Asymptotic Crack Tip Fields at Steady-Crack Growth and Vanishing Hardening

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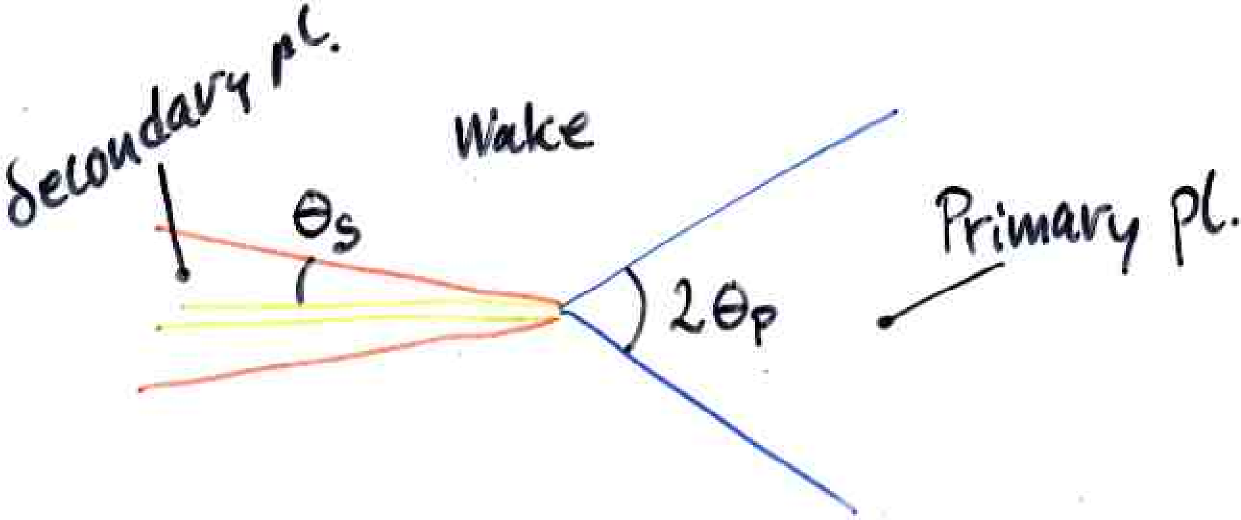
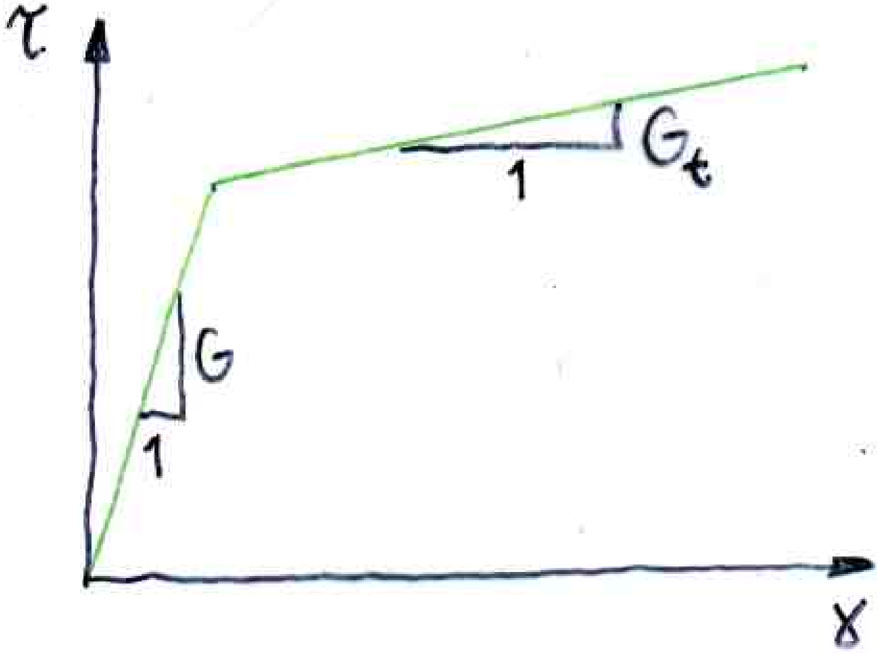
Asymptotic fields in steady crack growth with vanishing strain hardening

- Mode III
- Steady Growth
- Asymptotic field



- Linear Strain Hardening



$$\alpha = G_t / G$$

Vanishing Rates $\quad \alpha \to 0$

Serieutveckla r^s för små s.

