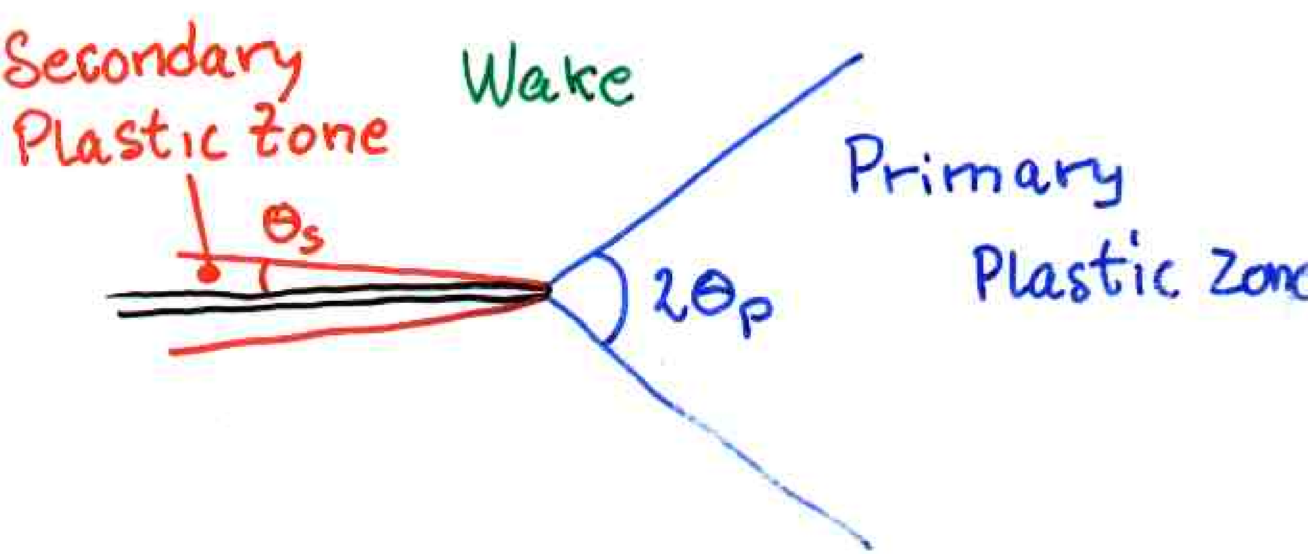
Dynamic Crack Growth at Vanishing Crack Growth Rate

Invited talk given at California Institute of Technology, CalTec, Cal. USA, Orationem Meam

Ståhle, P.

1988

- Mode III
- Steady State
- Vanishing Hardening Rate
- Vanishing Speed
- Asymptotic Field



