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## Asymptotic Crack Tip Fields at Steady-Crack Growth and Vanishing Hardening

Talk given at Nordic Mechanics Days in Trondheim, Norway, Orationem Meam

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1989

*Document Version:*

Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*

Ståhle, P. (1989). Asymptotic Crack Tip Fields at Steady-Crack Growth and Vanishing Hardening: Talk given at Nordic Mechanics Days in Trondheim, Norway, Orationem Meam.

*Total number of authors:*

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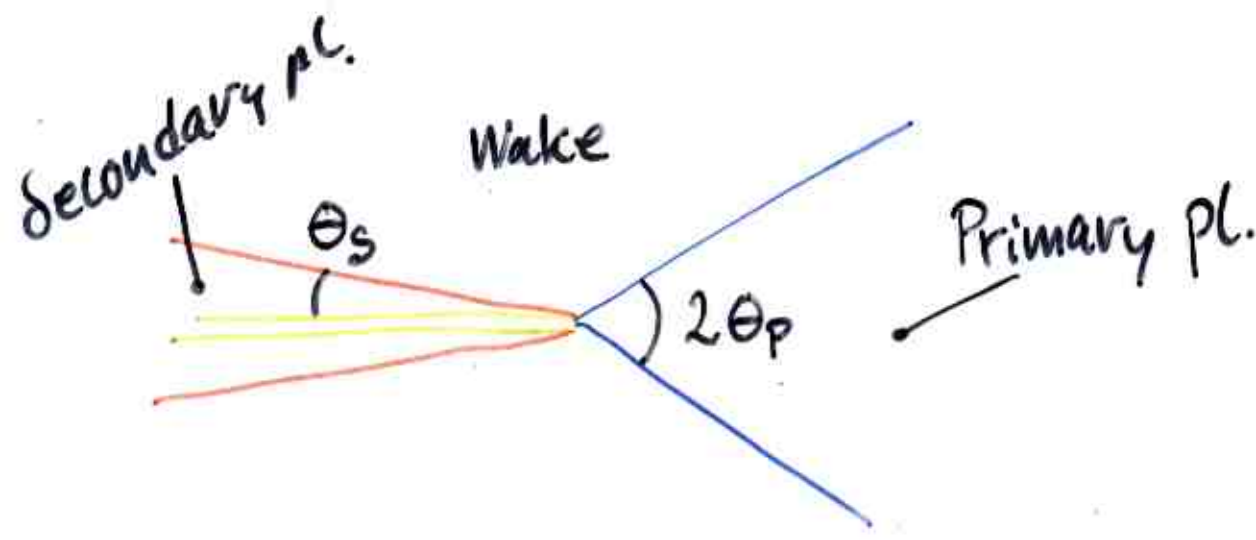
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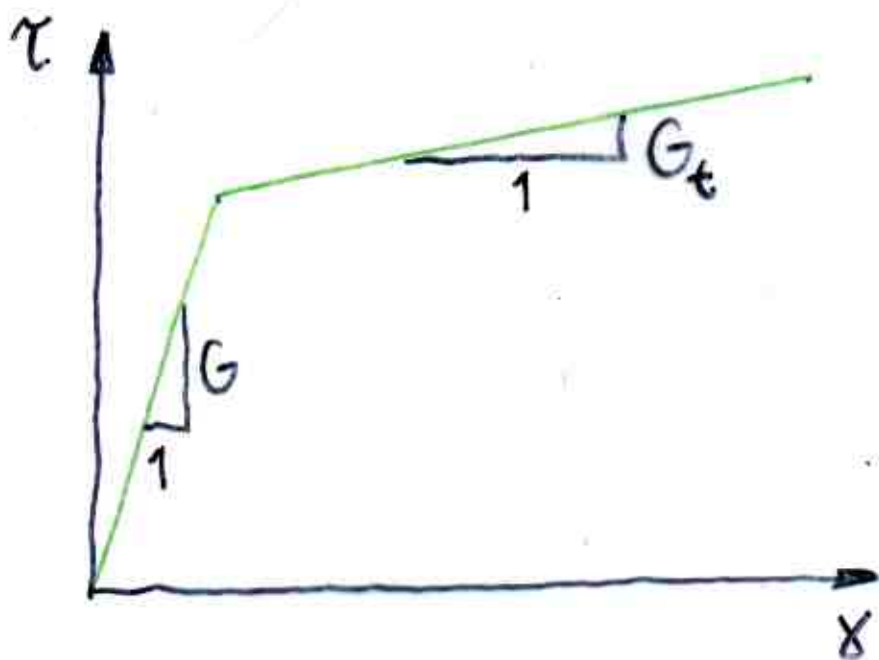
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# Asymptotic fields in steady crack growth with vanishing strain hardening

