



LUND UNIVERSITY

Spontaneous fracture at expanding phase transformation.

Spontant brott vid expanderande fasomvandling, SMD 2013. Orationem Meam.

Ståhle, P.

2013

Document Version:

Publisher's PDF, also known as Version of record

[Link to publication](#)

Citation for published version (APA):

Ståhle, P. (2013). Spontaneous fracture at expanding phase transformation. Spontant brott vid expanderande fasomvandling, SMD 2013. Orationem Meam.

Total number of authors:

1

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

SMD 2013

Spontant brott vid expanderande fasomvandling

P. Ståhle

Lunds tekniska högskola, Lund, Sverige

N. Lalanne-Aulet

Institut Francais de Mecanique Avancee, Clermont-Ferrand, France

R. N. Singh and S. Benerjee

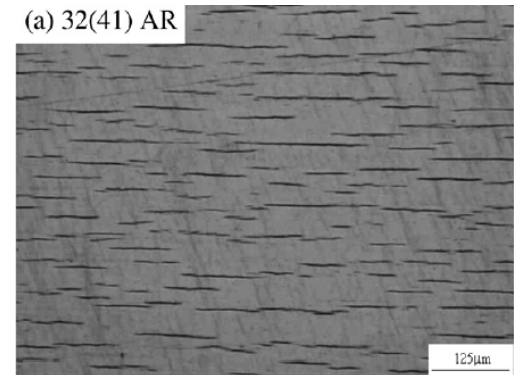
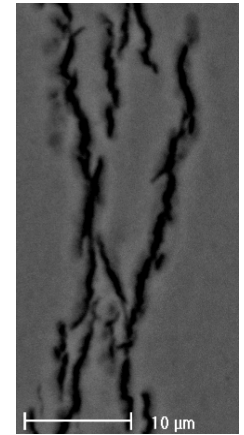
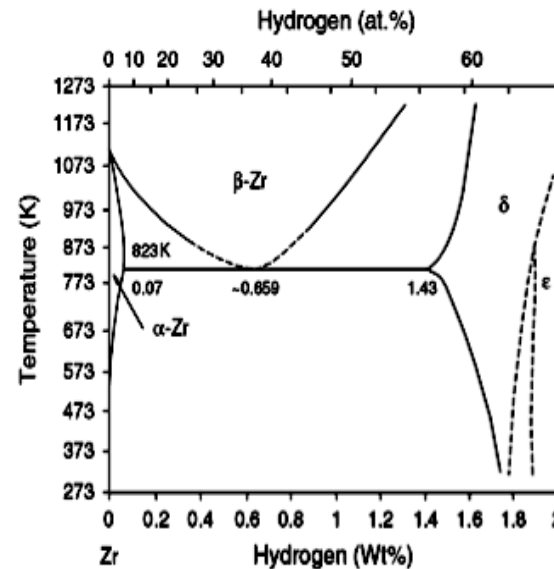
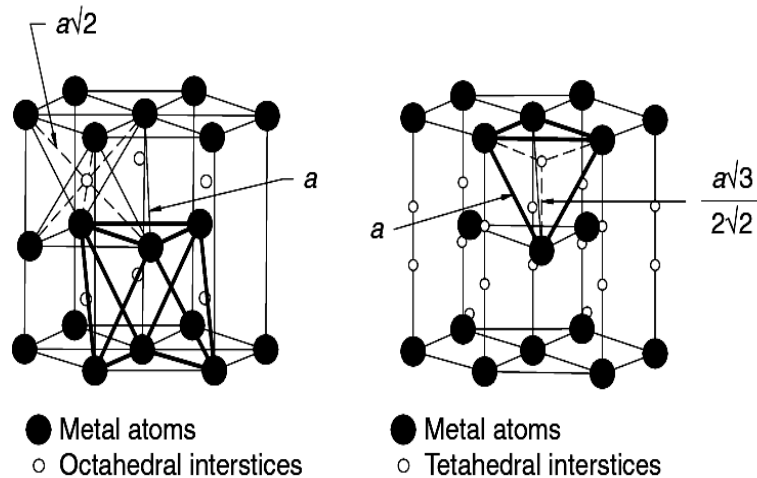
BARC, Atomic Research Centre, Mumbai, India

Zr-H solid solution and hydride

Hydrogen occupies tetrahedral sites in Zr-H solid solution

α -lattice distorts to match the γ and δ structures

Interstitial ordering results in periodic occupation of tetrahedral sites



Zr exhibits two allotropic modification: low T hcp (α) and high T bcc (β) phases

Two stable hydrides (δ and ϵ) and one metastable (γ) hydride forms in this system

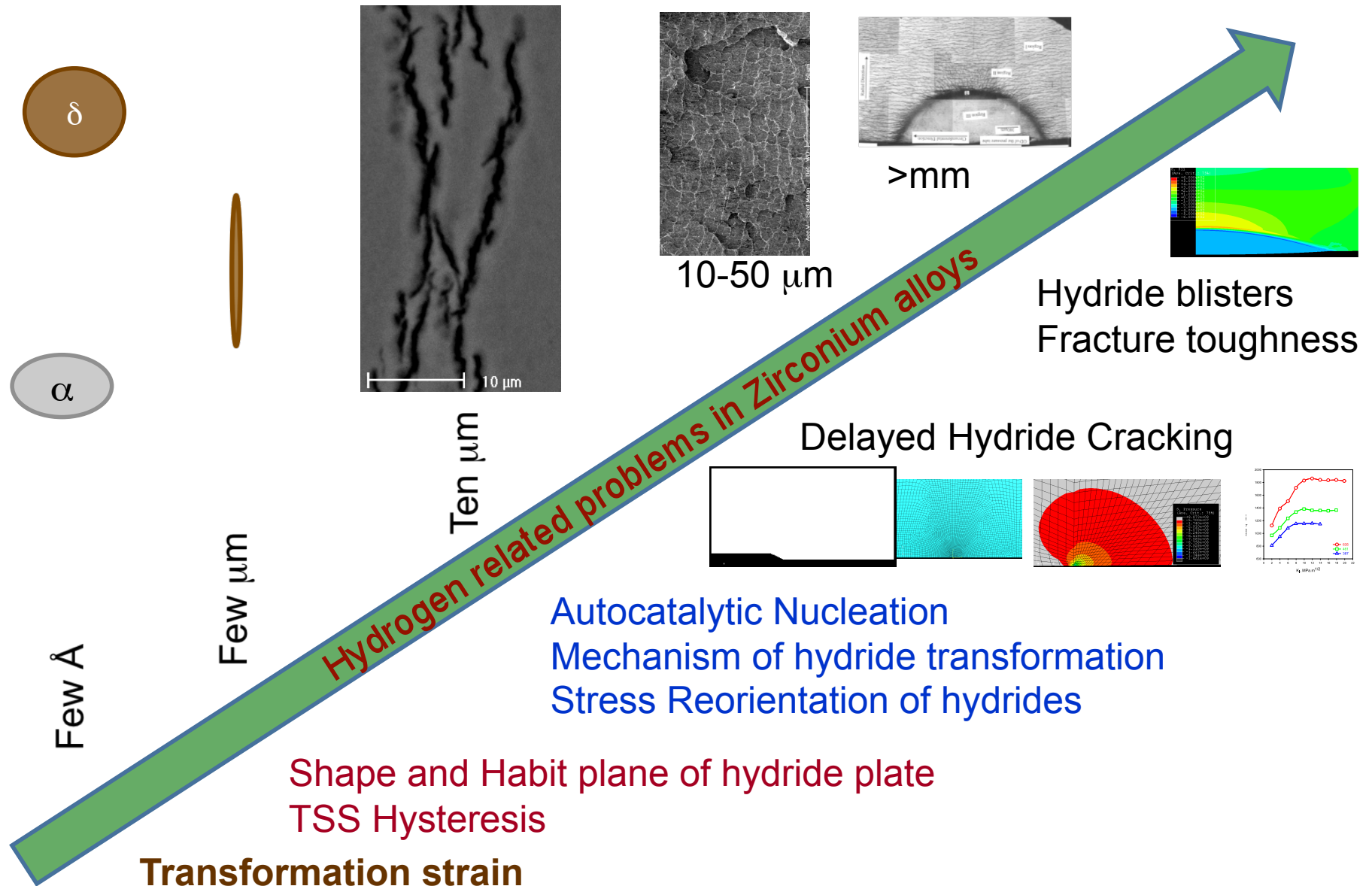
Under optical microscope the traces of hydrides appears as dark lines

At higher magnification the hydride plates reveal several smaller entities

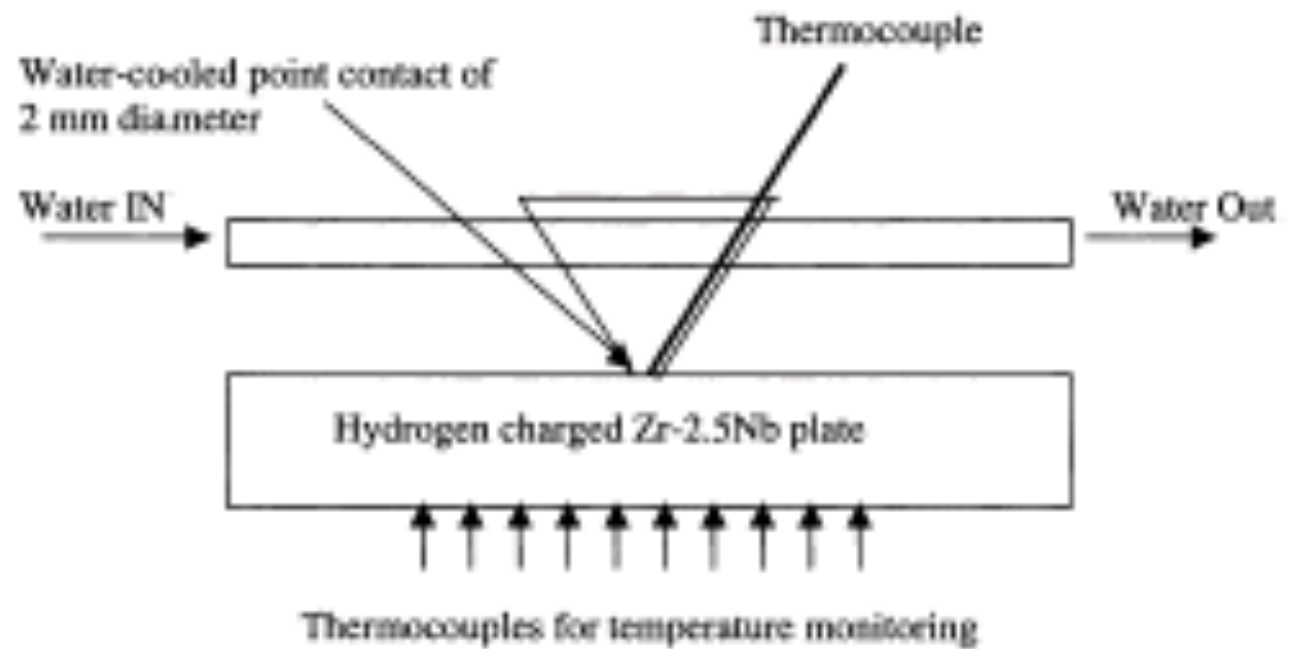
Hydride plate comprises of platelets stacked side by side

