I/\sigma_Y)^2$ i.e. the plastic zone is comparatively thin in the plane y=0.

A secondary plastic zone was developed in the wake adjacent to the crack surface. This zone is observed to increase in width and assumes its full width $y_S=0.0098\ (K_I/\sigma_Y)^2$ a distance about $0.030\ (K_I/\sigma_Y)^2$ behind the crack tip. The full width is covered by 4 elements. Plastic strains are observed to increase monotonically in the secondary plastic zone. This indicates that the width of this zone might be continuously increasing even though it was not revealed here.

Obviously an asymptotic solution should be found within the region where the regions of different material behaviour divide the near tip region in angular sectors. In the present study three distinctly different kinds of sectors are discovered e.g. active plastic (P), linearly elastic wake (W) and secondary plastic (S) sectors. The boundary between sectors (P) and (W) is found at $\theta_p=127^o$ and between sectors (W) and (S) is found at $\theta_s=173^o$. This can be compared with $\theta_p=123^o$ and $\theta_S=160^o$ found in [3]. [2] reports a larger value for the boundary between (P) and (W) sectors $\theta_p=154^o$. θ_S is not calculated at all in [2].

Stresses at $r=0.0035\ (K_I/\sigma_Y)^2$ is shown in fig. 4. The stress distribution corresponds well to [3]. The deviation from [2] is substantial for $\theta>90^o$. This is not all unexpected because of the neglected secondary plastic zone in [2].

The crack surface displacement v is shown in fig. 5. A solution on the form $v_a = kr^{\alpha}$ approximating v in the region $r < 0.017\ (K_I/\sigma_Y)^2$ - covering 12 nodes - was sought in the sence of least square method. $x = \sum_{i=1}^{12} (v^i - v_a^i)^2$, where v^i is the displacement at node i, found its minimum for $\alpha = 0.69$ and $k = 2.24\ \sigma_Y^{0.38} K_I^{0.62}/E$. The largest error $(v-v_a)/v$ is then 0.49% and occurs at $r = 0.0098\ (K_I/\sigma_Y)^2$. If a largest error of 3% is allowed the range of validity is extended to about $r = 0.0325\ (K_I/\sigma_Y)^2$.

Case 2

Fig. 3 shows the plastic zone for case 2. The shape is observed to be quite different from case 1. The largest extent in the y-direction $y_p = 0,103\ (K_I/\sigma_Y)^2$ i.e. only 65% of y_p for case 1. The largest extension in the x-direction is $0.054\ (K_I/\sigma_Y)^2$ which is 45% of the corresponding value for case 1. The width x_p is decreased to 45% of x_p for case 1 i.e. x_p is here $0,029\ (K_I/\sigma_Y)^2$. The width of the secondary plastic zone reaches a maximum value $y_S = 0.0079\ (K_I/\sigma_Y)^2$ making 81% of the case 1 value.

As opposed to the differences in the exterior of the plastic zone the stress distribution in the crack tip vicinity coincides almost exactly with the case 1 result.

The crack surface displacement coincides well with the result for case 1. The largest error for an approximation $v_a = 1.92\ \sigma_Y^{0,38}\ K_I^{0,62}\ r^{0,69}/E$ is 2.8% in the region $r < 0.017\ (K_I/\sigma_Y)^2$. With a largest allowable error of 3% v_a approximates v for $r < 0.018\ (K_I/\sigma_Y)^2$.

The conclusion is that the asymptotic field $v_a = k\ r^{0.69}$ is a good approximation for the crack surface displacements in the crack tip vicinity. It is also concluded that the stress distribution for the asymptotic field is given by fig. 3.

The ASTM-limit for linear fracture mechanics is chosen for examination in order to elucidate the result. This limit specifies:

$(K_I/\sigma_Y)^2 = a/2.5$

For a crack length 2a of say 20 mm the hight of the plastic zone y_p=0.63 mm and the width in the crack plane x_p=0.25 mm. The range for which the approximation $v_a=0.00305\ r^{0.69}$ is valid with a largest allowable error of 3% is r<0.13 mm. For the investigated material E=180000 MPa and σ_Y=160 MPa. Thus the crack opening 2v at r=0.13 mm is estimated to be 2v=15 µm.

