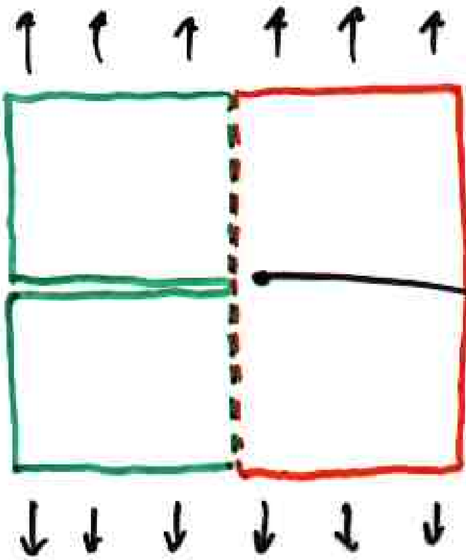
Crack Growing Across a Bi-material Interface

Invited Talk given at Tulane University, New Orleance, Lu, USA. Orationem Meam.

Ståhle, P.

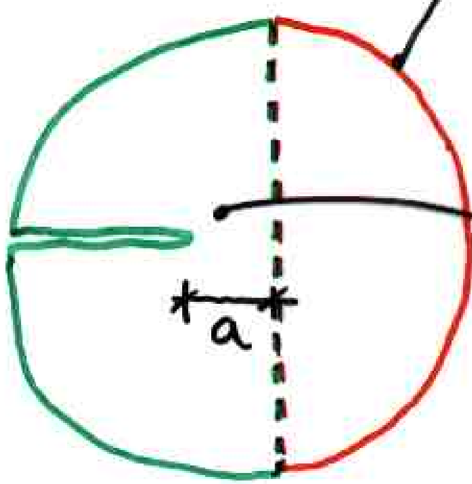
1989

Asymptotic Stress Fields



$$\sigma_{ij} = k\,\sigma_0 \left(\frac{r\sigma_0^2}{K_{Ic}^2}\right)^{-\lambda} g_{ij}(\theta)$$

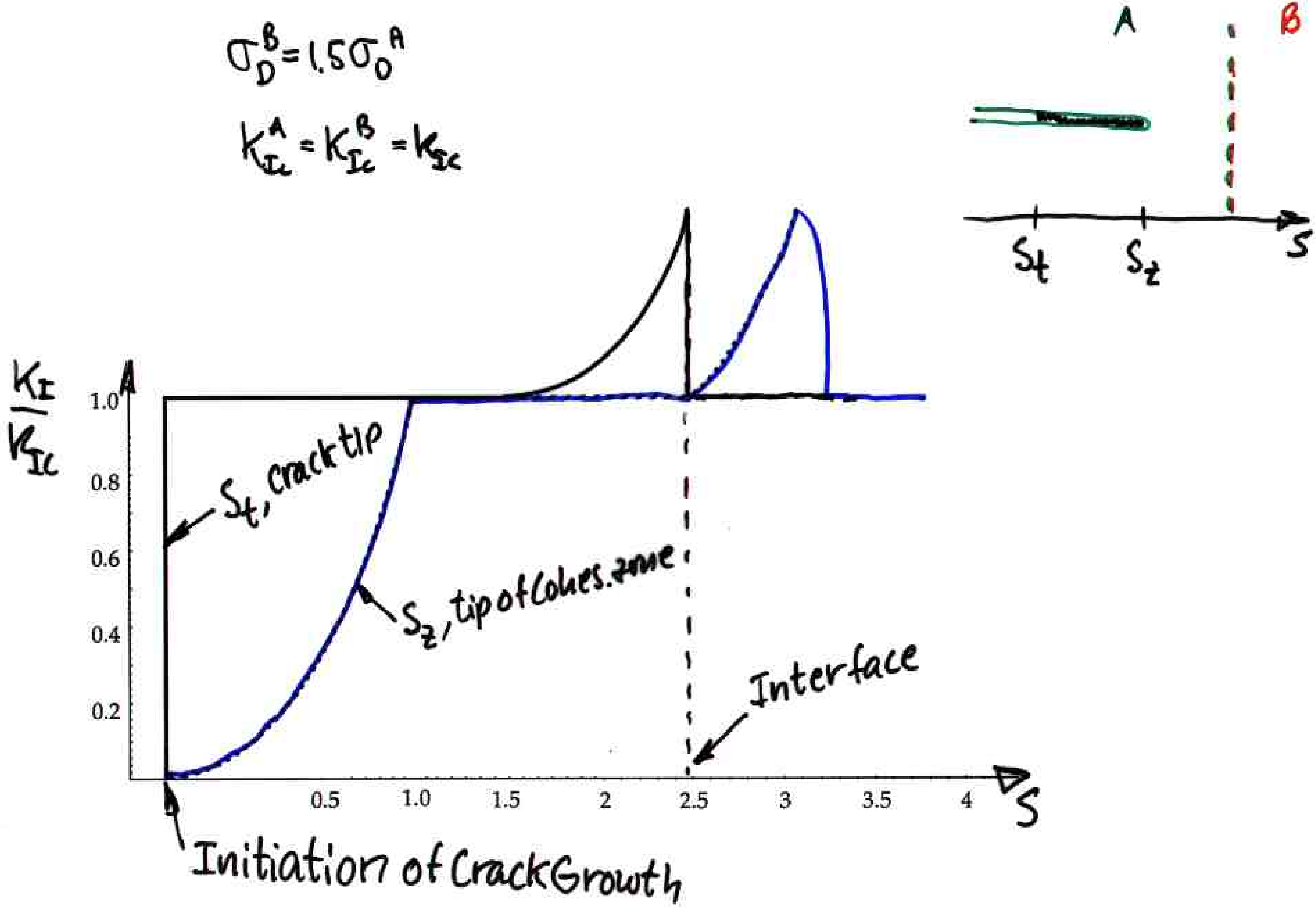
(Zak & Williams 63)



