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Crack Growing Across a Bi-material Interface

Invited Talk given at Tulane University, New Orleans, La, USA. Orationem Meam.
Ståhle, P.

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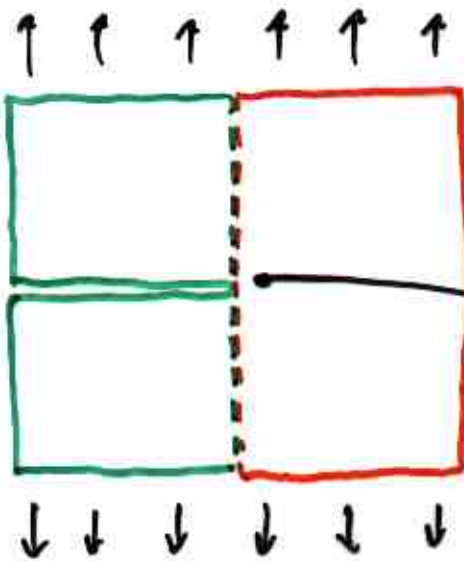
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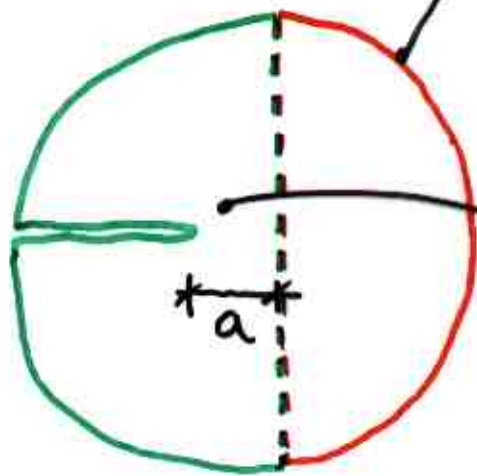
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Asymptotic Stress Fields



$$\sigma_{ij} = k \sigma_0 \left(\frac{r \sigma_0^2}{K_{Ic}^2} \right)^{-\lambda} f_{ij}(\theta)$$

(Zak & Williams 63)