$$r^s = e^{s\ln r} = 1 + s\ln r + \frac{s^2}{2}\ln^2 r + \dots$$

$$\phi = (\cos\theta + s f(\theta)) r^s \approx \cos\theta + s(\cos\theta \ln r + f(\theta))$$

$$\tau_r = s F(\theta) r^s \approx s F(\theta)$$

$$\dot{\tau} = -s(\cos\theta - F\sin\theta) r^{s-1} \approx -s(\cos\theta - F\sin\theta) r^{-1}$$

$$\dot{\gamma}_r = \dot{\tau}_r + \alpha^{-1}\dot{\tau}\tau_r\tau^{-1} = \left[-\sin\theta + \frac{s^2}{\alpha}(\cos\theta - F\sin\theta)F\right] r^{s-1}$$

$$\dot{\gamma}_r = \frac{\partial \dot{w}}{\partial r} \quad \Rightarrow$$

$$\dot{w} = \left[-\sin\theta + \frac{s^2}{\alpha}(\cos\theta - F\sin\theta)F\right]\left(-\frac{r^s}{s} + \frac{R^s(\theta)}{s}\right) \approx$$

$$\approx \left[-\sin\theta + \frac{s^2}{\alpha}(\cos\theta - F\sin\theta)F\right] \underline{\underline{\ln\frac{R(\theta)}{r}}}$$

jfr. Chitaley & McClintock (1971)

$$\dot{w} = -\sin\theta \ln\frac{R(\theta)}{r}$$

Spänningshastighetsfunktionen

separabel.

$$\phi = (\cos\theta + s f(\theta)) r^s$$

$$\tau_r = s F(\theta) r^s$$

$$\dot{\tau} = -s(\cos\theta - F\cos\theta) r^{s-1}$$

Kompatibilitet

$$-\alpha\cos\theta\, r^{s-1} - \overbrace{s\frac{\partial}{\partial r}(r\dot{\tau})}^{O(s^2)} +$$

$$+ s^2[F'(\cos\theta - 2F\sin\theta) - F^2\cos\theta - F\sin\theta] = 0$$

Störning $O(\alpha^{1/2})$. $s = -0.81\alpha^{1/2}$

Mode III. Störning av centered-fan-fält.

$$\phi = \cos\theta + s f(r,\theta)$$ spänningshast. funkt.

$$\tau_2 = s F(r,\theta)$$

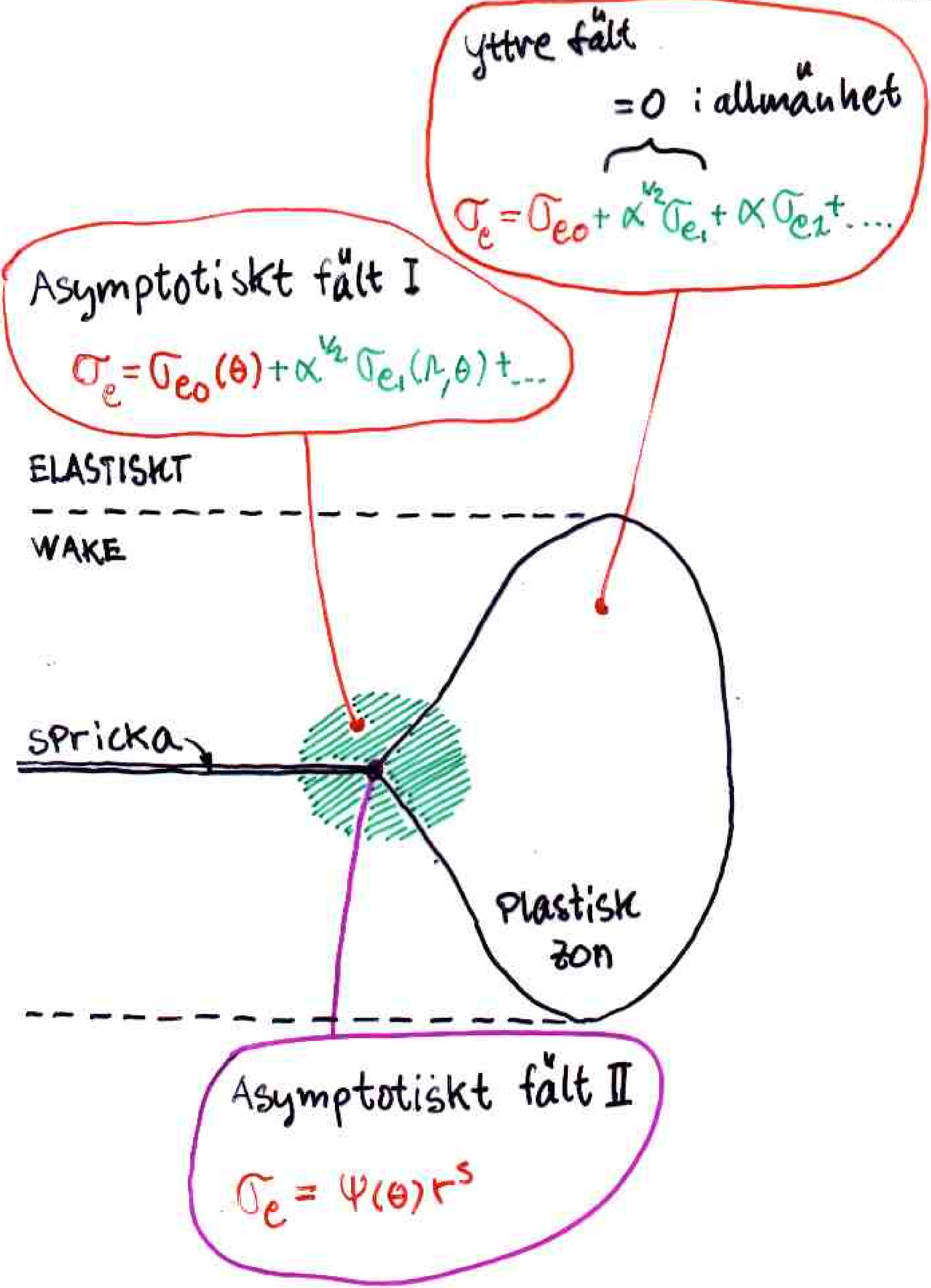
$$\dot{\tau} = s\left(\frac{\partial f}{\partial r} - F \sin\theta\, r^{-1}\right)$$