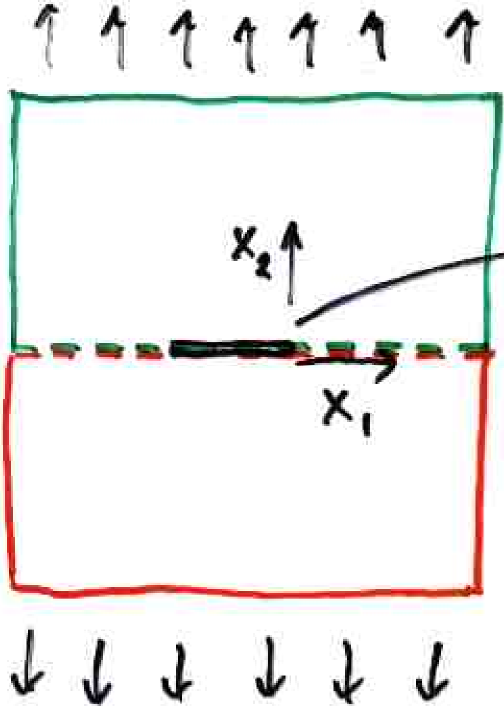
$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}}\, f_{ij}(\theta) =$$

$$= \underbrace{\frac{K_I}{\sqrt{2\pi}\,K_{Ic}}}_{k_0}\, \sigma_0 \left(\frac{r\sigma_0^2}{K_{Ic}^2}\right)^{-\frac{1}{2}} f_{ij}(\theta)$$

$$K_I = \xi\, k\, K_{Ic} \left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{\frac{1}{2}-\lambda}$$



$\sigma_D^B = 1.5\sigma_0^A$
$K_{Ic}^A = K_{Ic}^B = K_{Ic}$
A
B
S_t
S_z
S
K_I / K_{Ic}
1.0
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2
2.5
3
3.5
4
S
S_t, crack tip
S_z, tip of cohes. zone
Interface
Initiation of Crack Growth



Crack in interface

$$\sigma_{ij} = K(2\pi r)^{-\frac{1}{2}+i\varepsilon} f_{ij}(\theta, \varepsilon)$$

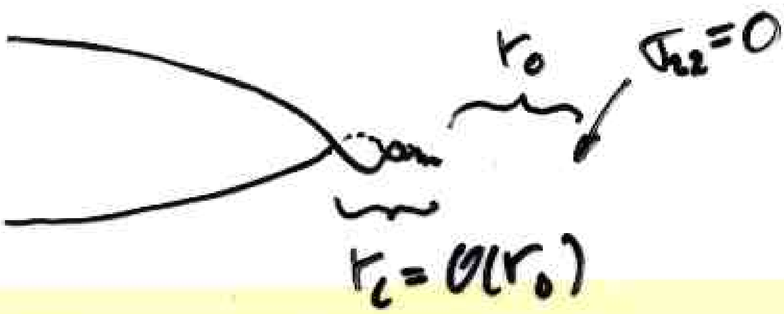
$$\varepsilon = \frac{1}{2\pi} \ln \frac{1-\beta}{1+\beta}$$

