



# LUND UNIVERSITY

## Fracture as a Moving Boundary Problem

STAMM XVIII, Haifa, Israel, 2012. Orationem Meam.

Ståhle, P.

2012

*Document Version:*

Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*

Ståhle, P. (2012, Sept 5). Fracture as a Moving Boundary Problem: STAMM XVIII, Haifa, Israel, 2012. Orationem Meam.

*Total number of authors:*

1

### General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# STAMM XVIII, Haifa 2012

## Fracture as a Moving Boundary Problem

Per Ståhle

Solid Mechanics,  
Lund University, Sweden

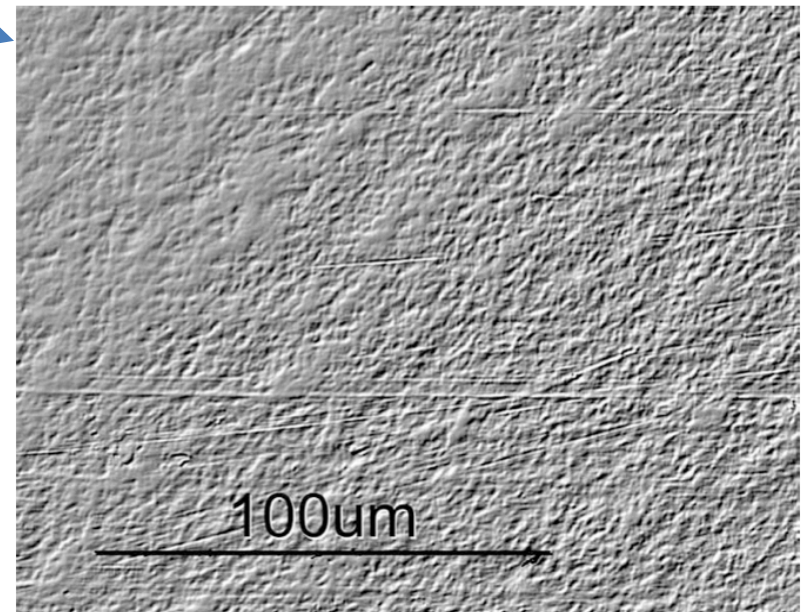
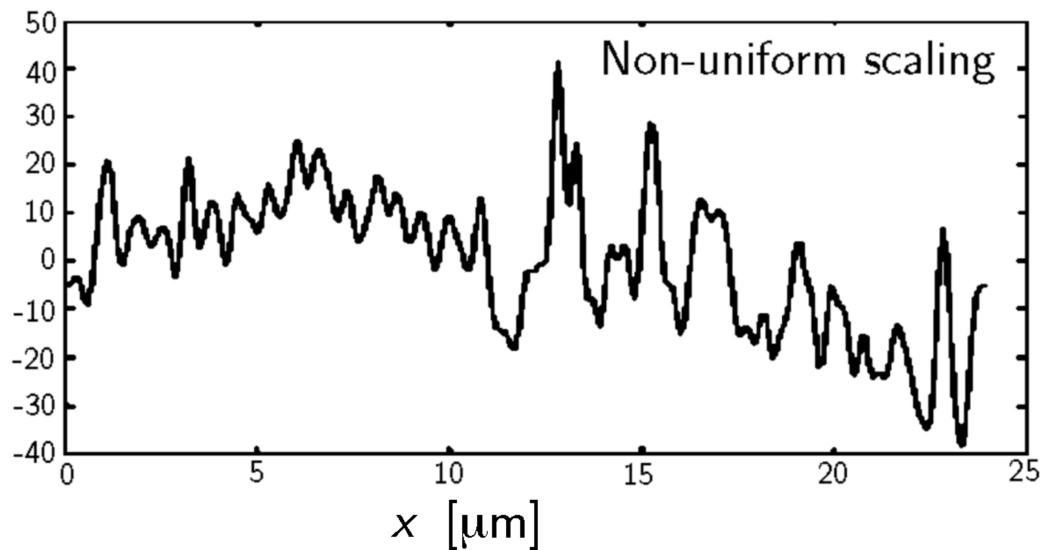
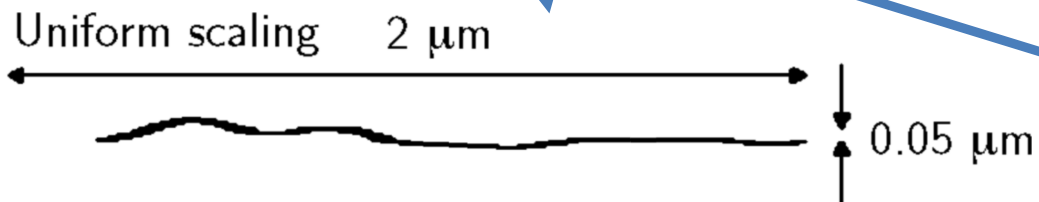
Co-workers: F. Nilsson, L. Ljungberg, L. Nelson,  
J. Gunnars, T.C. Wang, A.P. Jivkov, C. Bjerken,  
U. Hejman, E. Hansen

# Corroding environment leads to:

1. Continuous loss of mass
2. Pitting
- ... and with mechanical stress present
3. Surface roughening



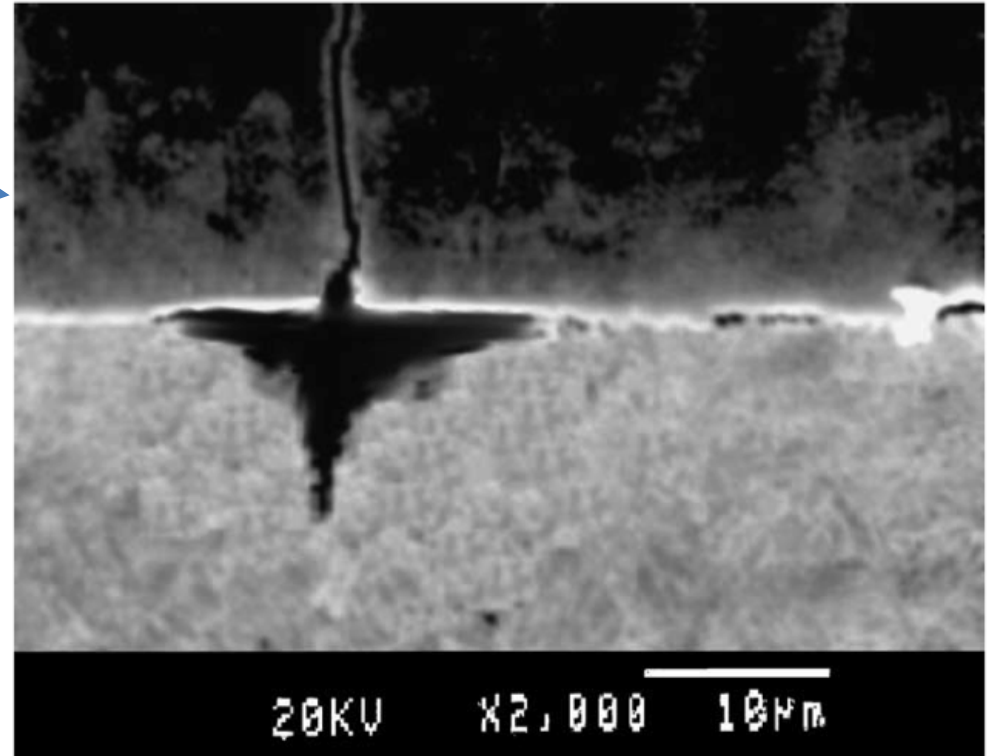
(from Kim *et al.* 2000)



4. Evolving pits
5. Formation of cracks
6. Crack growth
7. Crack branching

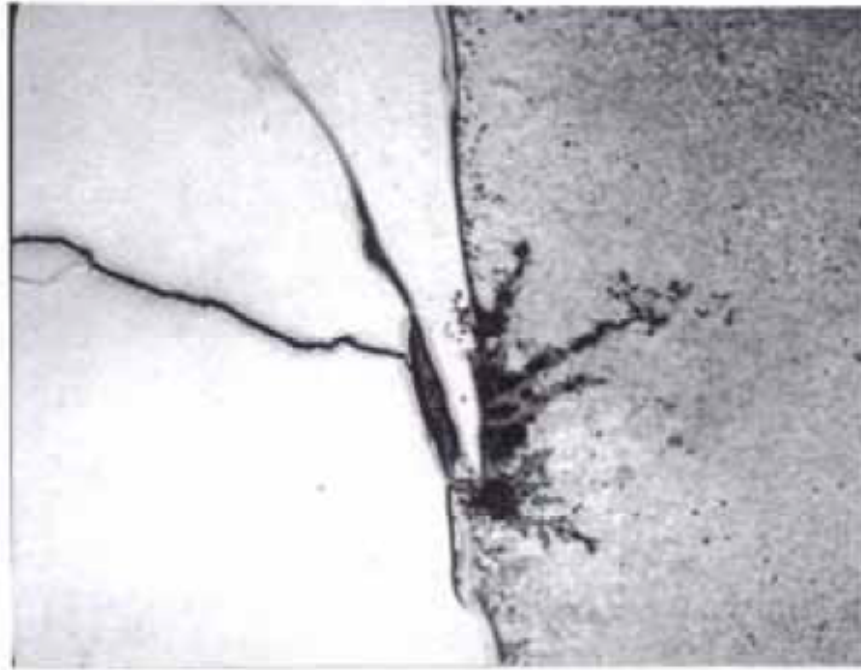


Growing crack in a polycarbonate exposed to acetone (Hejman 2011)

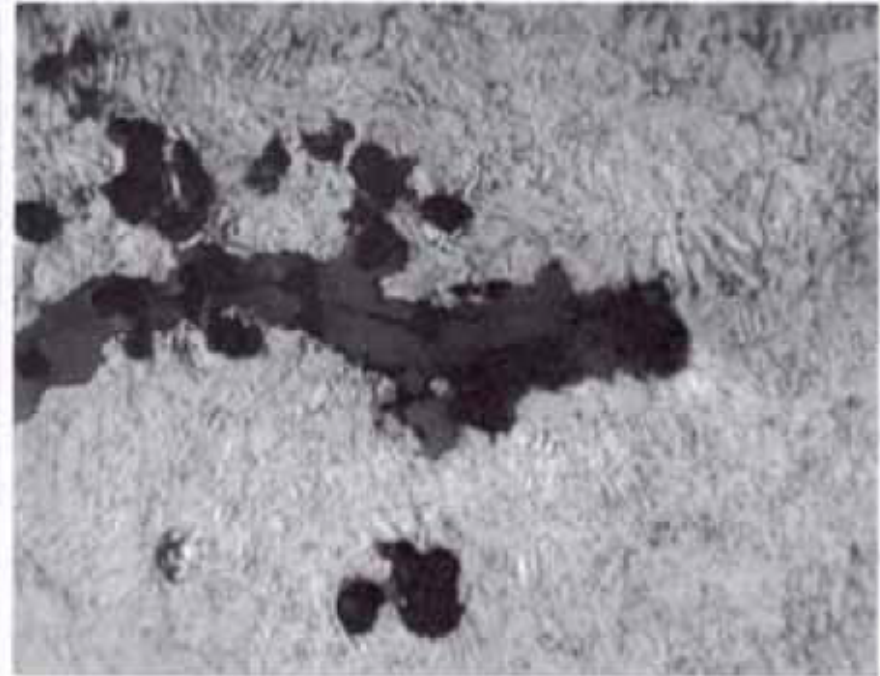


Cr/zone six charge related of land and groove substrate erosion through a micro-crack at the 12:00 bore origin. (Sopok *et al.* 2005)

# Corrosion Crack crossing a bi-material interface

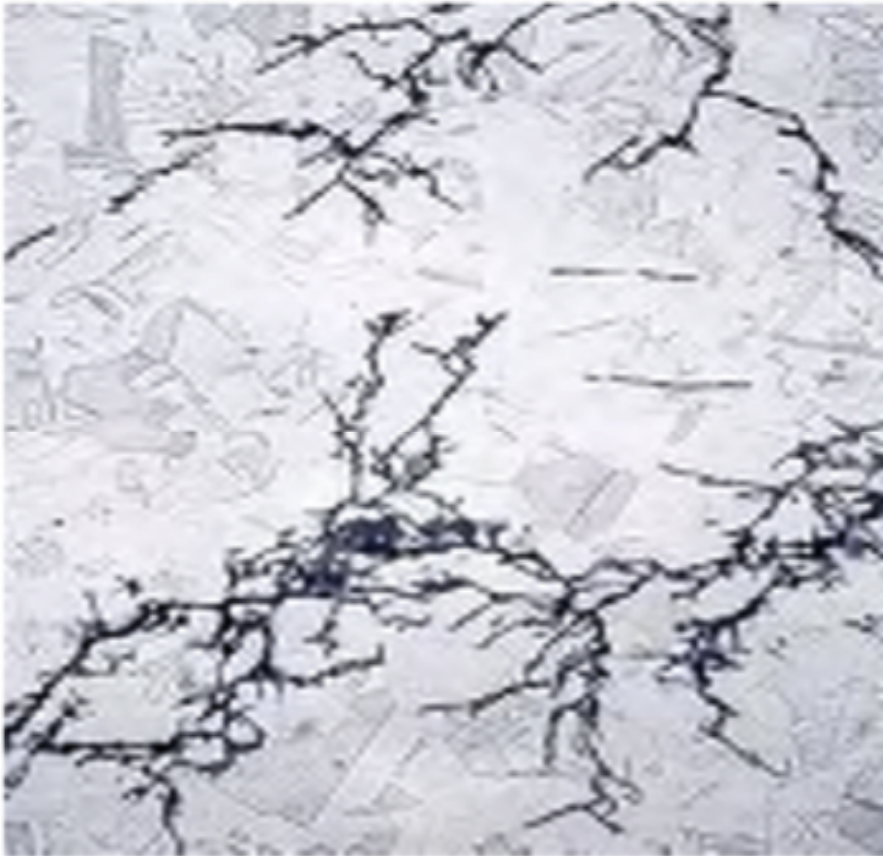


Corrosion crack penetrating a bimaterial interface between austenitic and pressure vessel steel of type SA533C11.