- Mode III
- Steady Growth
- Asymptotic field



- Linear Strain Hardening



$$\alpha = G_t / G$$

Vanishing Rates

$$\alpha \rightarrow 0$$

Serieutveckla  $r^s$  för små  $s$ .

$$r^s = e^{s \ln r} = 1 + s \ln r + \frac{s^2}{2} \ln^2 r + \dots$$

$$\phi = (\cos \theta + s f(\theta)) r^s \approx \cos \theta + s (\cos \theta \ln r + f(\theta))$$

$$\zeta_r = s F(\theta) r^s \approx s F(\theta)$$

$$\dot{\zeta} = -s (\cos \theta - F \sin \theta) r^{s-1} \approx -s (\cos \theta - F \sin \theta) r^{-1}$$

$$\dot{\chi}_r = \dot{\zeta}_r + \kappa^{-1} \dot{\zeta} \zeta_r \zeta^{-1} = \left[ -\sin \theta + \frac{s^2}{\alpha} (\cos \theta - F \sin \theta) F \right] r^{s-1}$$

$$\dot{\chi}_r = \frac{\partial \dot{W}}{\partial r} \Rightarrow$$

$$\dot{W} = \left[ -\sin \theta + \frac{s^2}{\alpha} (\cos \theta - F \sin \theta) F \right] \left( -\frac{r^s}{s} + \frac{R(\theta)}{s} \right) \approx$$

$$\approx \left[ -\sin \theta + \frac{s^2}{\alpha} (\cos \theta - F \sin \theta) F \right] \ln \frac{R(\theta)}{r}$$

ifr. Chitaley & McLintock (1971)

$$\dot{W} = -\sin \theta \ln \frac{R(\theta)}{r}$$

Spänningshastighetsfunktioner  
separabel.

$$\phi = (\cos \theta + s f(\theta)) r^s$$

$$\tau_r = s F(\theta) r^s$$

$$\dot{\tau} = -s(\omega \theta - F \omega \theta) r^{s-1}$$

Kompatibilitet  $\mathcal{O}(s^2)$

$$-\alpha \cos \theta r^{s-1} - s \frac{\partial}{\partial r} (r \dot{\tau}) +$$

$$+ s^2 [F'(\cos \theta - 2F \sin \theta) - F^2 \omega \theta - F \sin \theta] = 0$$

Störning  $\mathcal{O}(\alpha^{1/2})$ .  $s = -0.81 \alpha^{1/2}$



Mode III. Störning av centered-fan-fält.

$$\phi = \cos \theta + s f(r, \theta) \quad \text{spänningshest. funkt.}$$

$$\tau_{rz} = s F(r, \theta)$$

$$\dot{\zeta} = s \left( \frac{\partial f}{\partial r} - F \sin \theta r^{-1} \right)$$

Kompatibilitet

$$-\alpha \cos \theta r^{-1} - \frac{\partial}{\partial r} (r \dot{\zeta}) = 0$$

$$\Rightarrow \dot{\zeta} = \alpha \left( \cos \theta \frac{\ln r}{r} - \frac{(\cos \theta)}{r} \right)$$

Störning  $O(\alpha)$  ej separabel  
lösning.

yttre fält  
 = 0 : allmänhet  
 $\sigma_e = \sigma_{e0} + \alpha^{1/2} \sigma_{e1} + \alpha \sigma_{e2} + \dots$