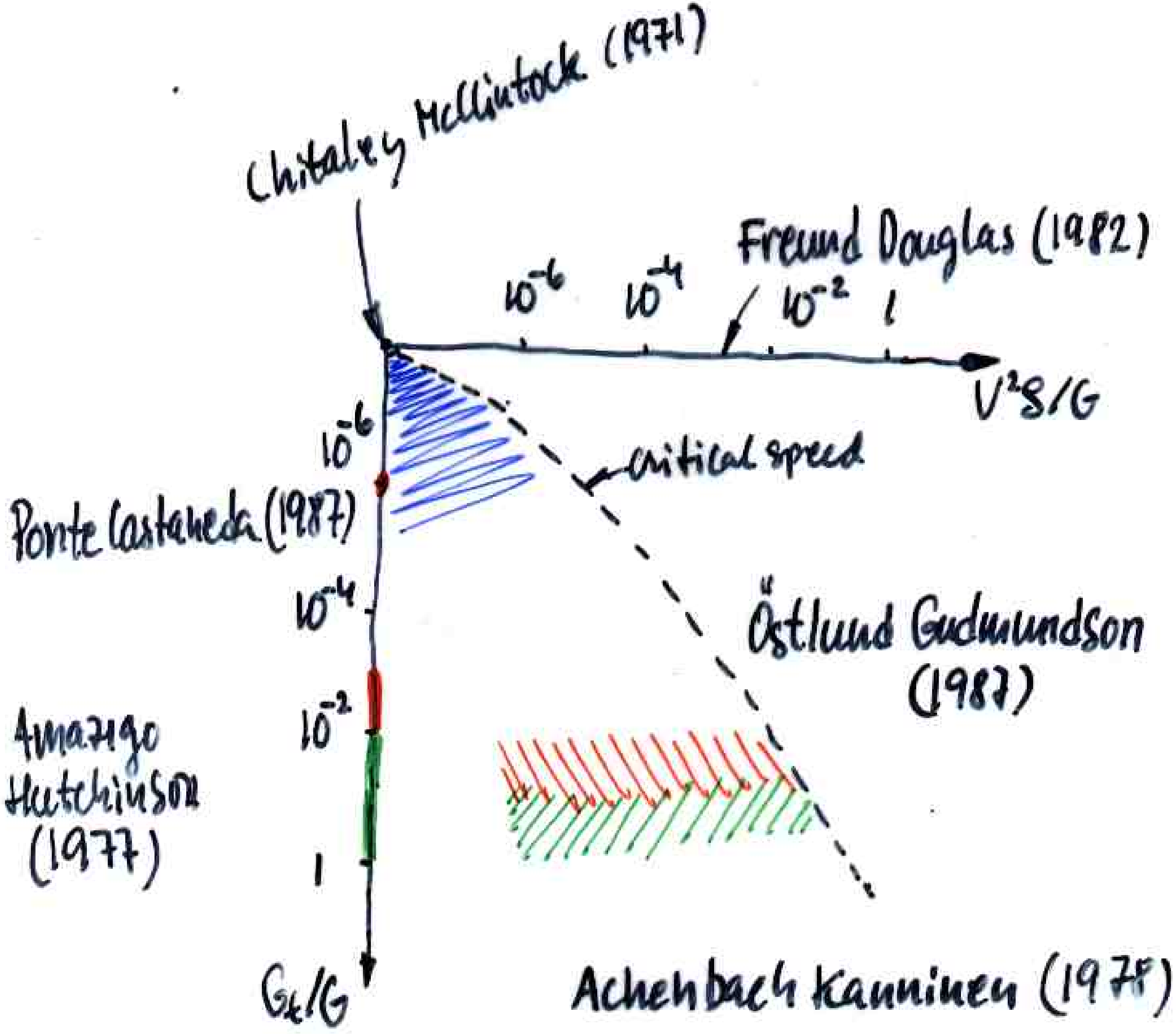
Kompatibilitet

$$-\alpha \cos\theta\, r^{-1} - \frac{\partial}{\partial r}(r\dot{\tau}) = 0$$

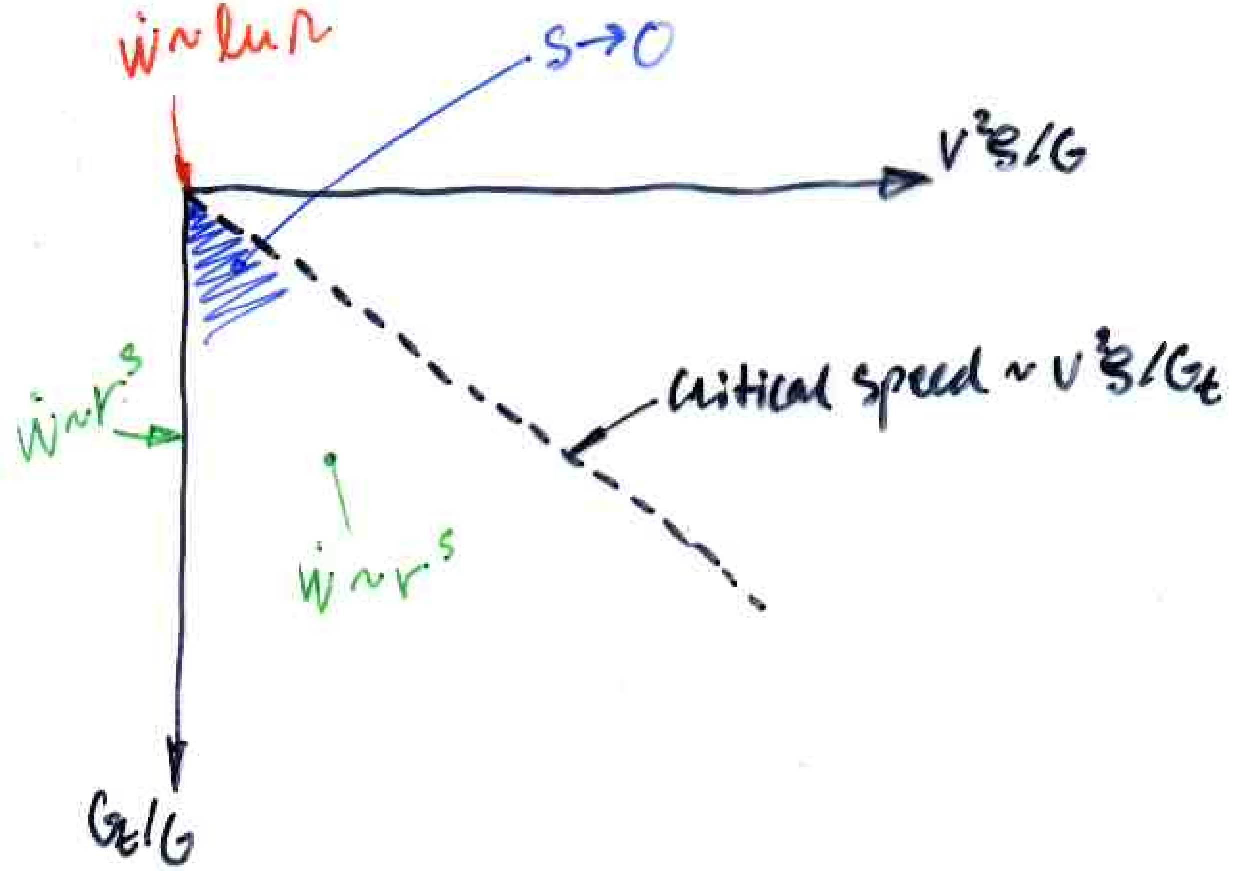
$$\Rightarrow \quad \dot{\tau} = \alpha\left(\cos\theta \frac{\ln r}{r} - \frac{c(\theta)}{r}\right)$$

Störning $\mathcal{O}(\alpha)$ ej separabel lösning.