Ahead of crack tip

$$(\sigma_{22} + i\sigma_{12})_{\theta=0} = \frac{K r^{i\varepsilon}}{\sqrt{2\pi r}}$$

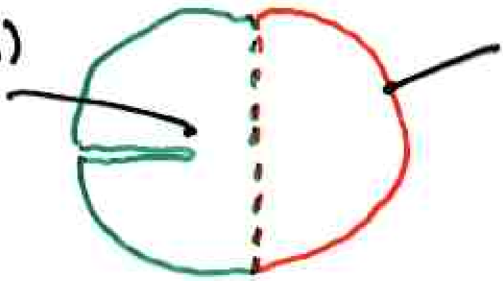
$$\sigma_{22}\sqrt{2\pi r} = \mathrm{Re}(K)\cos(\varepsilon \ln r) - \mathrm{Im}(K)\sin(\varepsilon \ln r)$$

$$(\sigma_{22})_{\theta=0} = 0 \Rightarrow \ln r_0 = \frac{1}{\varepsilon}\tan^{-1}\left(\frac{\mathrm{Re}\,K}{\mathrm{Im}\,K}\right)$$



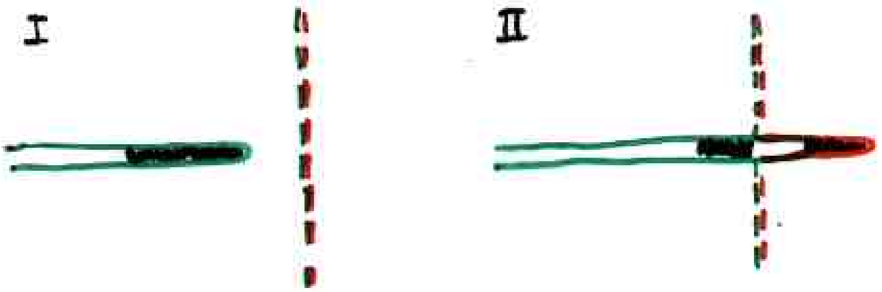
1. Elastic Straight Crack

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

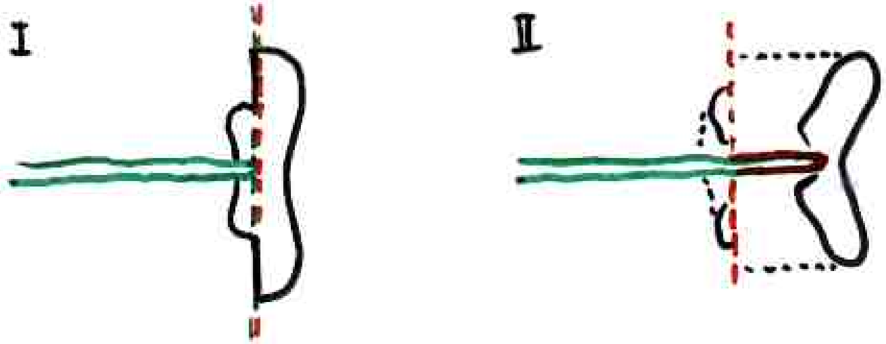


$$\sigma_{ij} = k\sigma_0 \left(\frac{r\sigma_0^2}{K_{Ic}^2}\right)^{-\lambda} g_{ij}(\theta)$$

