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## Mode III Dynamic Crack Growth and asymptotically Vanishing Hardening Rate

Talk given at the 7th International Congress on Fracture (ICF7), Houston, TX. USA, Orationem Meam

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1989

*Document Version:*

Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*

Ståhle, P. (1989). Mode III Dynamic Crack Growth and asymptotically Vanishing Hardening Rate: Talk given at the 7th International Congress on Fracture (ICF7), Houston, TX. USA, Orationem Meam.

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1

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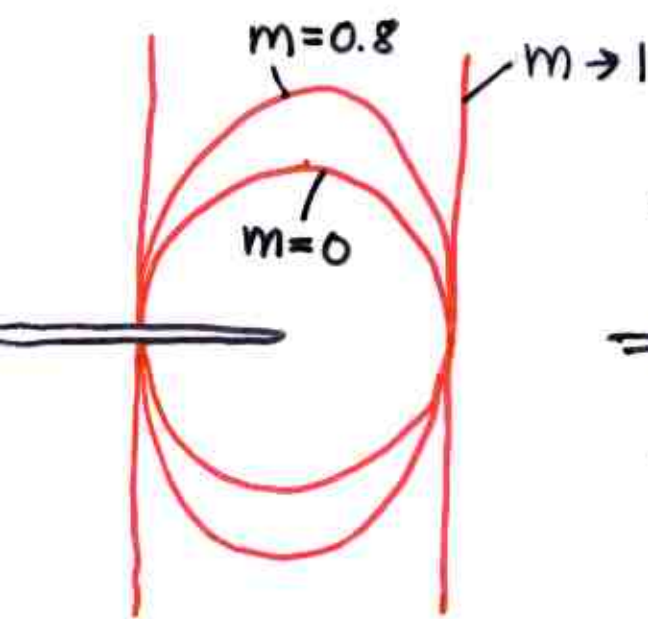
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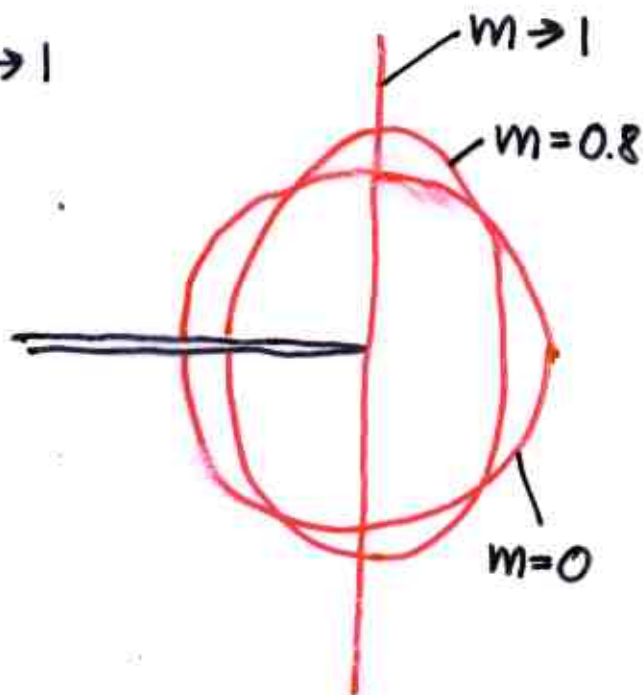
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# Mode III - High Strain Rates (Stähle, 1989)



Constant  $K_{III}$



Constant  $G$

Shape of the plastic zone:

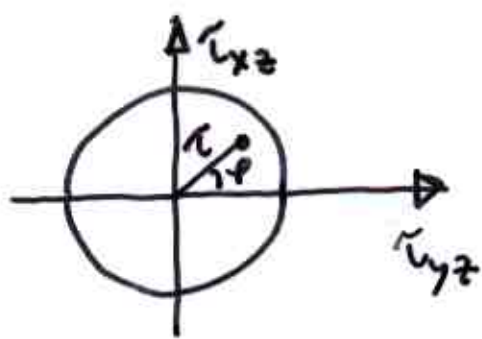
$$4\tau^4 x^4 - 4\kappa^2 \tau^2 x^3 + 4\tau^2 x^3 - 4\kappa^2 x^2 - y^2 - 2\kappa^2 y^2 + \kappa^4 y^2 + 4\tau^2 xy^2 - 4\kappa^2 xy^2 + 8\tau^4 x^2 y^2 + 4\tau^4 y^4 = 0$$