yttre fält
$\sigma_e = \sigma_{e0} + \overbrace{\alpha^{1/2}\sigma_{e1}}^{=0 \text{ i allmänhet}} + \alpha\sigma_{e2} + \ldots$
Asymptotiskt fält I
$\sigma_e = \sigma_{e0}(\theta) + \alpha^{1/2}\sigma_{e1}(r,\theta) + \ldots$
ELASTISKT
WAKE
spricka
Plastisk zon
Asymptotiskt fält II
$\sigma_e = \Psi(\theta) r^s$



$\dot{a}(\rho/G_t)^{1/2}$	$-s\alpha^{-1/2}$	Θ_p	$\pi-\Theta_s$
0	0.81	32.8	0.28
1	0.72	31.0	0.30
1.45	0.62	28.9	0.32



$$A(\theta)r^{s} - B(\theta)r^{-s} \to C(\theta)\ln(r/R)$$

as $s \to 0$

$$C(\theta) = s\,[A(\theta) + B(\theta)]$$

$$R(\theta) = \exp(A(\theta) - B(\theta))$$

$V(S/G_t)^{1/2}$	$-S\alpha^{-1/2}$	θ_p	θ_s
0	0.812	32.8	0.28
1	0.722	31.0	0.30
1.45	0.615	28.9	0.32

- Vanishingly small hardening rates may produce significant changes in stressdistribution.

- In practice, stress rates may be discontinuous for perfectly plastic materials.

Series expansion of r^s

$$r^s = e^{s\ln r} = 1 + s\ln r + \frac{s^2}{2}\ln^2 r + \ldots$$

$$\tau_{rz} = F(\theta) r^s \approx F(\theta)$$

$$\dot{\tau}_{rz} = -\sin\theta\, r^{s-1} \approx -\sin\theta\, r^{-1}$$

$$\dot{\gamma}_{rz} = \left[-\sin\theta + \frac{1}{\alpha}(s\cos\theta - F\sin\theta)F\right] r^{s-1}$$

$$\dot{\gamma}_{rz} = \frac{\partial \dot{w}}{\partial r}$$

$$\dot{w} = \left[-\sin\theta + \frac{1}{\alpha}(s\cos\theta - F\sin\theta)F\right] \underbrace{\left(-\frac{r^s}{s} + \frac{R^s(\theta)}{s}\right)}_{\approx \ln\frac{R(\theta)}{r}}$$

