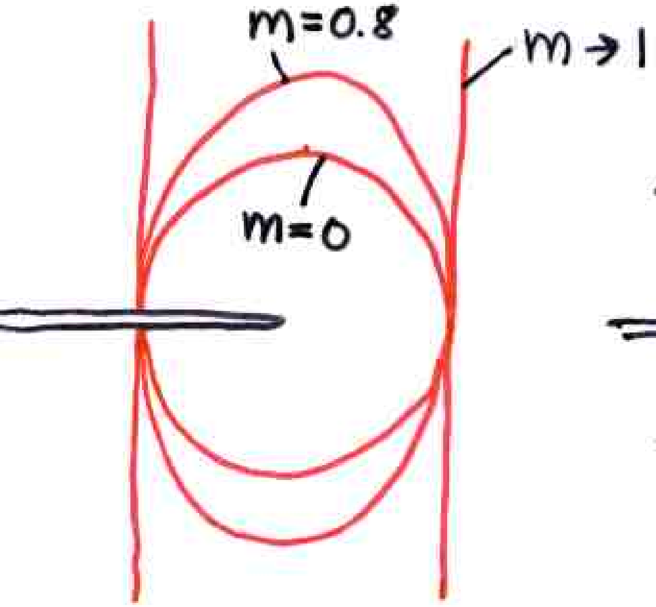
Mode III Dynamic Crack Growth and asymptotically Vanishing Hardening Rate

Talk given at the 7th International Congress on Fracture (ICF7), Houston, TX. USA, Orationem Meam

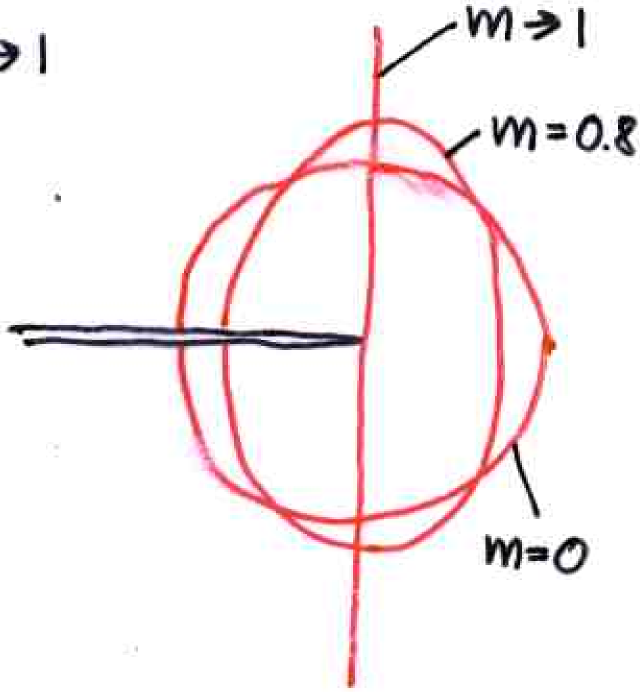
Ståhle, P.

1989

Mode III - High Strain Rates (Ståhle, 1989)



Constant K_{III}



Constant G

Shape of the plastic zone:

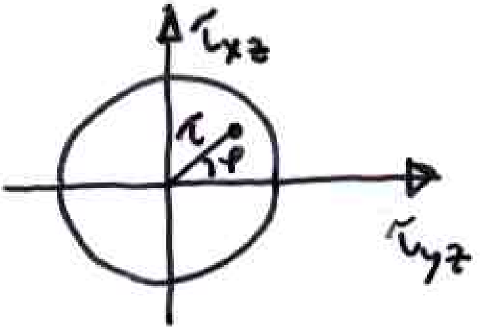
$$4\tau^4x^4 - 4\alpha^2\tau^2x^3 + 4\tau^2x^3 - 4\alpha^2x^2 - y^2 - 2\alpha^2y^2 +$$
$$+ \alpha^4y^2 + 4\tau^2xy^2 - 4\alpha^2xy^2 + 8\tau^4x^2y^2 + 4\tau^4y^4 = 0$$