The tip of one of the crack branches. Crack length 7 mm, notch width 10  $\mu\text{m}$ . *Reproduced with permission from Vattenfall AB.*

# Biocorrosion



Corrosion in stainless steel in the presence of Gallionella Bacillus

Known to cause SC

Anaerobic Bacteria:

*Desulfovibrio, -maculum, -monas*

Aerobic Bacteria: *Thiobacillus*

Fungi, Algae, Protozoans

Surface Morphology:  
Surface Wave Spectrum

Crack Initiation:  
Pit, Cusp and Crack

Crack Growth:  
Crack Growth, Blunting and Branching

# Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993), Srolovitz (1989), Freund (1995), Kim (2000)

Gibb's free energy

$$\Phi = U_c + U_e$$

where

$U_c$  is the free chemical energy and

$U_e$  is the free elastic energy



The free chemical energy

$$U_c = -\gamma \frac{\partial^2 h}{\partial x^2}$$

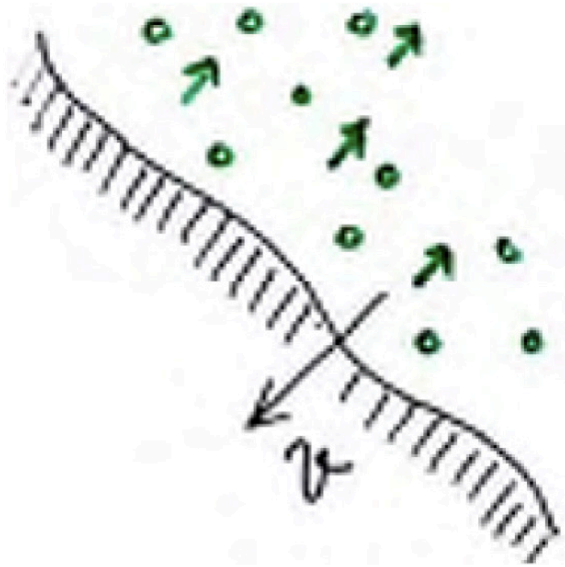
where

$h(x)$  gives the position of the surface  
 $\gamma$  is the surface energy density

The free elastic energy (Cerutti)

$$U_e = \frac{1}{2} \sigma_{ij} \epsilon_{ij} \sim \frac{1}{2} \mu \frac{\partial h}{\partial x}$$

# Evaporation-condensation



$$\frac{\partial h}{\partial t} = -L_1 \Phi$$

# Surface diffusion



$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2 \Phi}{\partial x^2}$$

Governing equations:

Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left( \gamma \frac{\partial^2 h}{\partial x^2} - \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

or surface diffusion

$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left( -\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2^\mu} \frac{\partial h}{\partial x} \right)$$

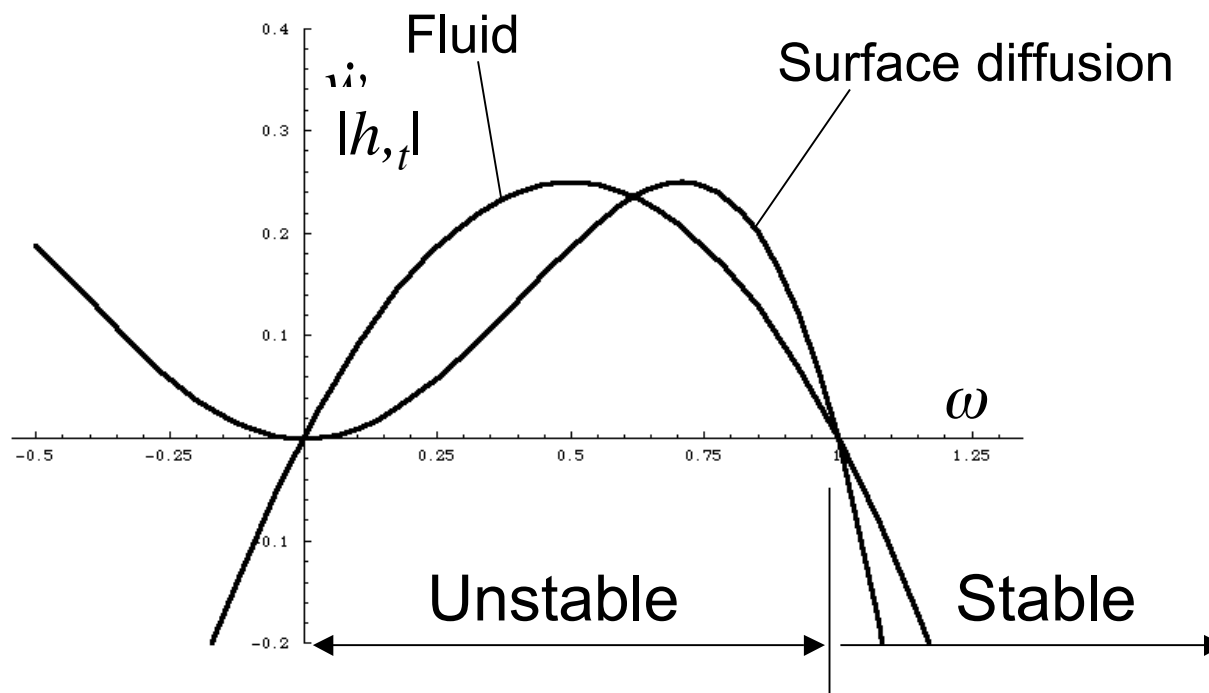
# 1. Nearly Plane Surface:

Fouriertransform, put  $h = A \sin \omega x$   $|A| \ll x$

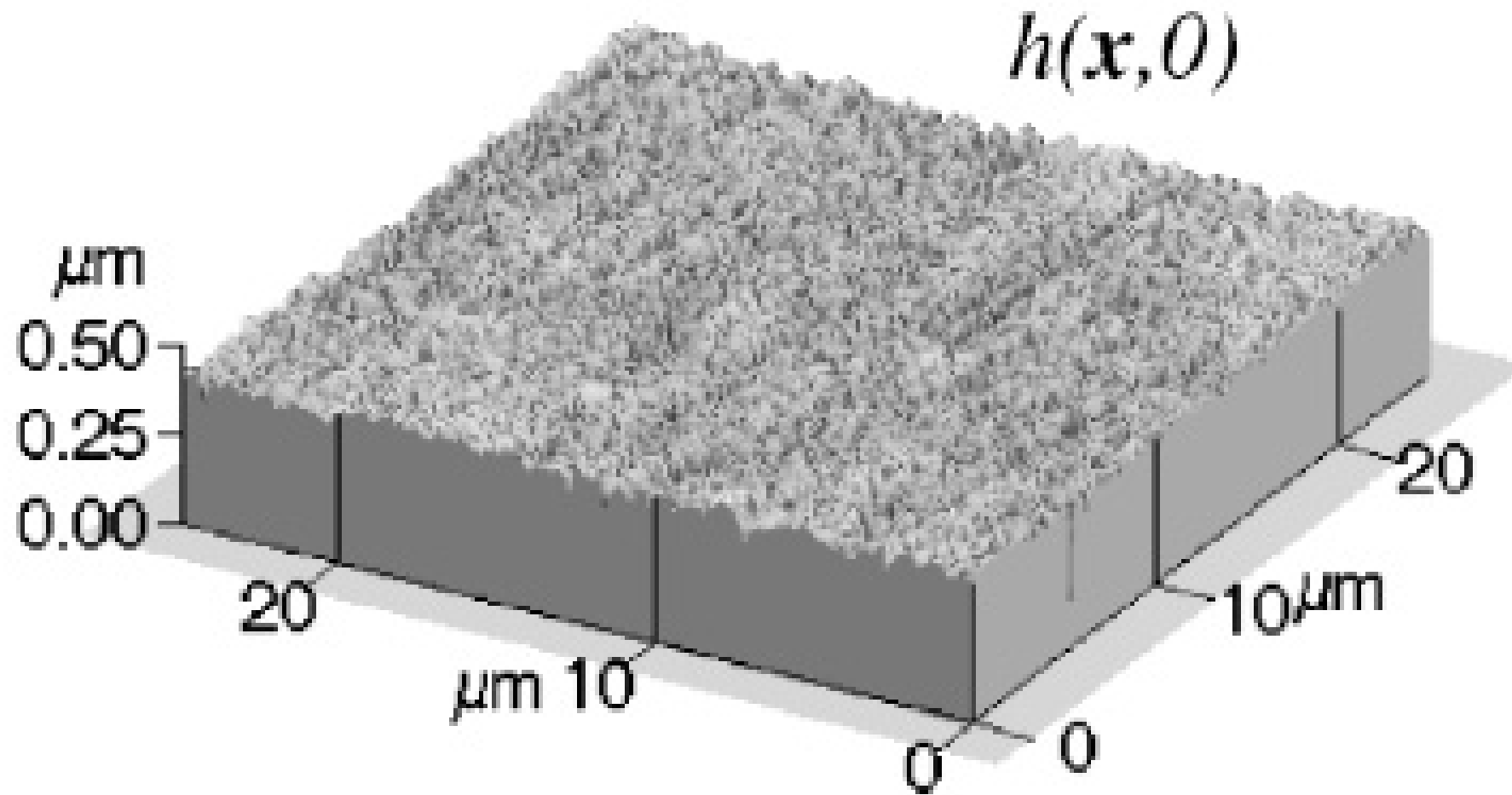
Surface Diffusion

Fluid

$$h_{,t} = D[(\mu/2)\varepsilon_0 \omega^3 - \gamma\omega^4]h \quad h_{,t} = k[(\mu/2)\varepsilon_0 \omega - \gamma\omega^2]h$$

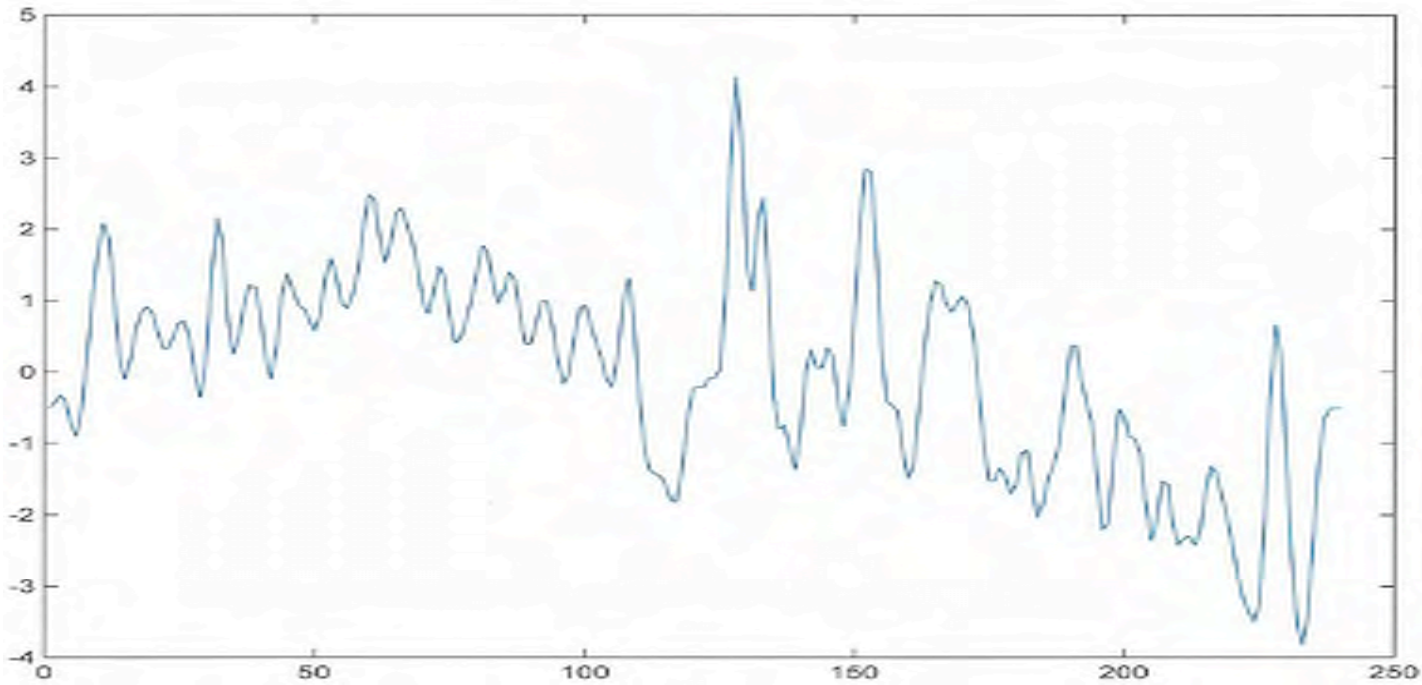
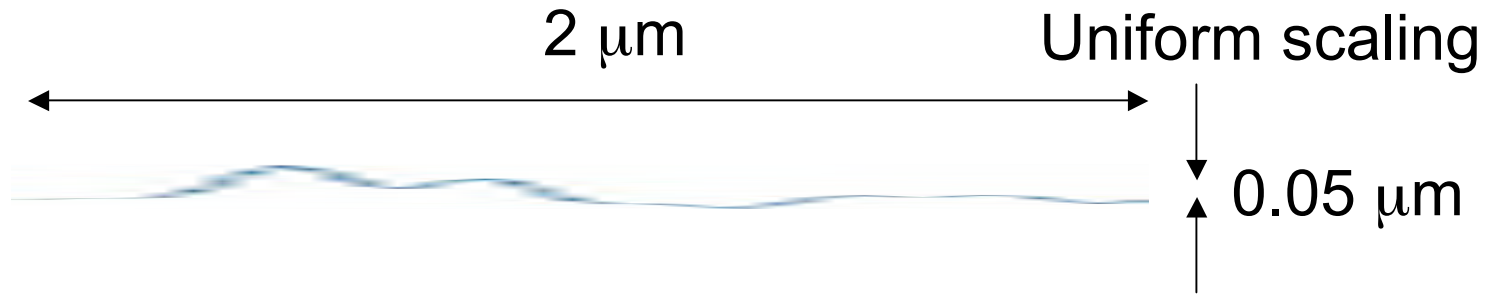


(verified by Kim et al., 2000)



AFM image of shallow etched aluminium (Kim *et al.* 2000)

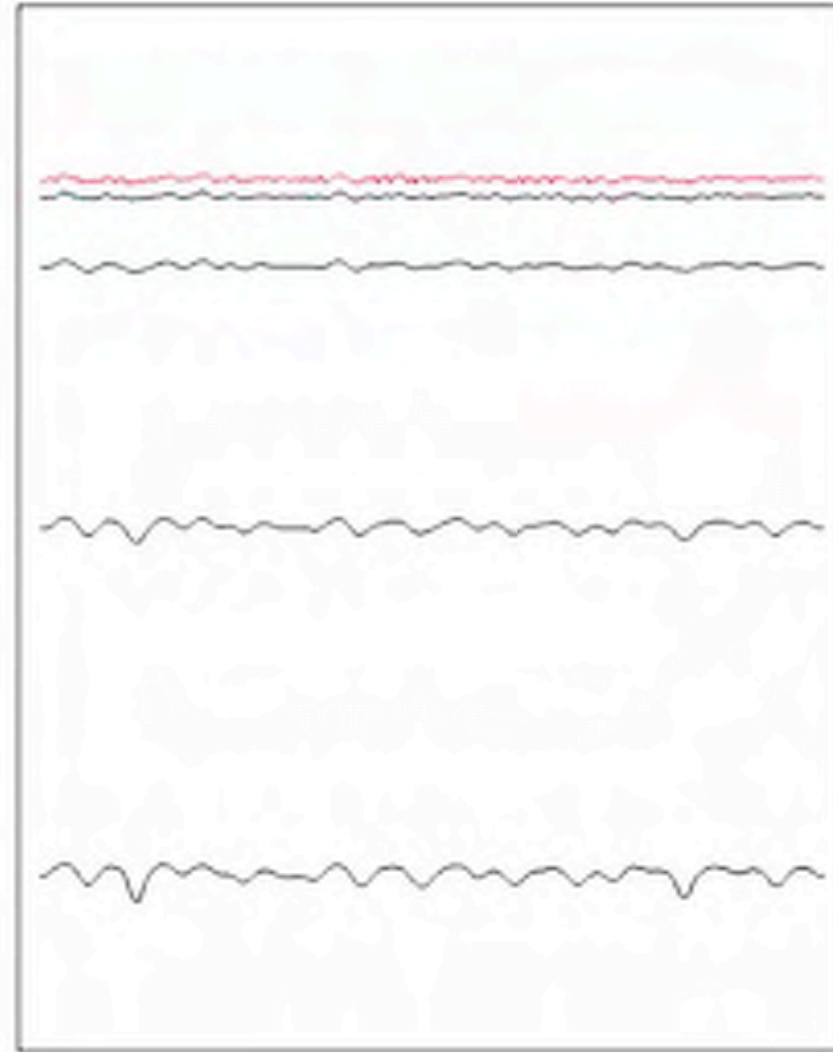
# Surface Roughness



(Kim *et al.*, 2000)

## FEM calculation of an evolving surface

Increasing time  
and depth



Original surface

## 2. Cusp solutions

Spencer and Meiron(94), Chiu and Gao (95),  
Yang Xiang and Weinan (02)