At unloading from a value of the loading expressed by K_I to another level $K_I-\Delta K_I$ the material will initially respond elastically. The displacement of the crack surfaces can thus be written

$$\Delta v = -\frac{2(1-v^2)}{\pi E}\ \Delta K_I\ (2\pi r)^{0,5} = -\frac{1,452}{E}\Delta K_I\ r^{0,5}$$

The resulting displacement is then the sum of the elastic-plastic steady-state part and the elastic unloading part.

$$v^{res} = \frac{1}{E}(2,24\ \sigma_Y^{0,38}\ K_I^{0,62}\ r^{0,19} - 1,452\Delta K_I)r^{0,5}$$

Since the first term within the parenthesis tends to zero as r → 0 the resulting expression is negative for small r-values. Thus, crack-closure occurs for any finite unloading. The contact zone may however be very small so that the practical significance of this is yet unclear. An estimate of the length of the closure zone is simply obtained by setting the parenthesis to zero. This yields the following result for r_s.

$$r_s = \left(\frac{0,6482\ \Delta K_I}{\sigma_Y^{0,38} K_I^{0,62}}\right)^{5,263} = 0,102\ \sigma_Y^{-2}\ K_I^{-3,263}\ \Delta K_I^{5,263}$$

In fig. 5, r_s is plotted as function of ΔK_I for some values of K_I for the case under consideration (σ_Y=160 MPa). It is seen that the closure regions are very small up to a ΔK_I of about 10 MPa√m when the closure region rapidly increases.