Asymptotiskt fält I

$$\sigma_e = \sigma_{e0}(\theta) + \alpha^{1/2} \sigma_{e1}(r, \theta) + \dots$$

ELASTISKT

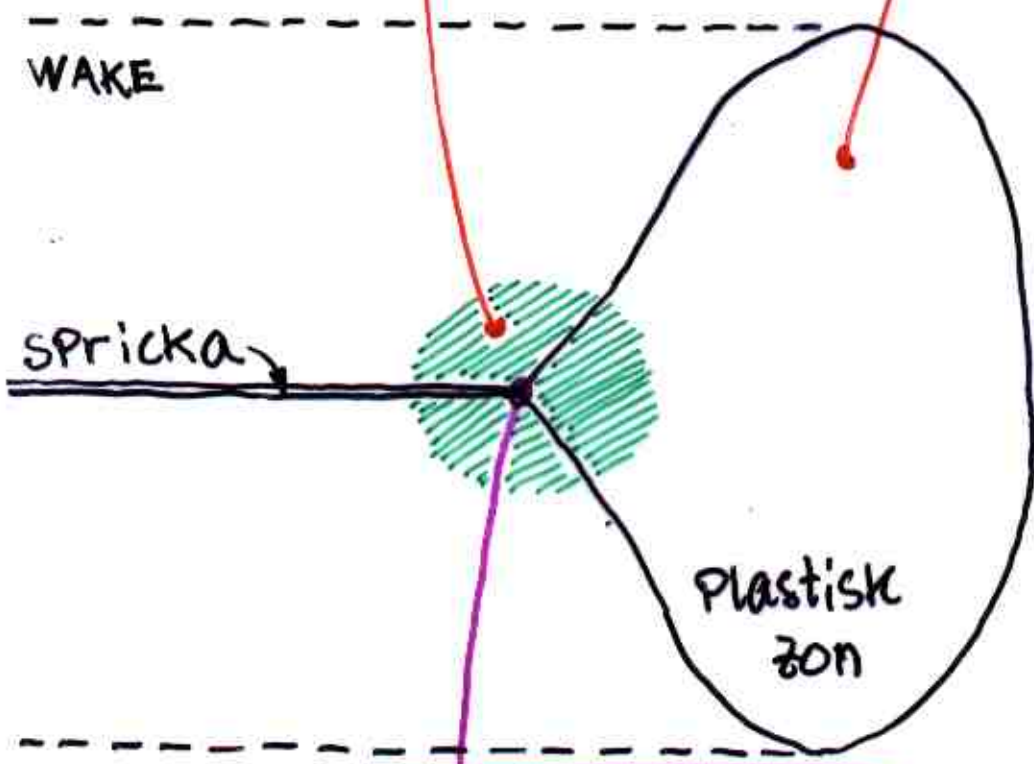
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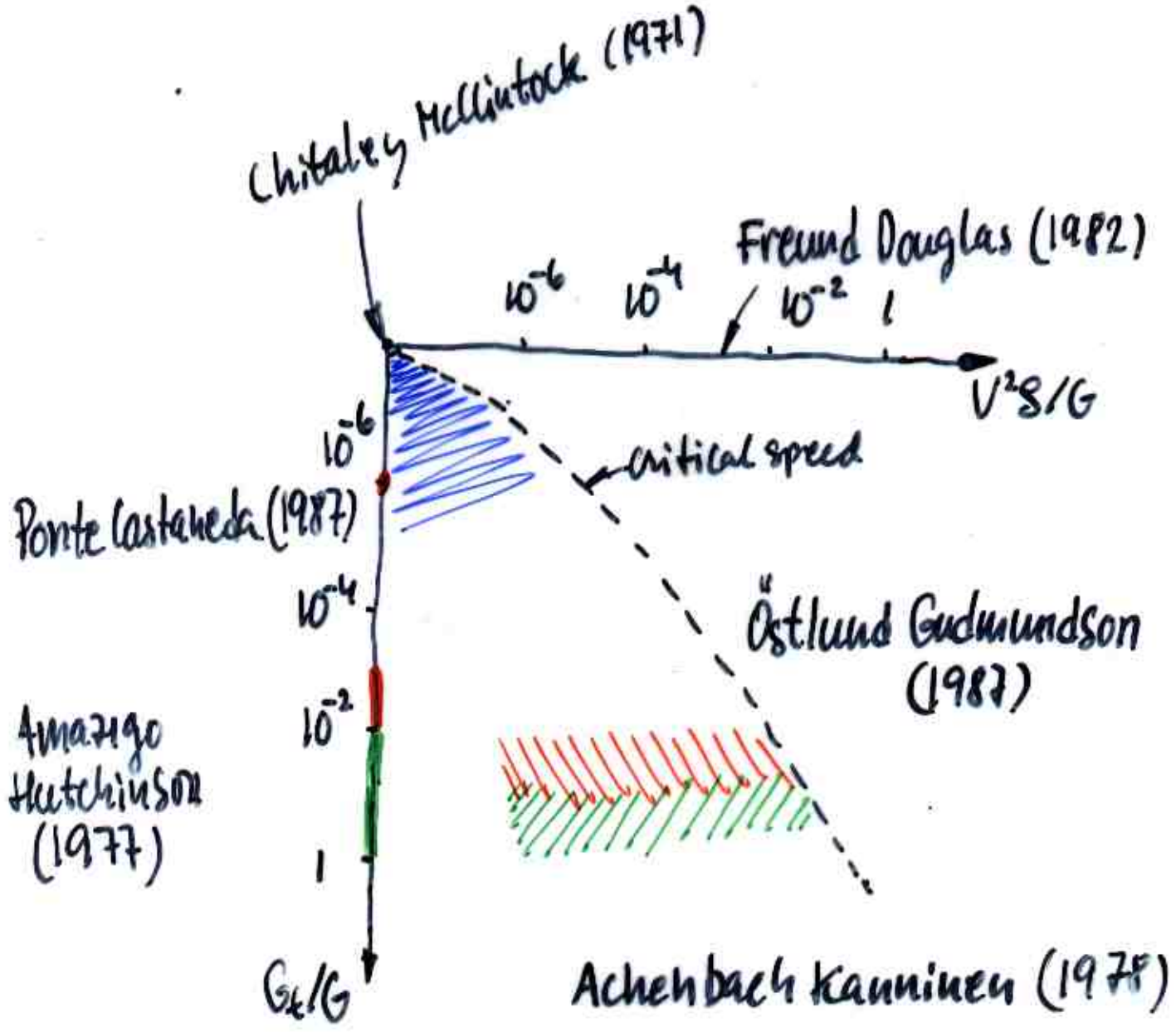
SPRICKA