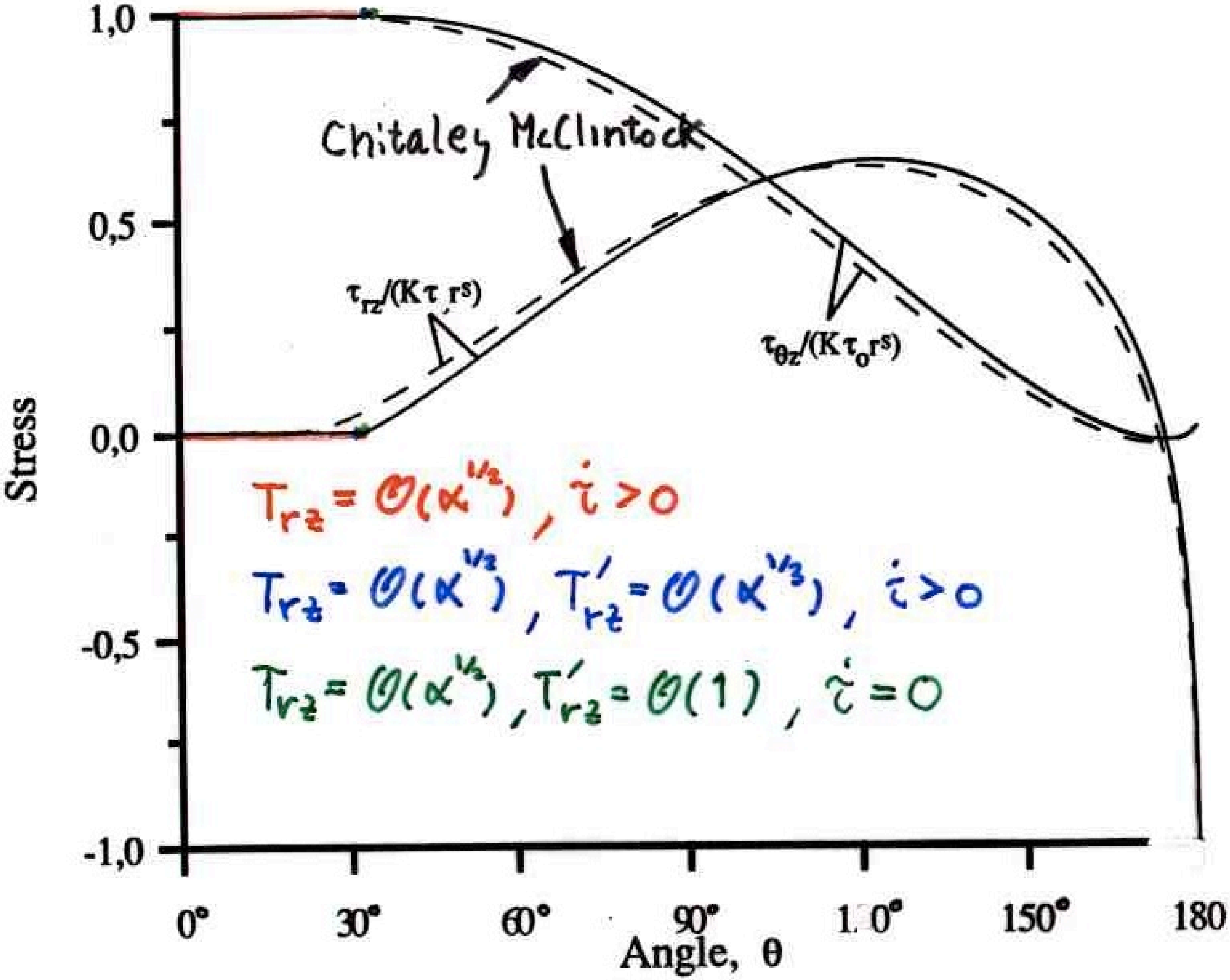
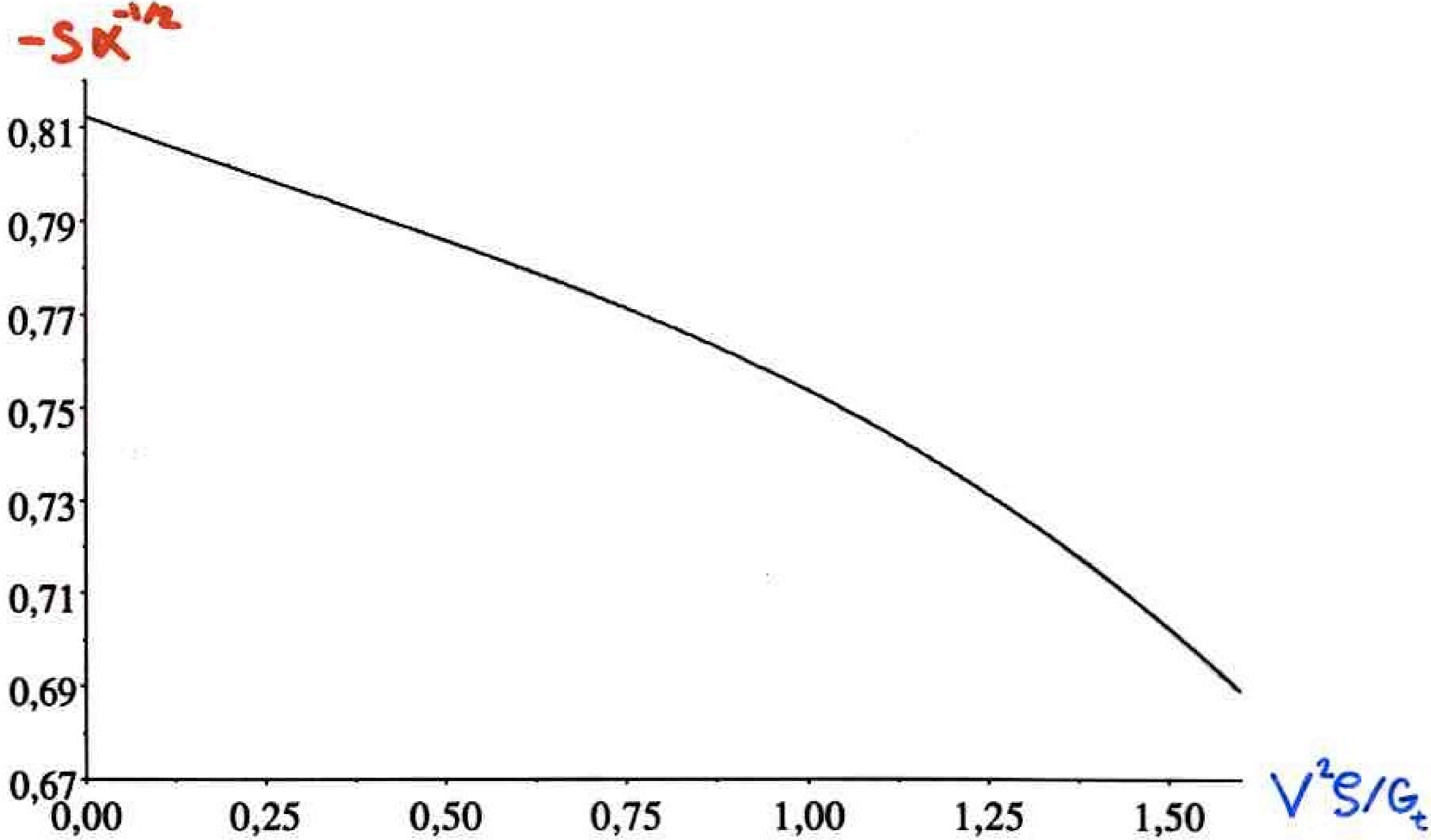


Fig. 4. Angular ditribution of the polar stress components τ_{rz} and $\tau_{\theta z}$. The stresses are normalized so that the effective stress equals unity straight ahead of the crack tip. Dashed curves show the result by CHITALEY and MCCLINTOCK [4].


$-s\kappa^{-1/2}$
0,81
0,79
0,77
0,75
0,73
0,71
0,69
0,67
0,00
0,25
0,50
0,75
1,00
1,25
1,50
$V^2 \rho / G_t$