Each platelet comprises of sub-platelets stacked end to end

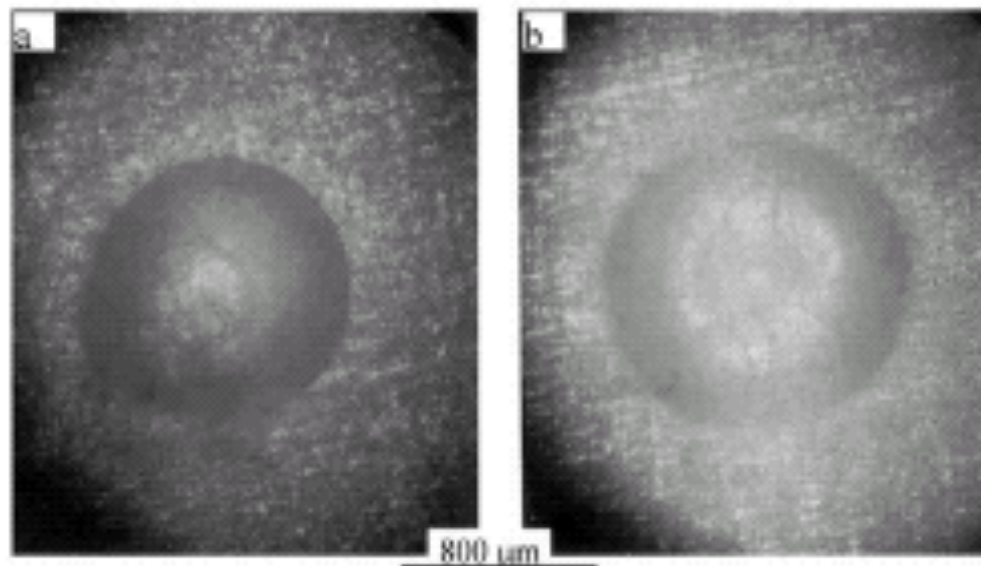
Multi-scale Structural Mechanics



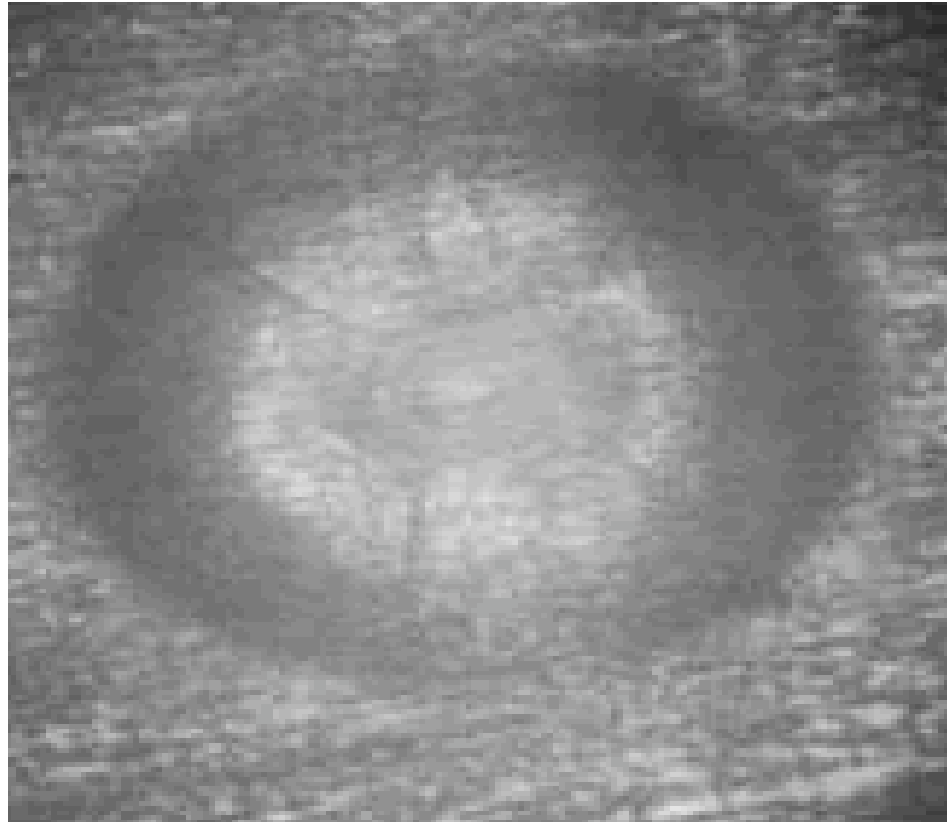
Experimental setup



Top view of
hydride blister

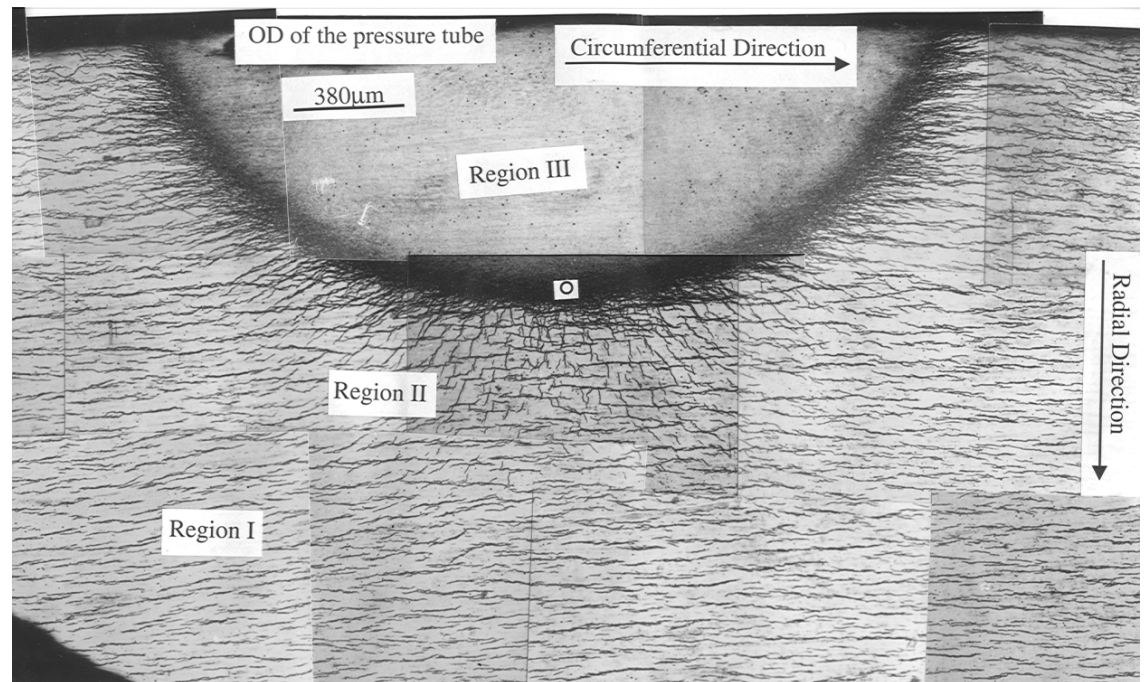
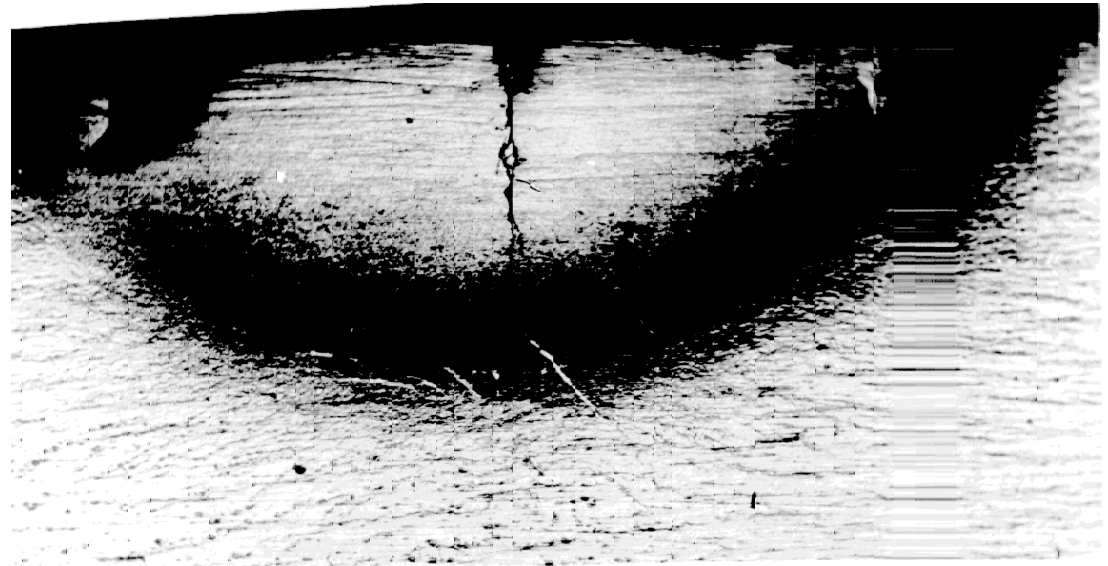


Top view of a hydride blister grown in
Zr–2.5wt%Nb pressure tube alloy (Singh *et al.*, 2001)