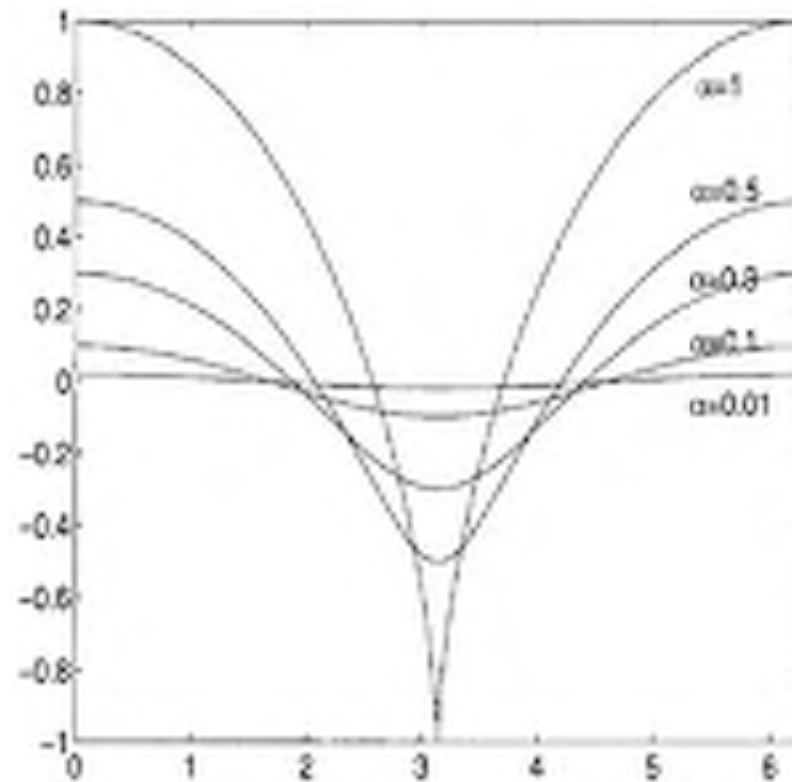
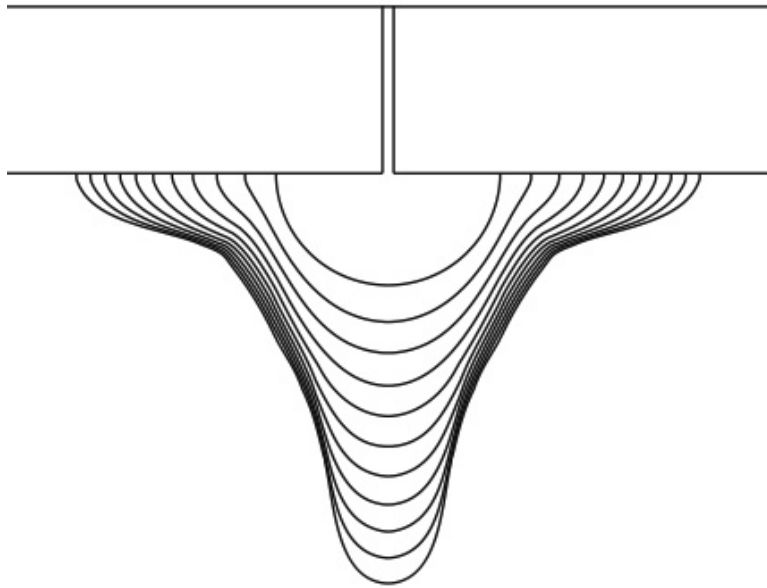
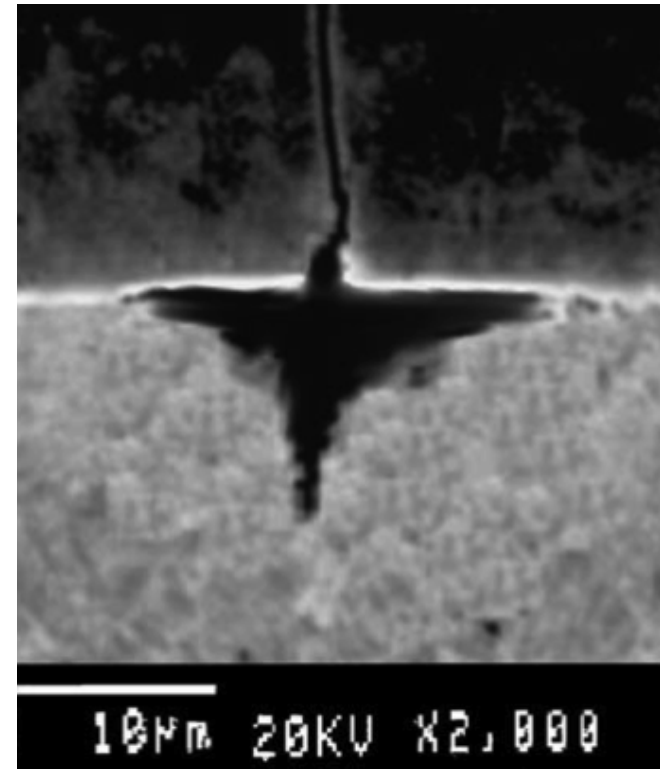


FIG. 1. Cycloid surfaces.



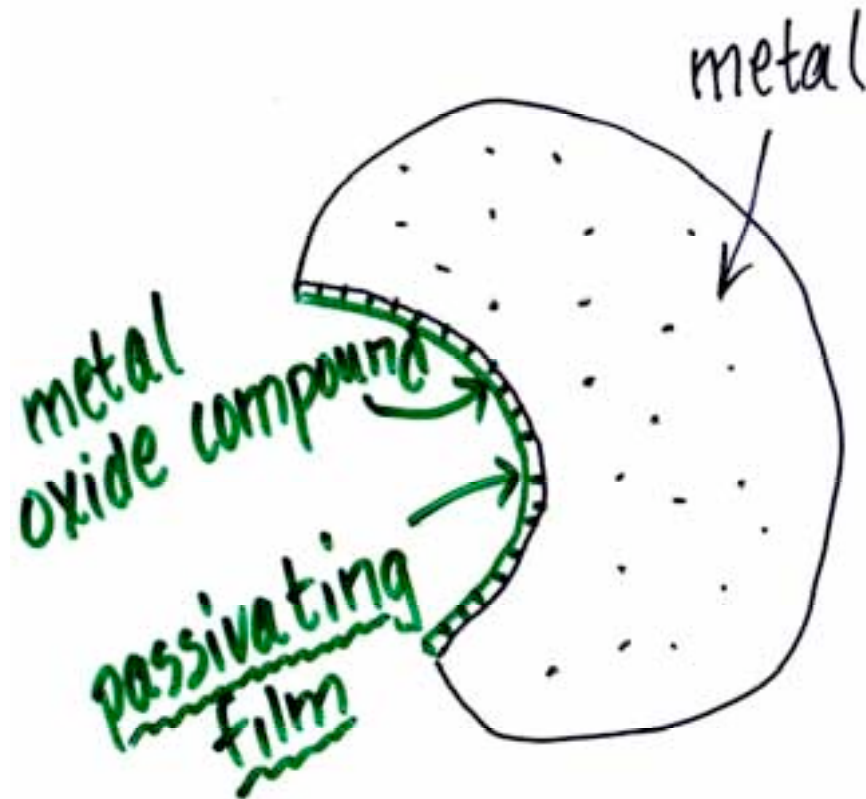


FEM simulation of material dissolution coating  
(Bjerkén and Ortiz, 2010)



SEM observation of corrosion-erosion crack in a canon bore  
(Sopok et al. 2005)

# Strain driven material dissolution

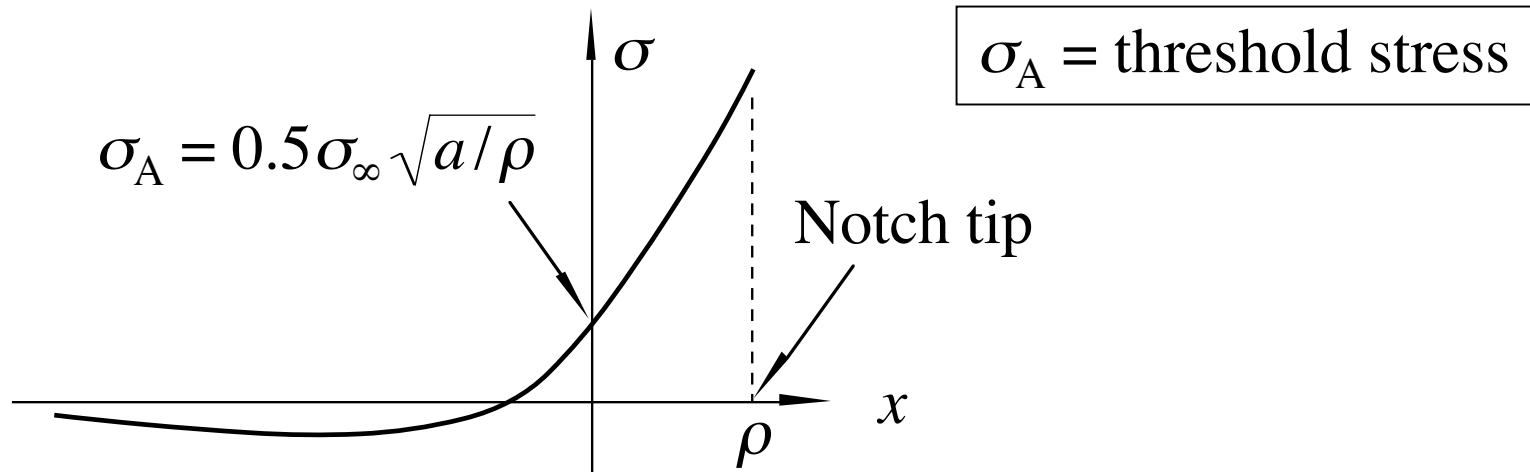
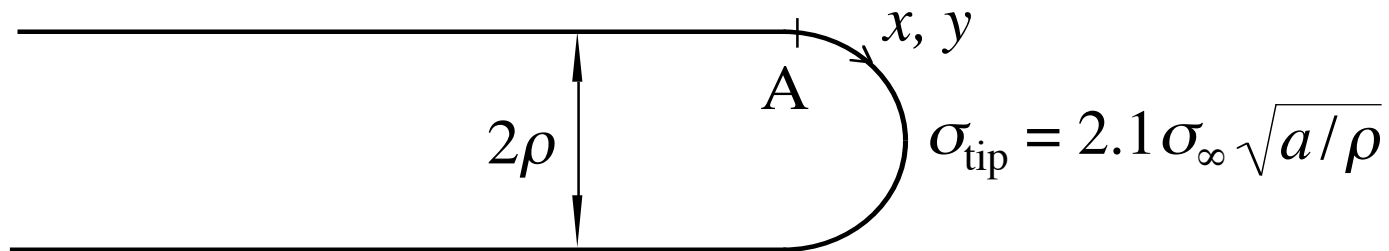


typical width < 10 nm

Cyclic load  
damaging the oxide  
film

$$U = k (\sigma - \sigma_f)^n$$

## Stress in a half circle notch



$$K_I = 1.12\sigma_{\infty}\sqrt{\pi a}, \sigma_{\text{th}} = \sigma_A \Rightarrow \rho = 0.625(K_I/\sigma_{\text{th}})^2$$

## 2. Cusp solutions

Spencer and Meiron(94), Chiu and Gao (95),  
Yang Xiang and Weinan (02)

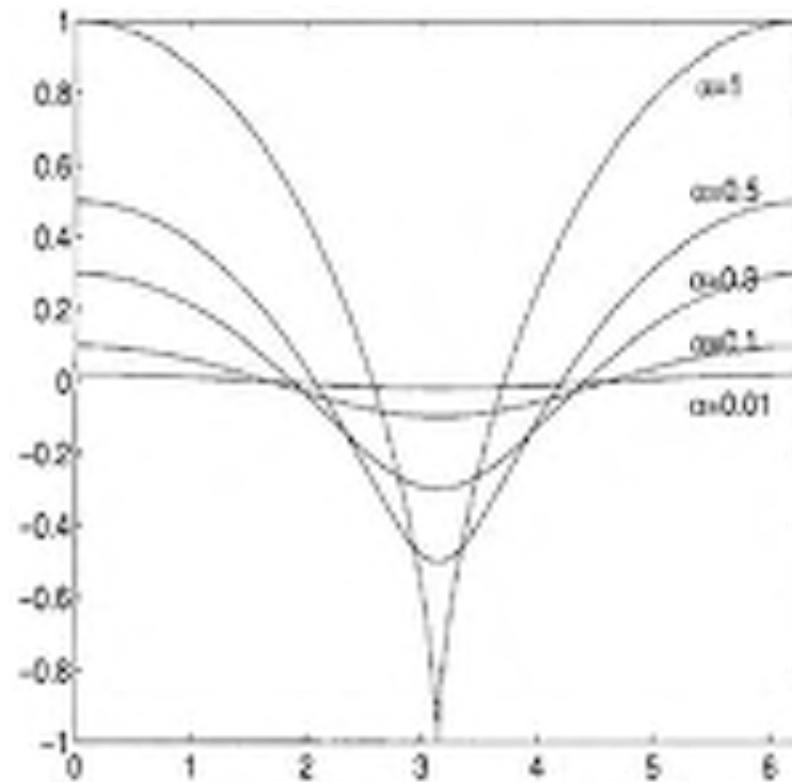
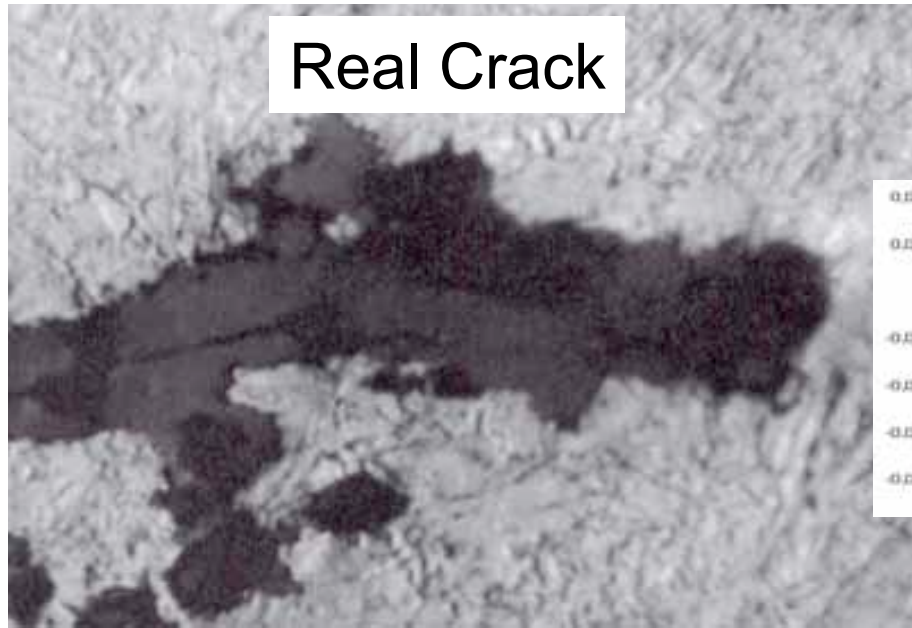
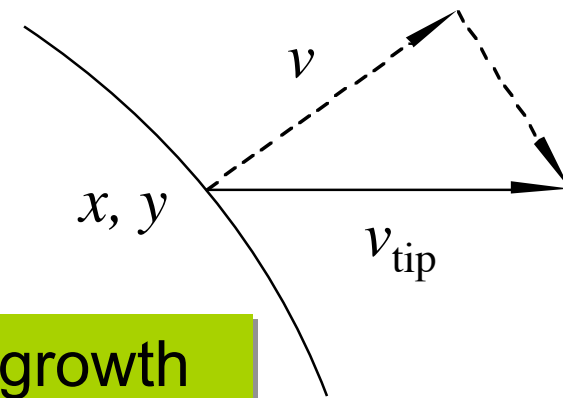
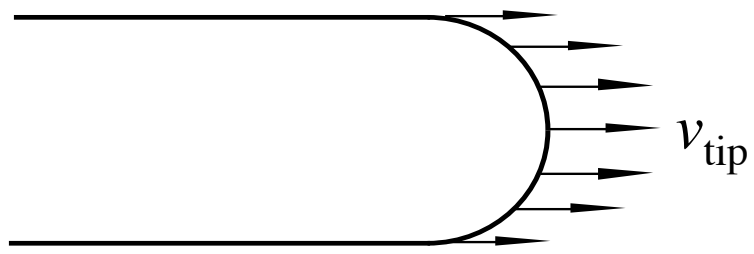
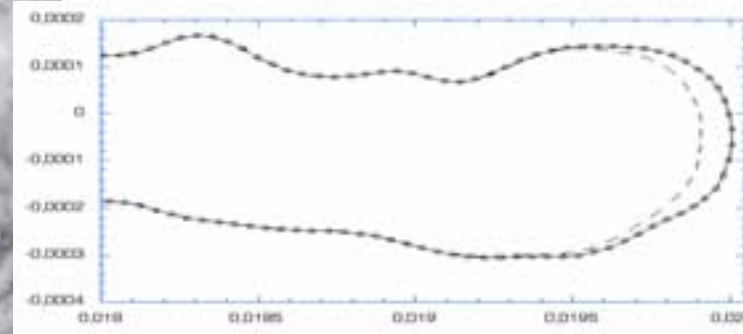


FIG. 1. Cycloid surfaces.