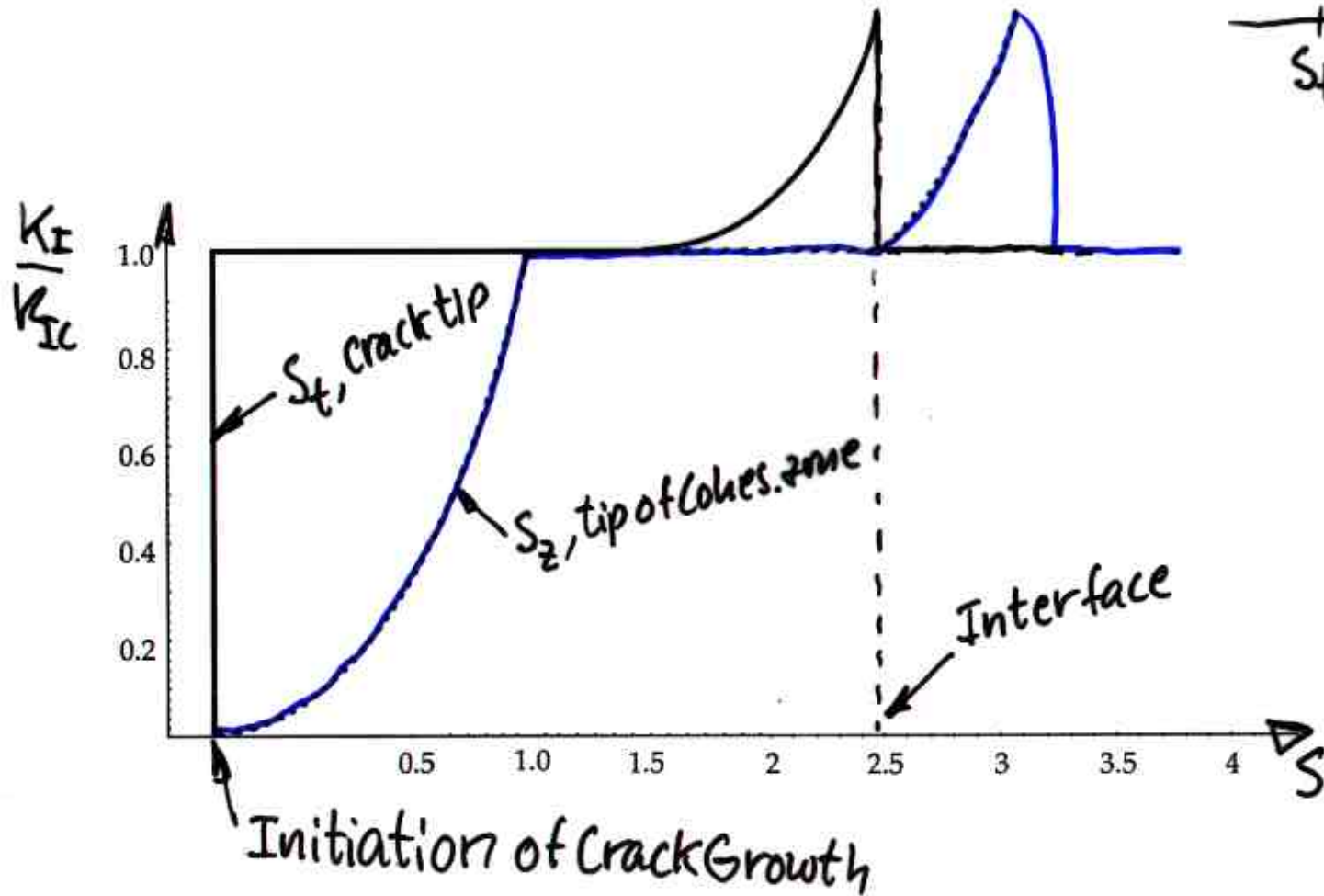
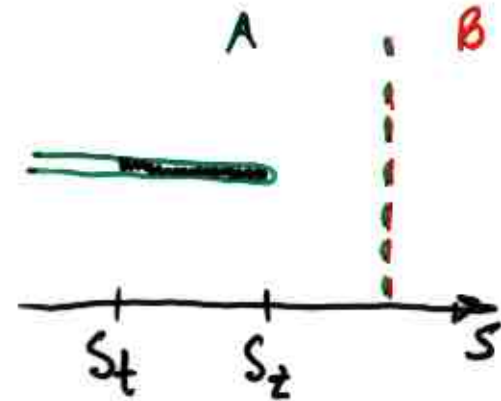
$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) =$$

$$= \underbrace{\frac{K_I}{\sqrt{2\pi} K_{Ic}}}_{k_0} \sigma_0 \left(\frac{r \sigma_0^2}{K_{Ic}^2} \right)^{-\frac{1}{2}} f_{ij}(\theta)$$

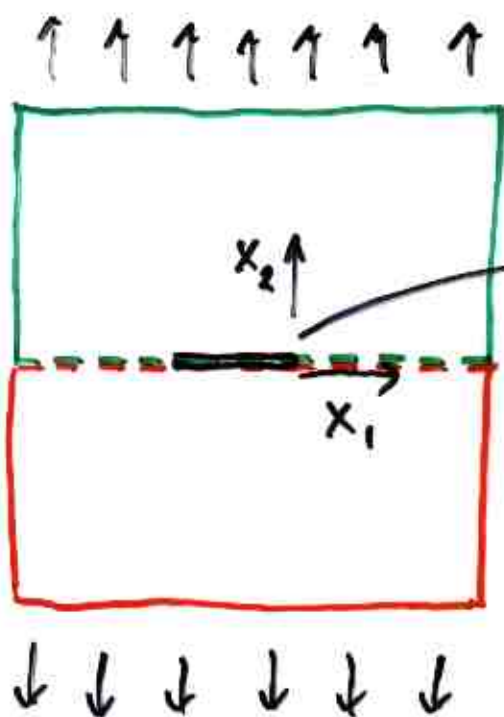
$$K_I = \sum k K_{Ic} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{\frac{1}{2} - \lambda}$$