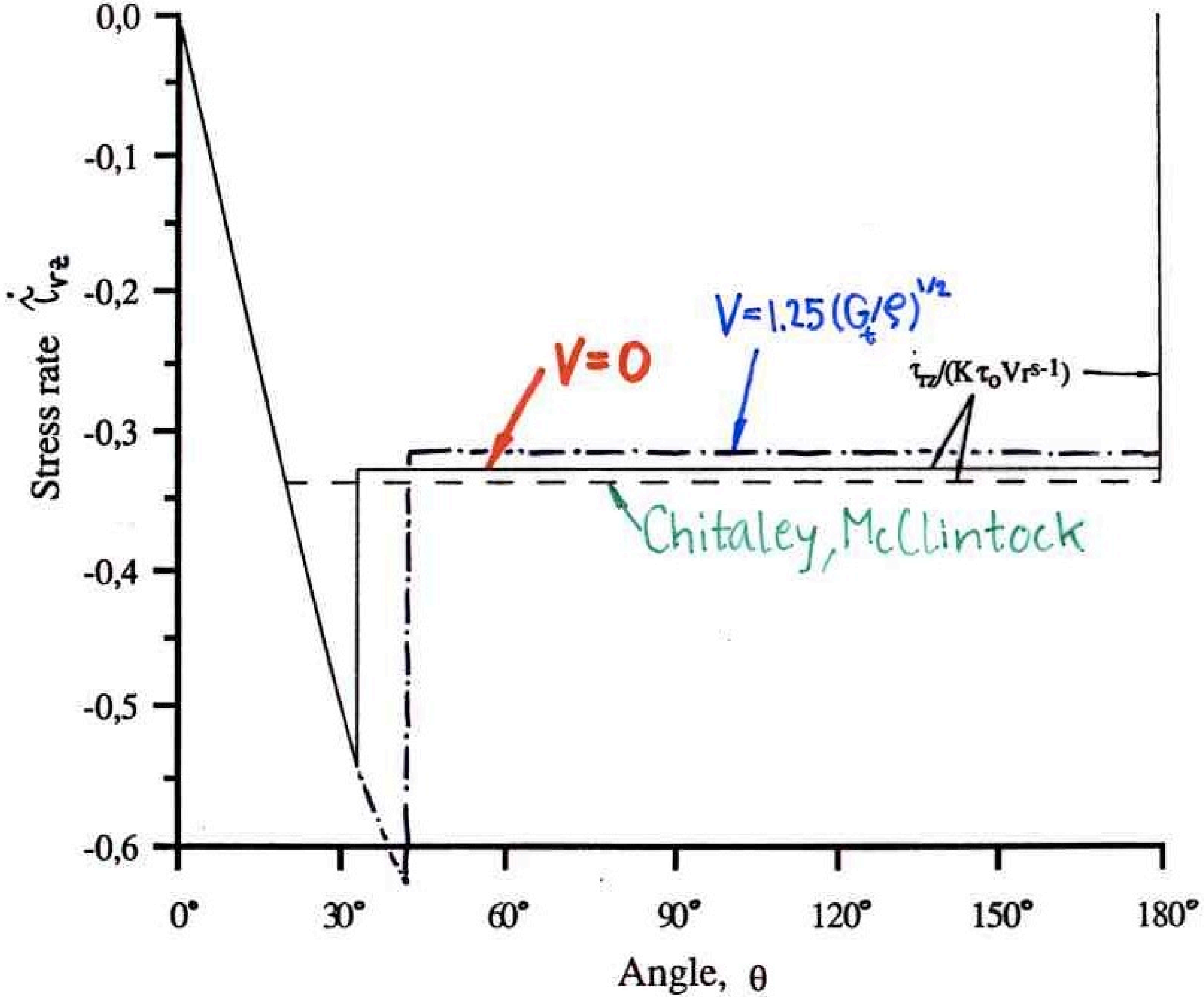
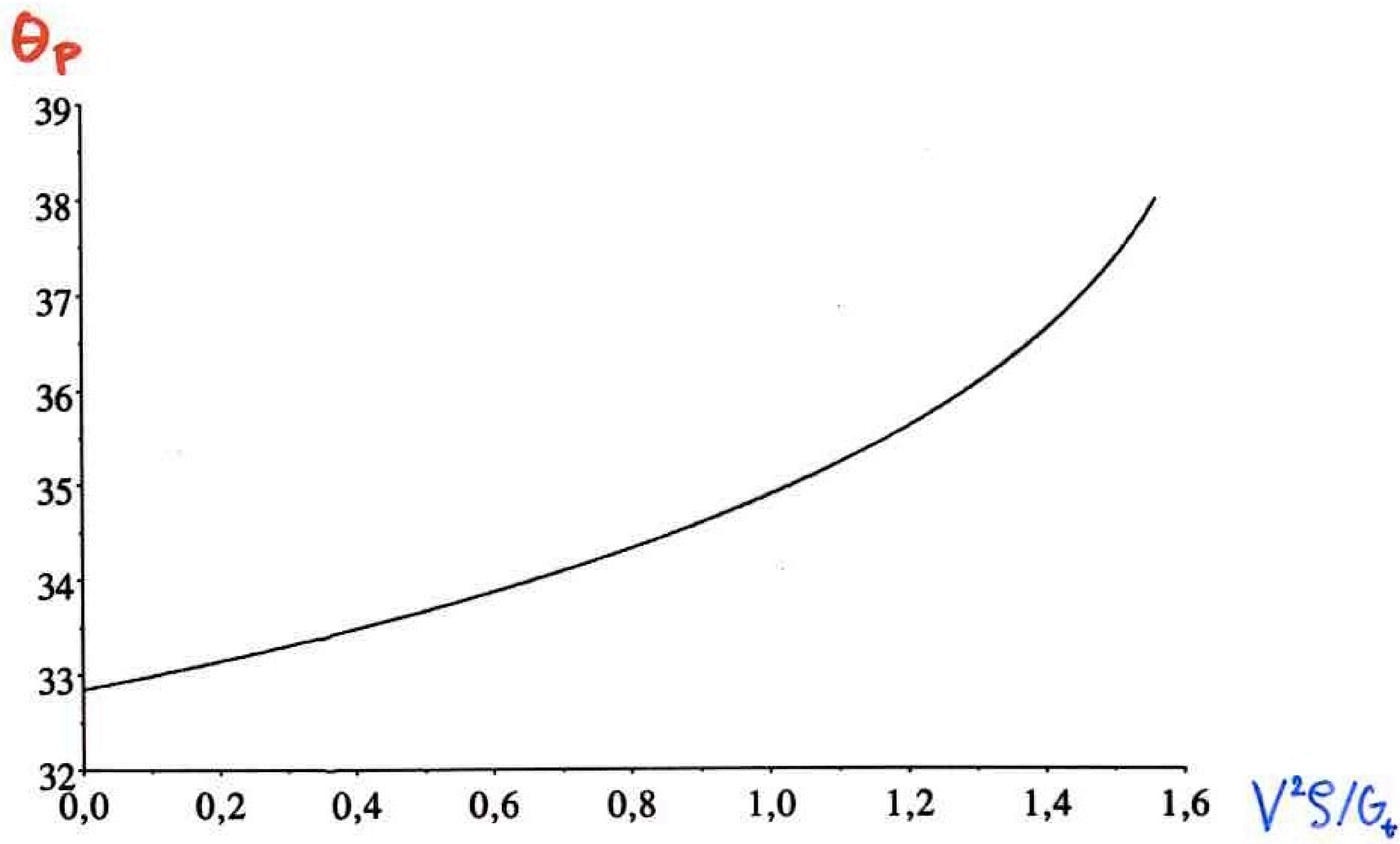