# Steady State Crack-tip Shape

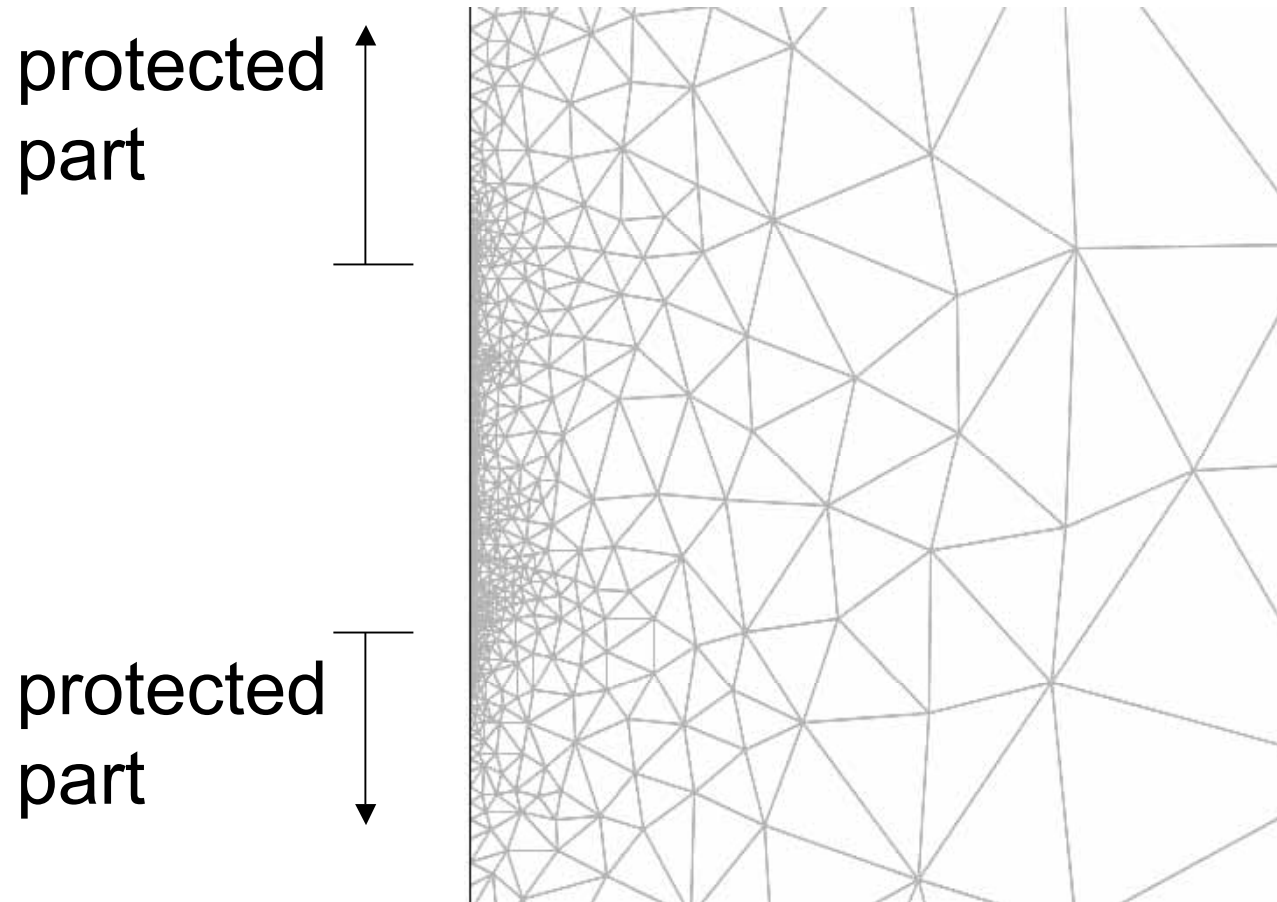


FEM result

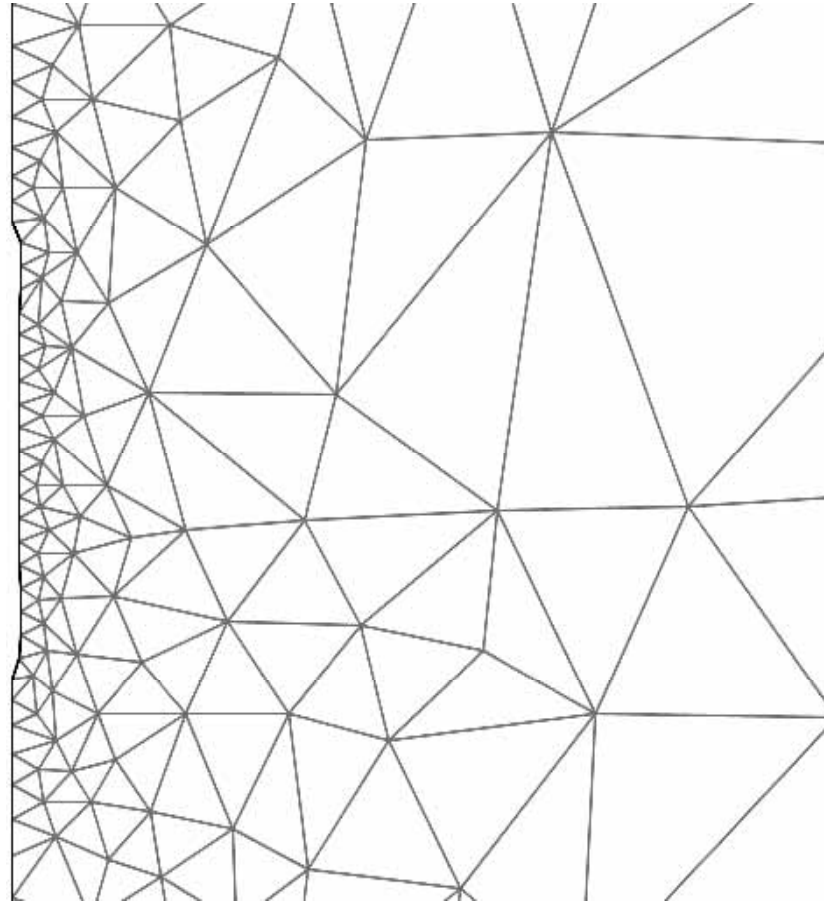


Constant Component in the growth direction:  $v = (x/\rho) v_{tip}$

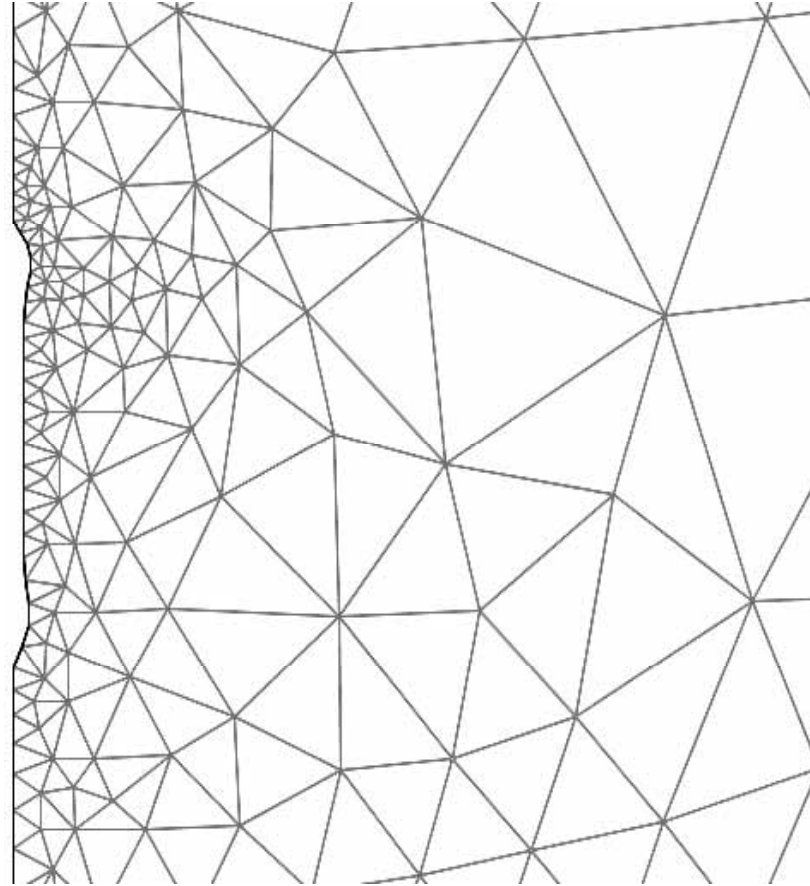
# Corroding Surface



# Corroding Surface

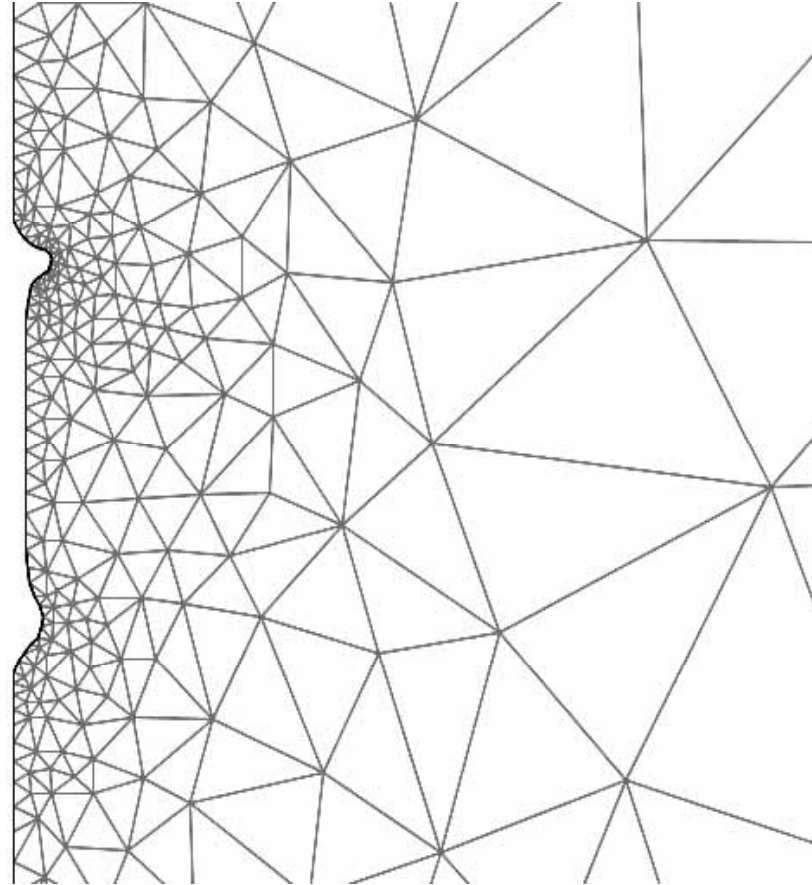