Hodograph transform:

$$\tau_{xz}(x,y),\ \tau_{yz}(x,y) \longrightarrow x(\tau_{xz}, \tau_{yz}),\ y(\tau_{xz}, \tau_{yz})$$

Elastic plastic boundary at:

$$\tau_{xz}^2 + \tau_{yz}^2 = \tau_Y^2$$



$$(1-m^2\sin^2\varphi)\,\Delta_\tau \dot{w} + m^2\left(\sin\varphi\left[\frac{\partial}{\partial\tau}-\frac{1}{\tau}\right]\frac{\partial\dot{w}}{\tau\,\partial\varphi} - \cos 2\varphi\,\frac{\partial^2\dot{w}}{\partial\tau^2}\right) =$$

$$= \frac{2\lambda(\tau-\tau_Y)^n}{\tau^4\tau_Y^n}\left\{\frac{(n-1)\tau+\tau_Y}{\tau-\tau_Y}\left(1-m^2\left[1+2\cos^2\varphi+\frac{m^2}{2}\sin^2 2\varphi\right]\right)+\right.$$

$$\left. + m^2\left[1-4\cos^2\varphi - m^2\sin\varphi\right]\right\}\sin\varphi$$

$n=1$

$$\dot{w} = \left\{2\dot{\gamma}_0 r + [3v+\dot{\gamma}_0(3r-R)]\left(\frac{R}{r}\right)^{1/2}\right\}\frac{\tau_Y\sin\theta/2}{3\mu}$$

$$+ m^2\{\quad\}\frac{\tau_Y\sin\theta/2}{3\mu} + m^2\{\quad\}\frac{\tau_Y\sin 3\theta/2}{3\mu} + O(m^4)$$

$n=2$

$$\dot{w} = \left\{2\dot{\gamma}_0(r+3R) + [3v+4\dot{\gamma}_0(3r-2R)]\left(\frac{R}{r}\right)^{1/2}\right\}\frac{\tau_Y\sin\theta/2}{3\mu} +$$