Acknowledgements

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Fig. 1

Dragprovdata för rostfritt austenitiskt stål

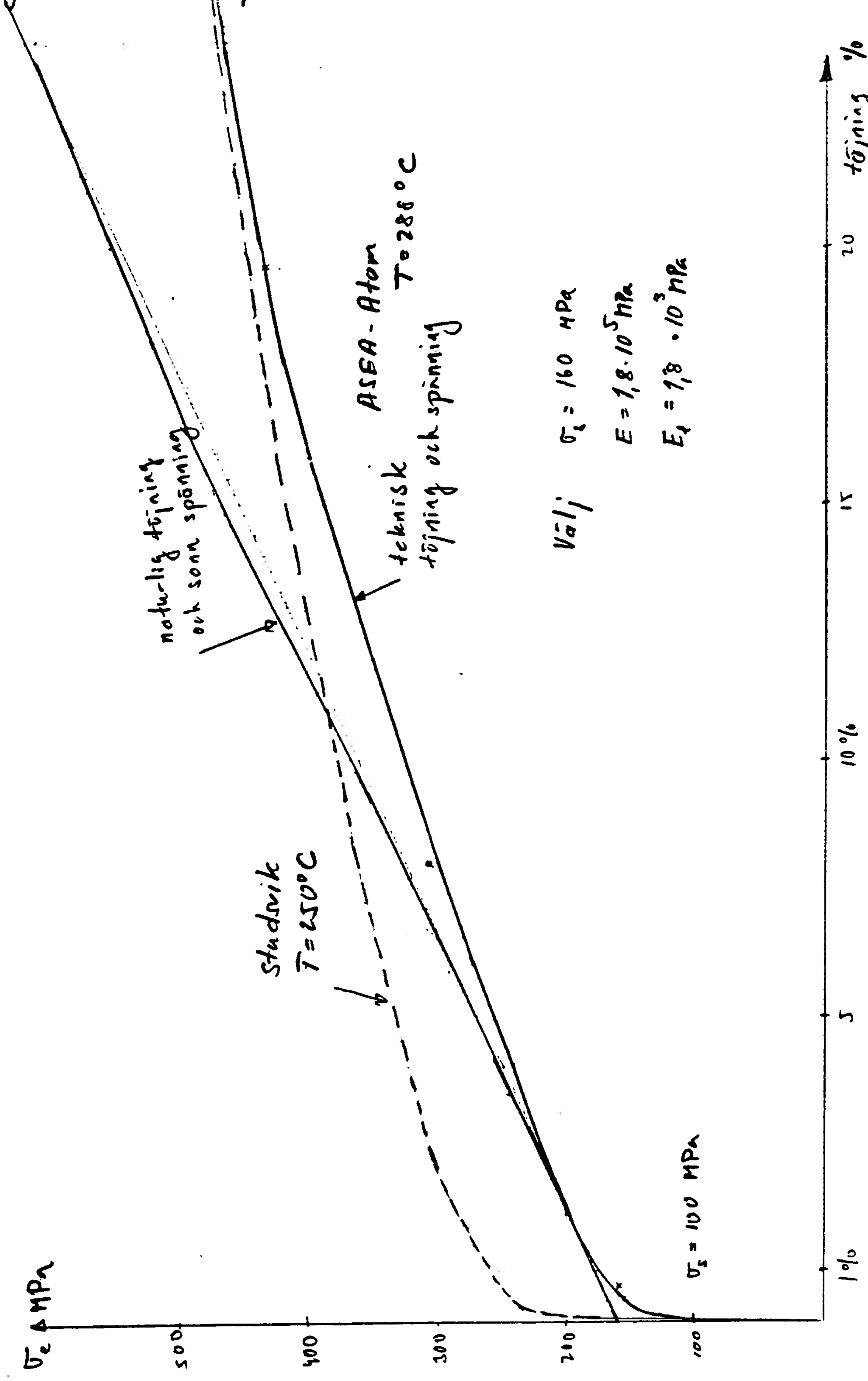

Fig. 2