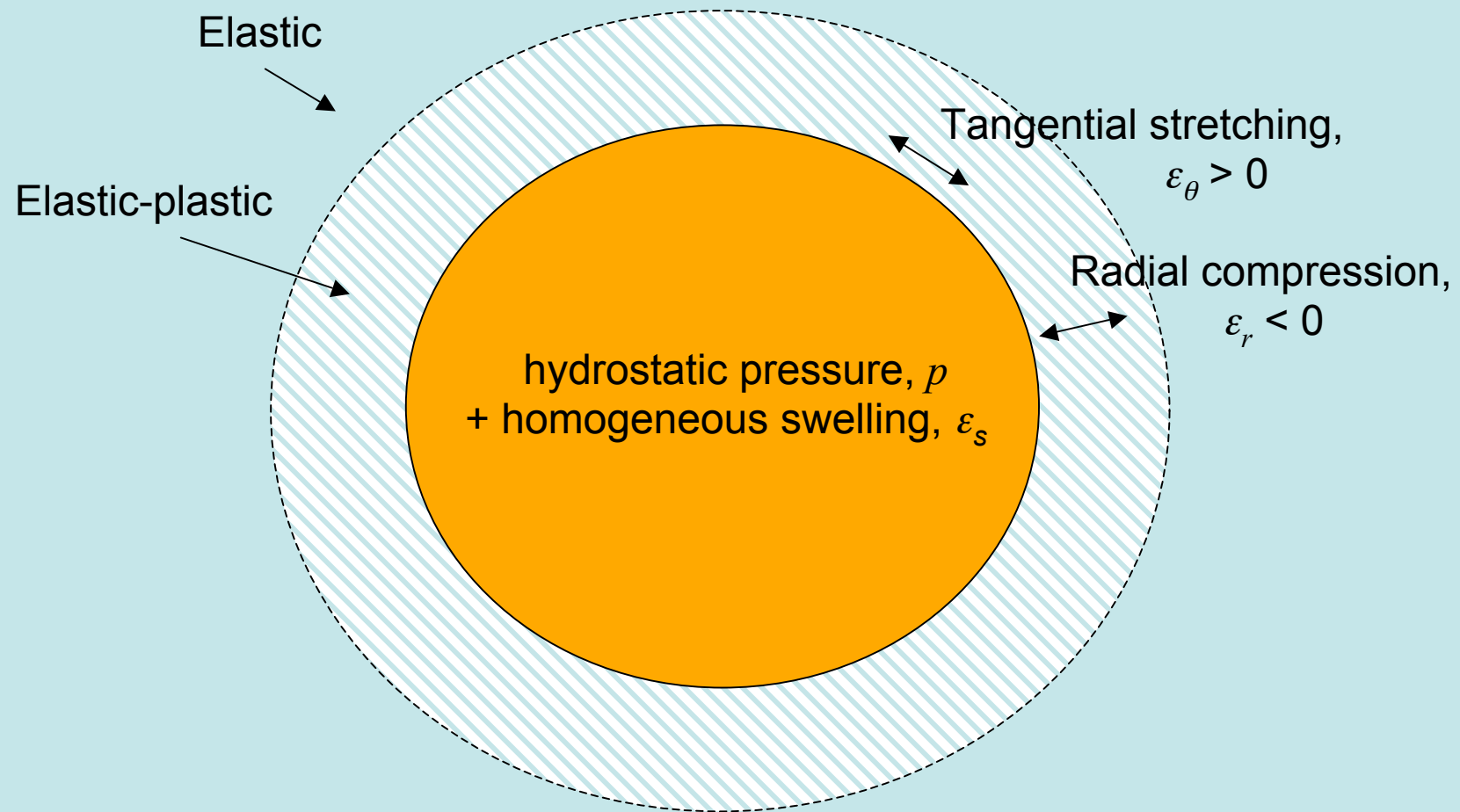


Examination of a section of blister (grown in Laboratory)

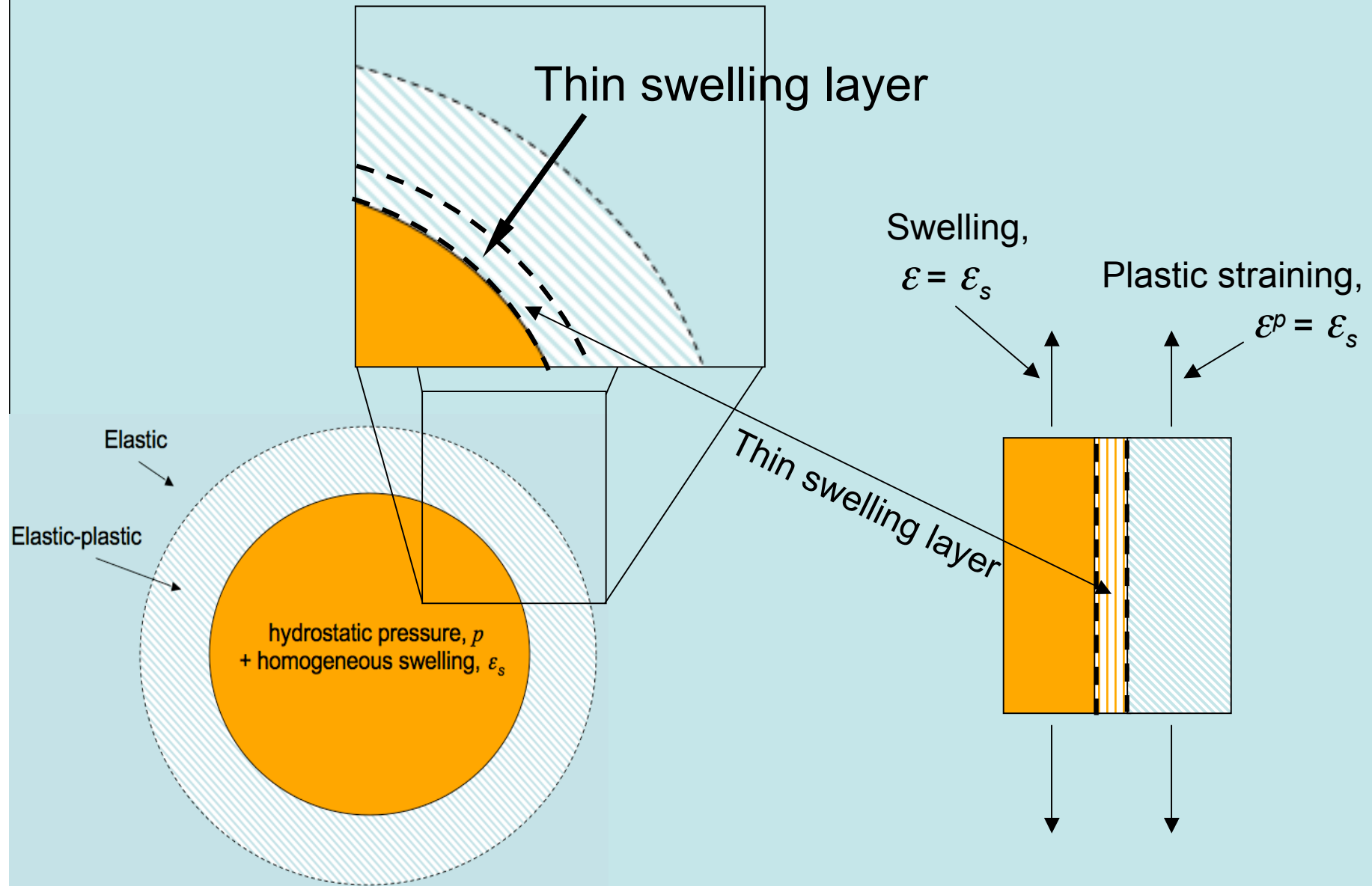
Figure : Optical micrograph of hydride blister section, grown in Zr-2.5wt.% Nb pressure tube material. Three regions - Region I - matrix & circumferential hydrides, region II - matrix containing both radial and circumferential hydrides and region III - mainly of δ -hydride.



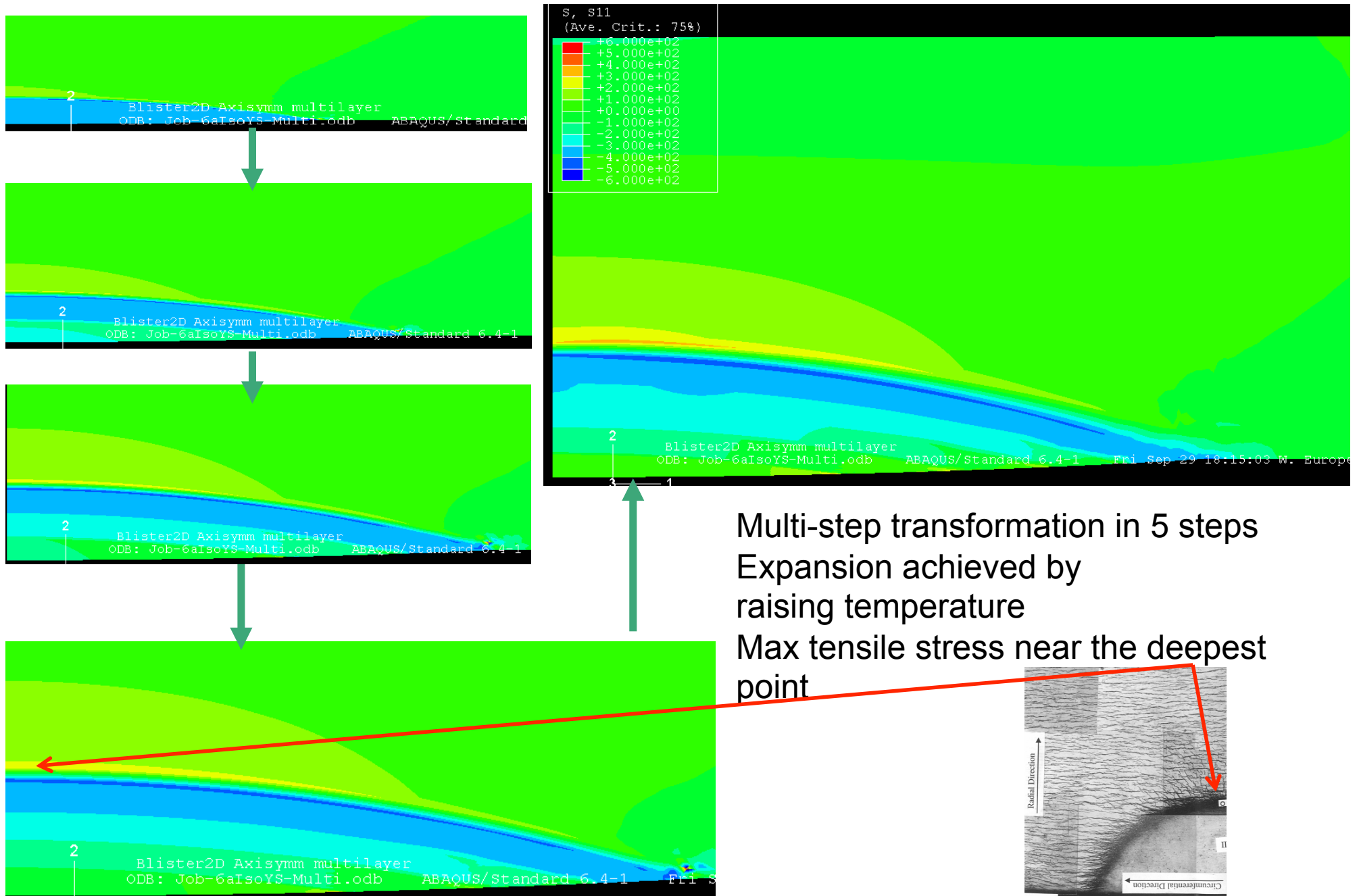
Expanding elastic-plastic inclusion (Hill, 1950)