$$\sigma_D^B = 1.5 \sigma_0^A$$

$$K_{Ic}^A = K_{Ic}^B = K_{Ic}$$



Crack in interface



$$\sigma_{ij} = K (2\pi r)^{-\frac{1}{2} + i\varepsilon} f_{ij}(\theta, \varepsilon)$$

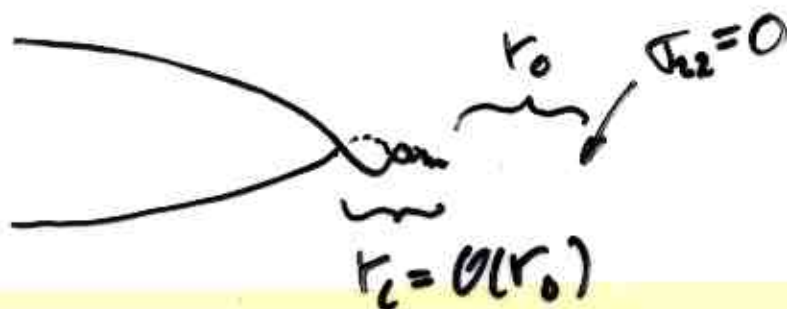
$$\varepsilon = \frac{1}{2\pi} \ln \frac{1-\beta}{1+\beta}$$

Ahead of crack tip

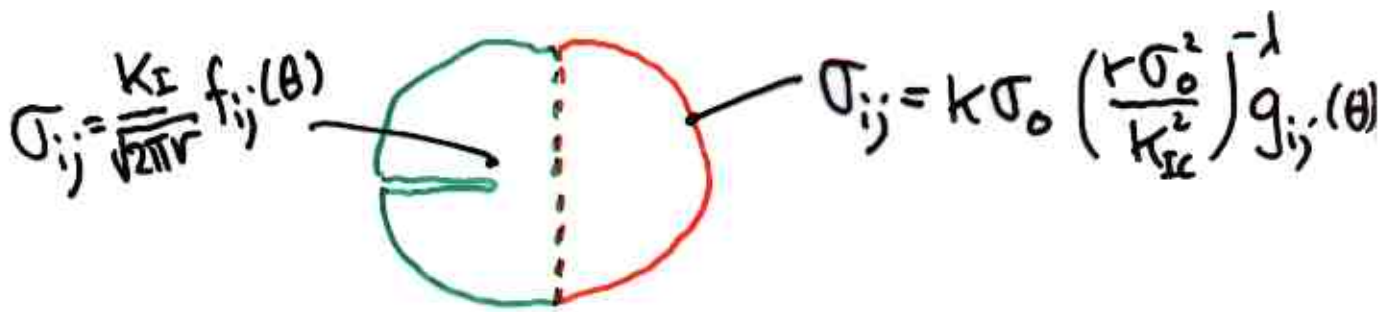
$$(\sigma_{22} + i\sigma_{12})_{\theta=0} = \frac{K r^{i\varepsilon}}{\sqrt{2\pi r}}$$

$$\sigma_{22} \sqrt{2\pi r} = \operatorname{Re}(K) \cos(\varepsilon \ln r) - \operatorname{Im}(K) \sin(\varepsilon \ln r)$$