Hodograph transform:

$$\tau_{xz}(x, y), \tau_{yz}(x, y) \rightarrow x(\tau_{xz}, \tau_{yz}), y(\tau_{xz}, \tau_{yz})$$

Elastic plastic boundary at:

$$\tau_{xz}^2 + \tau_{yz}^2 = \tau_Y^2$$



$$\begin{aligned}
 & (1 - m^2 \sin^2 \varphi) \Delta_{\tau} \dot{w} + m^2 \left( \sin \varphi \left[ \frac{\partial}{\partial \tau} - \frac{1}{\tau} \right] \frac{\partial \dot{w}}{\partial \varphi} - \cos 2\varphi \frac{\partial^2 \dot{w}}{\partial \tau^2} \right) = \\
 & = \frac{2\lambda (\tau - \tilde{\tau}_r)^n}{\tau^4 \tilde{\tau}_r^n} \left\{ \frac{(n-1)\tau + \tilde{\tau}_r}{\tau - \tilde{\tau}_r} (1 - m^2 [1 + 2\cos^2 \varphi + \frac{m^2}{2} \sin^2 2\varphi]) + \right. \\
 & \left. + m^2 [1 - 4\cos^2 \varphi - m^2 \sin \varphi] \right\} \sin \varphi
 \end{aligned}$$

$n=1$

$$\begin{aligned}
 \dot{w} &= \left\{ 2\dot{\delta}_0 r + [3v + \dot{\delta}_0 (3r - R)] \left(\frac{R}{r}\right)^{1/2} \right\} \frac{\tilde{\tau}_r \sin \theta/2}{3\mu} \\
 & + m^2 \left\{ \right\} \frac{\tilde{\tau}_r \sin \theta/2}{3\mu} + m^2 \left\{ \right\} \frac{\tilde{\tau}_r \sin 3\theta/2}{3\mu} + \mathcal{O}(m^4)
 \end{aligned}$$