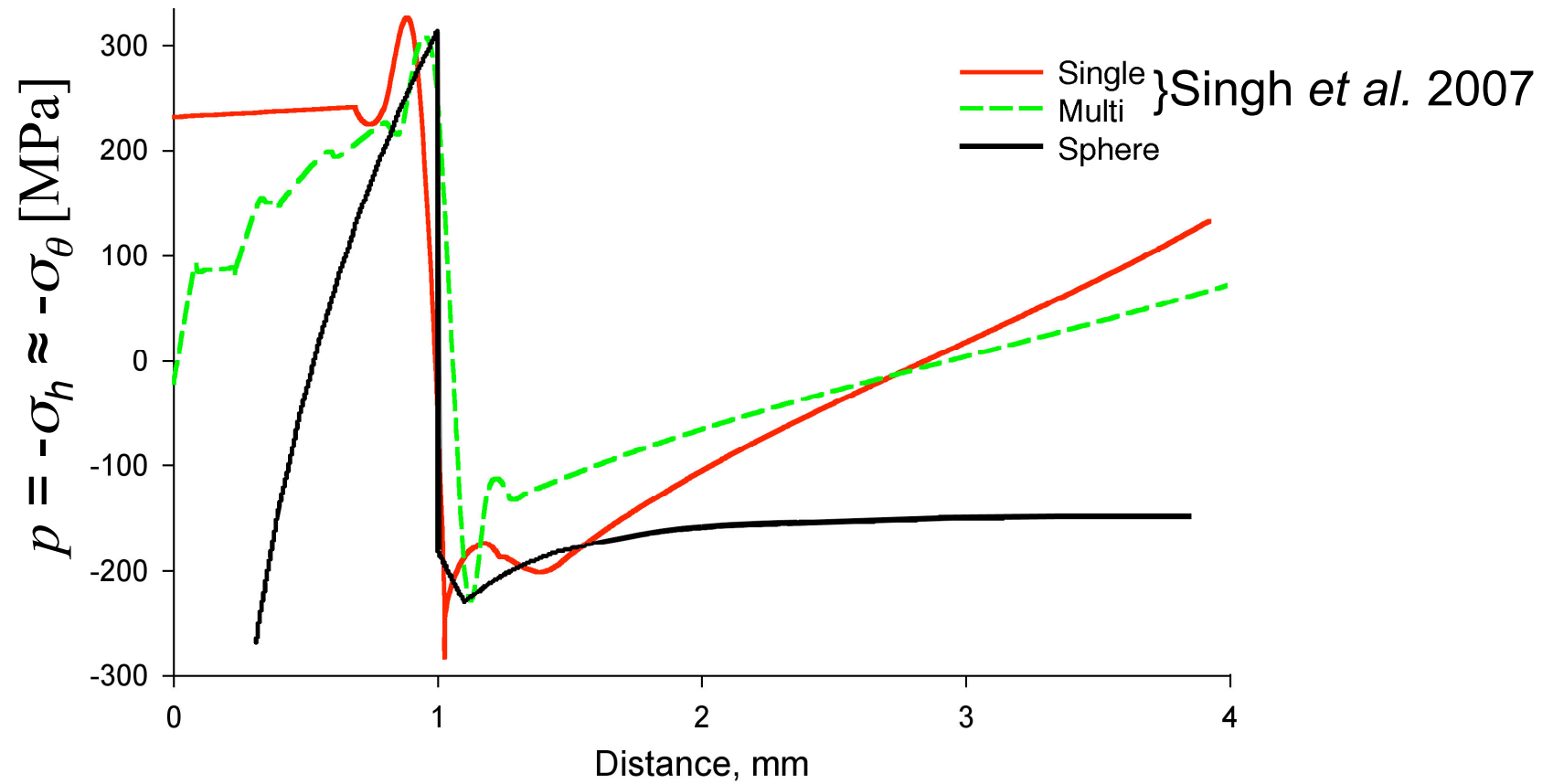
Thin layer increasing inclusion volume (and mass)



Blister formation - Stress field computation – Multi step



Essential result of FEM and present solution



Strains

$$\epsilon_r = \frac{\partial u_r}{\partial r}, \quad \epsilon_\theta = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r},$$

$$\epsilon_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta}.$$

Equilibrium

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r} (2\sigma_r - \sigma_\theta - \sigma_\psi + \sigma_{r\theta} \cot \theta) = 0.$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{1}{r} [(\sigma - \sigma_\psi) \cot \theta + 3\sigma_{r\theta}] = 0.$$

Decomposed strains

$$\epsilon_r = \epsilon_r^e + \epsilon_r^p + \epsilon^s,$$

$$\epsilon_\theta = \epsilon_\theta^e + \epsilon_\theta^p + \epsilon^s,$$

$$\epsilon_\psi = \epsilon_\psi^e + \epsilon_\psi^p + \epsilon^s \text{ and } \epsilon = \epsilon_{r\theta}^e + \epsilon_{r\theta}^p$$

Plast. strain prop. to deviatoric stress

$$\frac{\epsilon_r^p}{\sigma_r - \sigma_h} = \frac{\epsilon_\theta^p}{\sigma_\theta - \sigma_h} = \frac{\epsilon_\psi^p}{\sigma_\psi - \sigma_h} = \frac{\epsilon_{r\theta}^p}{\sigma_{r\theta}},$$

DE:
$$\frac{d^2 u_r}{dr^2} + 2 \frac{du_r}{r dr} - 2 \frac{u_r}{r^2} - \frac{1-2\nu}{1-\nu} \left(\frac{d\epsilon^p}{dr} + 3 \frac{\epsilon^p}{r} \right) = 0$$

Displacements:
$$u_r = \epsilon_s r - \frac{2(1-2\nu)}{3} \left(1 + 3 \ln \frac{r_p r}{R^2} \right) \frac{\sigma_o r}{E} \quad \text{in } r \leq R.$$

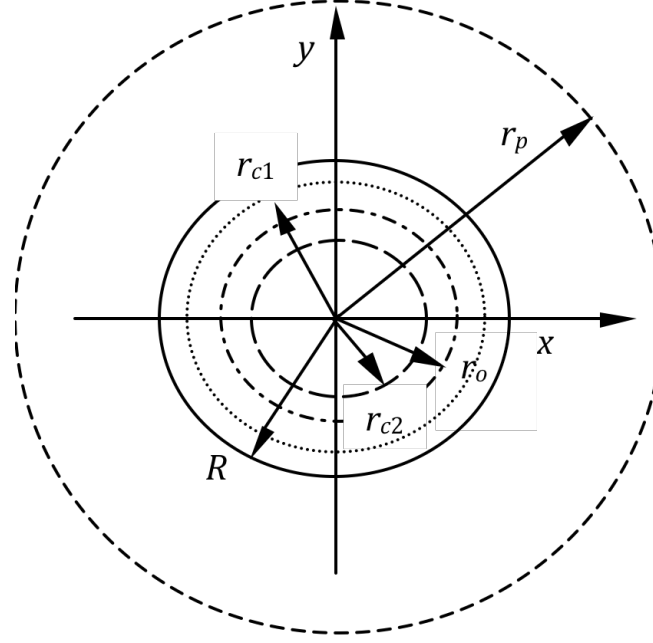
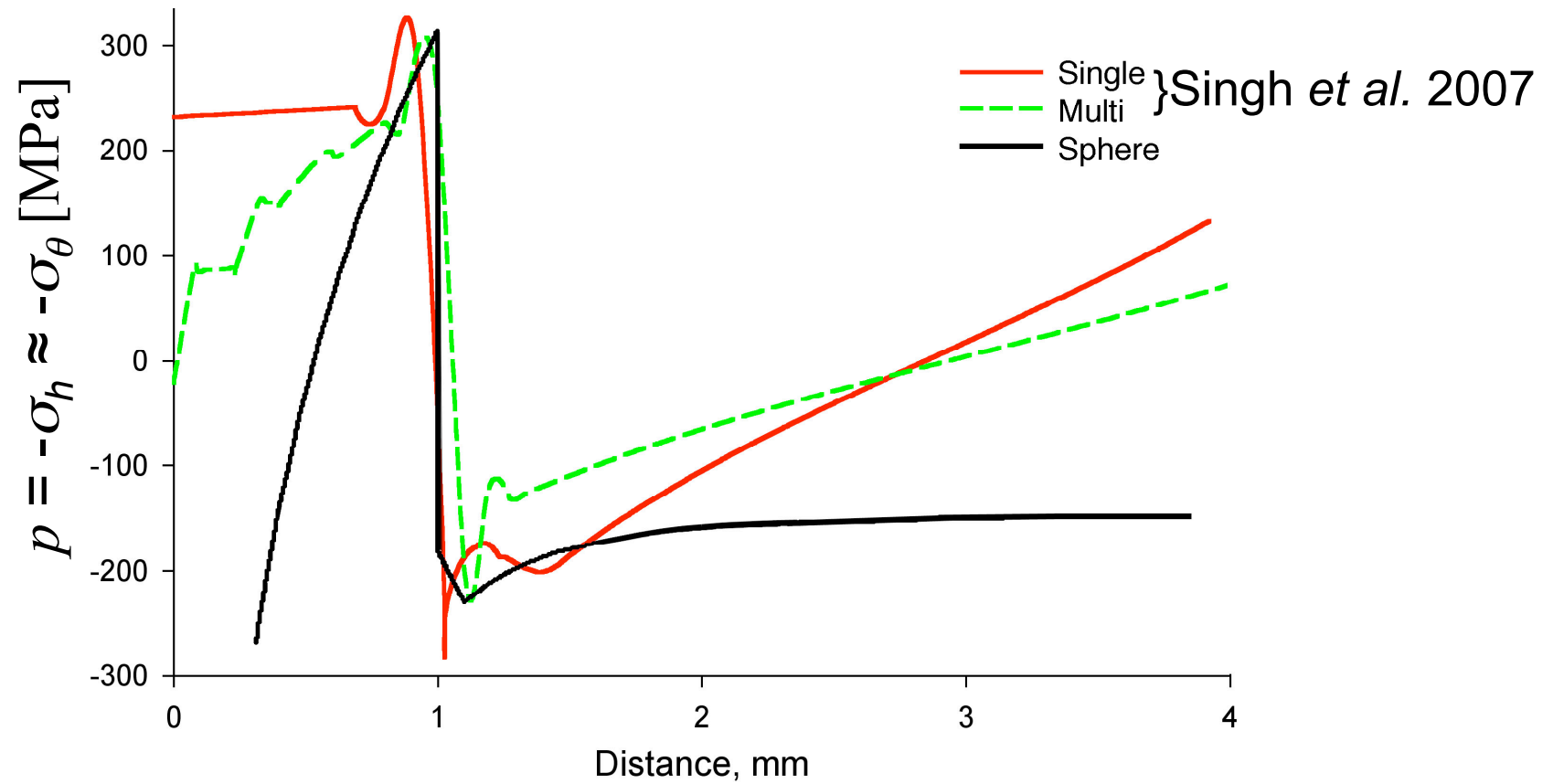