Plastisk  
zon

Asymptotiskt fält II

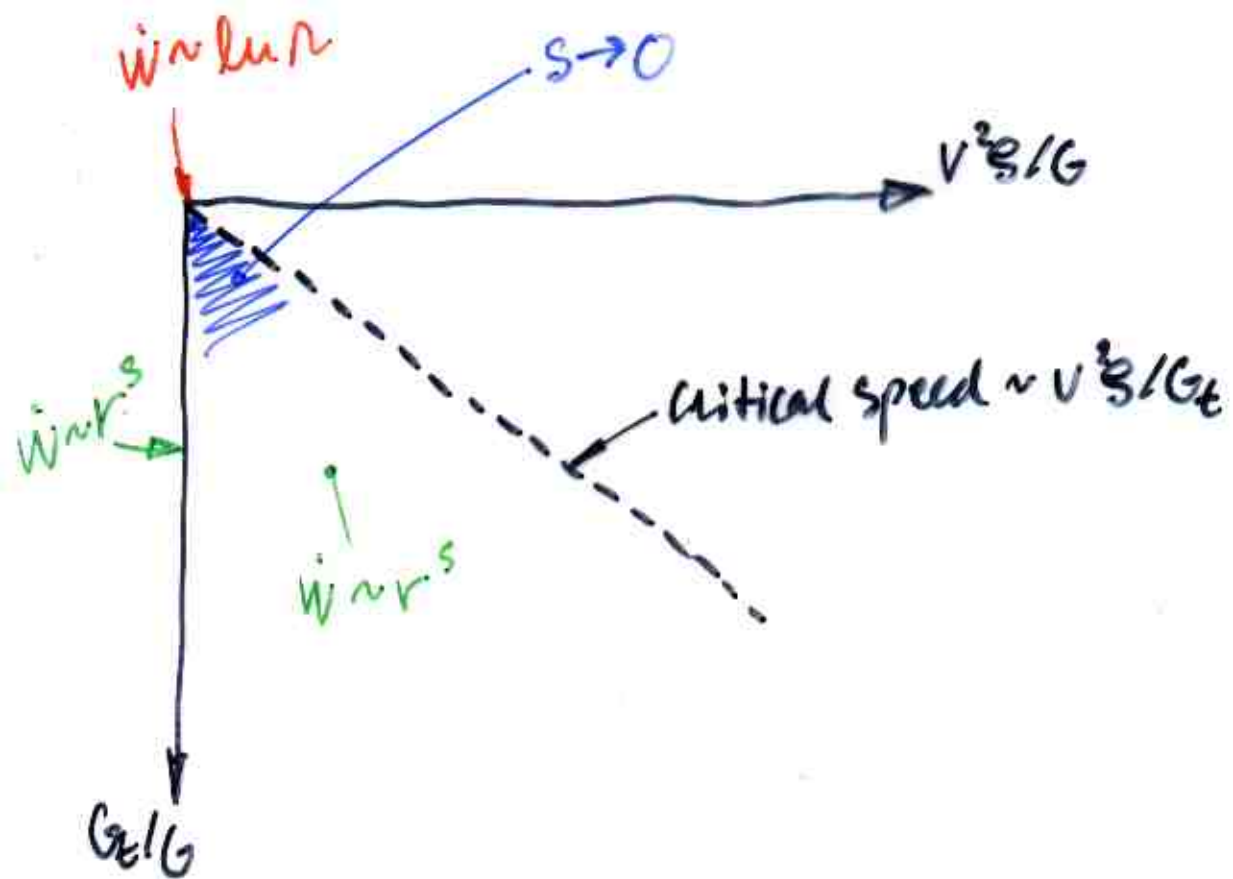
$$\sigma_e = \psi(\theta) r^s$$





$\dot{a} (S/G_x)^{1/2}$	$-S\alpha^{-1/2}$	$\Theta_p$	$\pi - \Theta_s$
0	0.81	32.8	0.28
1	0.72	31.0	0.30
1.45	0.62	28.9	0.32