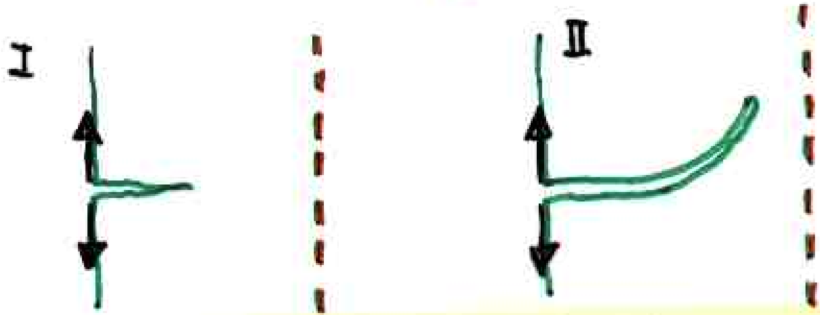
2. Straight Crack with Cohesive Zone



3. Initiation of Crack Growth from Interf.



4. Elastic Deflecting Crack

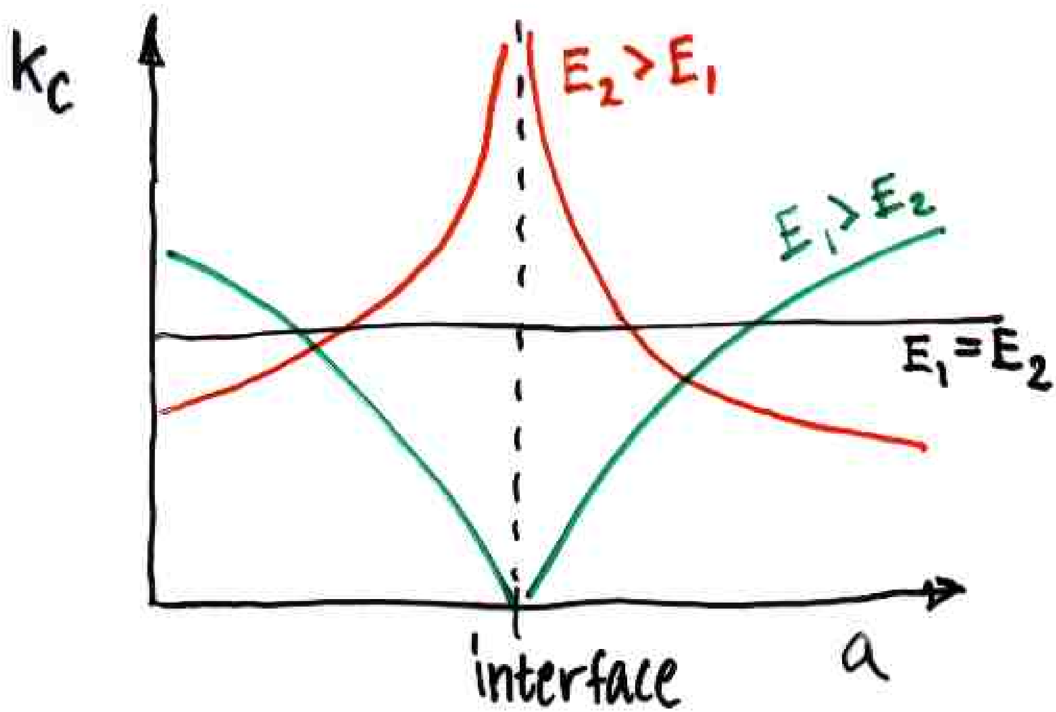


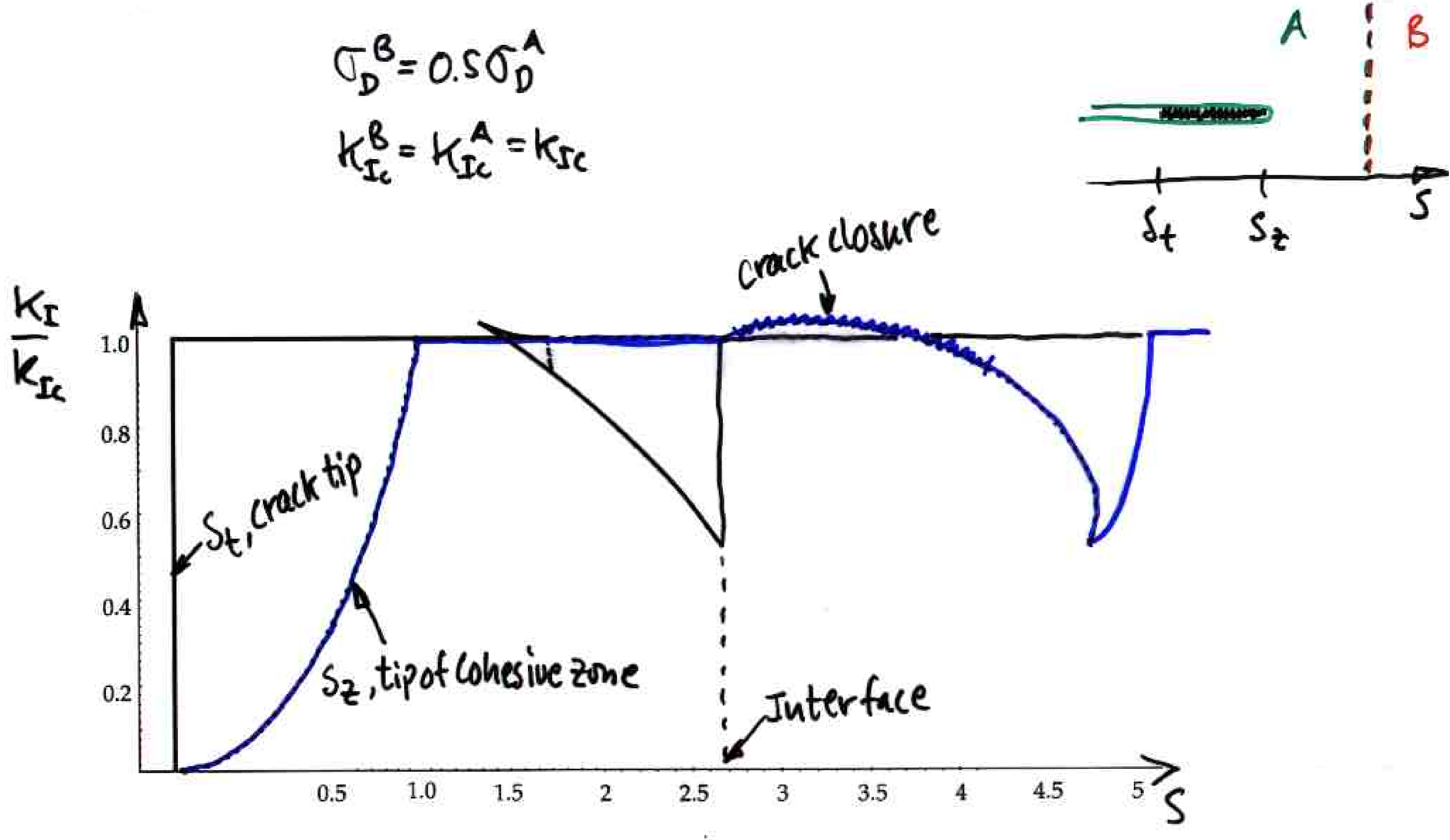
$$K_I = \xi k K_{Ic} \left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{\frac{1}{2}-\lambda}$$

Assume that

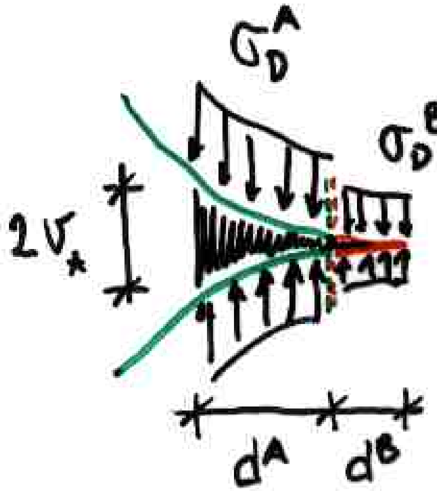
$$K_I = K_{Ic}$$

$$\Rightarrow \quad k = k_c = \frac{1}{\xi}\left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{-\frac{1}{2}+\lambda}$$





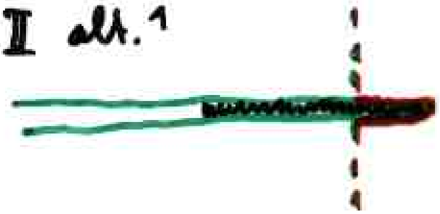
$\sigma_D^B = 0.5\sigma_D^A$
$K_{Ic}^B = K_{Ic}^A = K_{Ic}$
A
B
s_t
s_z
s
crack closure
$\frac{K_I}{K_{Ic}}$
1.0
0.8
0.6
0.4
0.2
s_t, crack tip
s_z, tip of cohesive zone
Interface
0.5
1.0
1.5
2
2.5
3
3.5
4
4.5
5
s



$$K_I = 2\sqrt{\frac{2}{\pi}}\left[\sqrt{d^A + d^B}\,\sigma_D^A - \sqrt{d^B}\left(\sigma_D^A - \sigma_D^B\right)\right]$$