The mesh consisting of 882 isoparametric elements. $c = 0.625(K_I/\sigma_Y)^2$

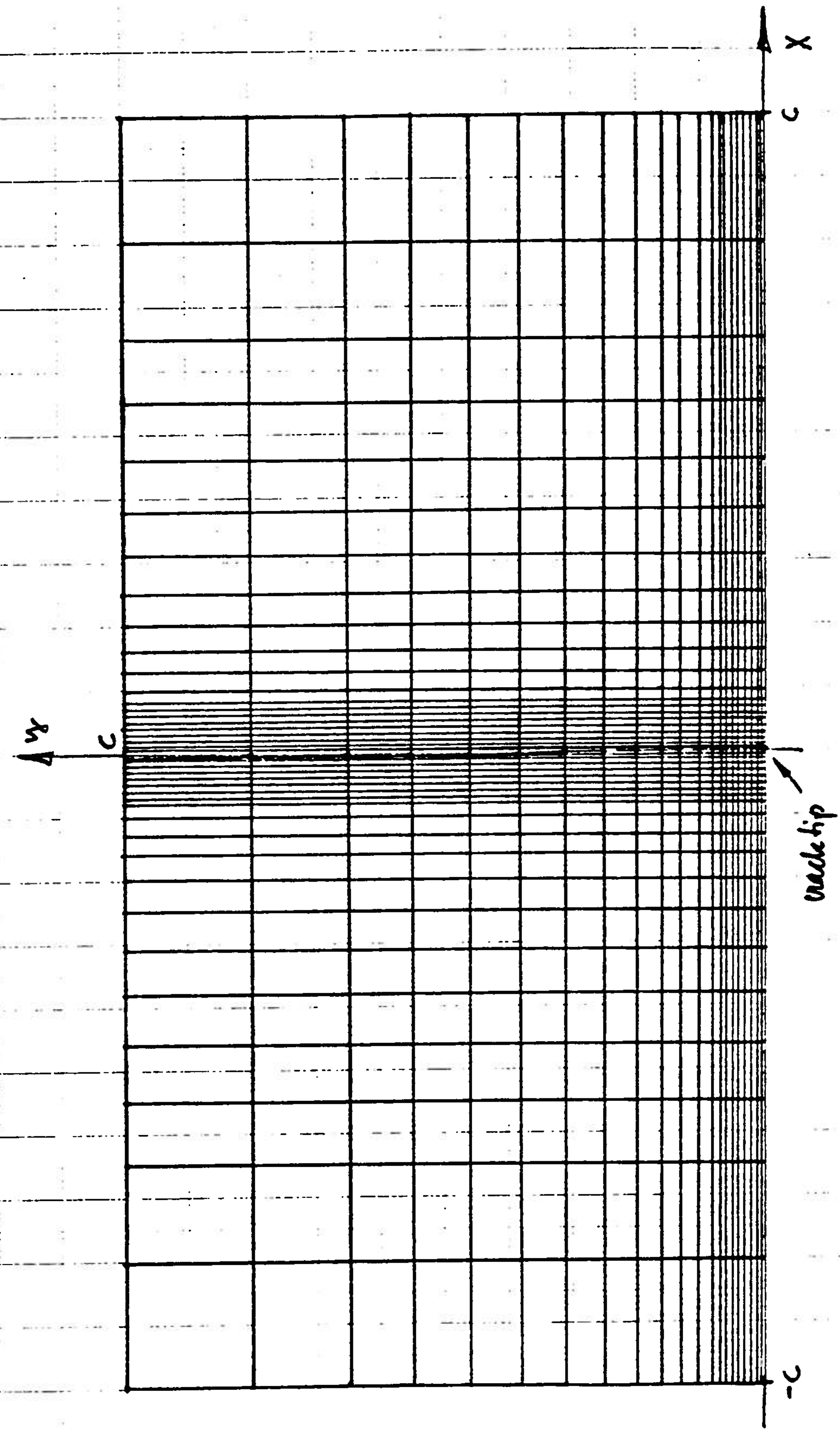

Fig. 3 The shapes of plastic zone, secondary plastic zone and wake for case 1 and 2.