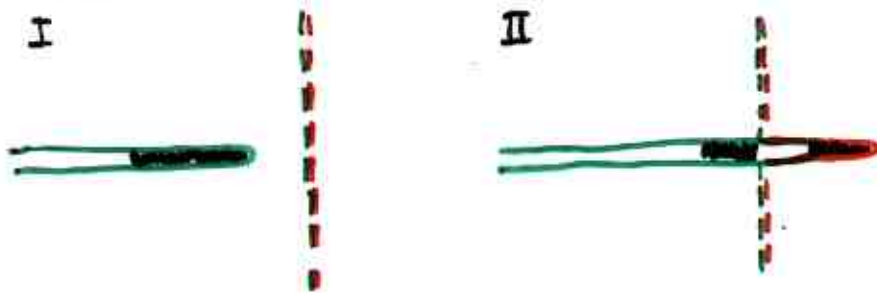
$$(\sigma_{22})_{\theta=0} = 0 \Rightarrow \ln r_0 = \frac{1}{\varepsilon} \tan^{-1} \left(\frac{\operatorname{Re} K}{\operatorname{Im} K} \right)$$



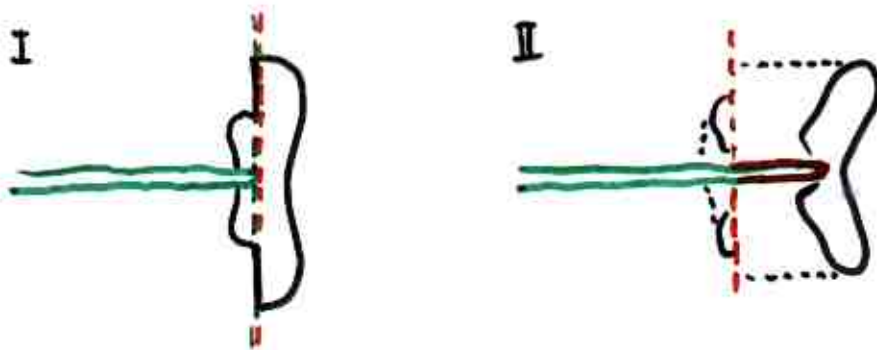
1. Elastic Straight Crack



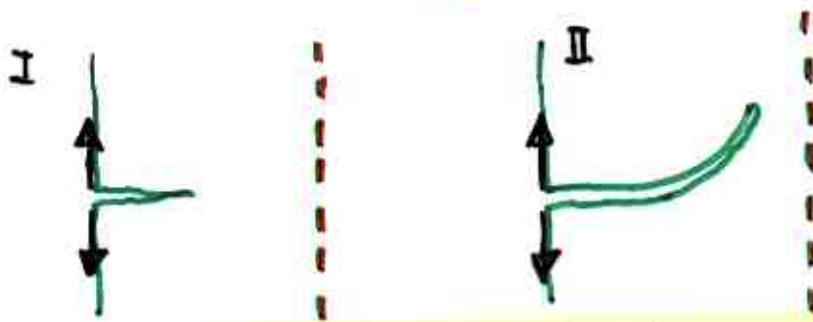
2. Straight Crack with Cohesive Zone



3. Initiation of Crack Growth from Interf.



4. Elastic Deflecting Crack

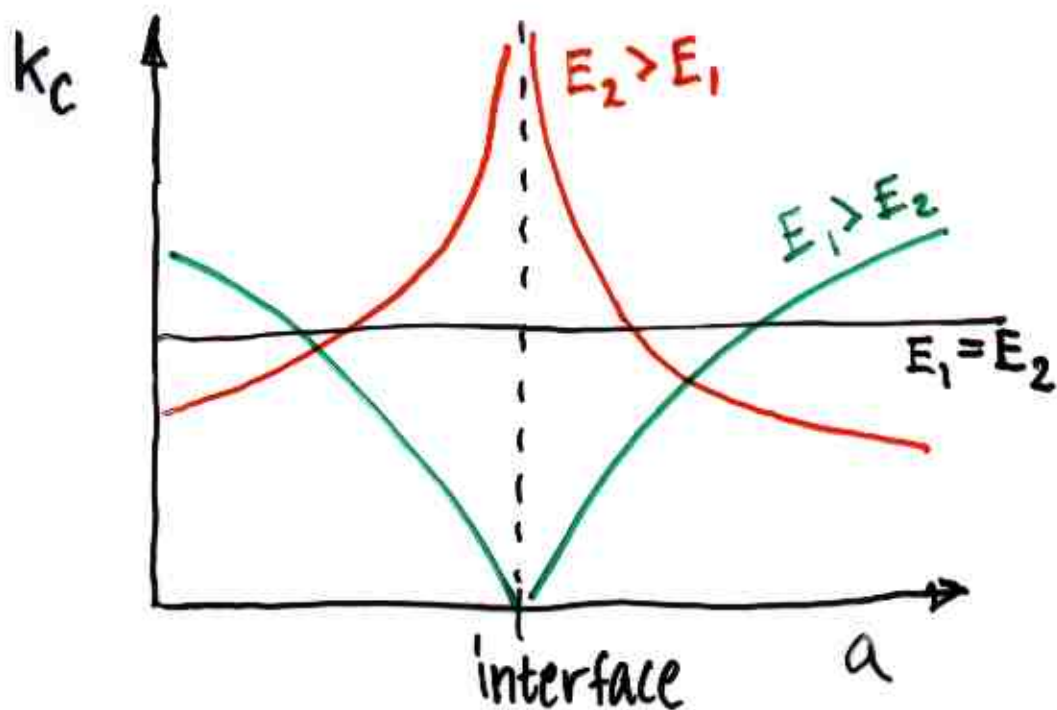


$$K_I = \int k K_{Ic} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{\frac{1}{2} - \lambda}$$