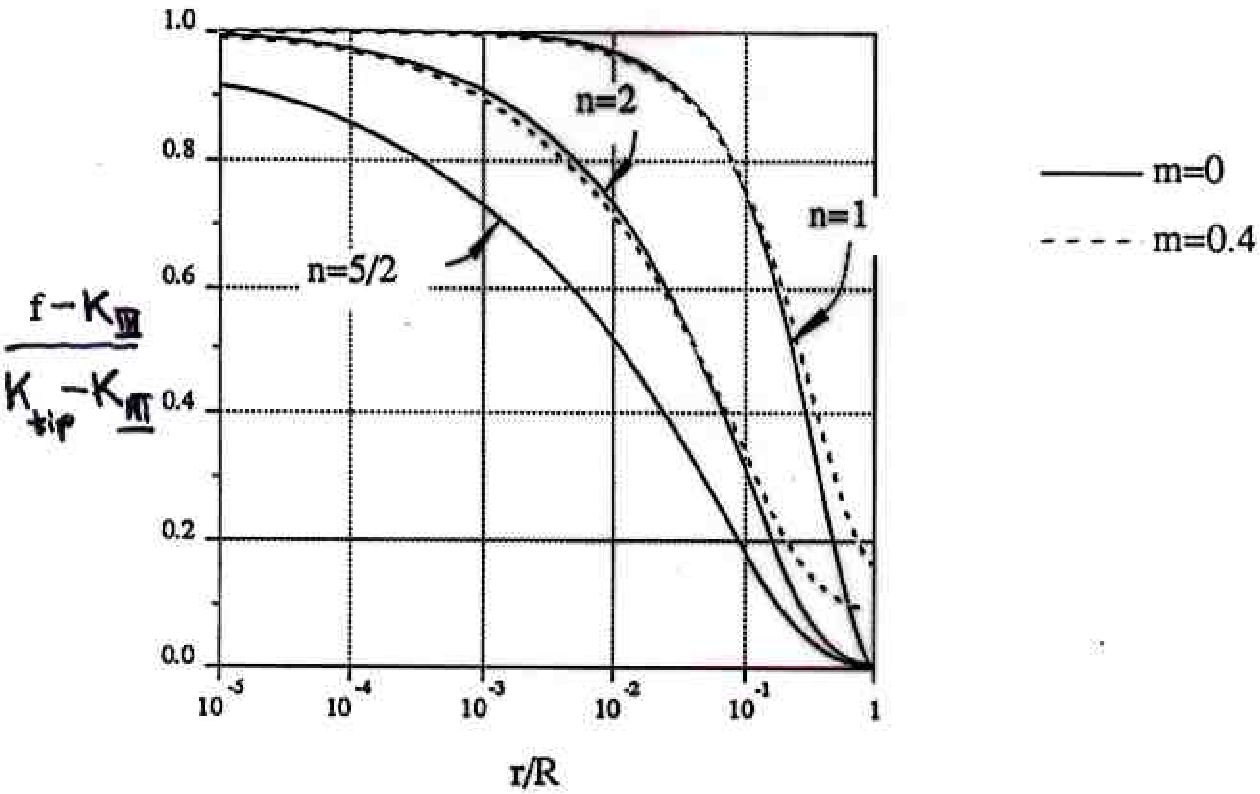
$$+ O(m^2)$$

$n=2.5$

$$\dot{w} = \{\dot{\gamma}_0 R(33z^5+40z^3+15z) + (12v - 15\dot{\gamma}_0 R\tan^{-1}z)\left(\frac{R}{r}\right)^{1/2}\frac{\tau_Y\sin\theta/2}{3\mu} +$$

$$+ O(m^2) \quad ; \quad z = \left[(R/r)^{1/2}-1\right]^{1/2}$$

$$f = \frac{\dot{w}\sqrt{2\pi r}}{v \sin\theta/2} \Rightarrow f \to \begin{cases} K_{III} & \text{as } r \to \infty \text{ and } \theta \to 0 \\ K_{tip} & \text{as } r \to 0 \text{ and } \theta \to 0 \end{cases}$$