I

II alt.1

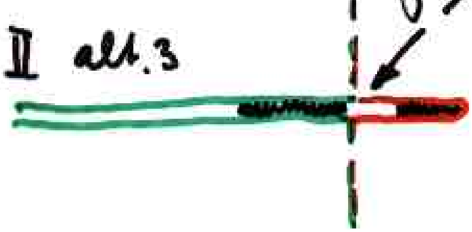


II alt.2

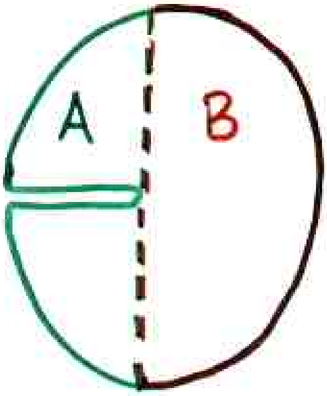
$\sigma_D^B < \sigma_D^A$

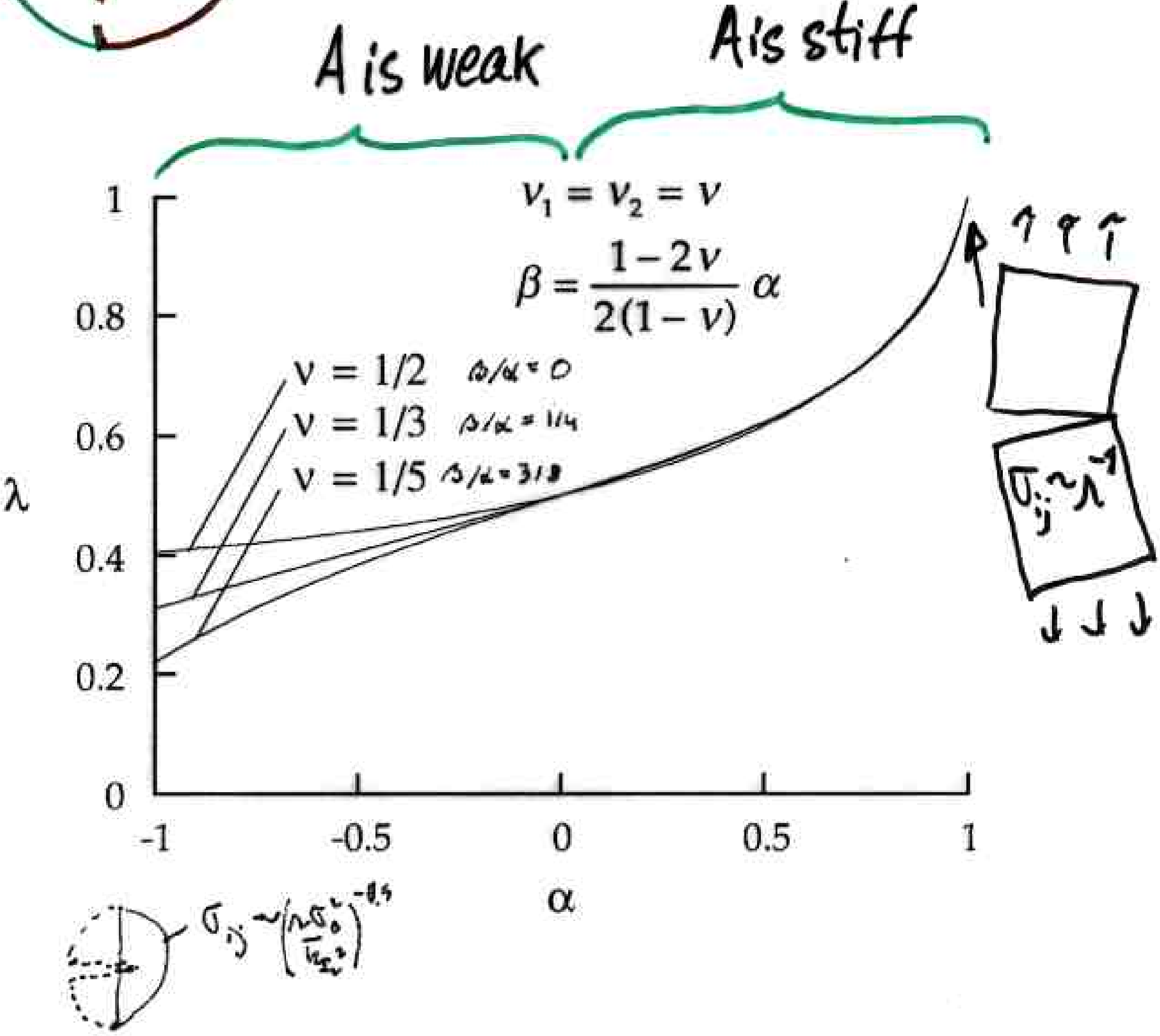
$v > v_*^B \Rightarrow \sigma_D = 0$

II alt.3



$v_*^B < v_*^A$





$$\alpha = \frac{E_A/(1-\nu_A^2) - E_B/(1-\nu_B^2)}{E_A/(1-\nu_A^2) + E_B/(1-\nu_B^2)}$$

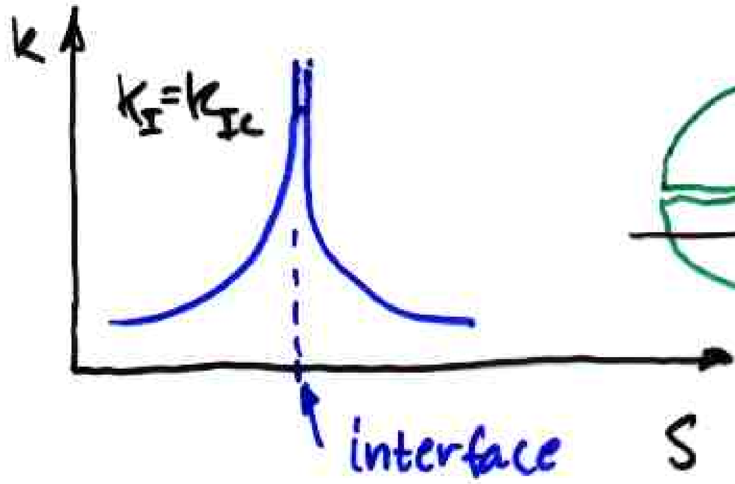
Cracks in Bimaterials

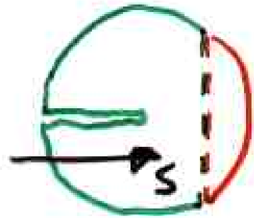
Dunders' parameters

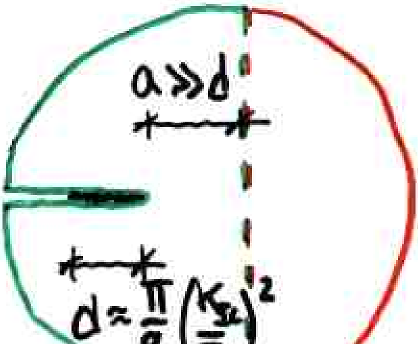
$$\alpha = \frac{G_1(\varkappa_2+1) - G_2(\varkappa_1+1)}{G_1(\varkappa_2+1) + G_2(\varkappa_1+1)}$$