cf. Chitaley & McClintock

$$\dot{w} = -\sin\theta \ln\frac{R(\theta)}{r}$$

Solution

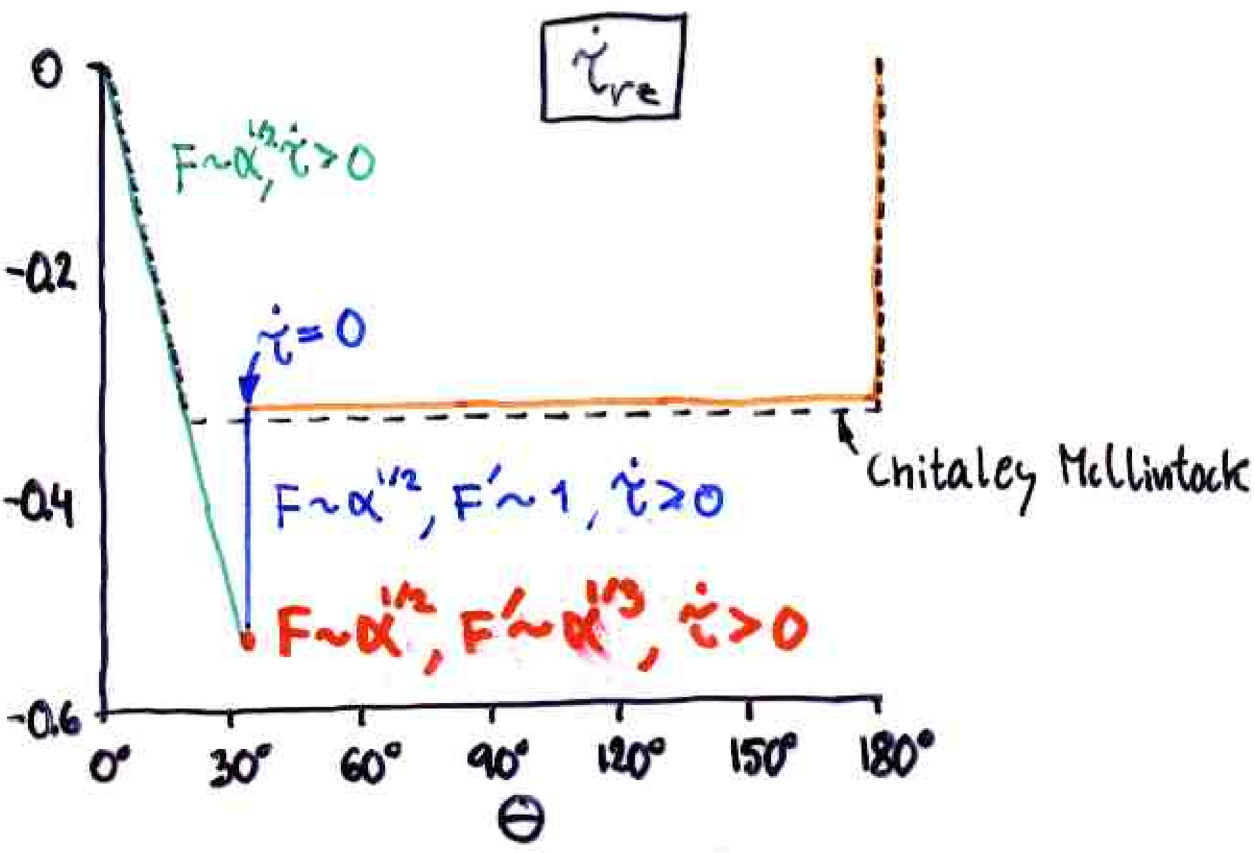
Assumed

$$\tau_{\alpha z} \sim r^{s}$$

Small deviation from Centered fan straight ahead

$$\tau_{rz} = F(\theta) r^{s} \qquad \alpha = G_t / G$$

$$s \sim \alpha^{1/2}$$



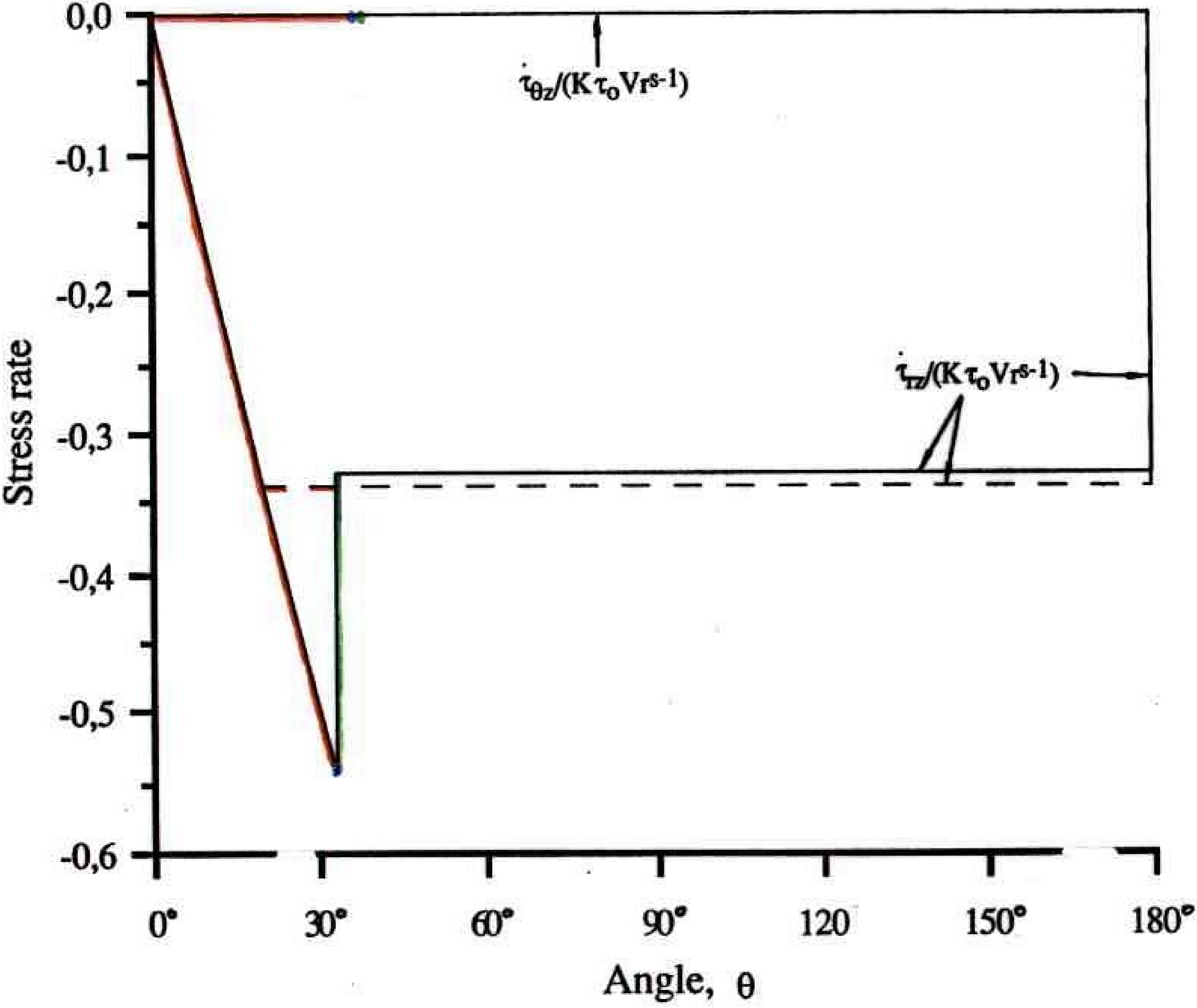


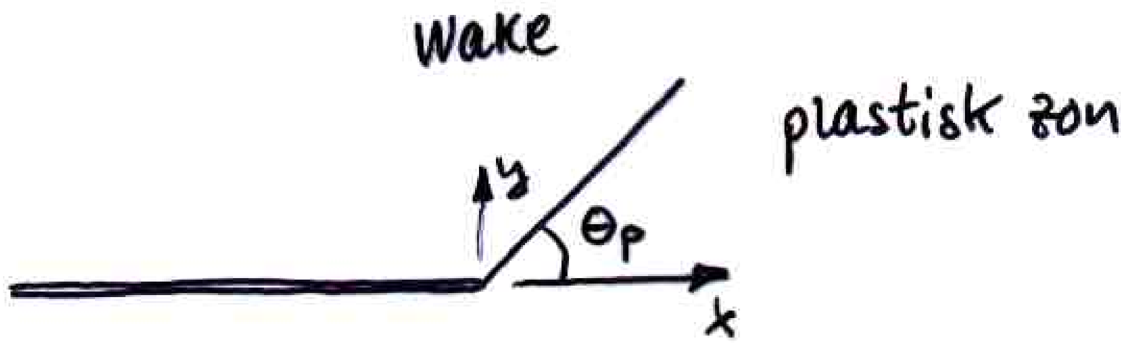
Fig. 7. Angular distribution of the polar stress rate components $\dot{\tau}_{rz}$ and $\dot{\tau}_{\theta z}$. The stress rates are normalized so that the effective stress equals unity straight ahead of the crack tip. Dashed curves show the result by CHITALEY and MCCLINTOCK [4].