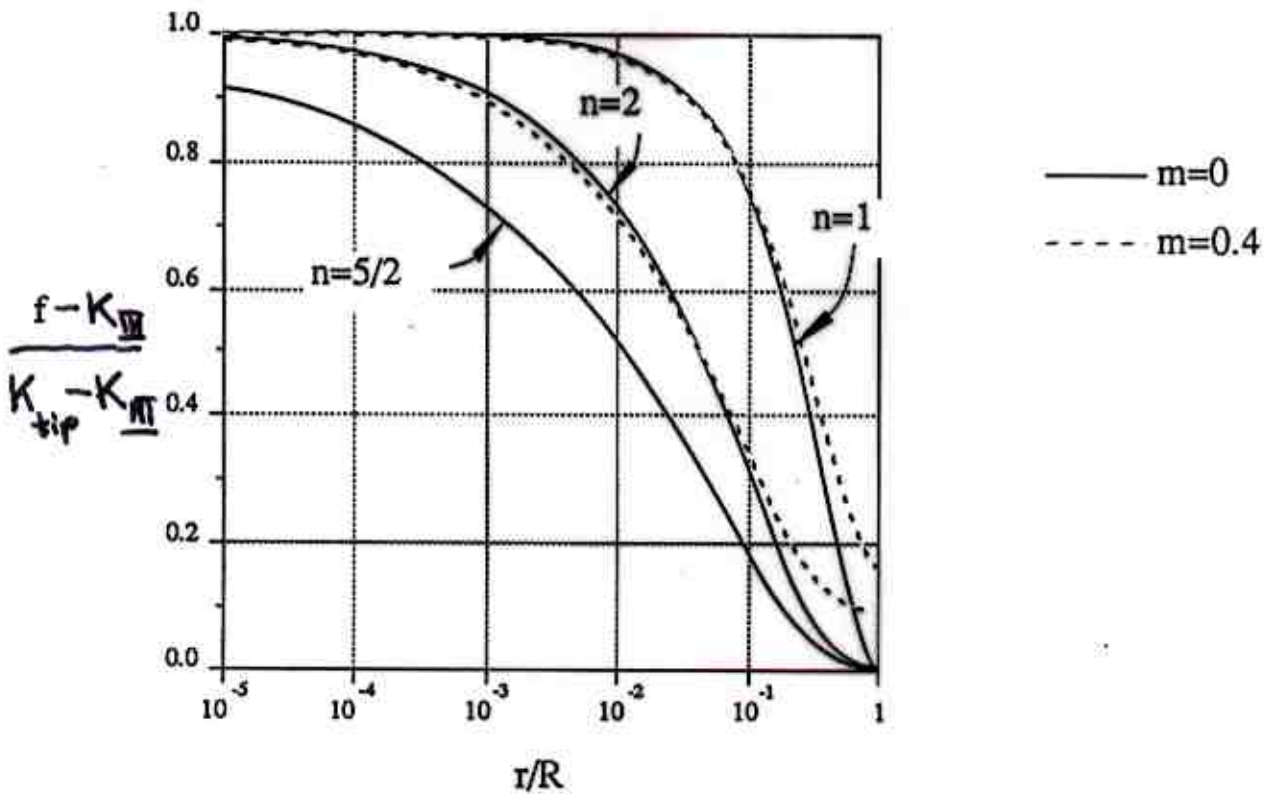
$n=2$

$$\begin{aligned}
 \dot{w} &= \left\{ 2\dot{\delta}_0 (r + 3R) + [3v + 4\dot{\delta}_0 (3r - 2R)] \left(\frac{R}{r}\right)^{1/2} \right\} \frac{\tilde{\tau}_r \sin \theta/2}{3\mu} + \\
 & + \mathcal{O}(m^2)
 \end{aligned}$$

$n=2.5$

$$\begin{aligned}
 \dot{w} &= \left\{ \dot{\delta}_0 R (33z^5 + 40z^3 + 15z) + (12v - 15\dot{\delta}_0 R \tan^{-1} z) \left(\frac{R}{r}\right)^{1/2} \frac{\tilde{\tau}_r \sin \theta/2}{3\mu} + \right. \\
 & \left. + \mathcal{O}(m^2) \right\} ; z = \left[ \left(\frac{R}{r}\right)^{1/2} - 1 \right]^{1/2}
 \end{aligned}$$

$$f = \frac{\dot{w}\sqrt{2\pi r}}{v \sin \theta/2} \Rightarrow f \rightarrow \begin{cases} K_{III} & \text{as } r \rightarrow \infty \text{ and } \theta \rightarrow 0 \\ K_{tip} & \text{as } r \rightarrow 0 \text{ and } \theta \rightarrow 0 \end{cases}$$