Assume that

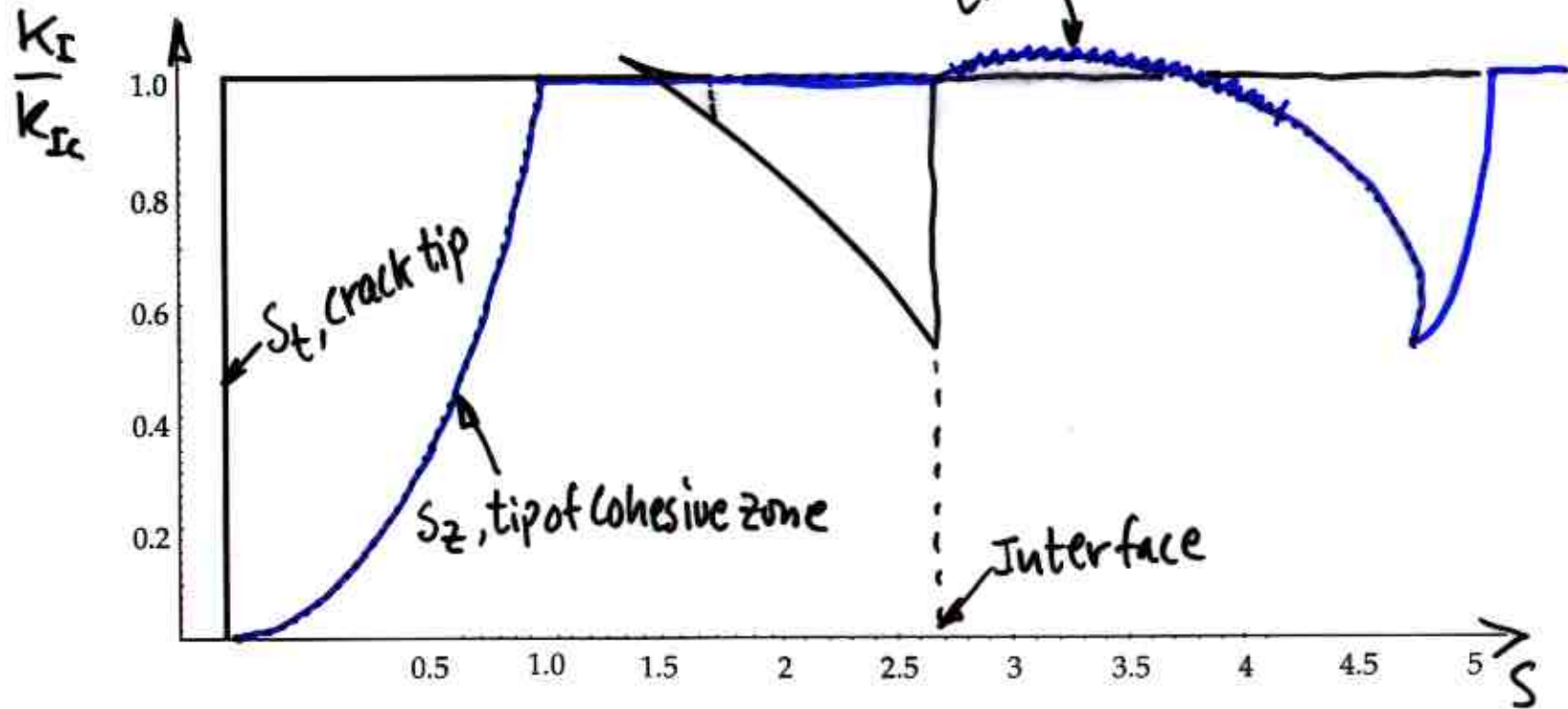
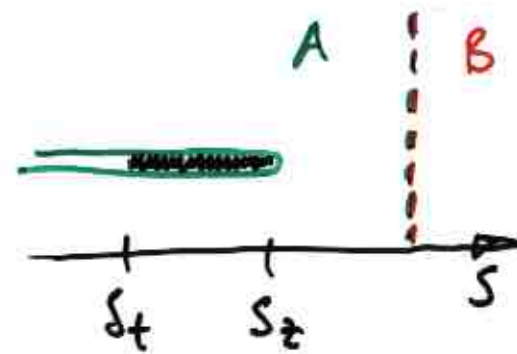
$$K_I = K_{Ic}$$

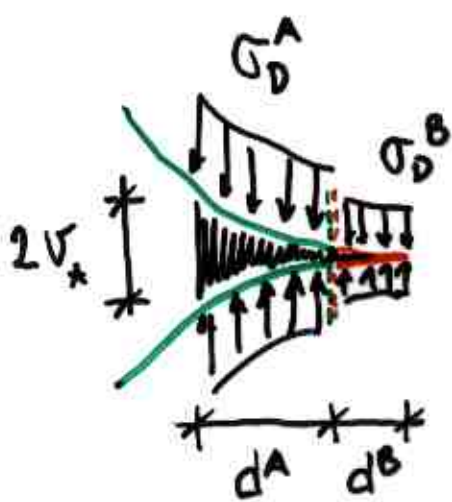
$$\Rightarrow k = k_c = \frac{1}{\int} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{\frac{1}{2} + \lambda}$$



$$\sigma_D^B = 0.5 \sigma_D^A$$

$$K_{Ic}^B = K_{Ic}^A = K_{Ic}$$





$$K_I = 2\sqrt{\frac{2}{\pi}} \left[\sqrt{d^A + d^B} \sigma_D^A - \sqrt{d^B} (\sigma_D^A - \sigma_D^B) \right]$$