- Idealplastiska gränslösningar har stor betydelse.

- Grad av hårdnande har ringa betydelse.

- Dynamiska effekter även när $\dot{a} \to 0$.

- Val av hårdnande?



Ideal plastiskt.

$\dot{\sigma}_e = 0$ i den plastiska zonen.
Plastisk deformation obestämd.
I allmänhet $\dot{\varepsilon}_e \neq 0$ när $\theta \to \theta_p$
i den plastiska zonen.

Mode I	Drugan, Rice & Sham (1982)
Mode II	Ponte Castañeda (1986)
Mode III	Chitaley & McClintock (1971)

Hårdnande material

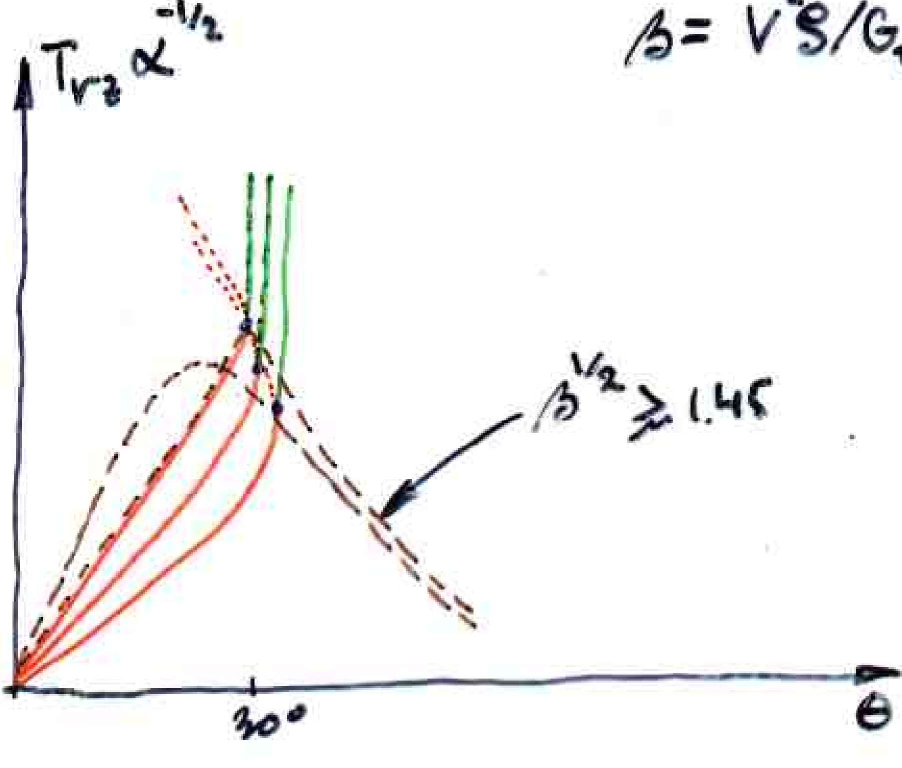
$\dot{\sigma}_e \to 0 \Rightarrow \dot{\varepsilon}_e \to 0$ när $\theta \to \theta_p$