90% obtained at

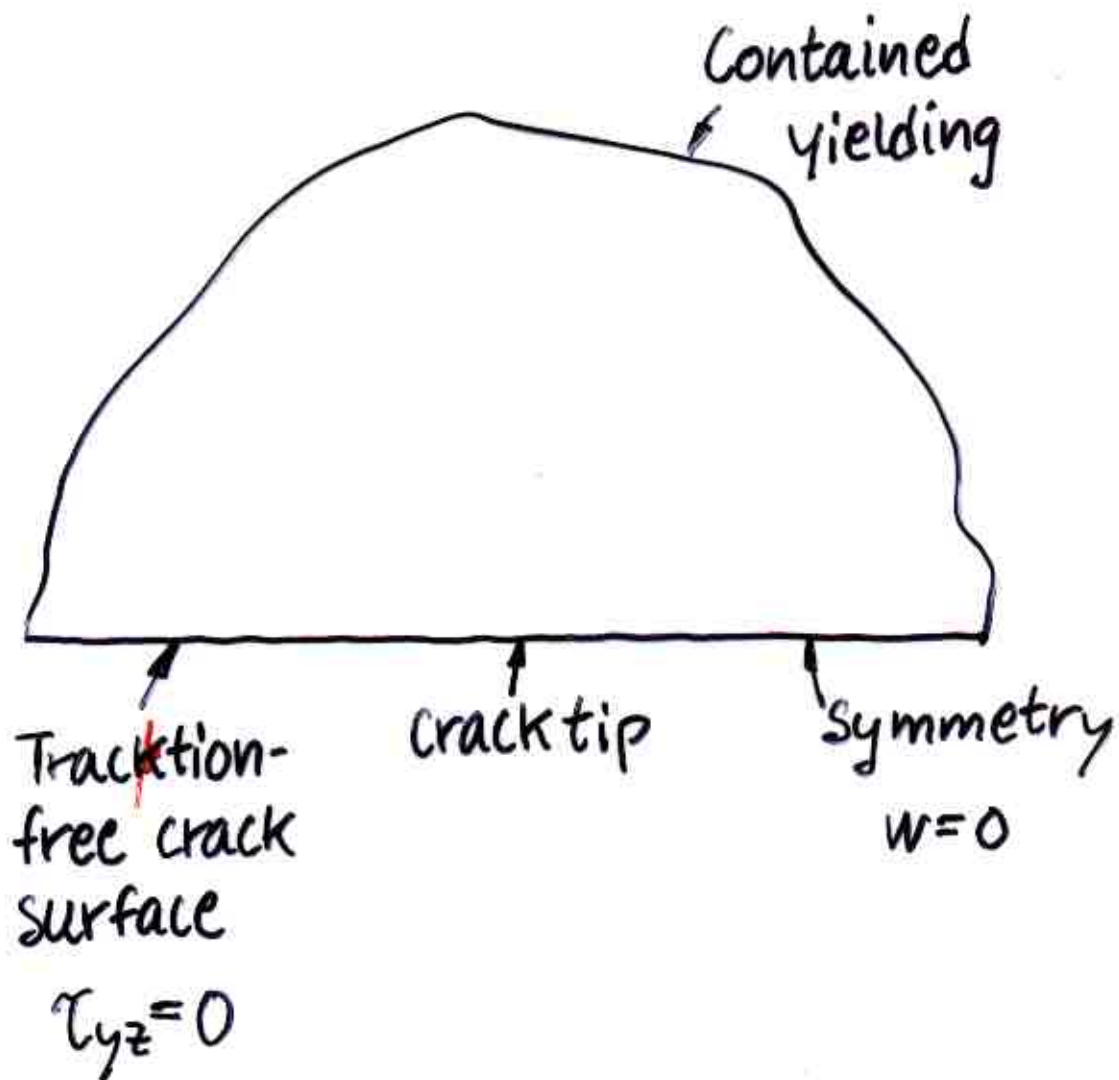
$R/30$  for  $n=1$

$R/1000$  for  $n=2$

$R/10^5$  for  $n=2.5$

- Mode III
- Asymptotic Field
- Steady Quasi Static Growth
- Inertia Terms Included

# Boundary Conditions





Steady state

$$(\dot{\phantom{x}}) \equiv -V \frac{\partial}{\partial x}$$

Linear strain hardening

$$G_t \dot{\gamma}_{xz} = \alpha \dot{\tau}_{xz} + (1-\alpha) \tau_{xz} \dot{\tau}^{-1}$$

$$G_t \dot{\gamma}_{yz} = \alpha \dot{\tau}_{yz} + (1-\alpha) \tau_{yz} \dot{\tau}^{-1}$$

Equation of motion

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho \ddot{w}$$

Compatibility

$$\frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x} = 0$$

# Primary plastic zone

Perfectly plastic

$$v_t = 0$$

$$\tau_{rz} = 0$$

$$\dot{\tau}_{rz} = -kr^{-1} \sin \theta$$

$$\tau_{\theta z} = k$$

$$\dot{\tau}_{\theta z} = 0$$

Here

$$\tau_{rz} = kr^s T_{rz}$$

$$\dot{\tau}_{rz} = kr^{s-1} (-\sin \theta + \dot{t}_{rz})$$

$$\tau_{\theta z} = kr^s (1 + T_{\theta z})$$

$$\dot{\tau}_{\theta z} = kr^{s-1} t_{\theta z}$$



Continuity conditions

$\dot{w}, w$

$\dot{L}_{12}, L_{12}$