# Corroding Surface

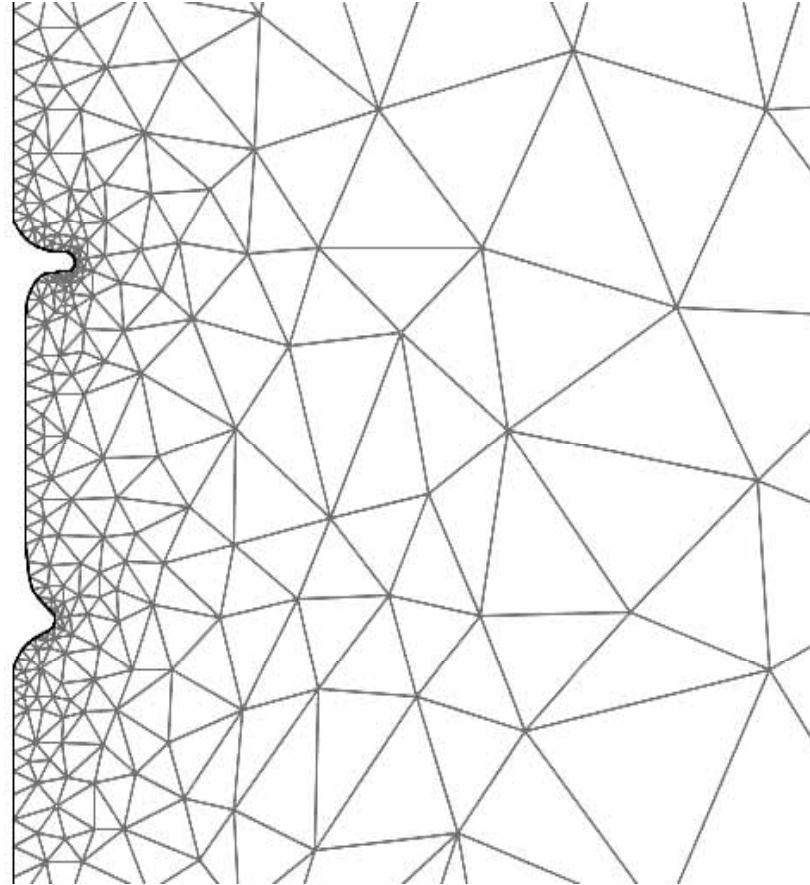




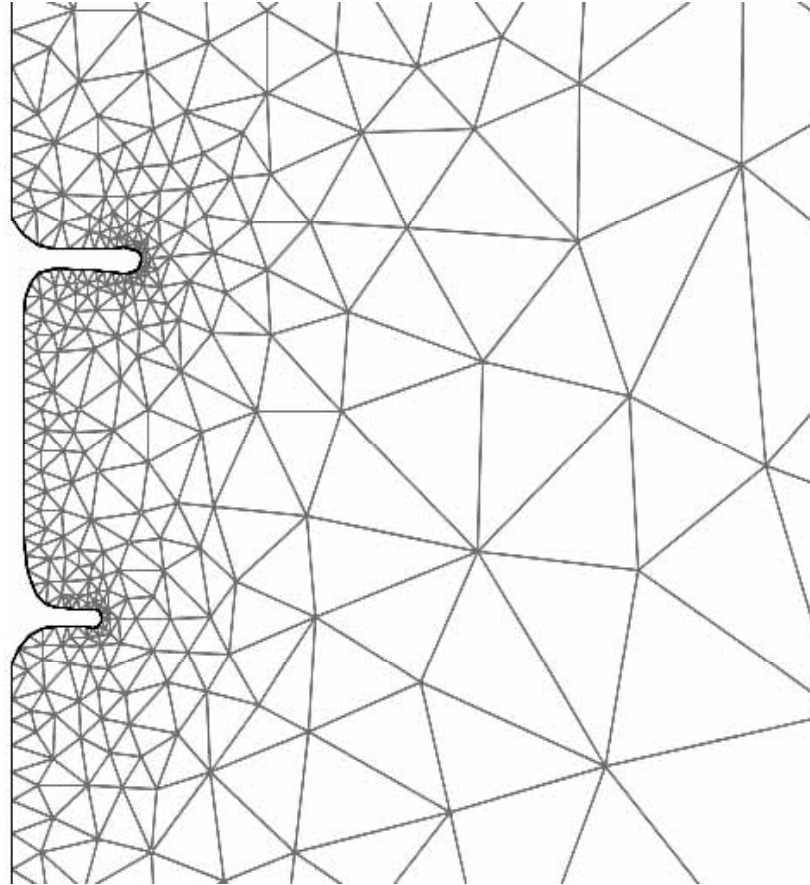
# Corroding Surface



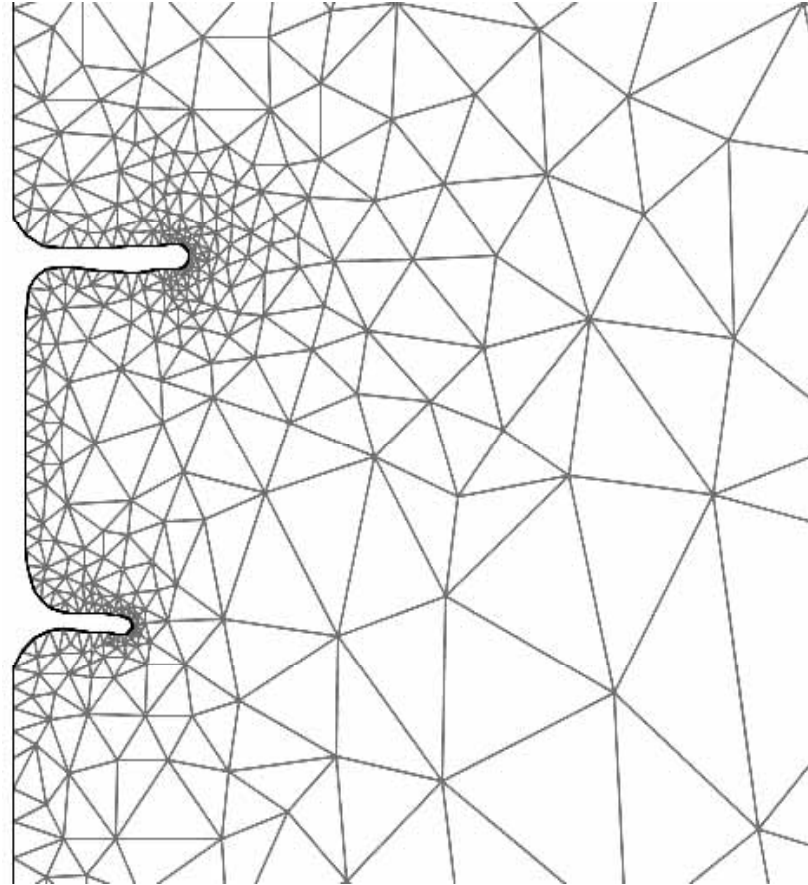
# Corroding Surface



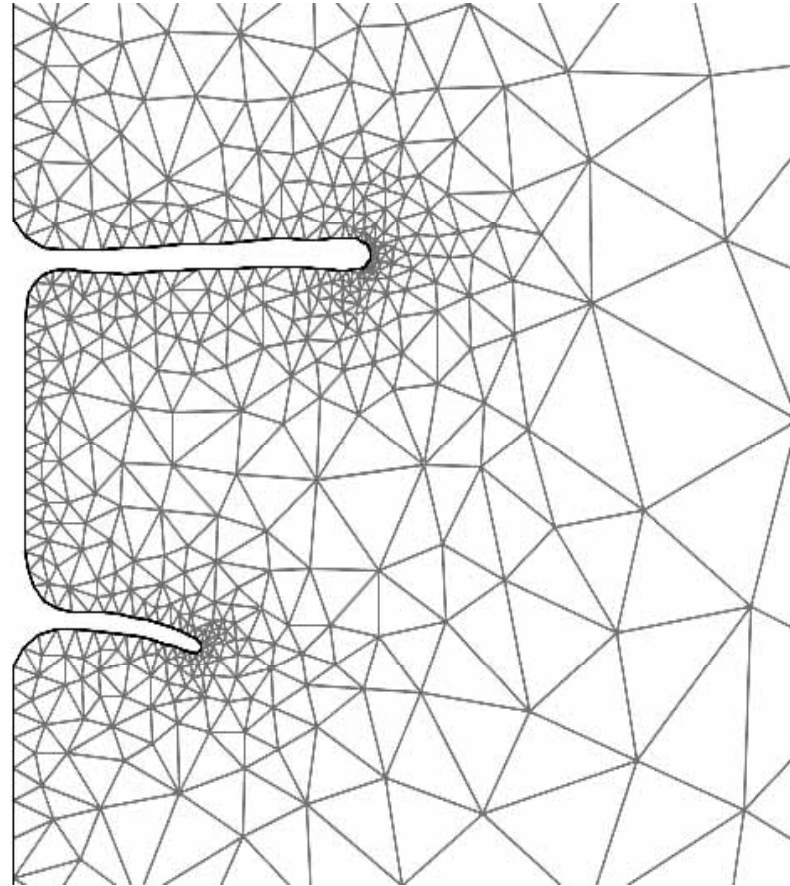