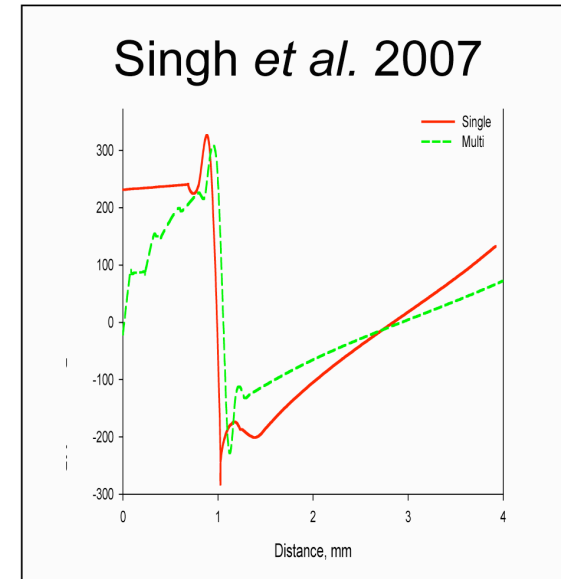
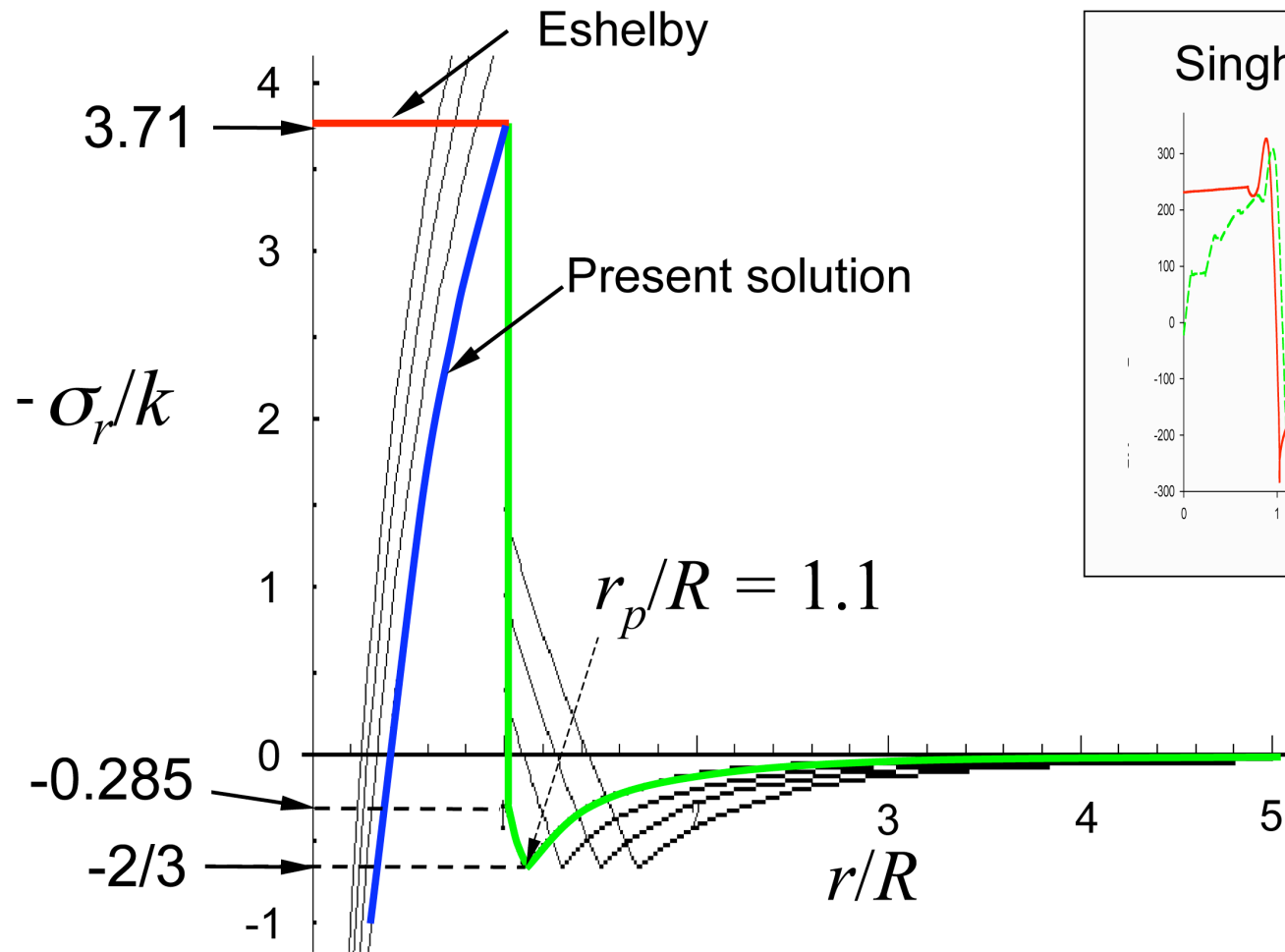
Fig. 2 Different characteristic regions of the solution. In $r > r_p$ the material is elastic; in $r \leq r_p$ the material is plastic; in $r_{c1} < r < R$ no tensile stress; in $r_o < r < r_{c1}$ tensile radial stress; in $r_{c2} < r < r_o$ tensile radial stress and hydrostatic stress; in $r < r_{c2}$ all stresses are tensile.

$$r < r_{c1} = \frac{R^2}{r_p e^{1/3}}, \quad r < r_o = \frac{R^2}{r_p e^{2/3}} \quad \text{and} \quad r < r_{c2} = \frac{R^2}{r_p e^{5/6}}$$

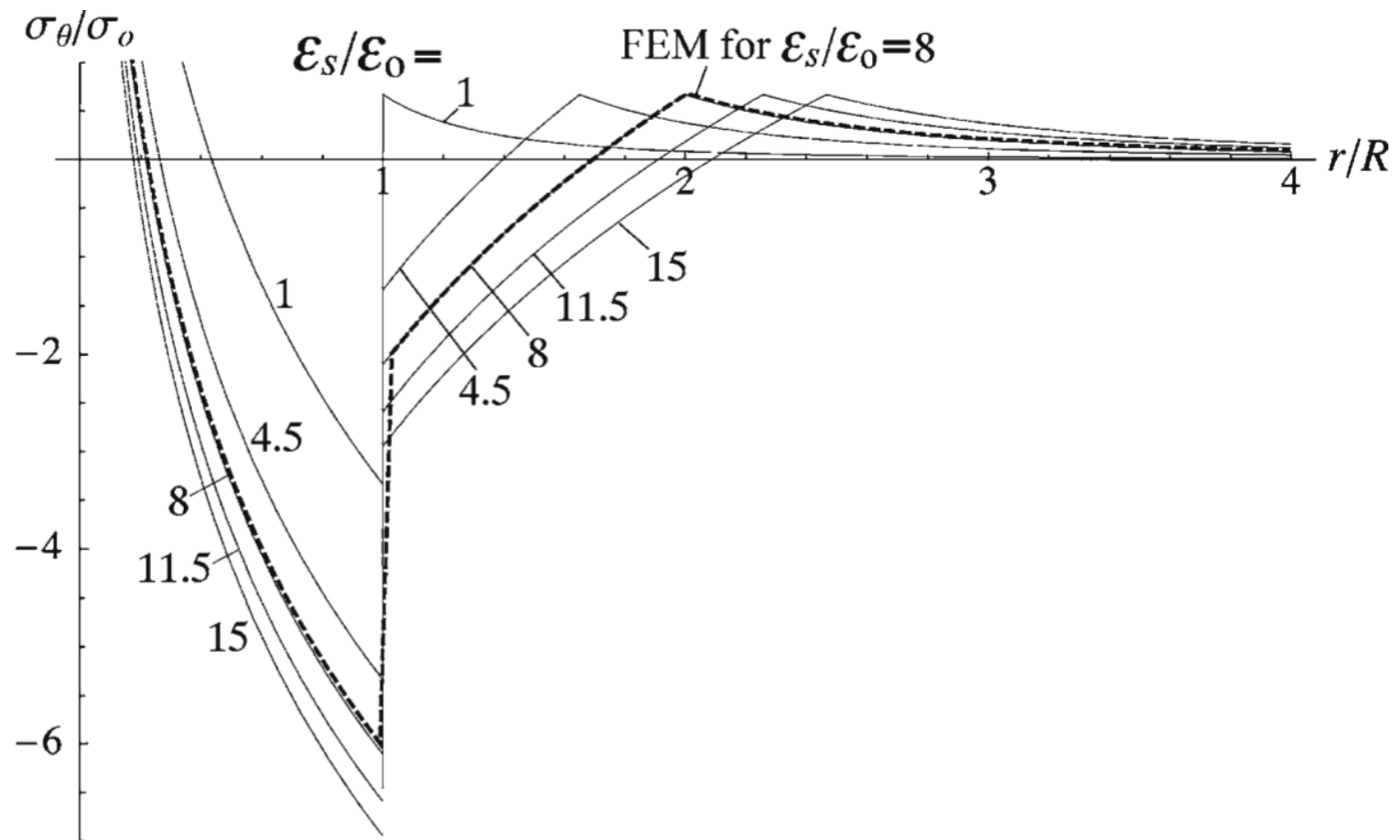
Essential result of FEM and present solution



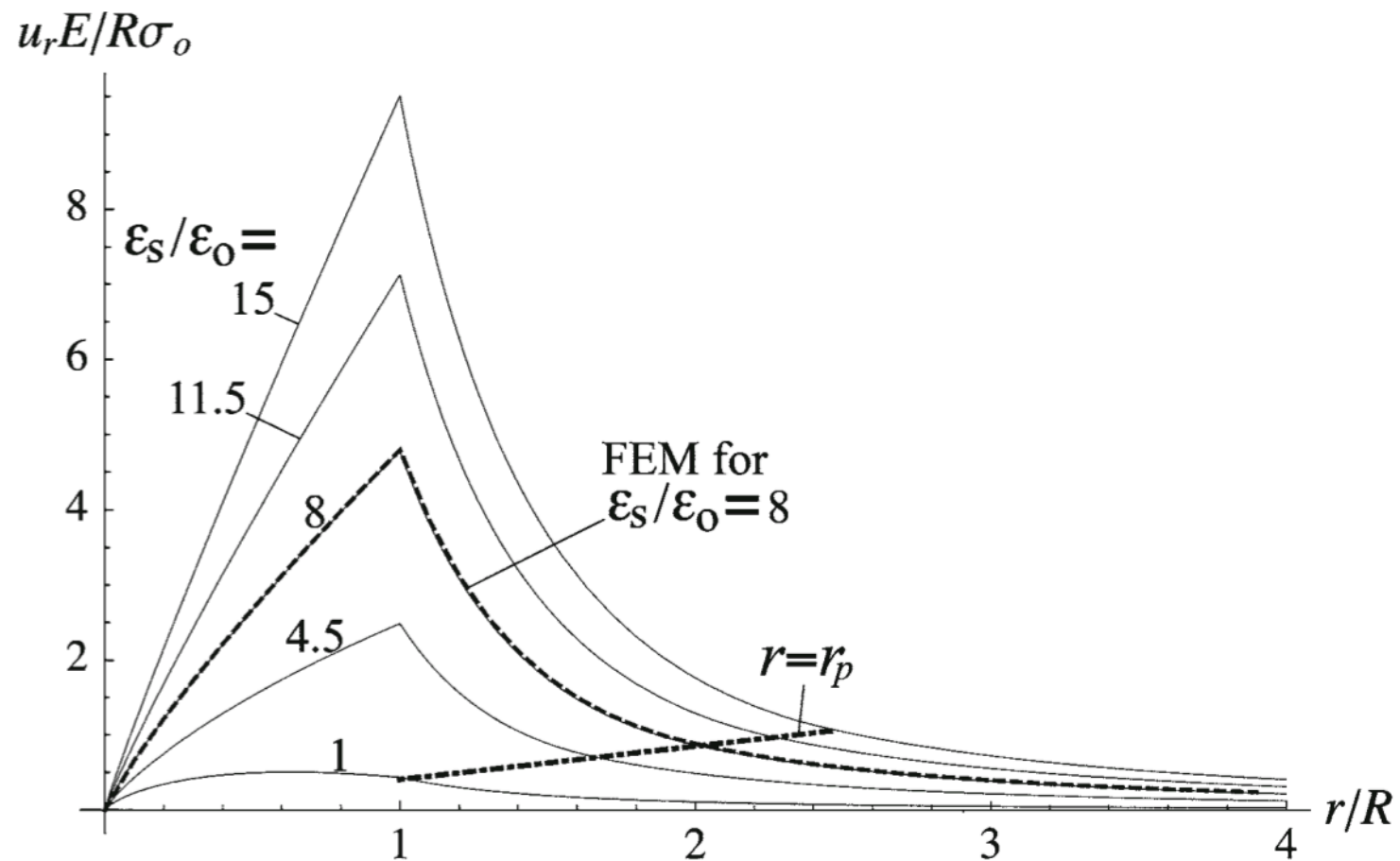
Radial pressure, $-\sigma_r/k$, versus vs. distance r/R