$$A(\theta)r^s - B(\theta)r^{-s} \rightarrow C(\theta) \ln(r/R)$$

as  $s \rightarrow 0$

$$C(\theta) = s [A(\theta) + B(\theta)]$$

$$R(\theta) = \exp(A(\theta) - B(\theta))$$

$V(\dot{S}/G_t)^{1/2}$	$-S\alpha^{-1/2}$	$\theta_p$	$\theta_s$
0	0.812	32.8	0.28
1	0.722	31.0	0.30
1.45	0.615	28.9	0.32

- Vanishingly small hardening rates may produce significant changes in stress distribution.
- In practice, stress rates may be discontinuous for perfectly plastic materials.

## Series expansion of $r^s$

$$r^s = e^{s \ln r} = 1 + s \ln r + \frac{s^2}{2} \ln^2 r + \dots$$

$$\zeta_{r^s} = F(\theta) r^s \approx F(\theta)$$

$$\dot{\zeta}_{r^s} = -\sin \theta r^{s-1} \approx -\sin \theta r^{-1}$$

$$\dot{\chi}_{r^s} = \left[ -\sin \theta + \frac{1}{\alpha} (s \cos \theta - F \sin \theta) F \right] r^{s-1}$$

$$\dot{\chi}_{r^s} = \frac{\partial \dot{w}}{\partial r}$$

$$\dot{w} = \left[ -\sin \theta + \frac{1}{\alpha} (s \cos \theta - F \sin \theta) F \right] \underbrace{\left( -\frac{r^s}{s} + \frac{R(\theta)}{s} \right)}_{\approx \ln \frac{R(\theta)}{r}}$$

cf. Chitale & Mullintock

$$\dot{w} = -\sin \theta \ln \frac{R(\theta)}{r}$$

# Solution

Assumed

$$\tilde{r}_{rz} \sim r^5$$

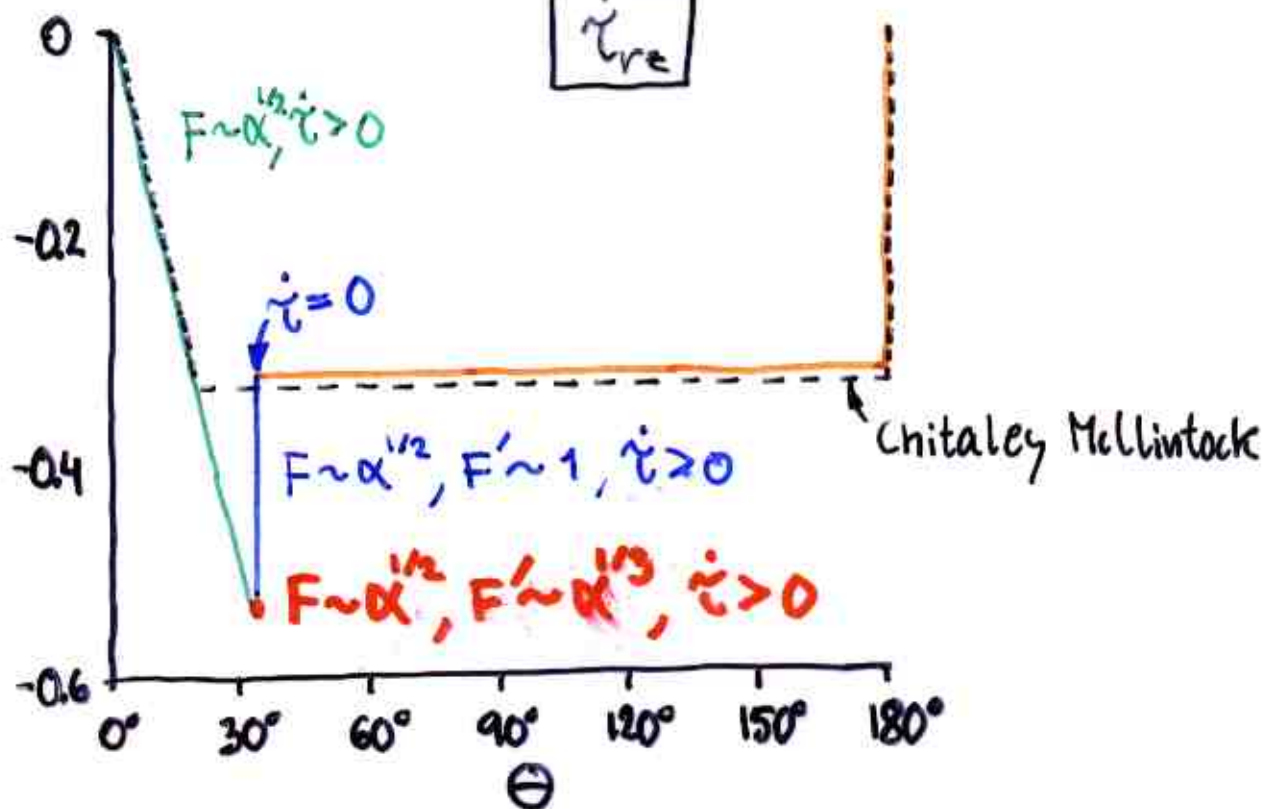
Small deviation from centered fan straight ahead