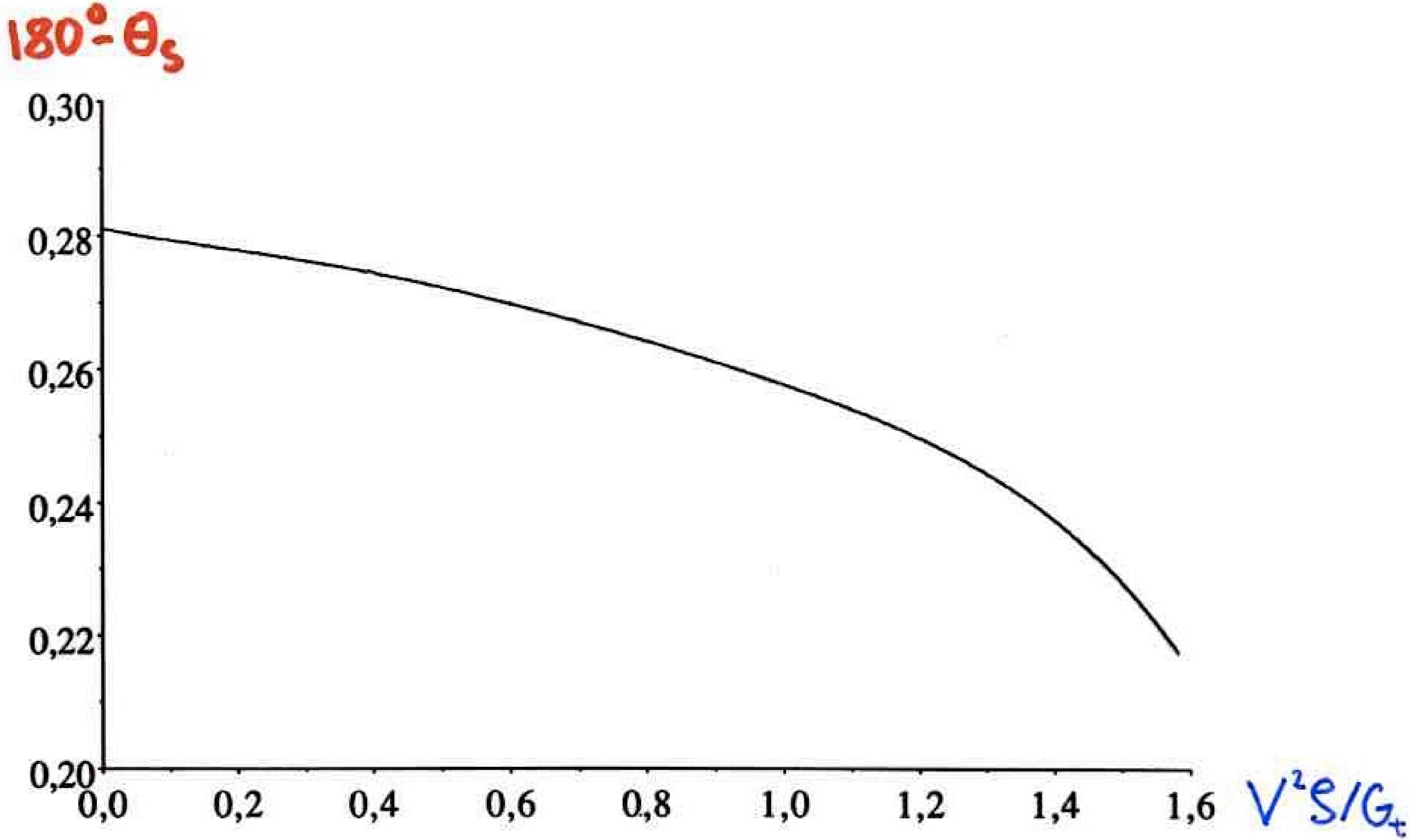