I



I alt. 1



I alt. 2



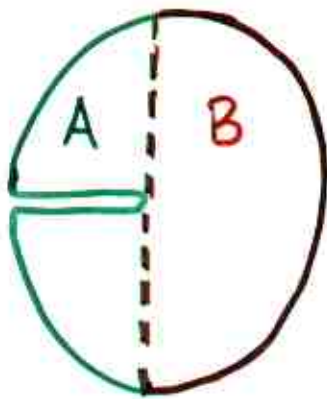
$$\sigma_D^B < \sigma_D^A$$

II alt. 3

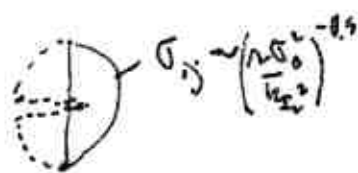
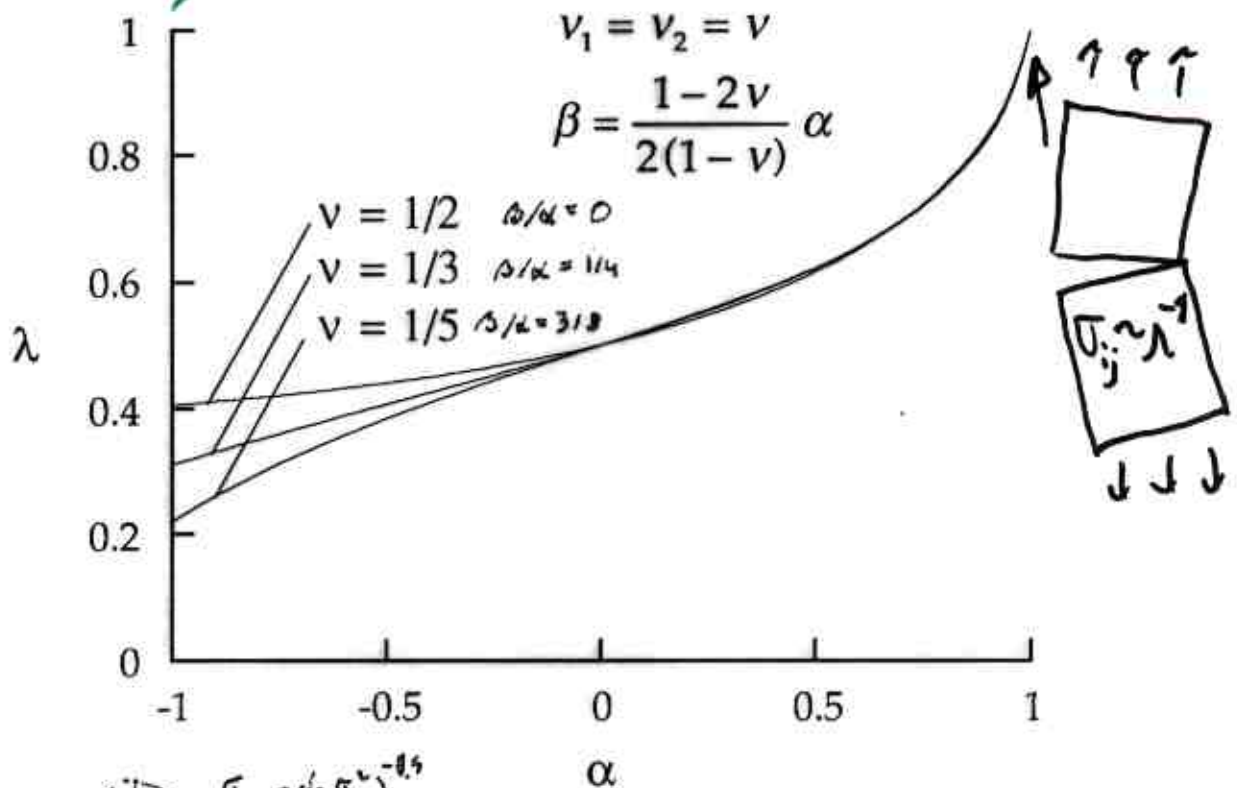


$$V > V_*^B \Rightarrow \sigma_D = 0$$

$$V_*^B < V_*^A$$



A is weak A is stiff



$$\alpha = \frac{E_A / (1-\nu_A^2) - E_B / (1-\nu_B^2)}{E_A / (1-\nu_A^2) + E_B / (1-\nu_B^2)}$$