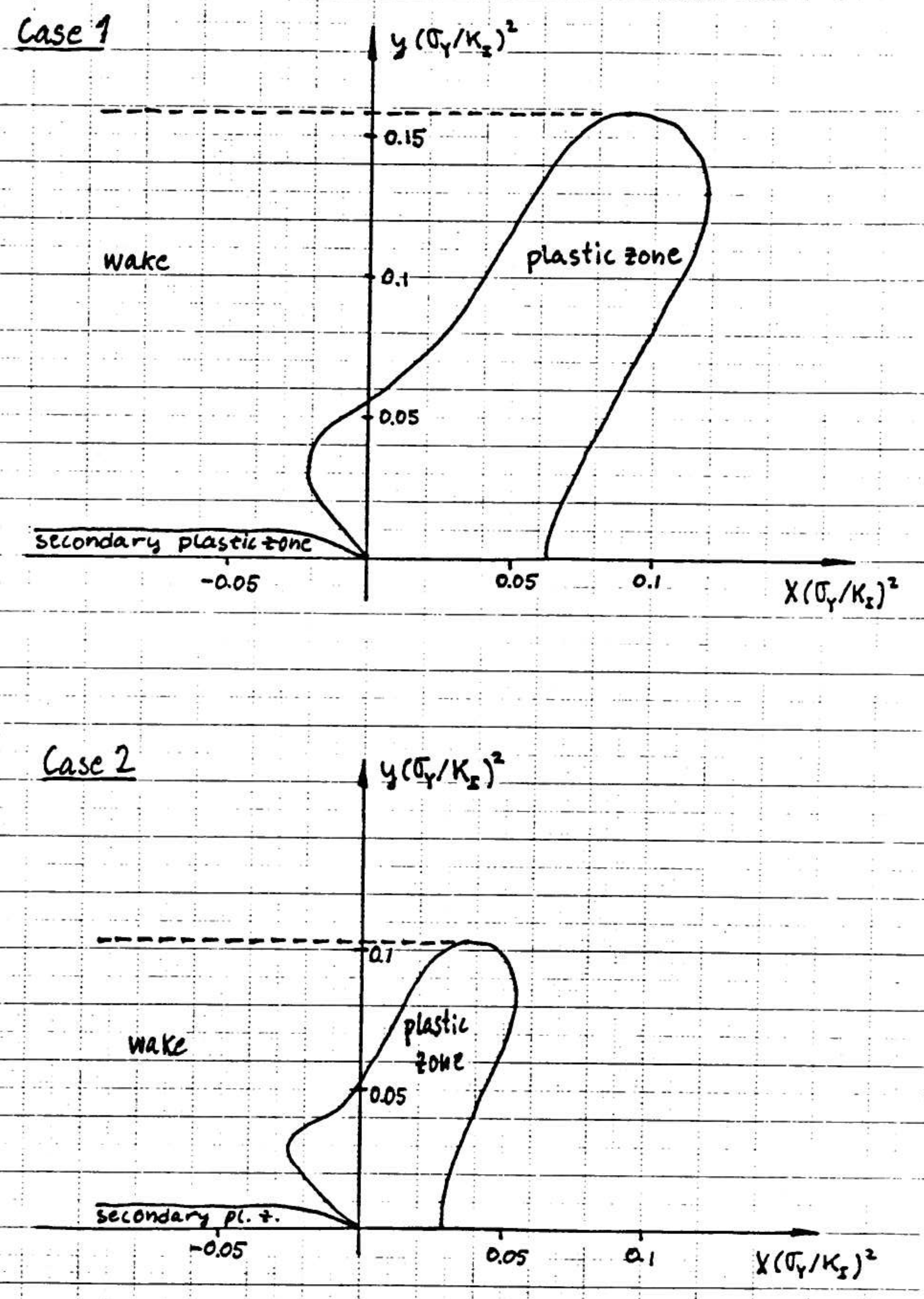

Fig. 4   Stressess at $r = 0.035(K_I/\sigma_Y)^2$

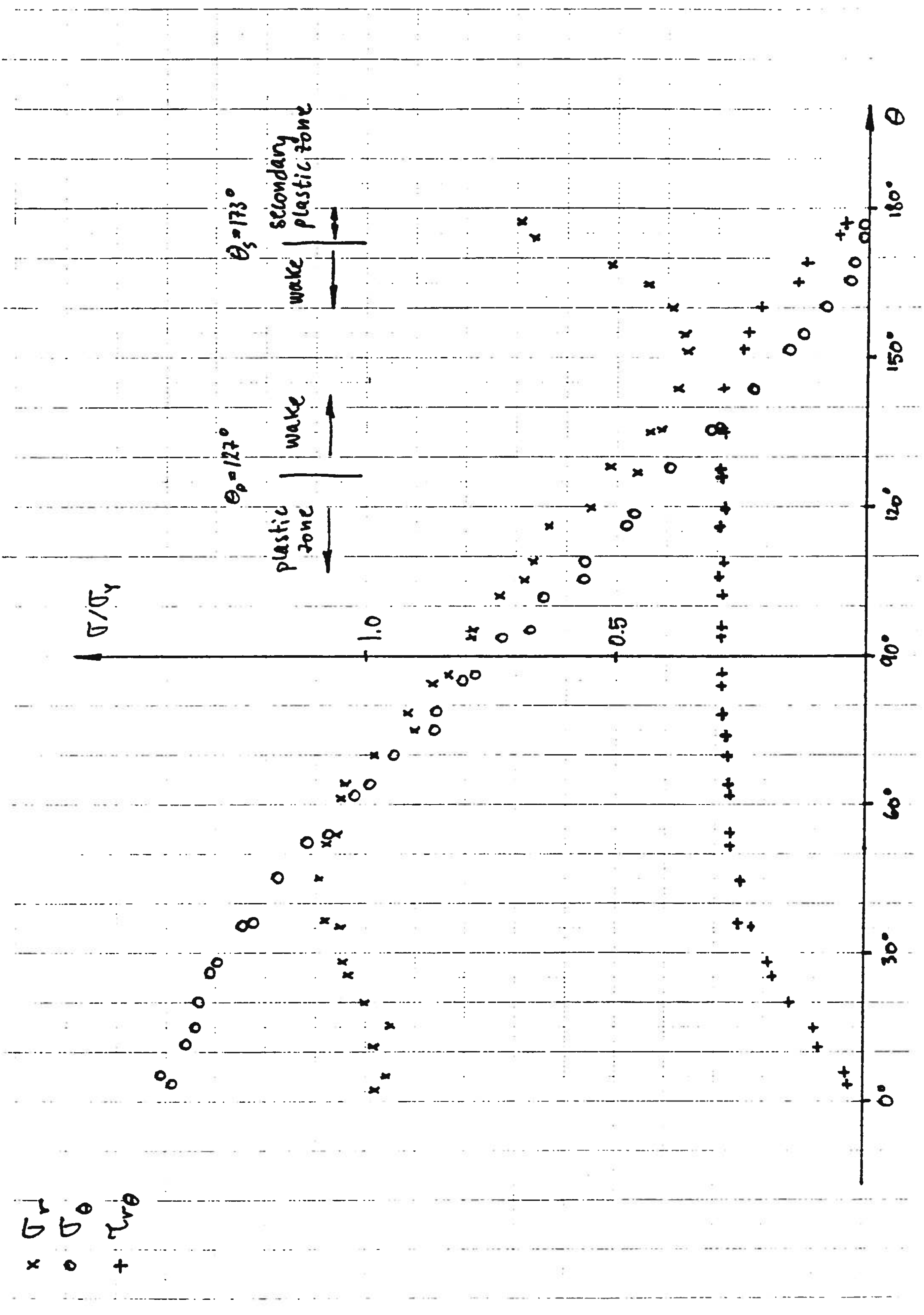

Fig. 5

The crack surface displacement v for case 1 and the approximation $v_a = 2.24\sigma_Y^{-0.38} K_I^{0.62} r^{0.69}/E$