Fig. 7. Angular distribution of the polar stress rate components $\dot{\tau}_{rz}$ and $\dot{\tau}_{\theta z}$. The stress rates are normalized so that the effective stress equals unity straight ahead of the crack tip. Dashed curves show the result by CHITALEY and MCCLINTOCK [4]



θ_P
39
38
37
36
35
34
33
32
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
$V^2\rho/G_t$



180°- θs
0,30
0,28
0,26
0,24
0,22
0,20
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
V²S/Gt

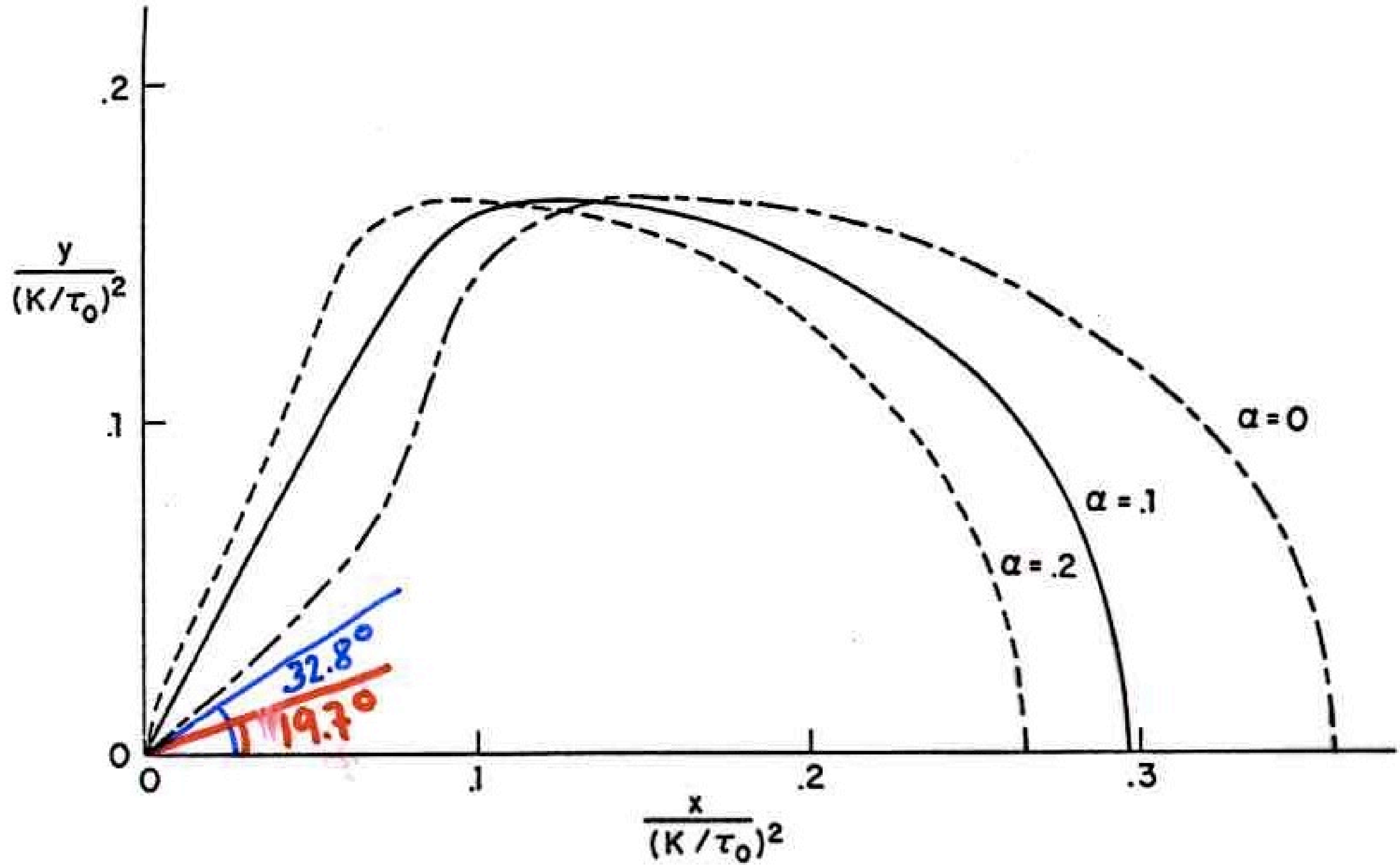


Fig. 6 Effect of linear hardening parameter α on active plastic zone shape in Mode III.

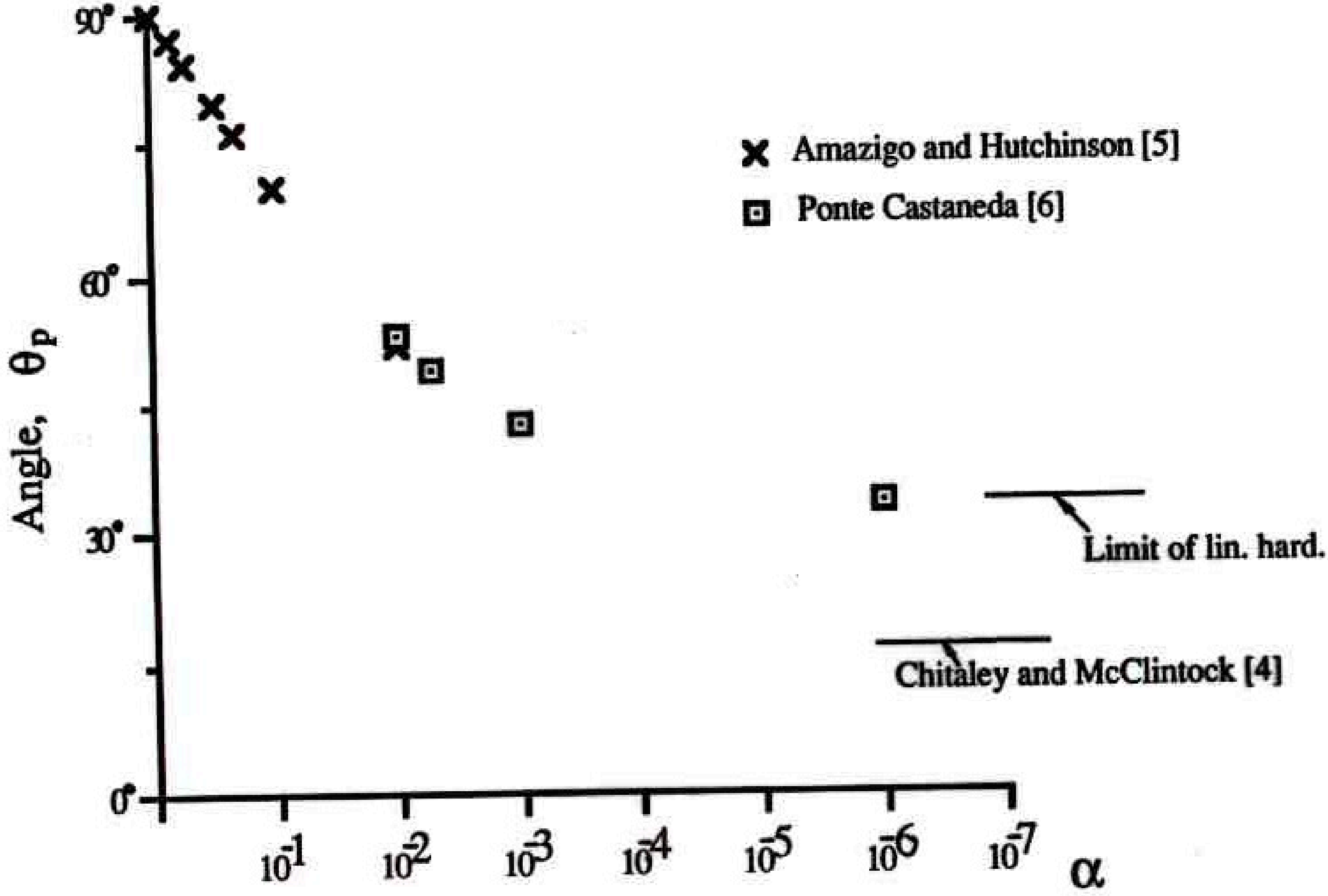


Fig. 9. Unloading angle θ_p for materials with different hardening rates $\alpha=G_t/G$. Results obtained by AMAZIGO and HUTCHINSON [5] and PONTE CASTANEDA [6] compared with the present result for the limit as $\alpha \to 0$ and the result obtained by CHITALEY and MCCLINTOCK [4]