90% obtained at

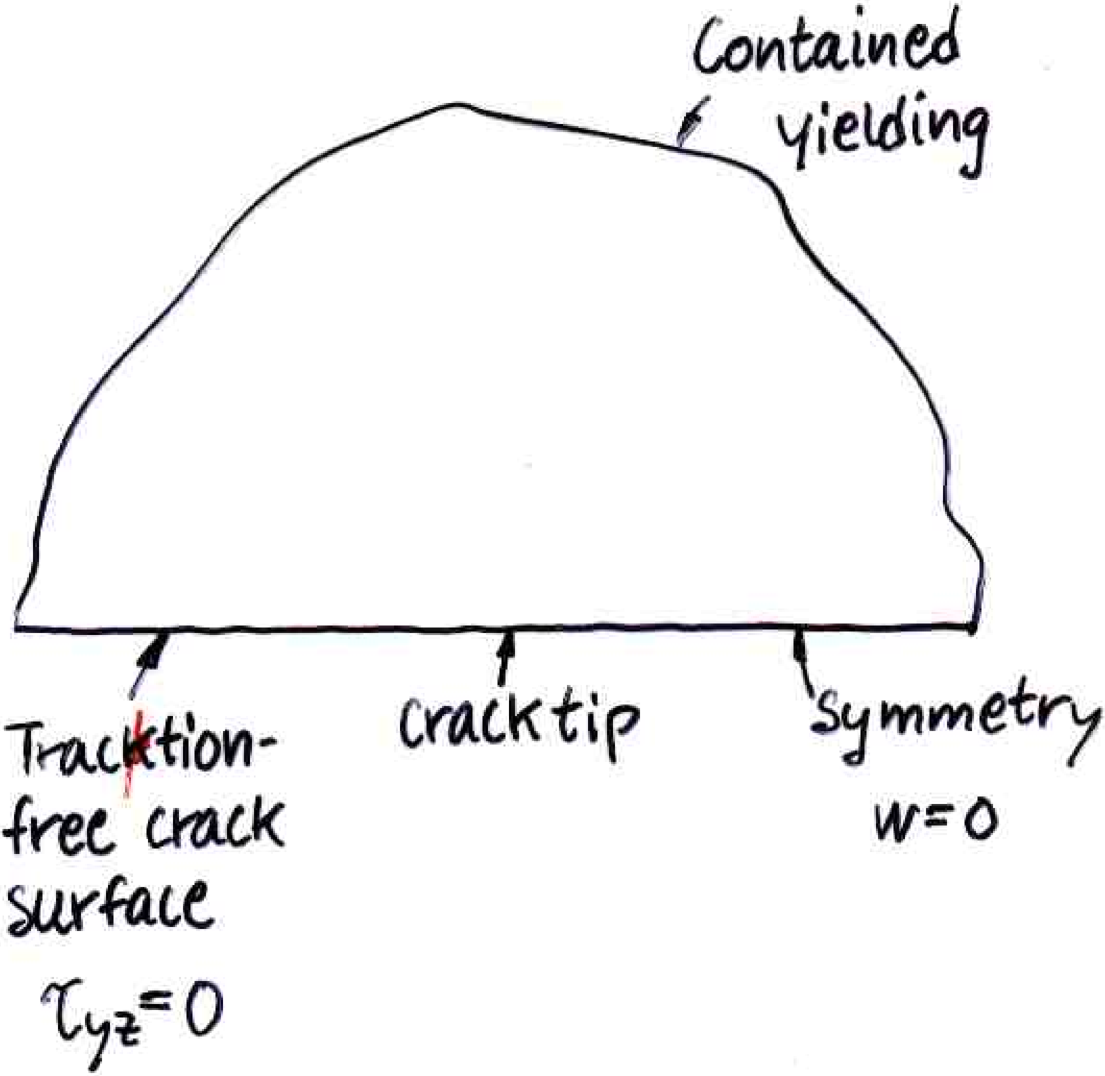
$R/30$ for $n=1$

$R/1000$ for $n=2$

$R/10^5$ for $n=2.5$

- Mode III
- Asymptotic Field
- Steady Quasi Static Growth
- Inertia Terms Included

Boundary Conditions



Steady state

$$(\dot{\ }) \equiv -V\frac{\partial}{\partial x}$$

Linear strain hardening

$$G_t\dot{\gamma}_{xz} = \alpha\dot{\tau}_{xz} + (1-\alpha)\tau_{xz}\dot{\tau}\tau^{-1}$$

$$G_t\dot{\gamma}_{yz} = \alpha\dot{\tau}_{yz} + (1-\alpha)\tau_{yz}\dot{\tau}\tau^{-1}$$

Equation of motion

$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} = \rho\ddot{w}$$

Compatibility

$$\frac{\partial\gamma_{xz}}{\partial y} - \frac{\partial\gamma_{yz}}{\partial x} = 0$$

Primary plastic zon

Perfectly plastic $\sigma_t = 0$

$\tau_{rz} = 0$ $\dot{\tau}_{rz} = -kr^{-1}\sin\theta$

$\tau_{\theta z} = k$ $\dot{\tau}_{\theta z} = 0$

Here

$\tau_{rz} = kr^s T_{rz}$ $\dot{\tau}_{rz} = kr^{s-1}(-\sin\theta + \dot{t}_{rz})$

$\tau_{\theta z} = kr^s(1 + T_{\theta z})$ $\dot{\tau}_{\theta z} = kr^{s-1} t_{\theta z}$

Continuity conditions

$\dot{w}$, w

$\dot{\tau}_{rz}$, τ_{rz}