# Corroding Surface



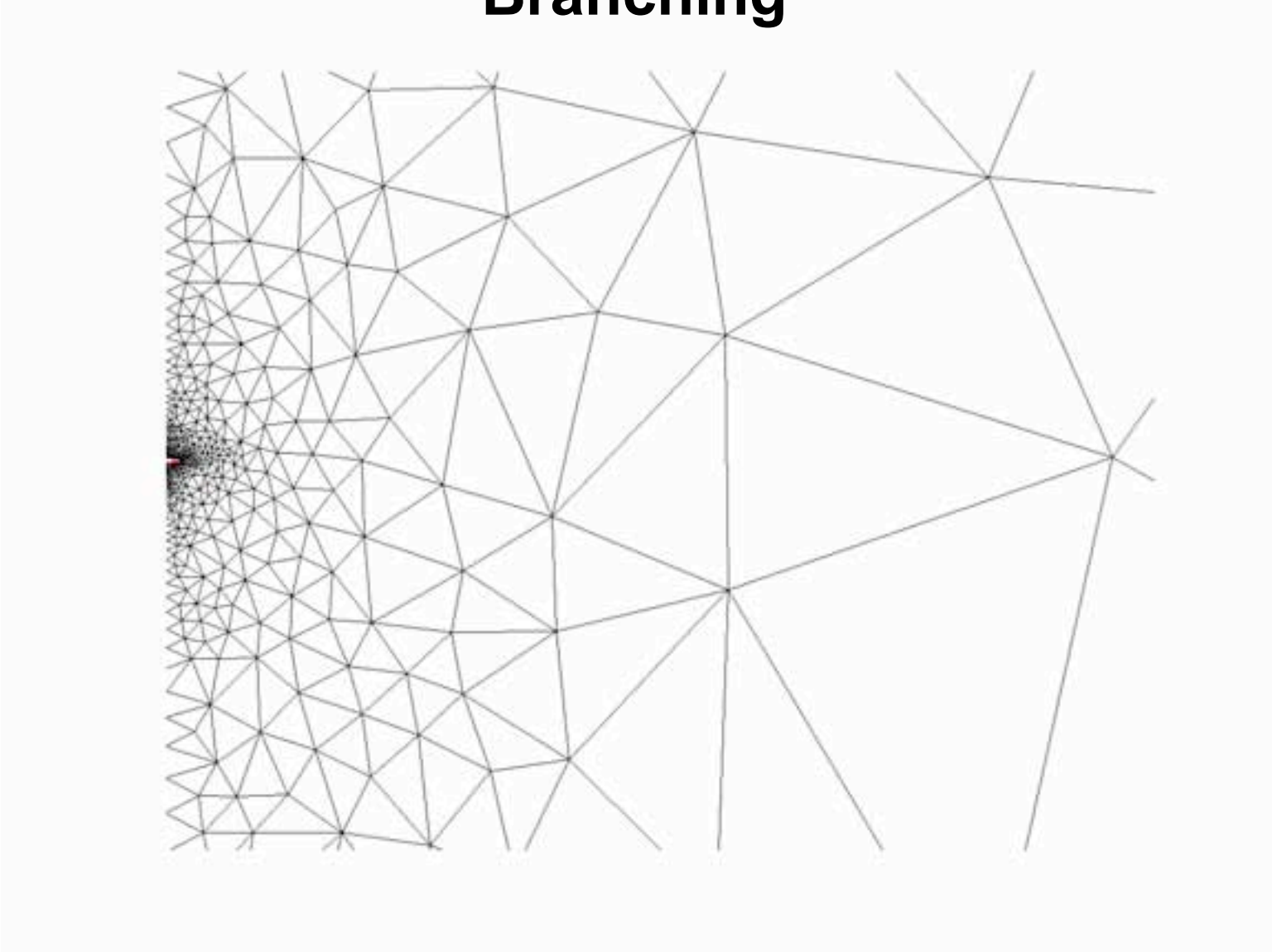
# Corroding Surface



# Corroding Surface



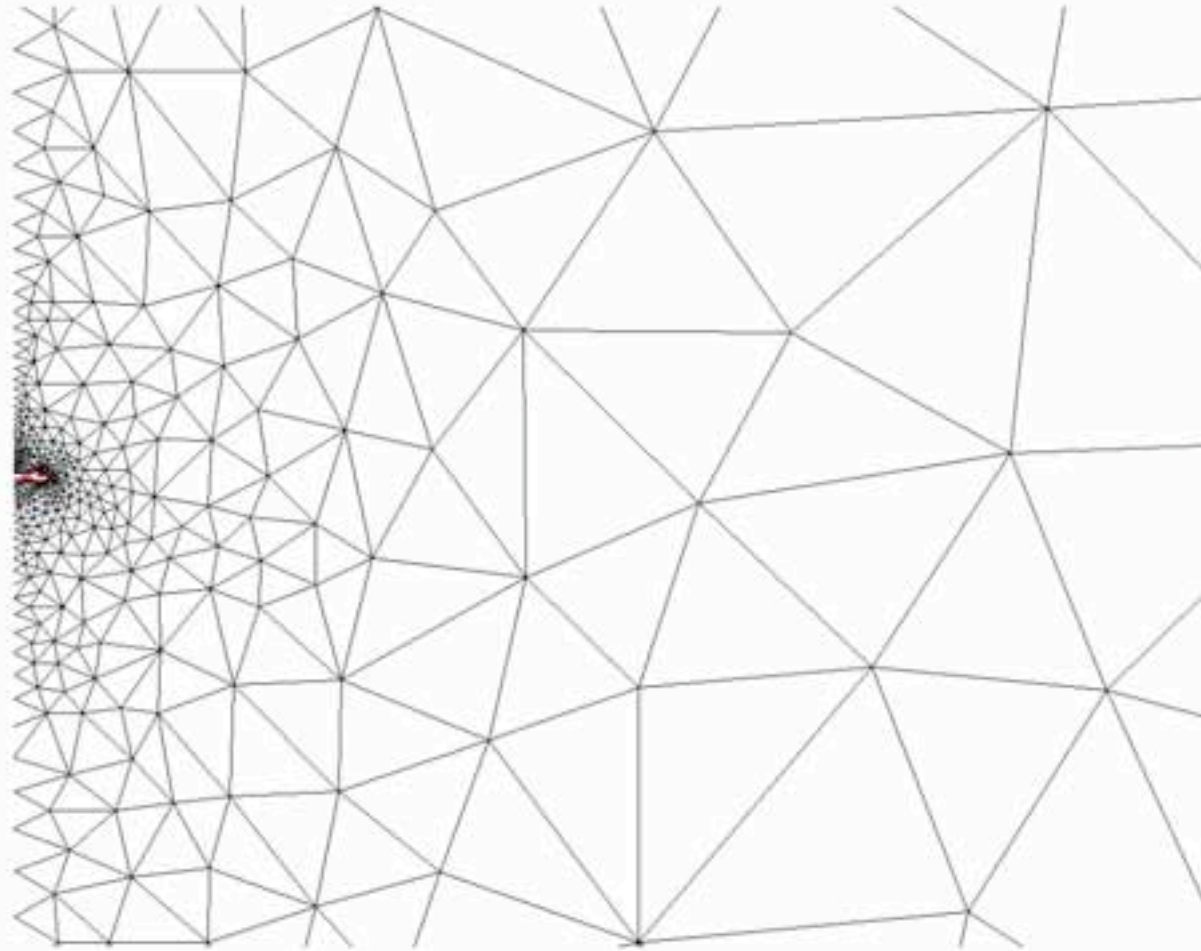
# Branching



# Branching

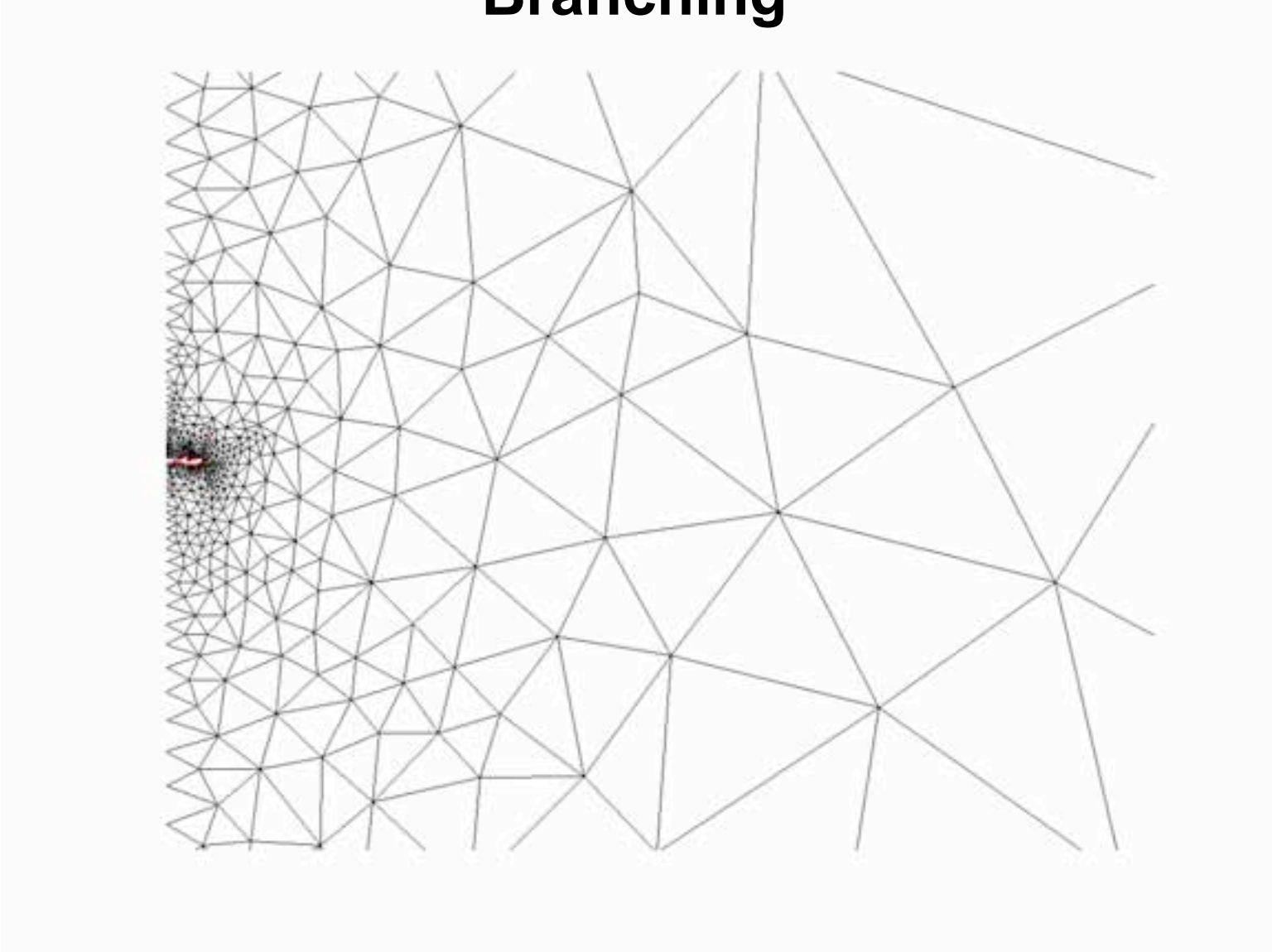


# Branching

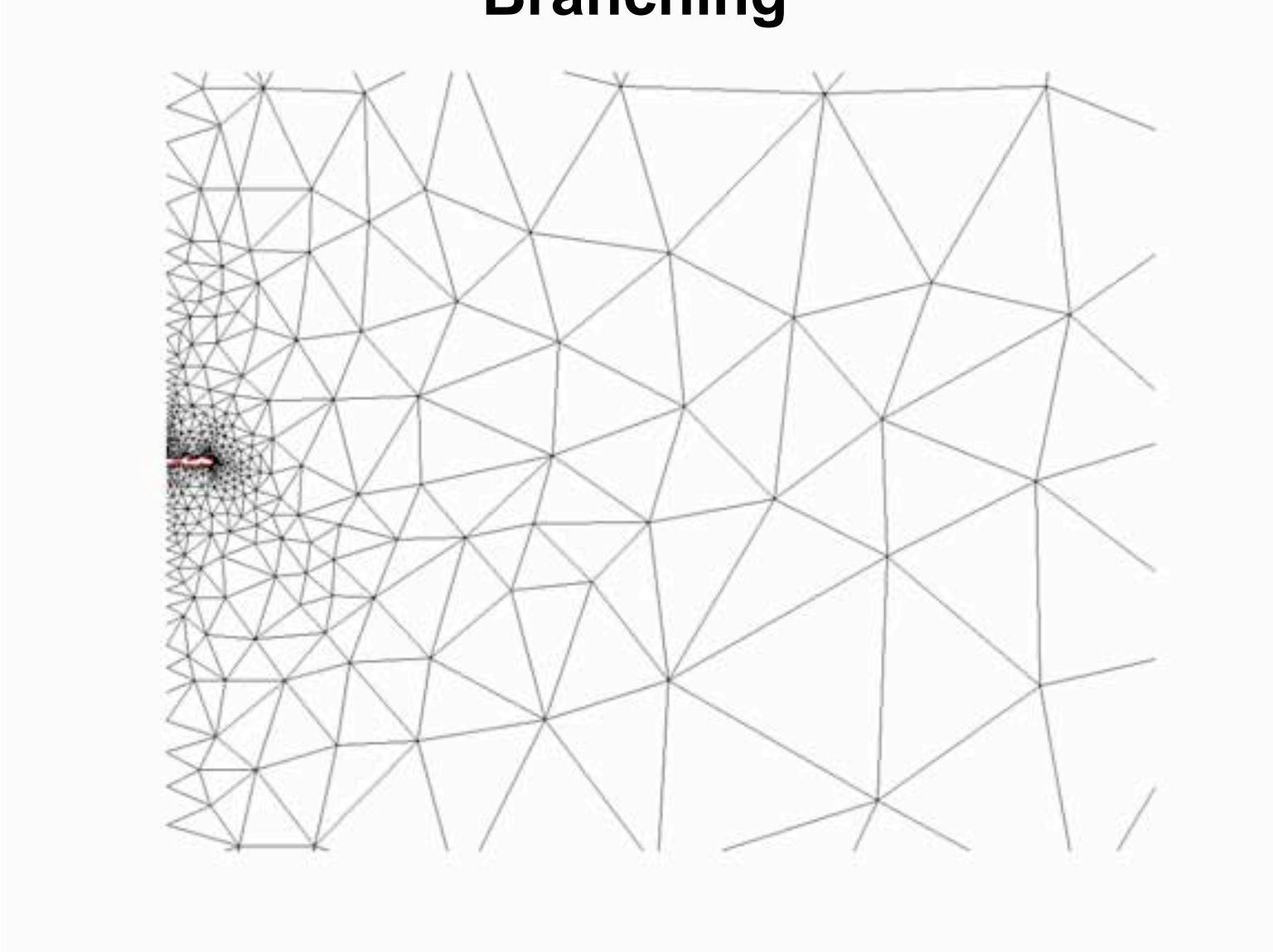




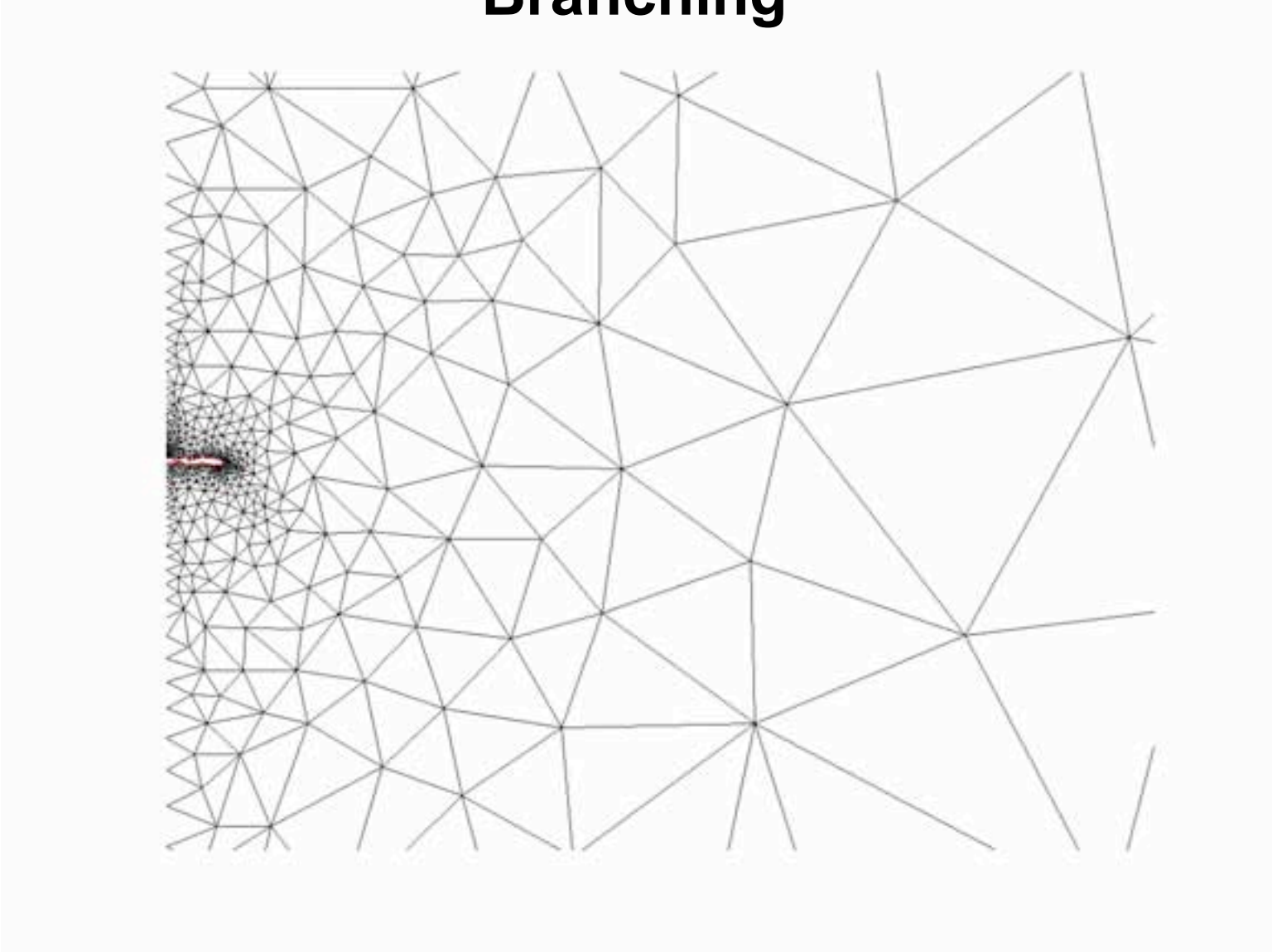