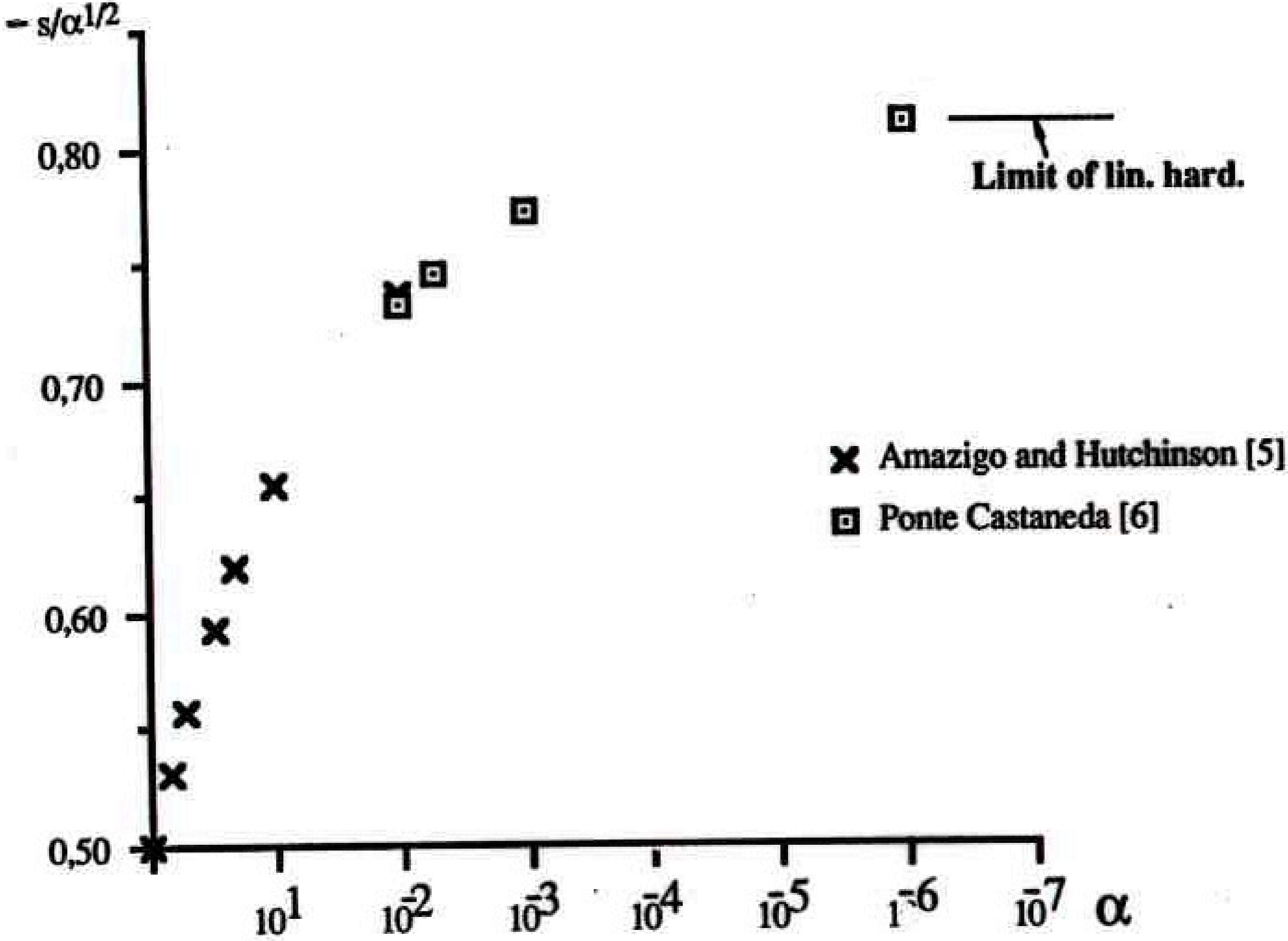
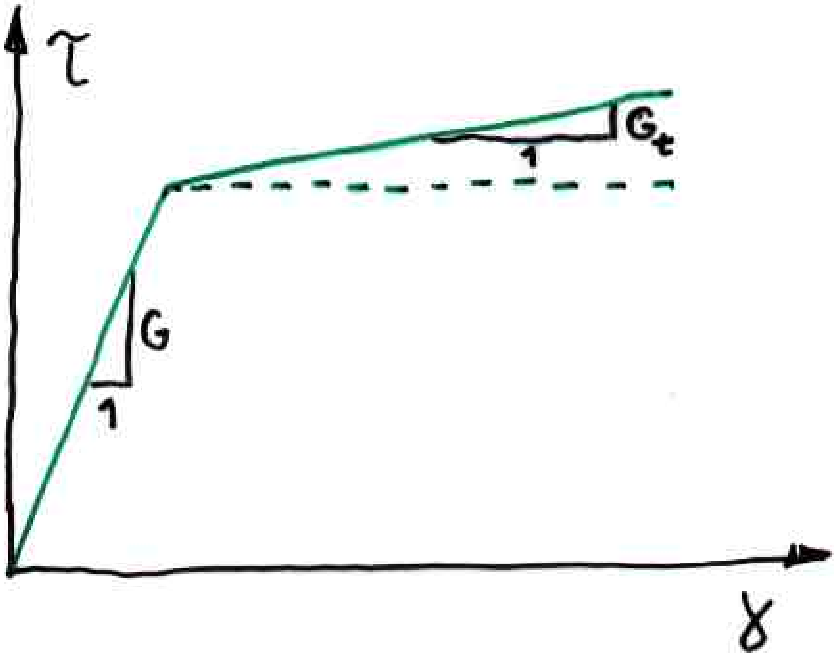


Fig. 10. Ratio $s/\alpha^{1/2}$ for materials with different hardening rates $\alpha=G_t/G$. Results obtained by AMAZIGO and HUTCHINSON [5] and PONTE CASTANEDA [6] compared with the present result for the limit as $\alpha \to 0$.

Linear Hardening



$$\alpha = \frac{G_t}{G}$$

Assumed: $\alpha \to 0$

$$V\sqrt{\frac{\rho}{G_t}} \text{ - finite}$$

Equations

Steady state $\quad \dot{(\,)} = -V(\,)_{,x}$

Equation of motion $\quad \tau_{xz,x} = \rho \ddot{w}$

Compatibility $\quad \gamma_{xz} = \dot{w}_{,x}$

Material $\quad \dot{\gamma}_{xz} = G^{-1}\dot{\tau}_{xz} + (G_t^{-1} - G^{-1})\,\tau_{xz}\,\dot{\tau}\,\tau^{-1}$