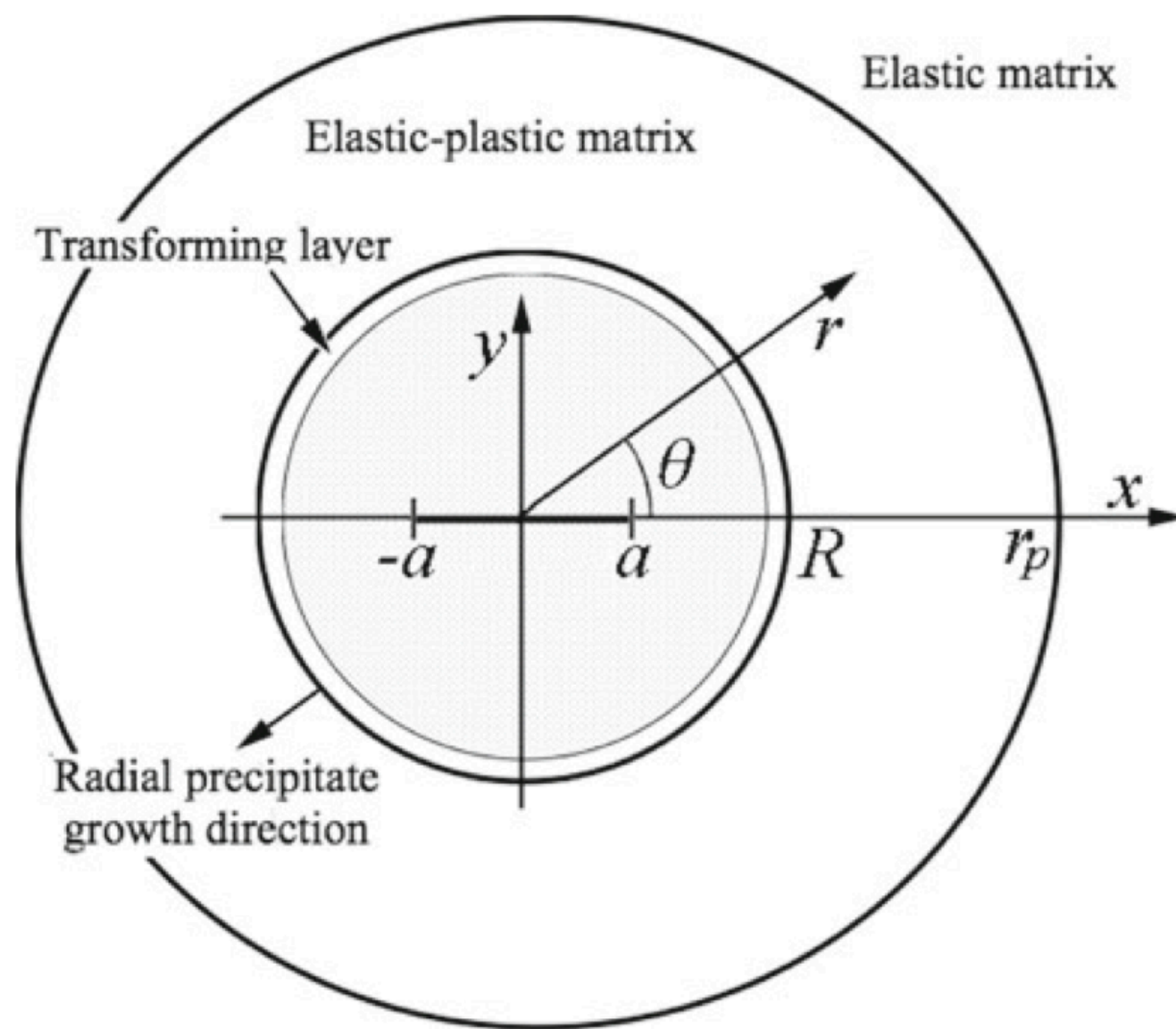


Hoop Stress

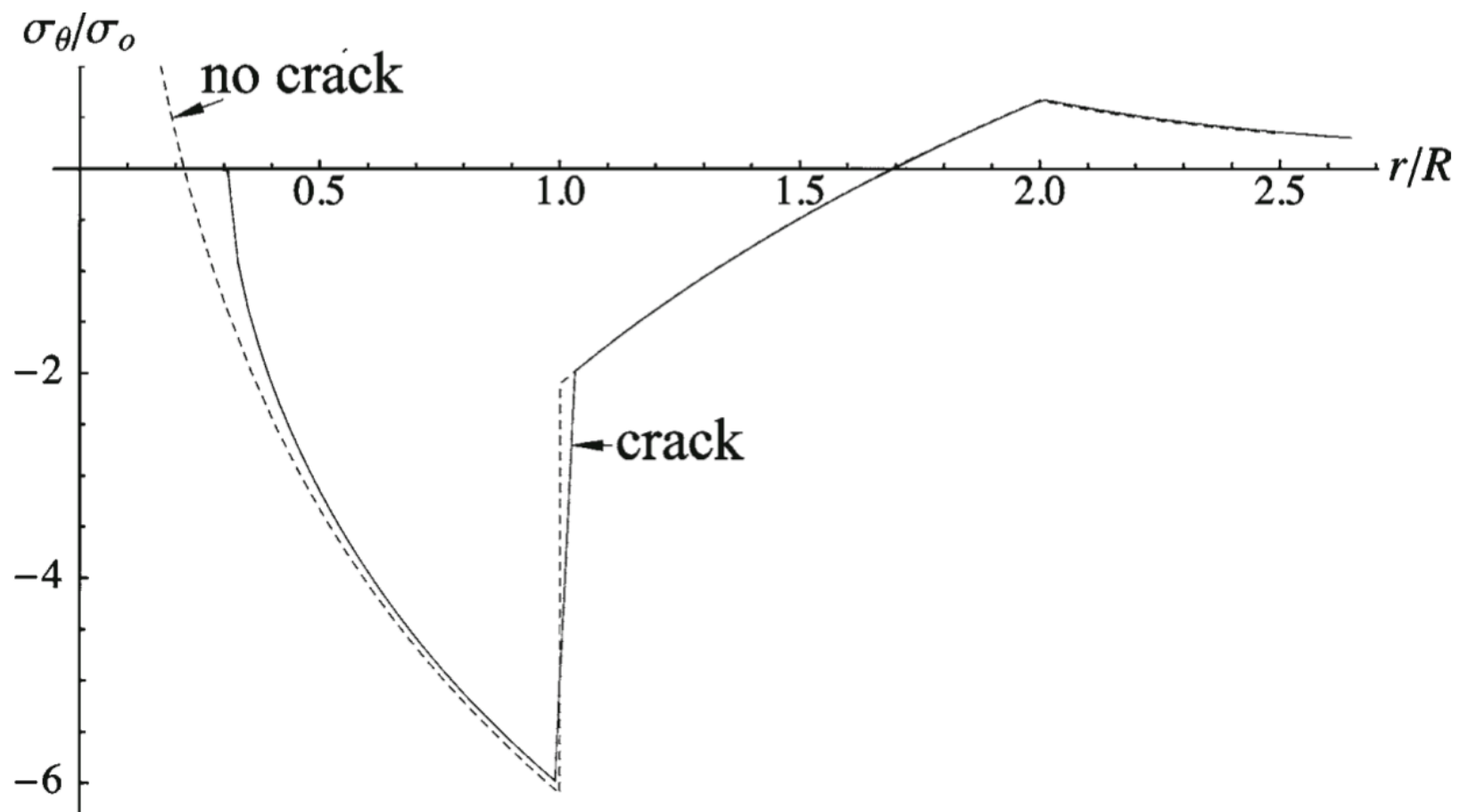


Radial Displacement

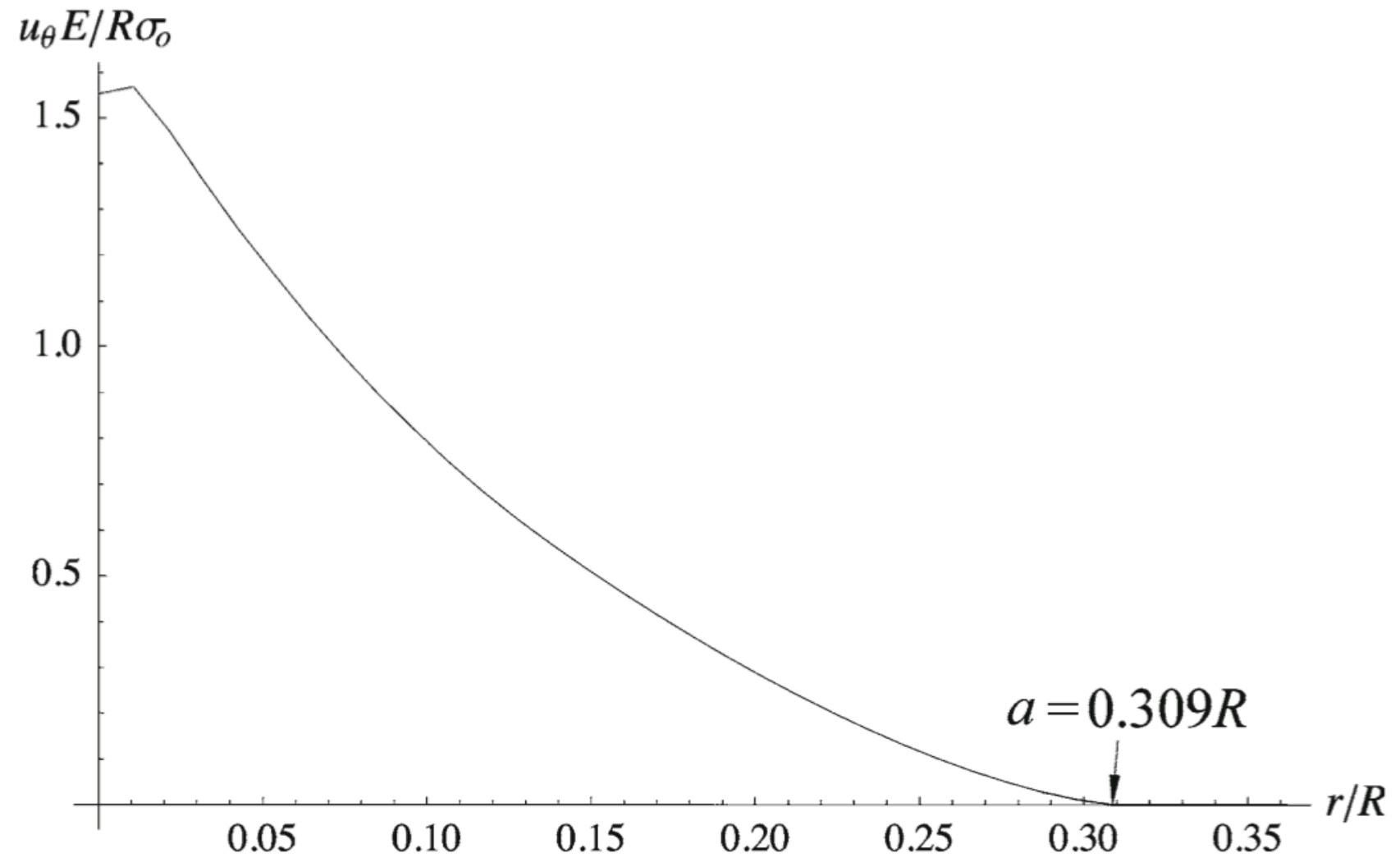




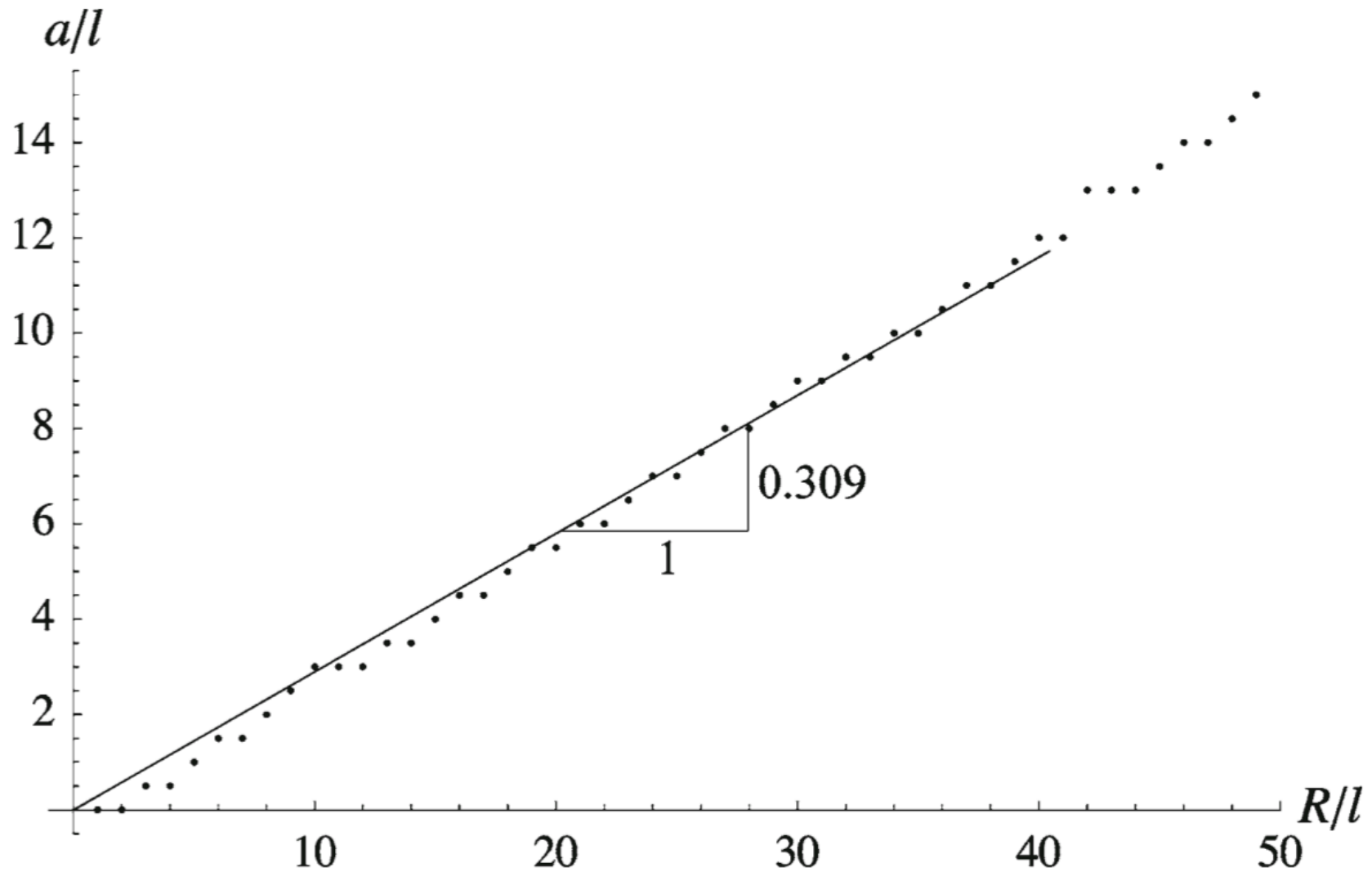
Hoop Stress



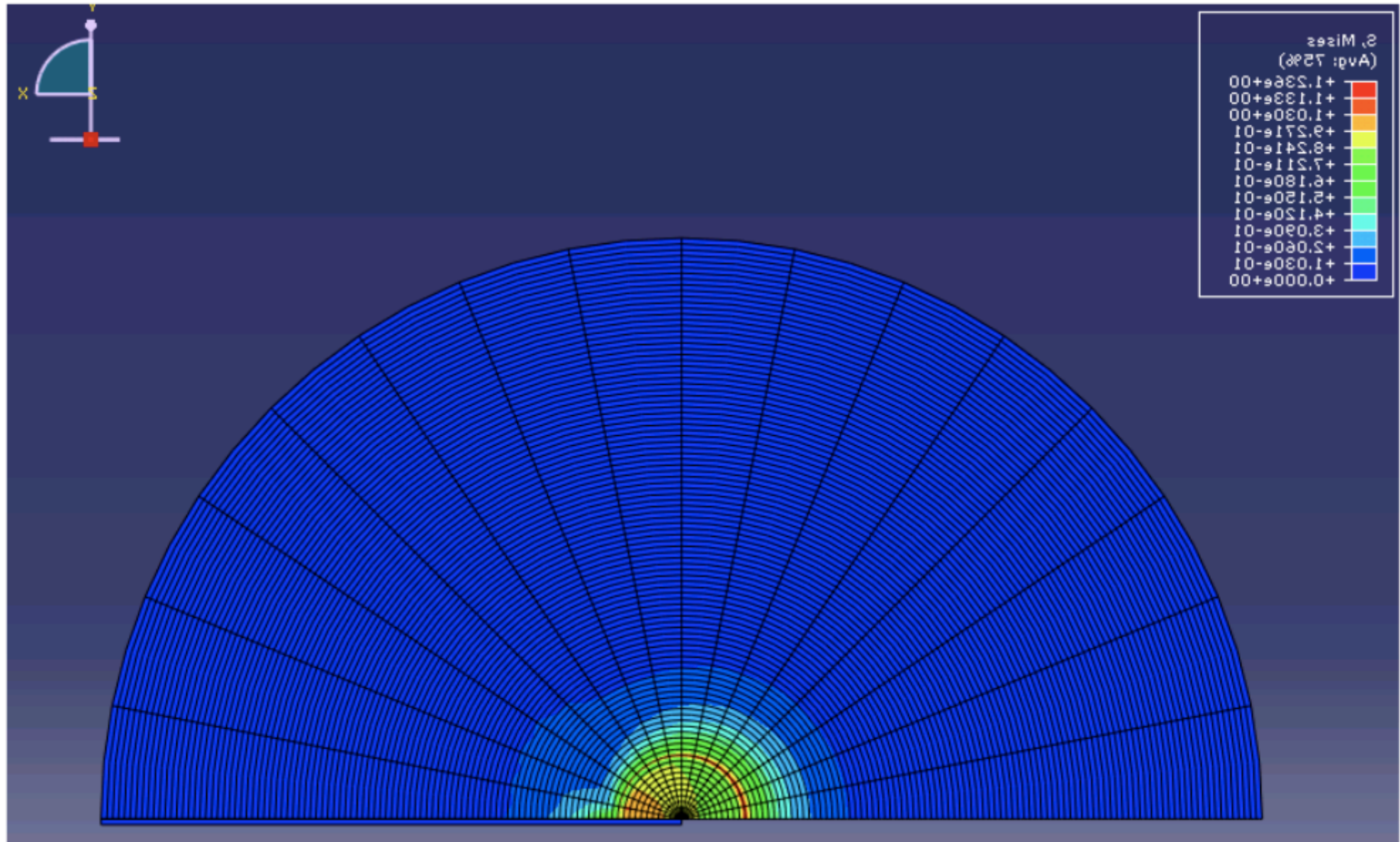
Crack Surface Displacement



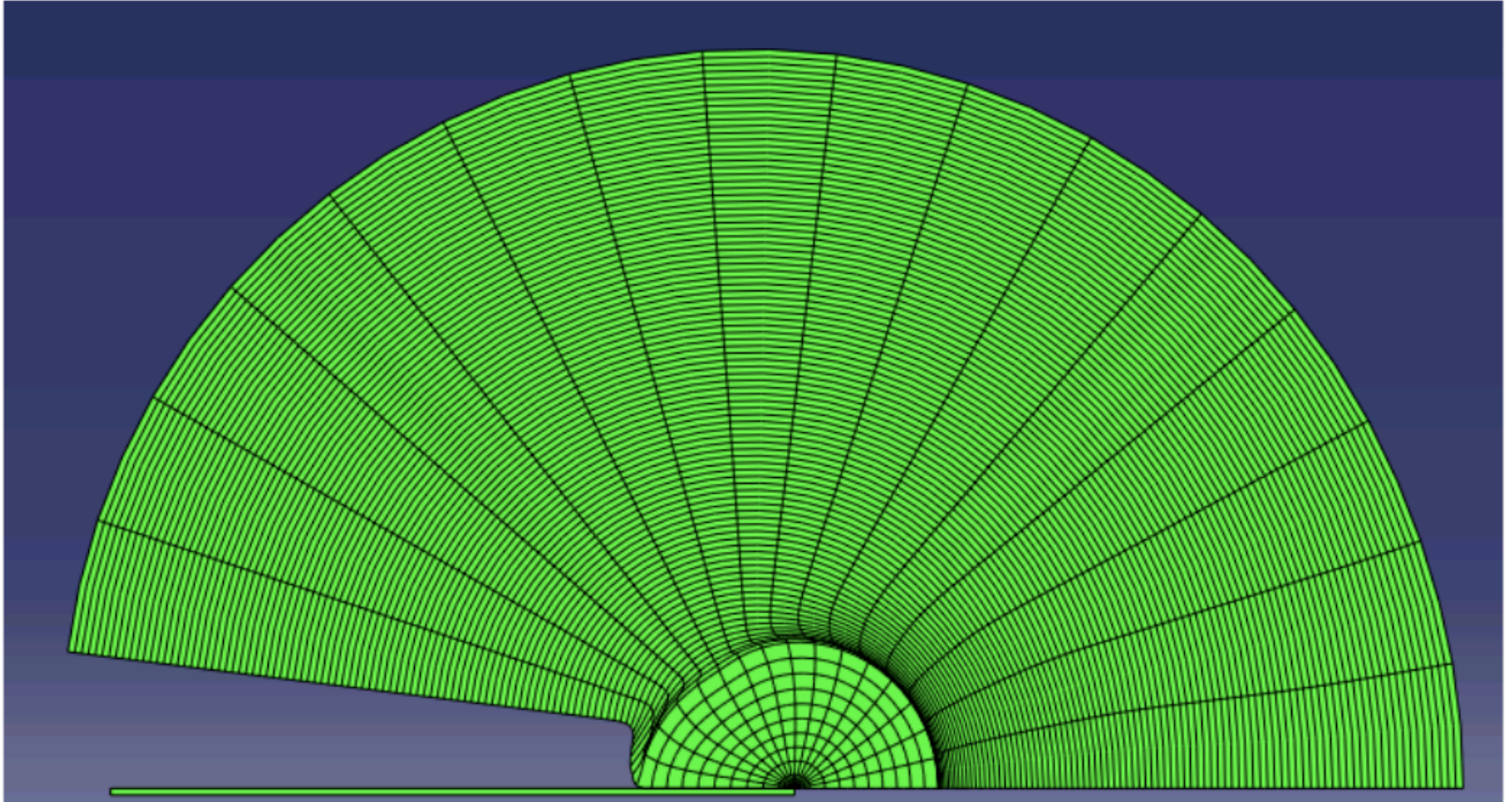
Crack Length vs. Inclusion Radii