# Branching



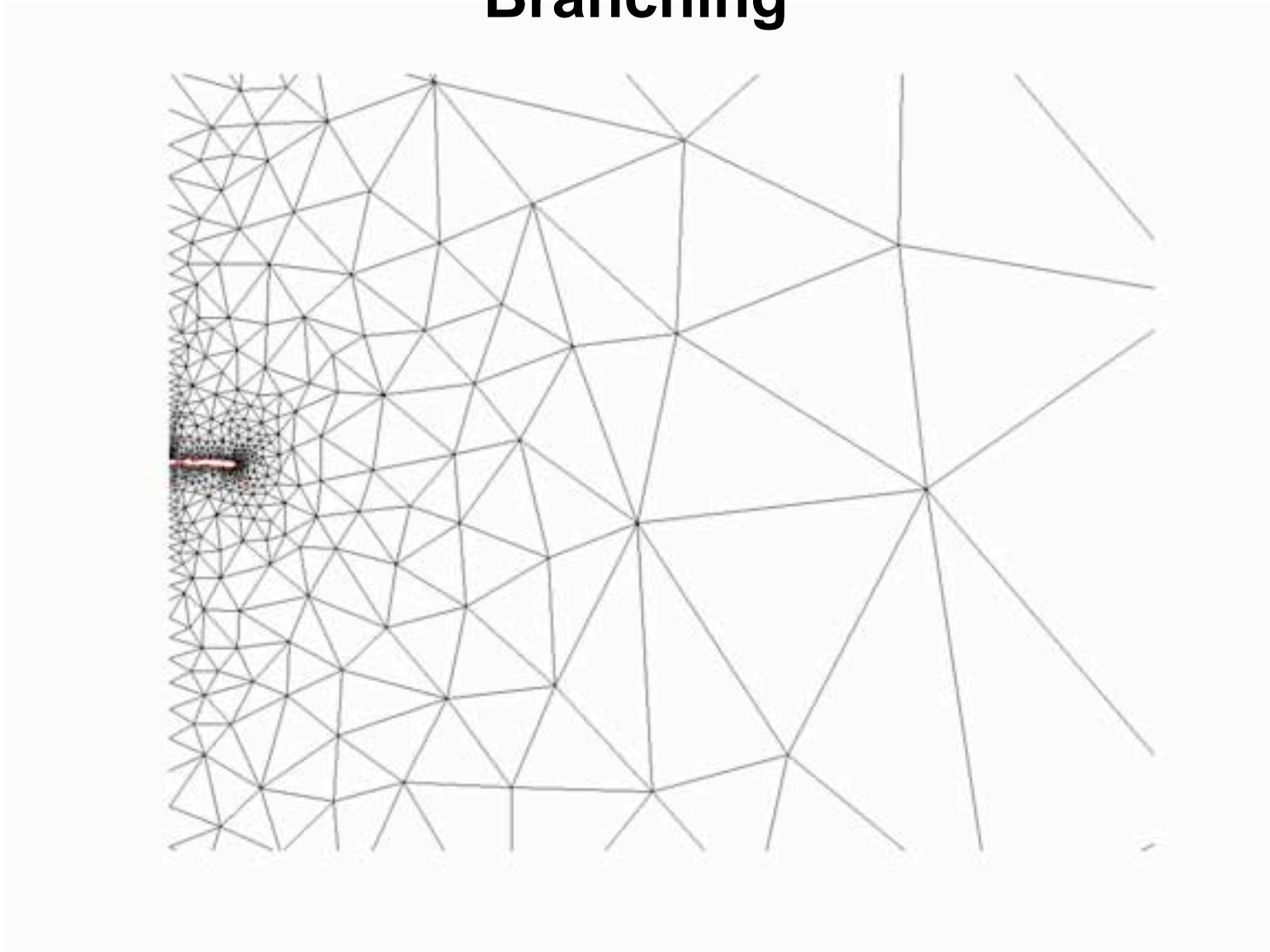
# Branching



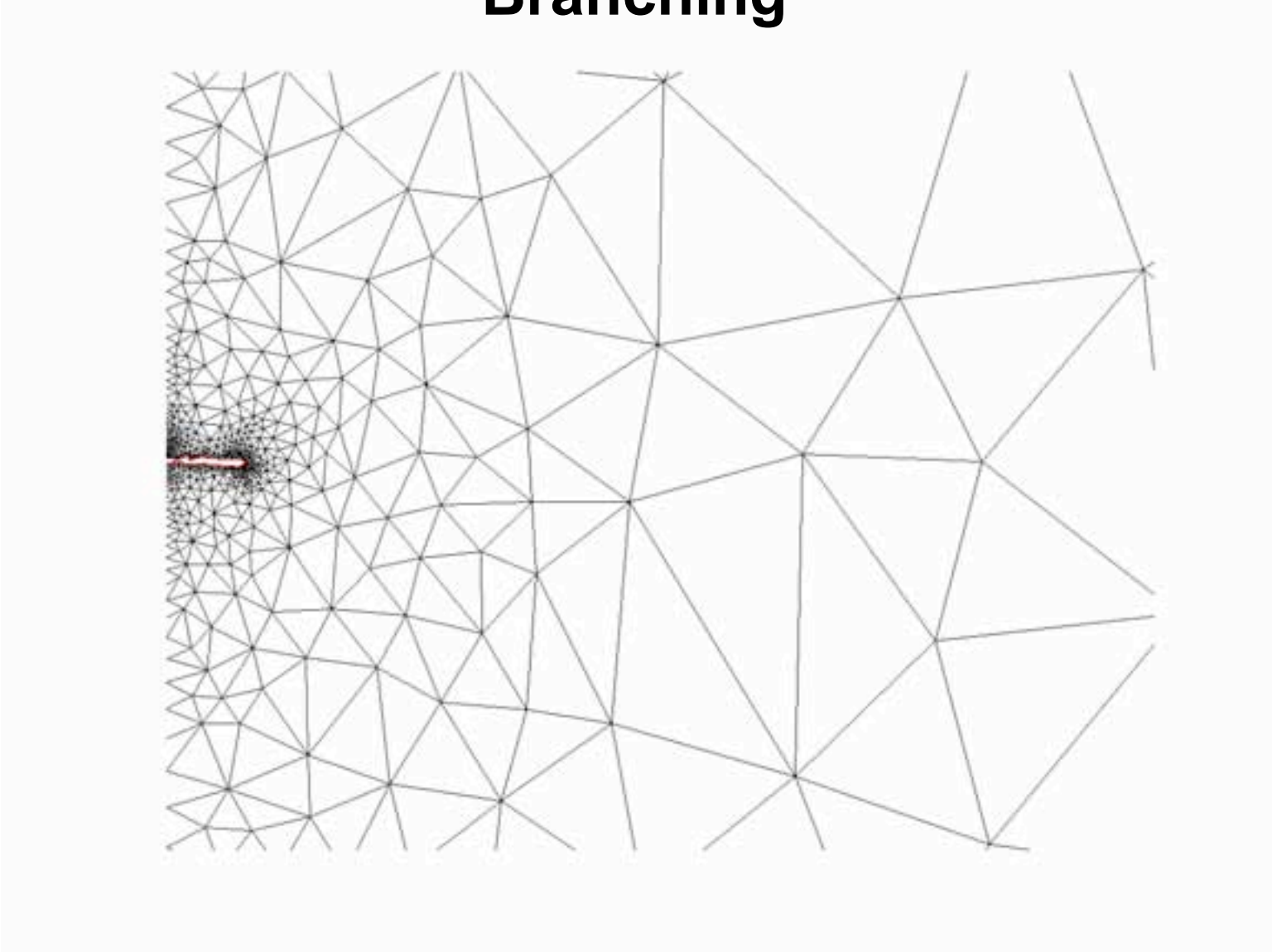
# Branching



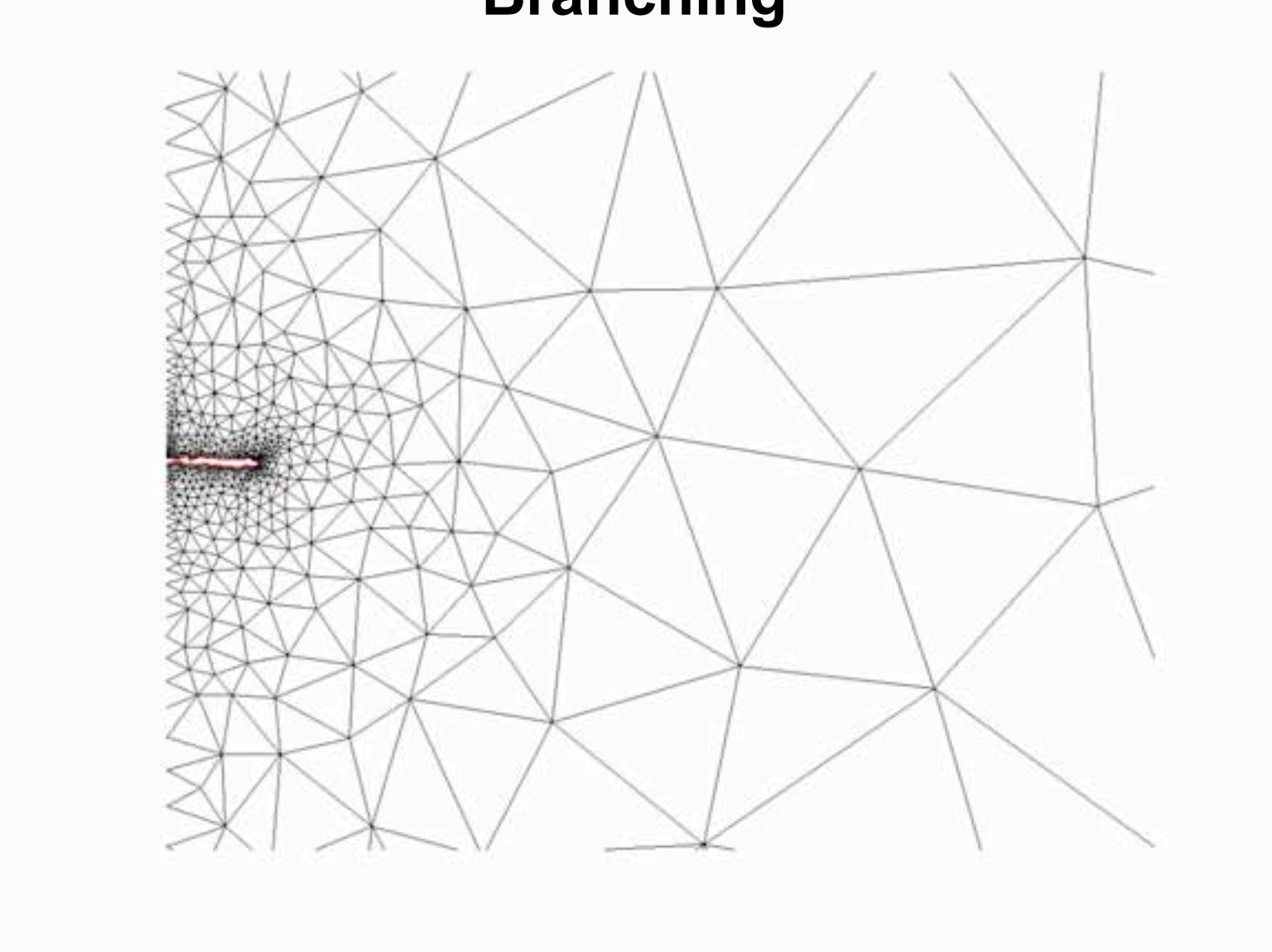
# Branching



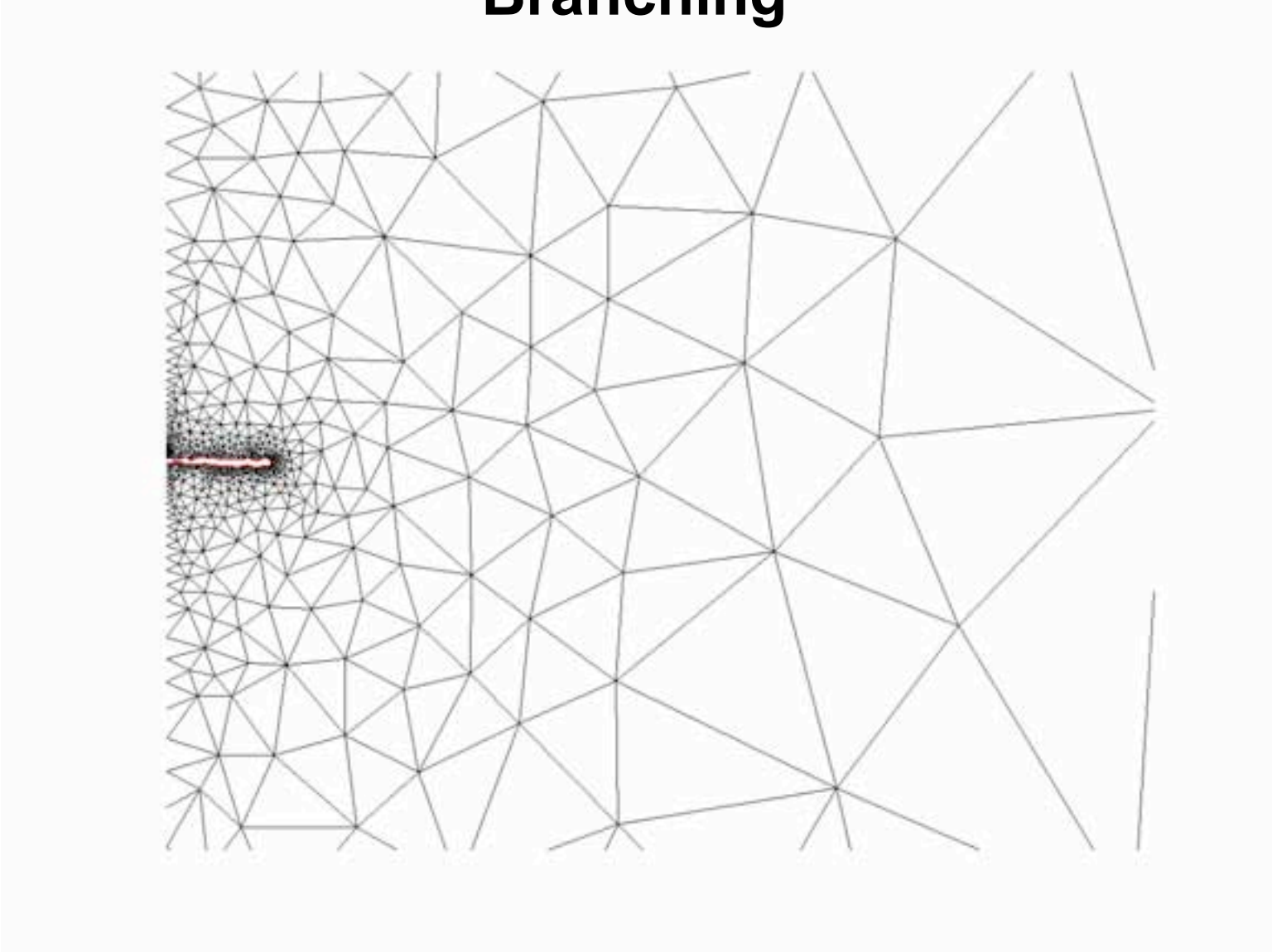
# Branching



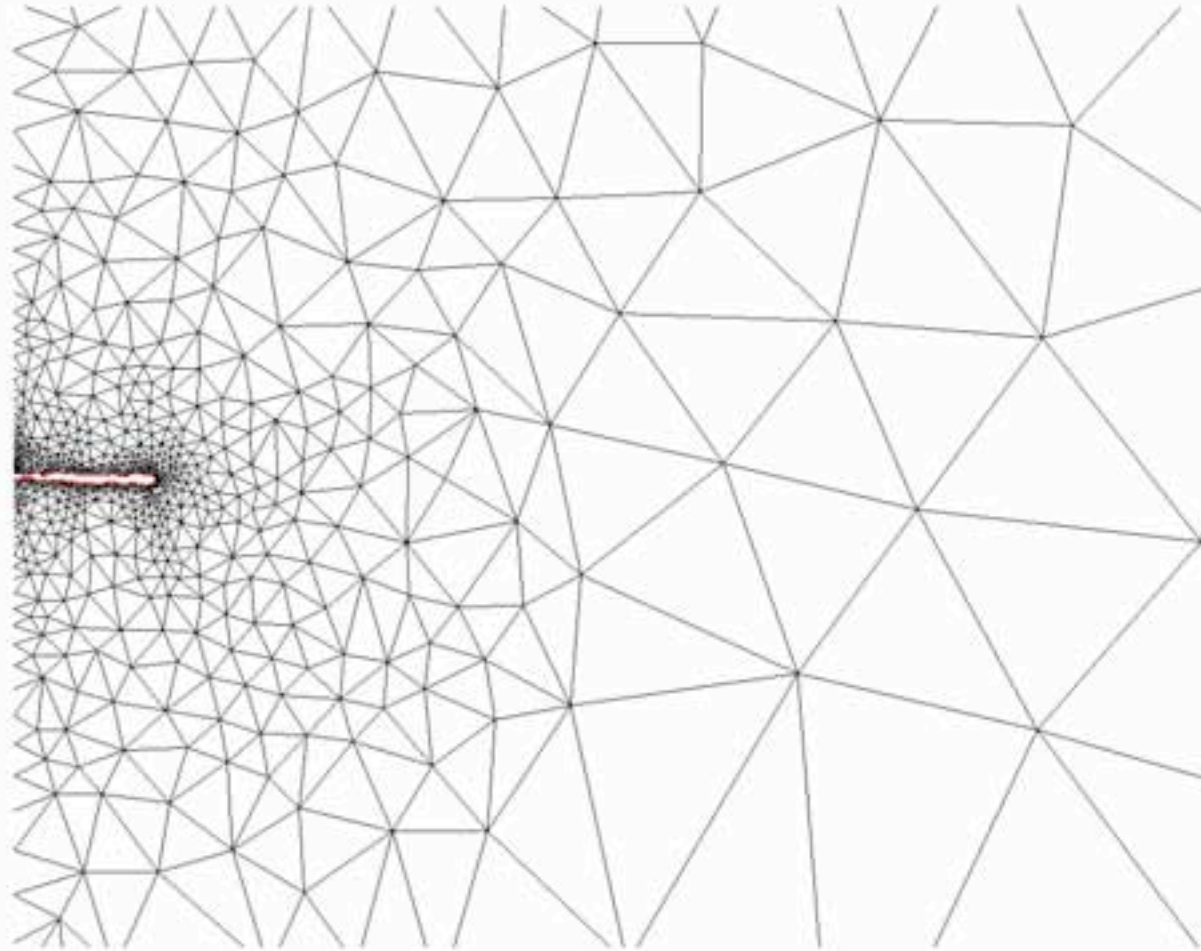