Continuity Conditions

Full stress continuity

$$[\tau_{xz}] = 0$$

Drugan, Rice 1982

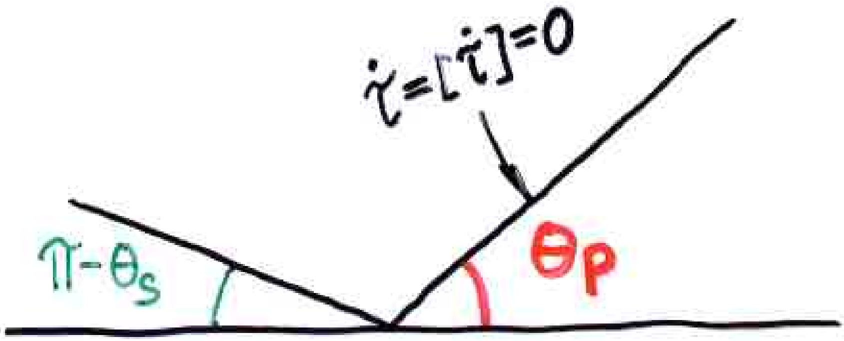
Hardening material

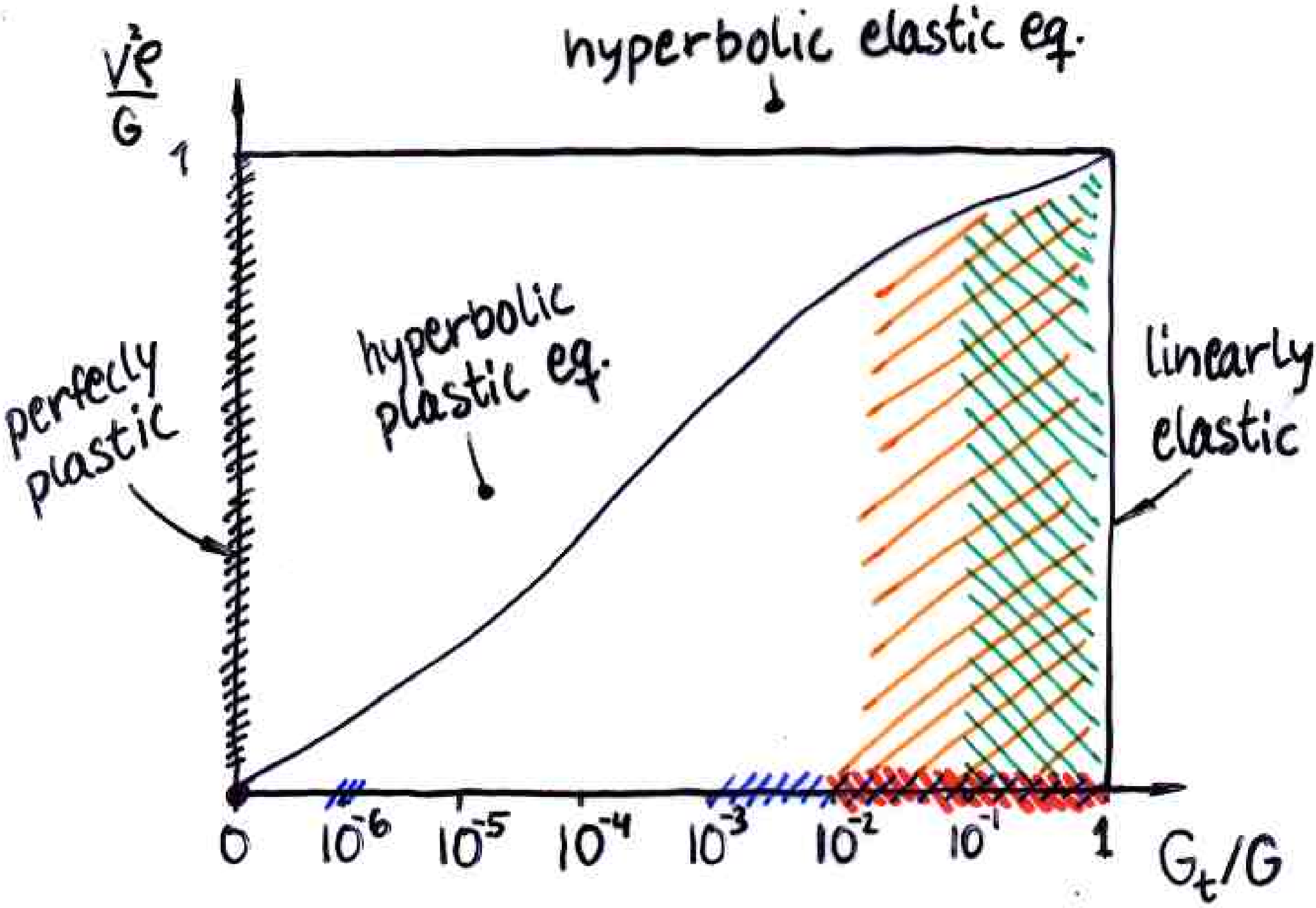
$$[\dot{\tau}_{xz}] = 0$$

Amazigo, Hutchinson 1977

Unloading

$$\dot{\tau} = 0$$





• Chitaley, McClintock 1971

Amazigo, Hutchinson 1977

Achenbach, Kanninen 1978

Ponte Castañeda 1987

Freund, Douglas 1982

Östlund, Gudmundson 1987

Method

Assumed:

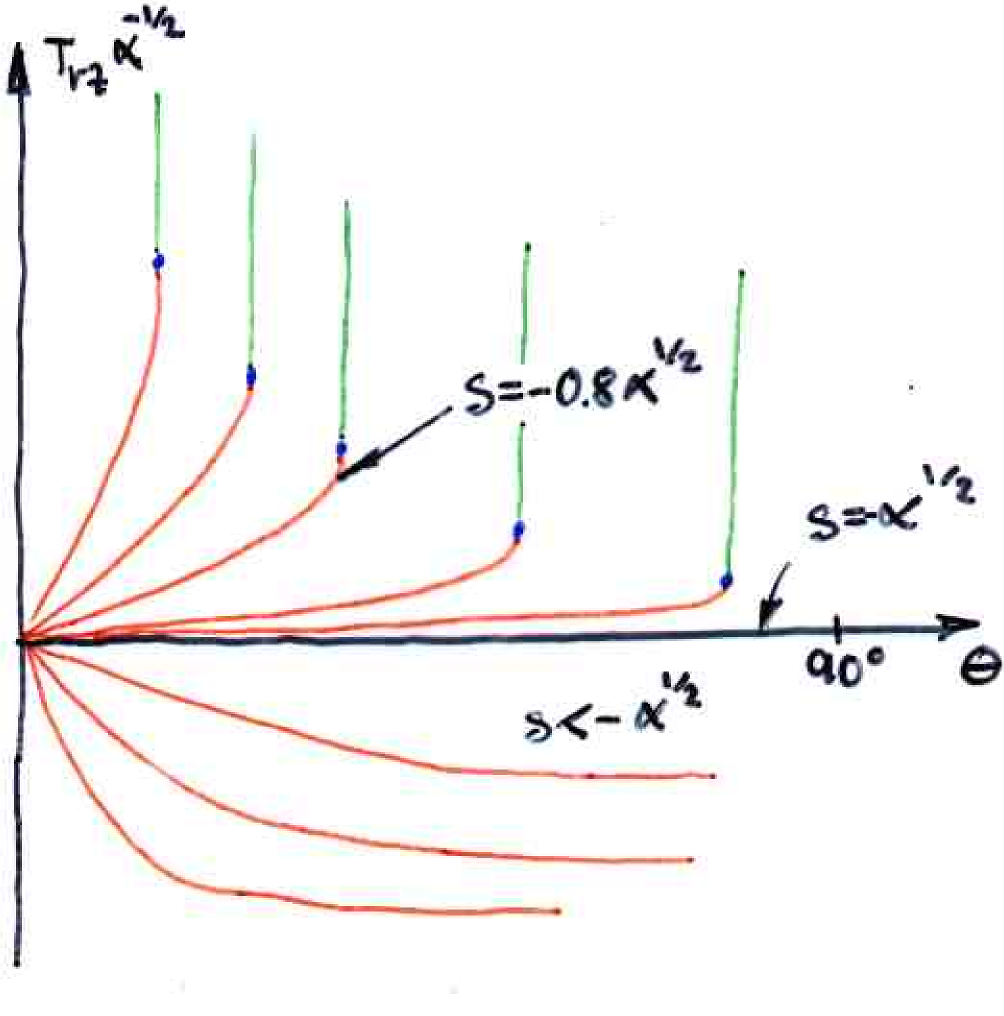
$$\tilde{\tau}_{rz} = T_{rz}(\theta)\, r^{s}$$