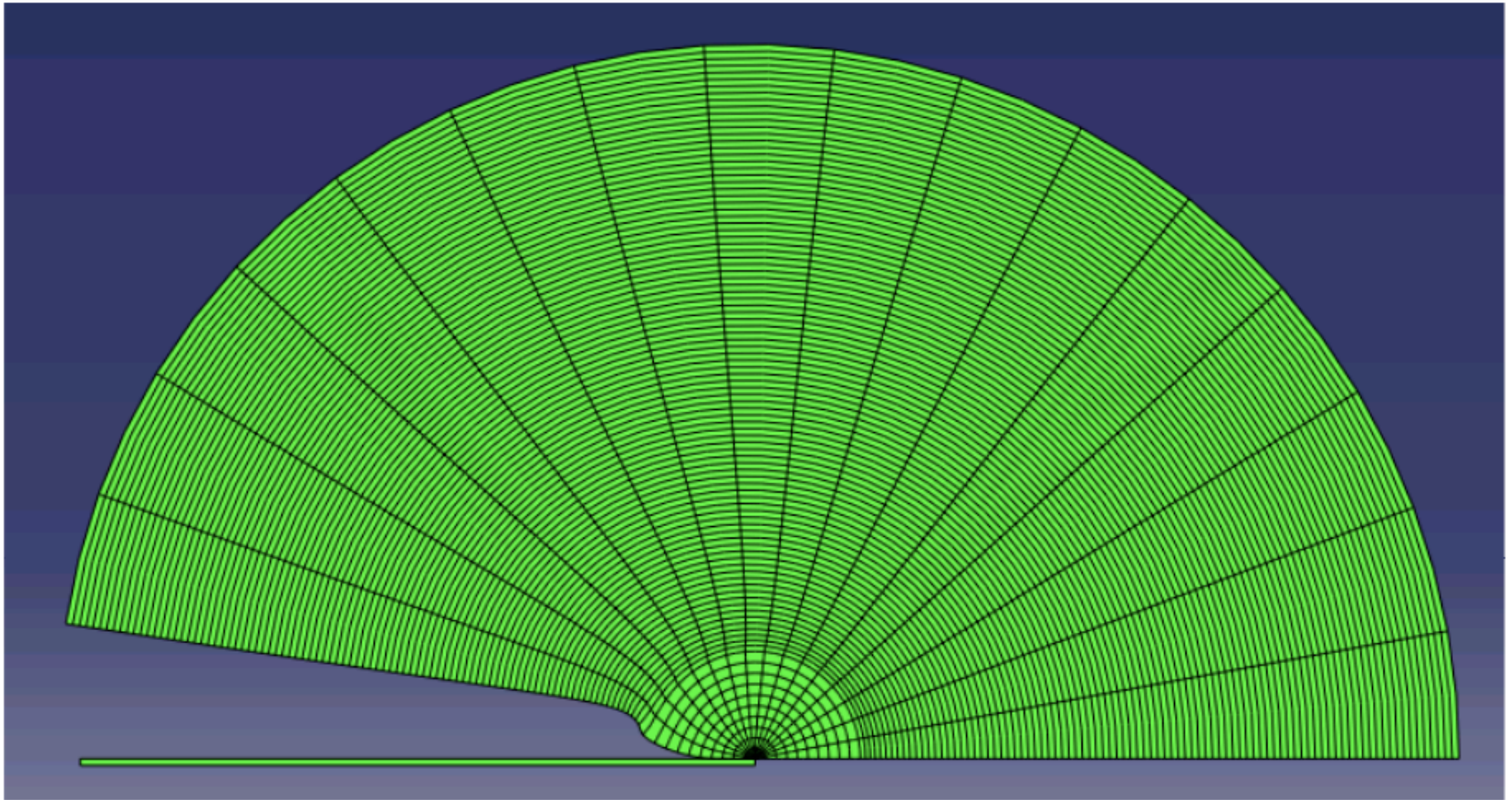
v Mises Stress, Incremental Growth Expanding Cylinder ahead of a Crack



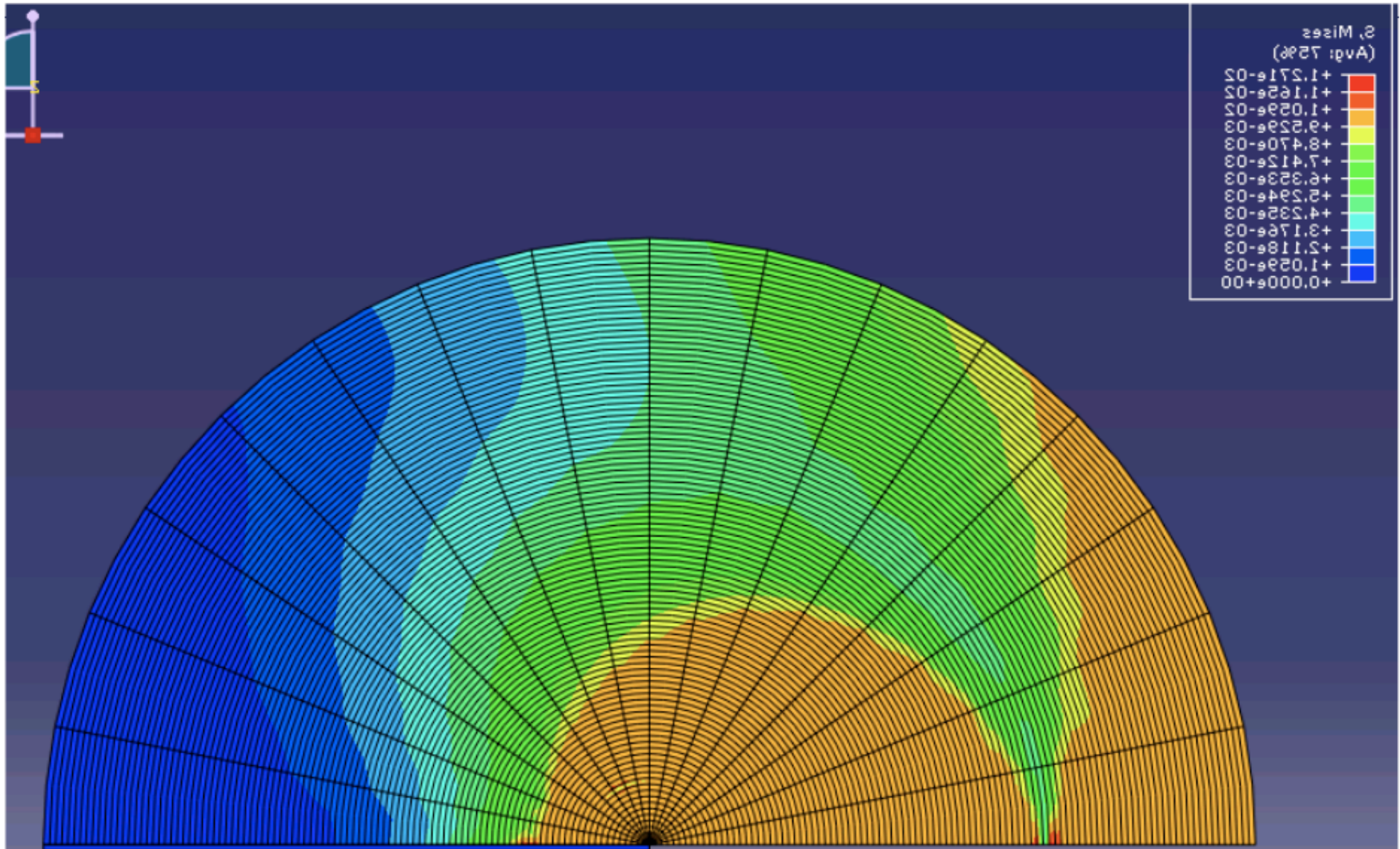
Deformation, Simultaneous Growth
Expanding Cylinder ahead of a Crack



Deformation, Incremental Growth Expanding Cylinder ahead of a Crack



v Mises Stress, Simultaneous Growth Expanding Cylinder ahead of a Crack



Conclusions

- Precipitates that grow at its edges obtain reduced pressure in its interior.
- At self-similar growth, stresses become logarithmically singular for all precipitates shapes.
- The mechanical state of the matrix of homogeneously growing precipitates matches the present solution exactly but for larger expansion.
- Spontaneous crack growth may occur