$$\tilde{r}_{rz} = F(\theta)r^5$$

$$\alpha = G_t/G$$

$$S \sim \alpha^{1/2}$$

$$\tilde{r}_{rz}$$



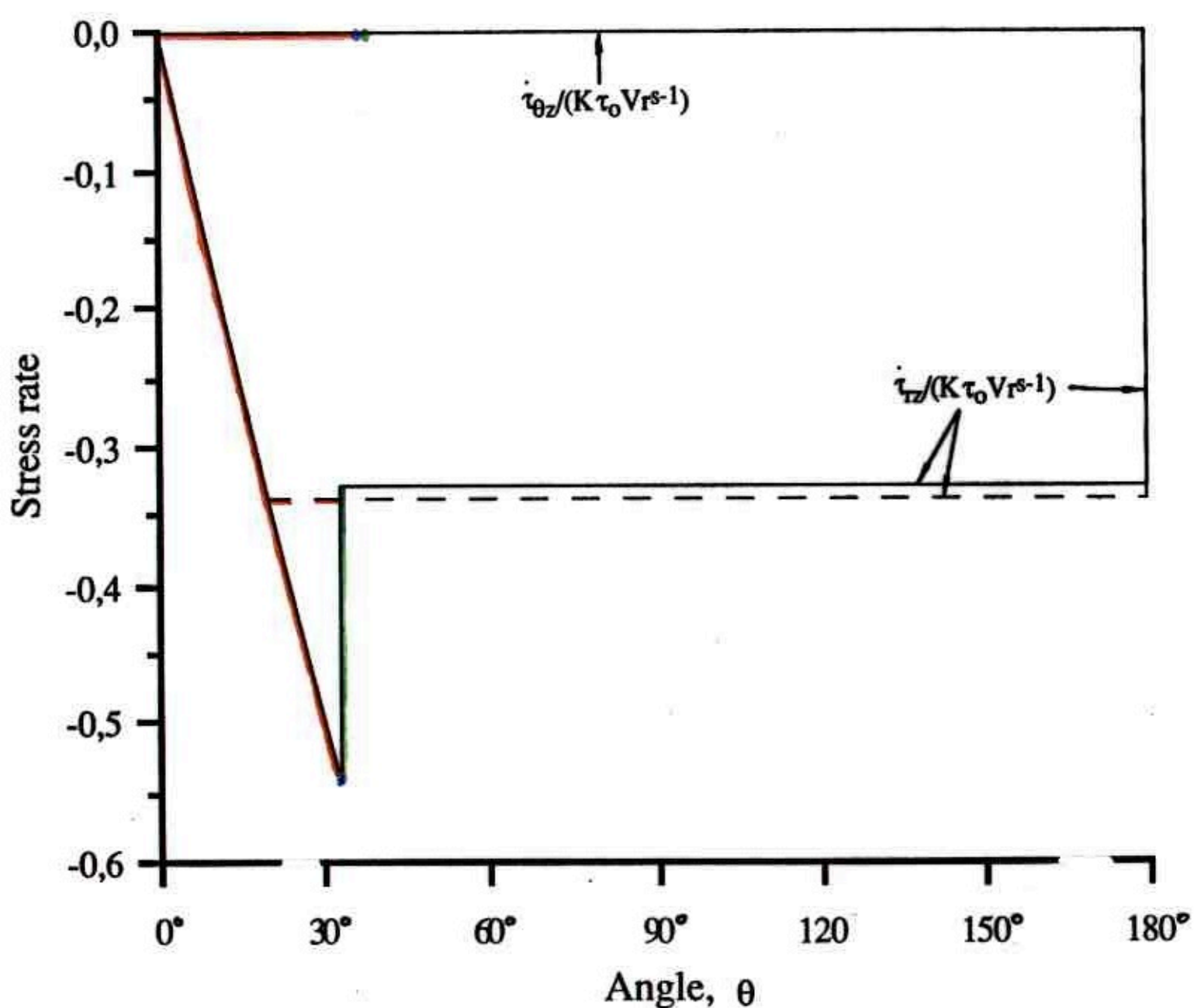
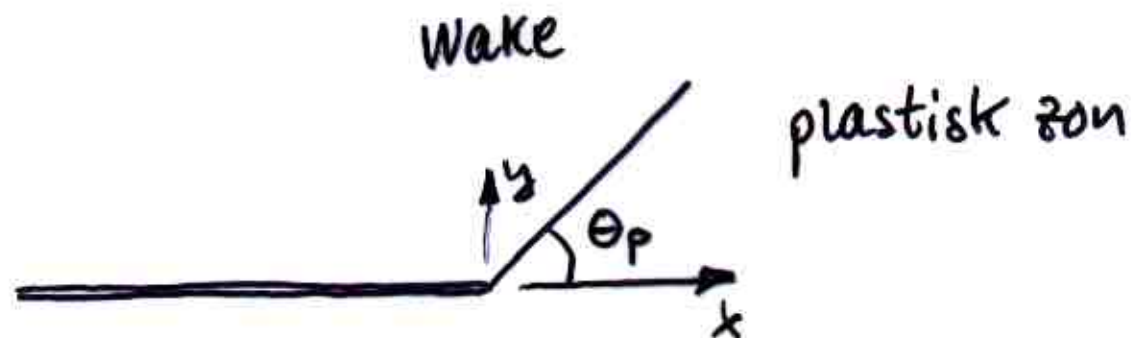