$$\beta = \frac{G_1(\varkappa_2-1) - G_2(\varkappa_1-1)}{G_1(\varkappa_2+1) + G_2(\varkappa_1+1)}$$

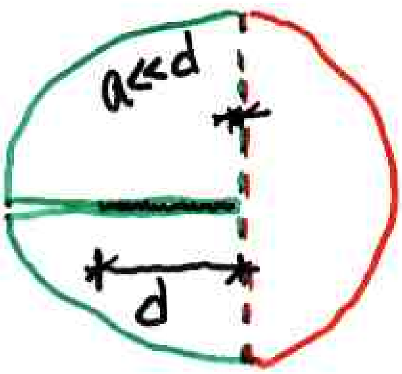
$$K_I = \xi k K_{Ic} \left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{\frac{1}{2}-\lambda} \Rightarrow \quad k = \xi^{-1}\left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{\lambda-\frac{1}{2}}$$







$$\sigma_{ij} = \xi^{-1}\sigma_0\left(\frac{a}{r}\right)^{\lambda} \left(\frac{a\sigma_0^2}{K_{Ic}^2}\right)^{-\frac{1}{2}}$$



$$d \rightarrow d_0 = \left[\frac{\pi}{2}(1-\lambda)k\right]^{\frac{1}{1-\lambda}} \left(\frac{K_{Ic}}{\sigma_0}\right)^2$$