Small deviation from Centered fan:

$$T_{rz} \to 0 \quad \text{as} \quad \alpha \to 0 \quad \text{in} \quad \theta < \theta_{p}$$

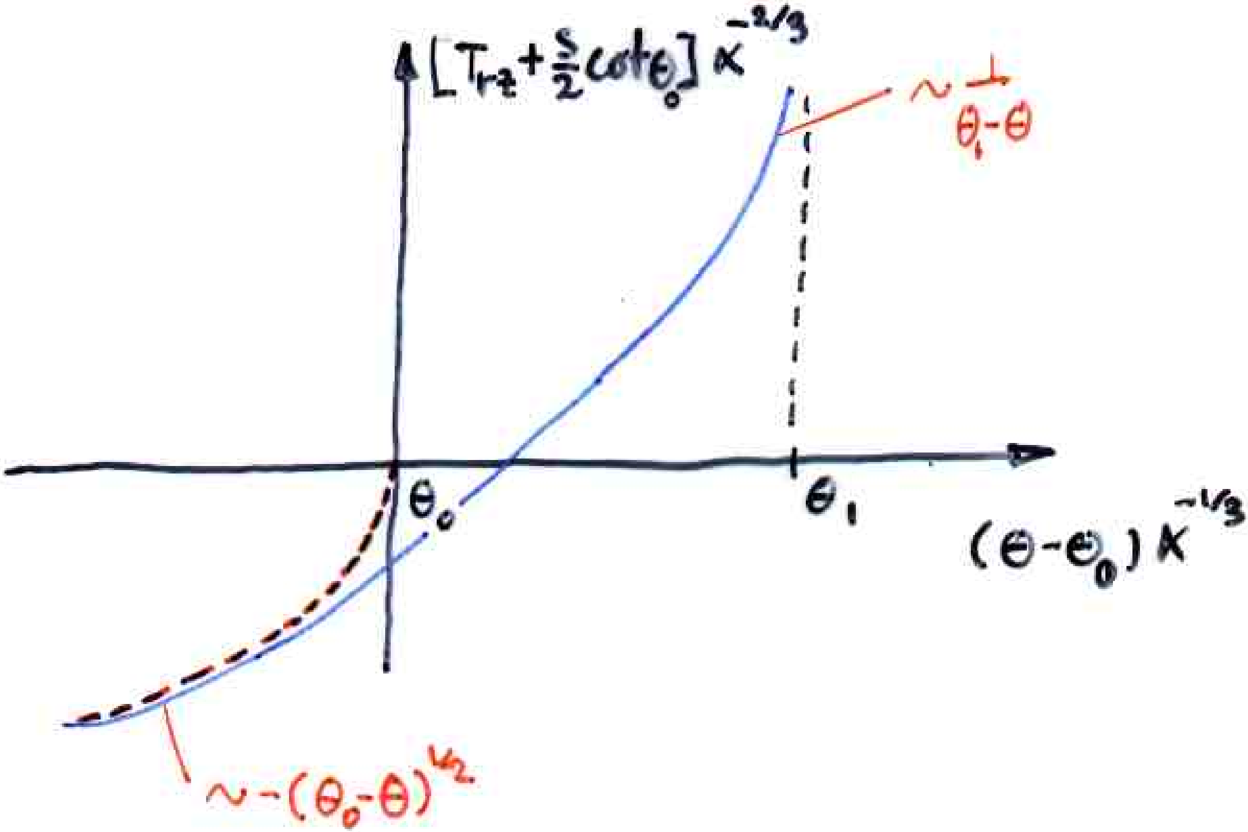
$$T'_{rz}(s + 2T_{rz}\tan\theta) - 2sT_{rz}\tan\theta + T_{rz}^2 + \alpha - s^2 = 0$$

$$s = \mathcal{O}(\alpha^{1/2}) \qquad T'_{rz}, T_{rz} = \mathcal{O}(\alpha^{1/2})$$



$$T_{rz}'' = \mathcal{O}(1) \qquad T_{rz}' = \mathcal{O}(\alpha^{1/3}) \qquad T_{rz} = -\frac{s}{2}\cot\theta_0 + \mathcal{O}(\alpha^{2/3})$$

$$(T_{rz}'' + \cot\theta_0)(T_{rz}^2 + \alpha) - 2T_{rz}'(T_{rz} + \frac{s}{2}\cot\theta_0) = 0$$



$$T''_{rz} = \mathcal{O}(\alpha^{-1/2}) \qquad T'_{rz} = \mathcal{O}(1) \qquad T_{rz} = -\frac{s}{2}\cot\theta_0 - \mathcal{O}(\alpha^{1/2})$$

$$T''_{rz}(T_{rz}^2 + \alpha) + 2T_{rz}T'_{rz}(T'_{rz} - 1) - sT'_{rz}\cot\theta_0 = 0$$