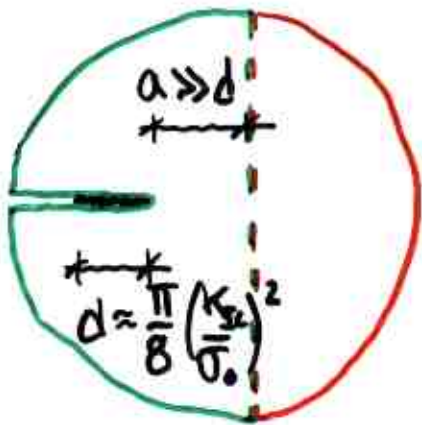
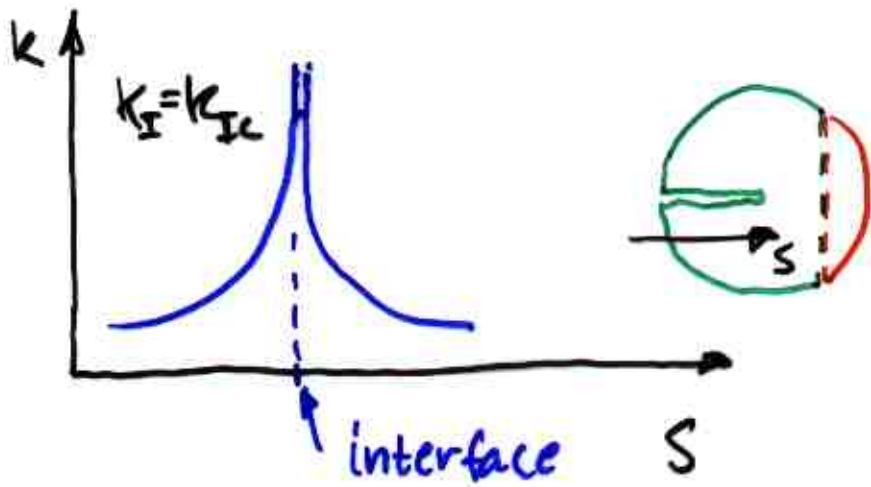
Cracks in Bimaterials

Dunders' parameters

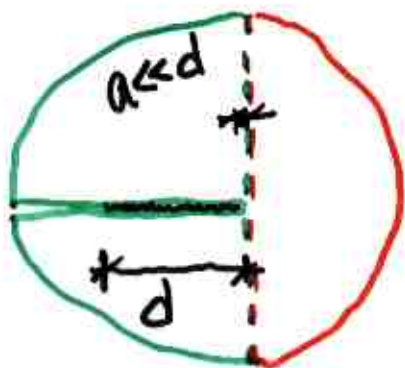
$$\alpha = \frac{G_1(\nu_2 + 1) - G_2(\nu_1 + 1)}{G_1(\nu_2 + 1) + G_2(\nu_1 + 1)}$$

$$\beta = \frac{G_1(\nu_2 - 1) - G_2(\nu_1 - 1)}{G_1(\nu_2 + 1) + G_2(\nu_1 + 1)}$$

$$K_I = \xi k K_{Ic} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{\frac{1}{2} - \lambda} \Rightarrow k = \xi^{-1} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{\lambda - \frac{1}{2}}$$



$$\sigma_{ij} = \xi^{-1} \sigma_0 \left(\frac{a}{r} \right)^{\lambda} \left(\frac{a \sigma_0^2}{K_{Ic}^2} \right)^{-\frac{1}{2}}$$



$$d \rightarrow d_0 = \left[\frac{\pi}{2} (1 - \lambda) k \right]^{\frac{1}{1 - \lambda}} \left(\frac{K_{Ic}}{\sigma_0} \right)^2$$