Fig. 7. Angular distribution of the polar stress rate components  $\dot{\tau}_{rz}$  and  $\dot{\tau}_{\theta z}$ . The stress rates are normalized so that the effective stress equals unity straight ahead of the crack tip. Dashed curves show the result by CHITALEY and MCCLINTOCK [4].

- Idealplastiska gränslösningar har stor betydelse.
- Grad av hårdnande har ringa betydelse.
- Dynamiska effekter även när  $\dot{a} \rightarrow 0$ .
- Val av hårdnande?



## Ideal plastiskt.

$\dot{\sigma}_e = 0$  i den plastiska zonen.

Plastisk deformation obestämd.

I allmänhet  $\dot{\epsilon}_e \neq 0$  när  $\theta \rightarrow \theta_p$   
i den plastiska zonen.

Mode I Druggan, Rice & Sham (1982)

Mode II Ponte Castañeda (1986)

Mode III Chitaley & McClinton (1971)

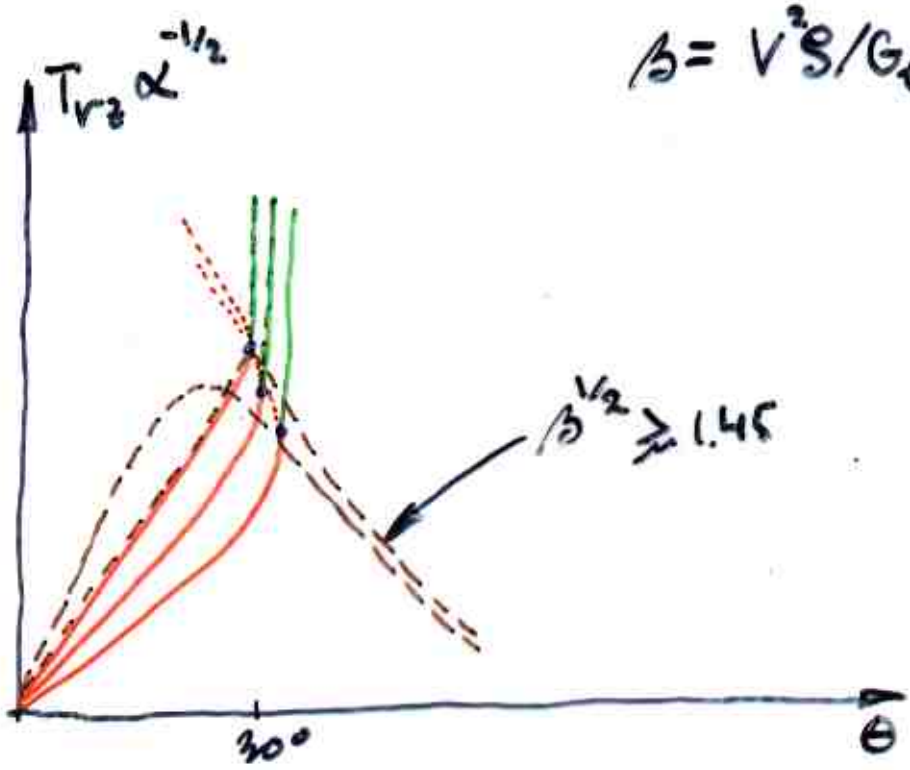
## Hårdnande material

$\dot{\sigma}_e \rightarrow 0 \Rightarrow \dot{\epsilon}_e \rightarrow 0$  när  $\theta \rightarrow \theta_p$

$$T_{r2}' (s + 2T_{r2} \tan \theta) - (sT_{r2} \tan \theta - T_{r2}^2 - \alpha)(1 - \beta \sin^2 \theta) -$$

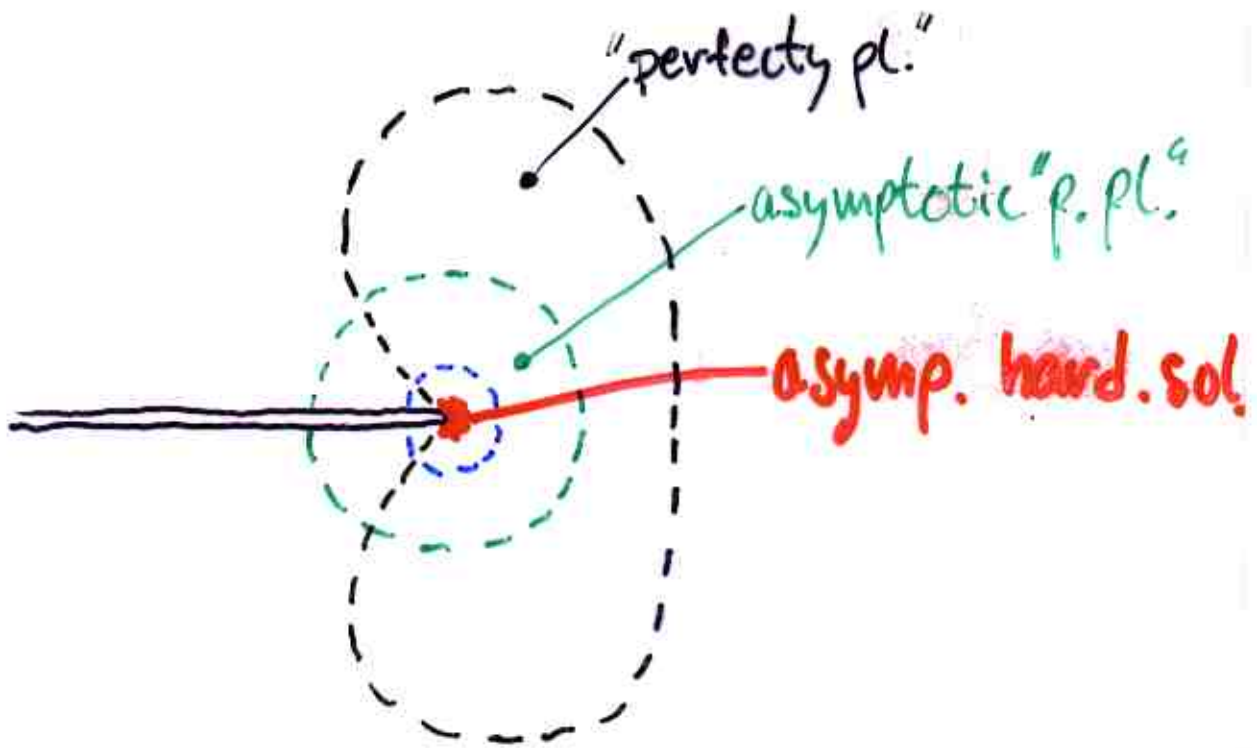
$$-s^2 - sT_{r2} \tan \theta + \beta T_{r2} \sin^2 \theta (s \tan \theta - 3T_{r2}) -$$

$$-2\beta \sin \theta T_{r2} (s \cos \theta + \beta T_{r2} \sin^3 \theta)(1 - \beta \sin^2 \theta)^{-1} = 0$$





rate independent metallic materials



Stationary cracks OK

growing mode I  $v \neq 0$  OK

- " - - " -  $v = 1/2$  ?

- " - mode III not OK

- Gränslösning för försvinnande lågt hårdnande.

- Spricktillväxt

- Fortvarighet