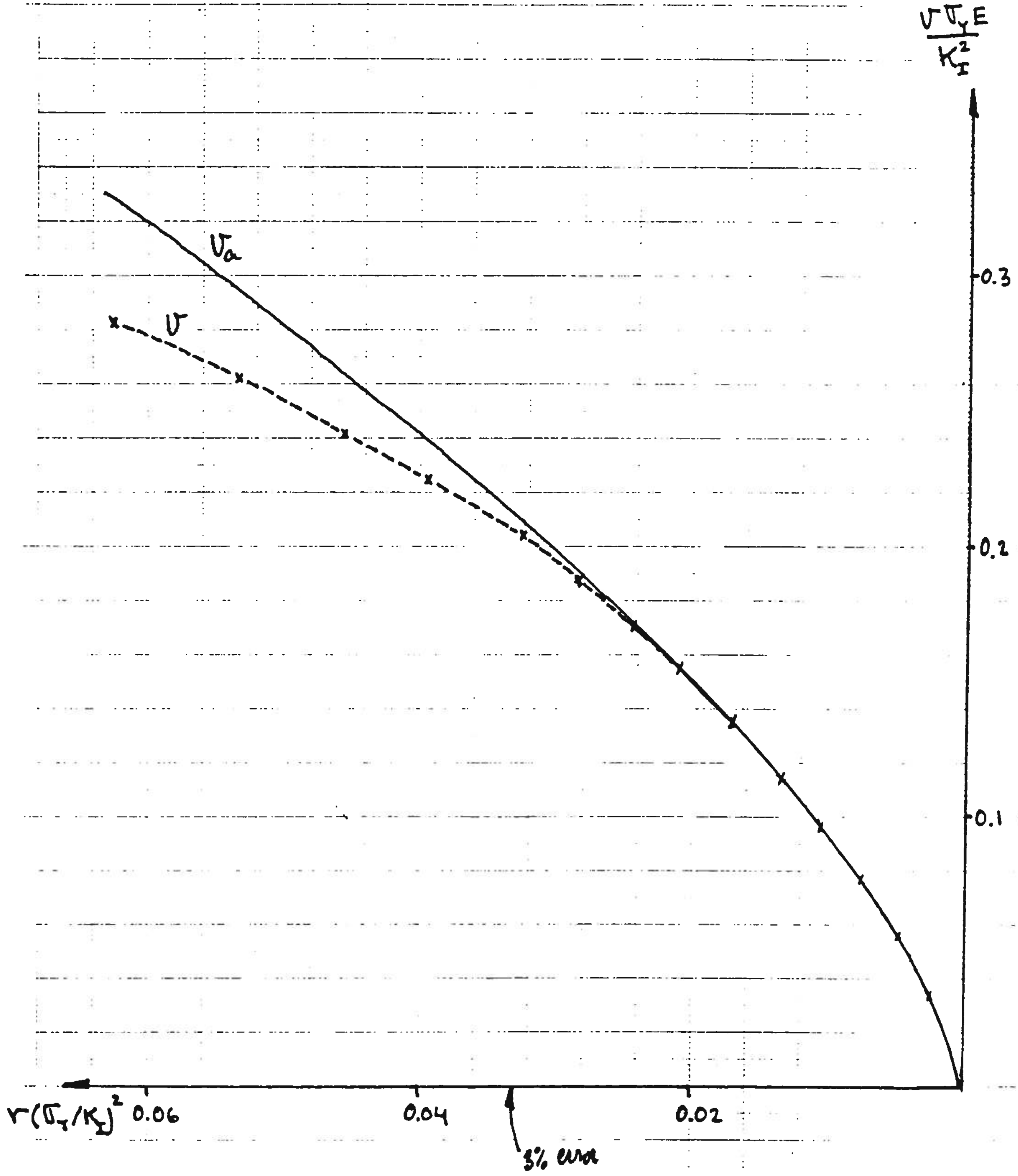

Fig. 6 The size of the crack closure region as function of ΔK_I