$$T_{rz}'(s+2T_{rz}\tan\theta)-(sT_{rz}\tan\theta-T_{rz}^2-\alpha)(1-\beta\sin^2\theta)-$$

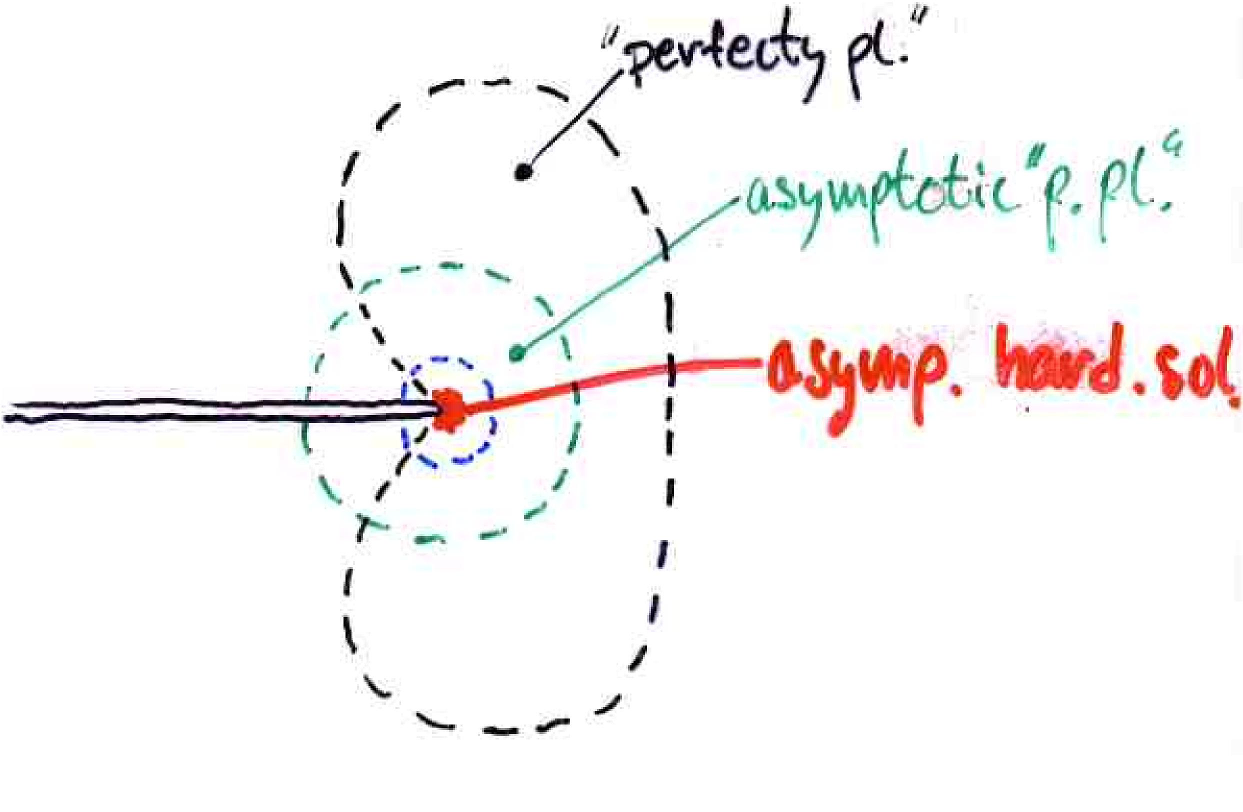
$$-s^2-sT_{rz}\tan\theta+\beta T_{rz}\sin^2\theta\,(s\tan\theta-3T_{rz})-$$

$$-2\beta\sin\theta\,T_{rz}(s\cos\theta+\beta T_{rz}\sin^3\theta)(1-\beta\sin^2\theta)^{-1}=0$$

$\beta = V^2\rho/G_t$



rate independent metallic materials



Stationary cracks OK

growing mode I $\nu \neq 0$ OK

— " — — " — $\nu = 1/2$?

— " — mode III not OK

- Gränslösning för försvinnande lågt hårdnande.

- Spricktillväxt

- Fortvarighet