# Branching



# Branching

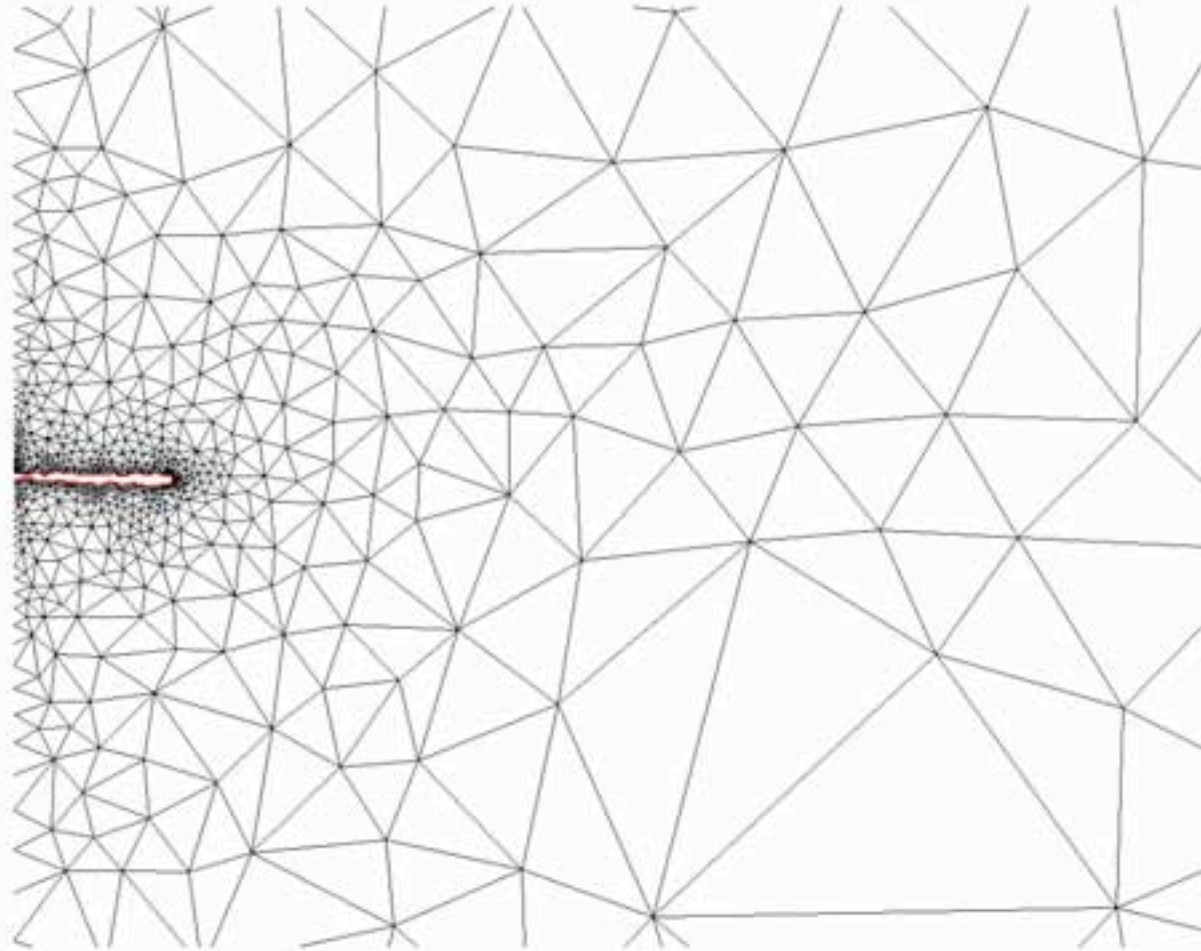


# Branching

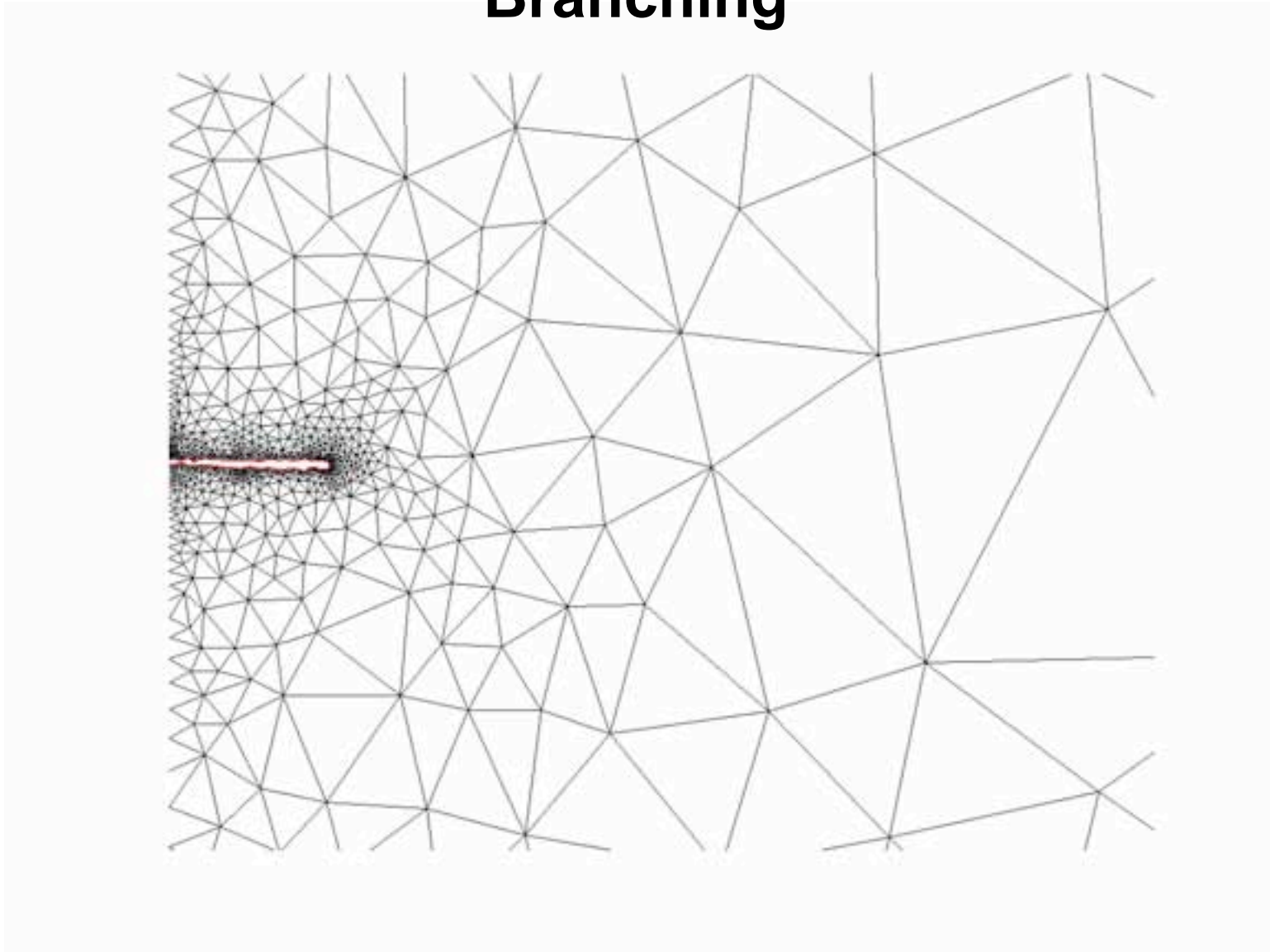




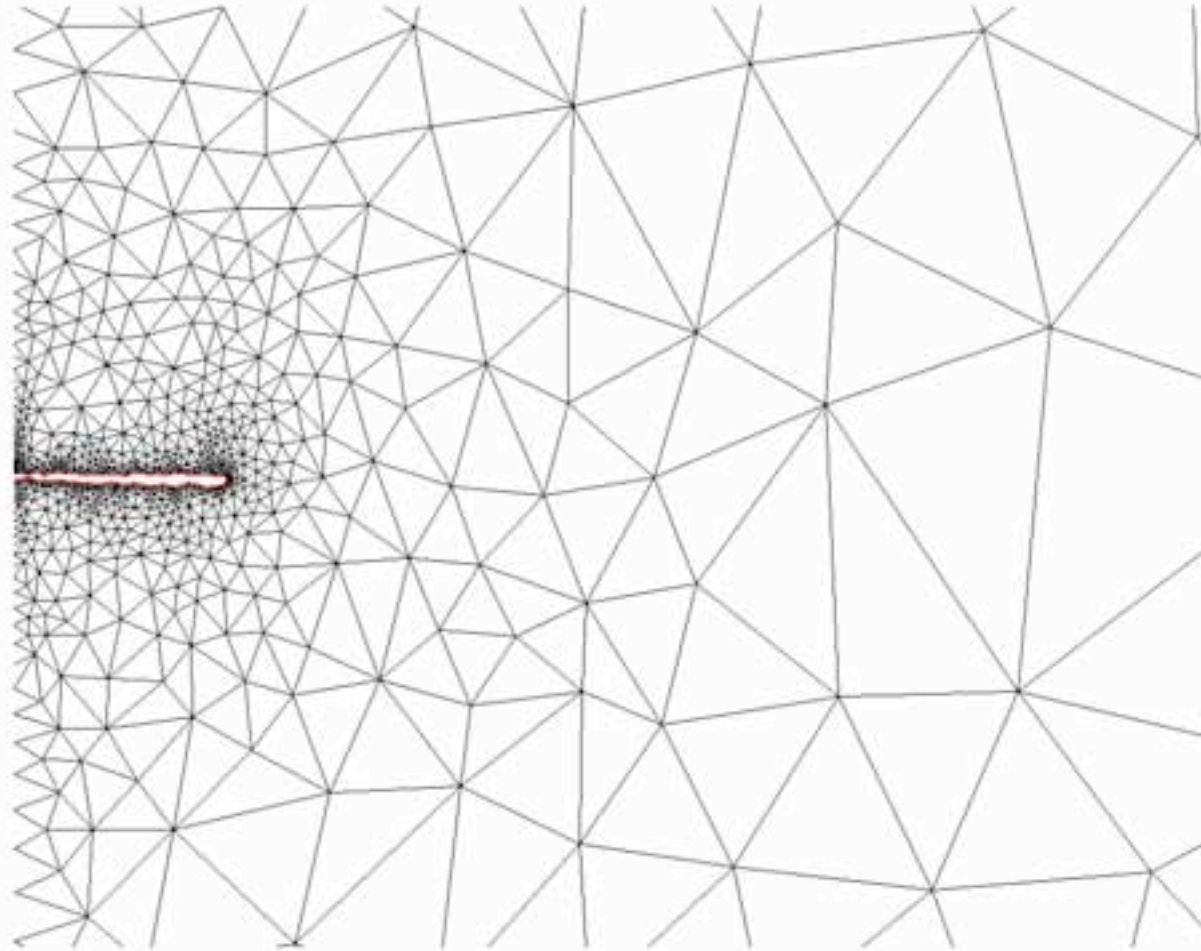
# Branching



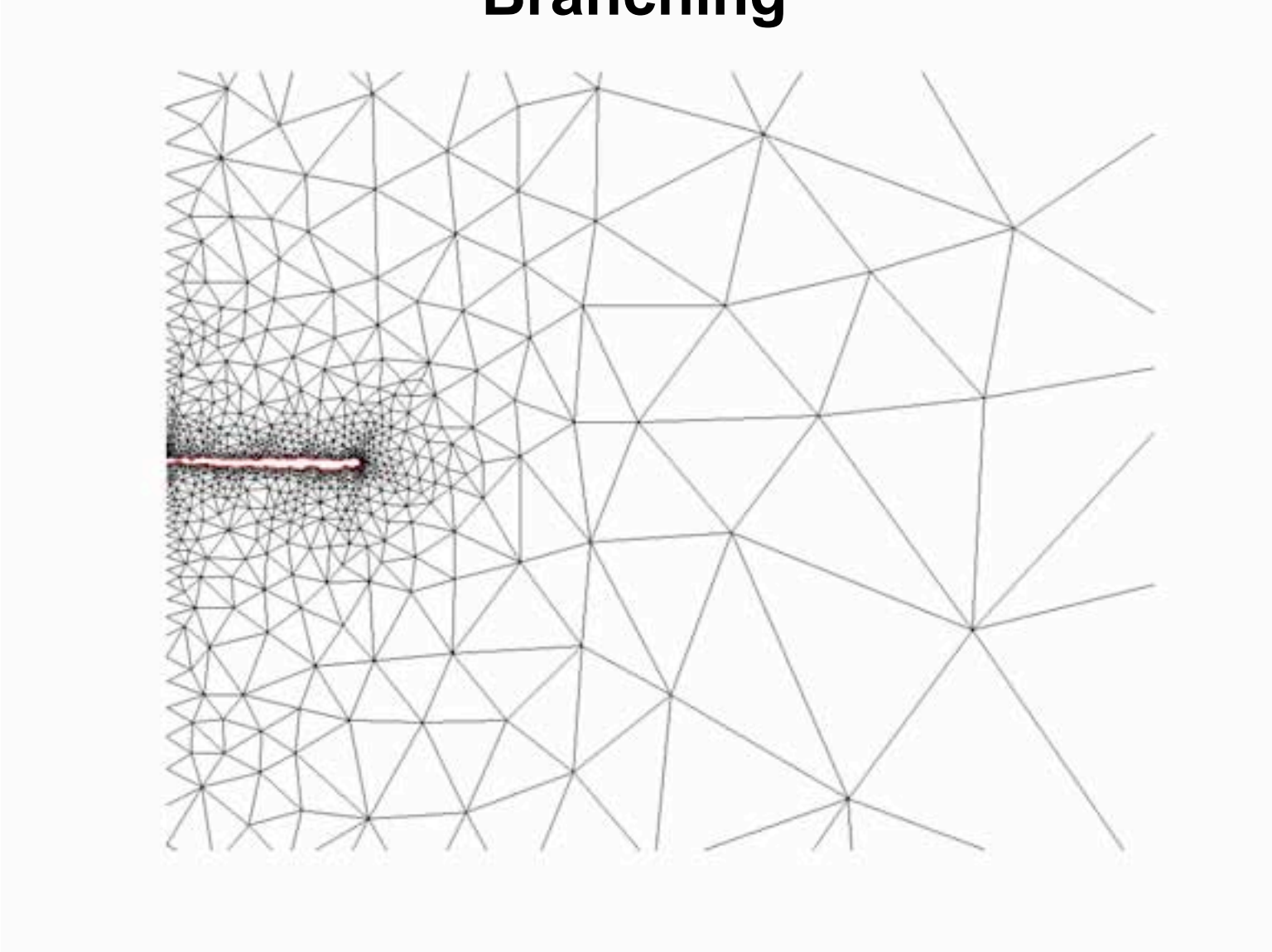
# Branching



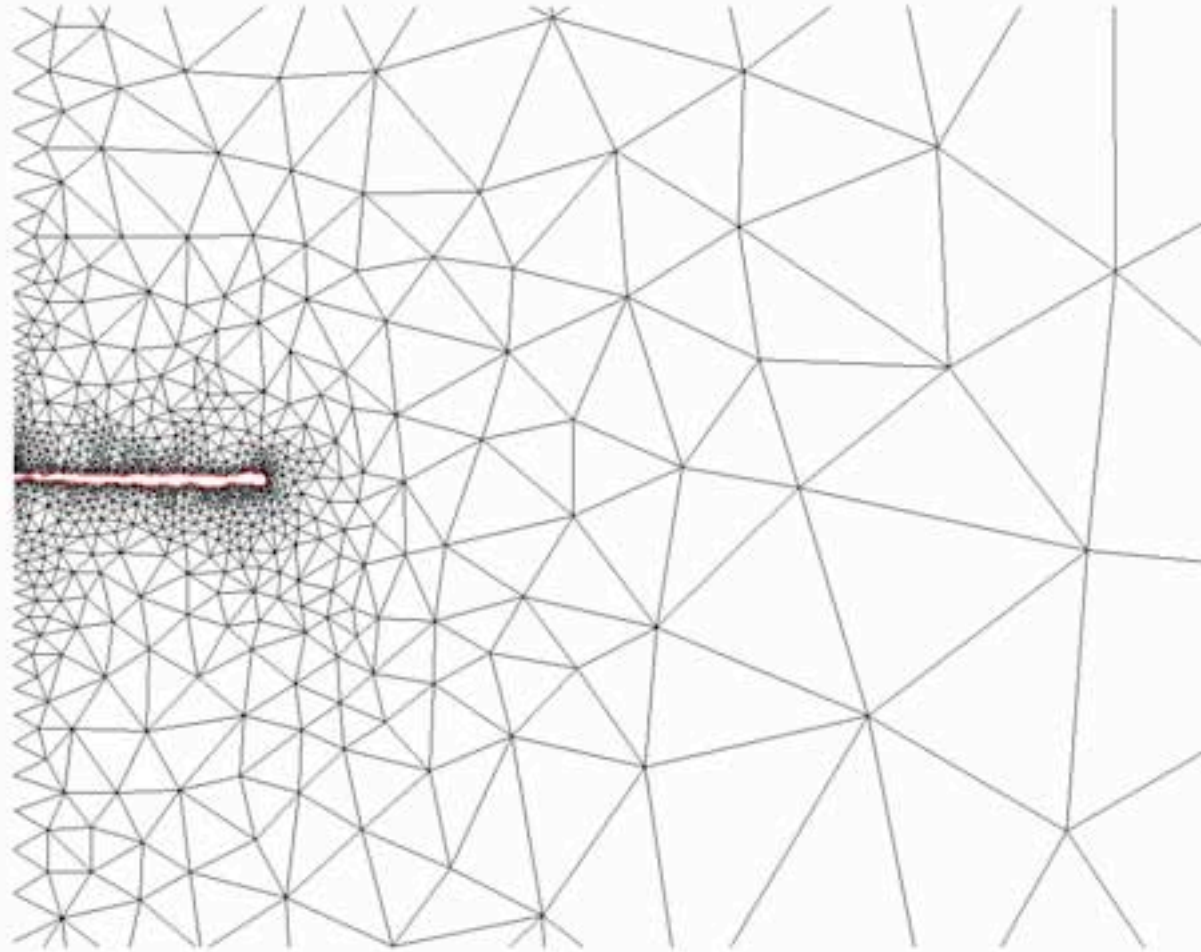
# Branching



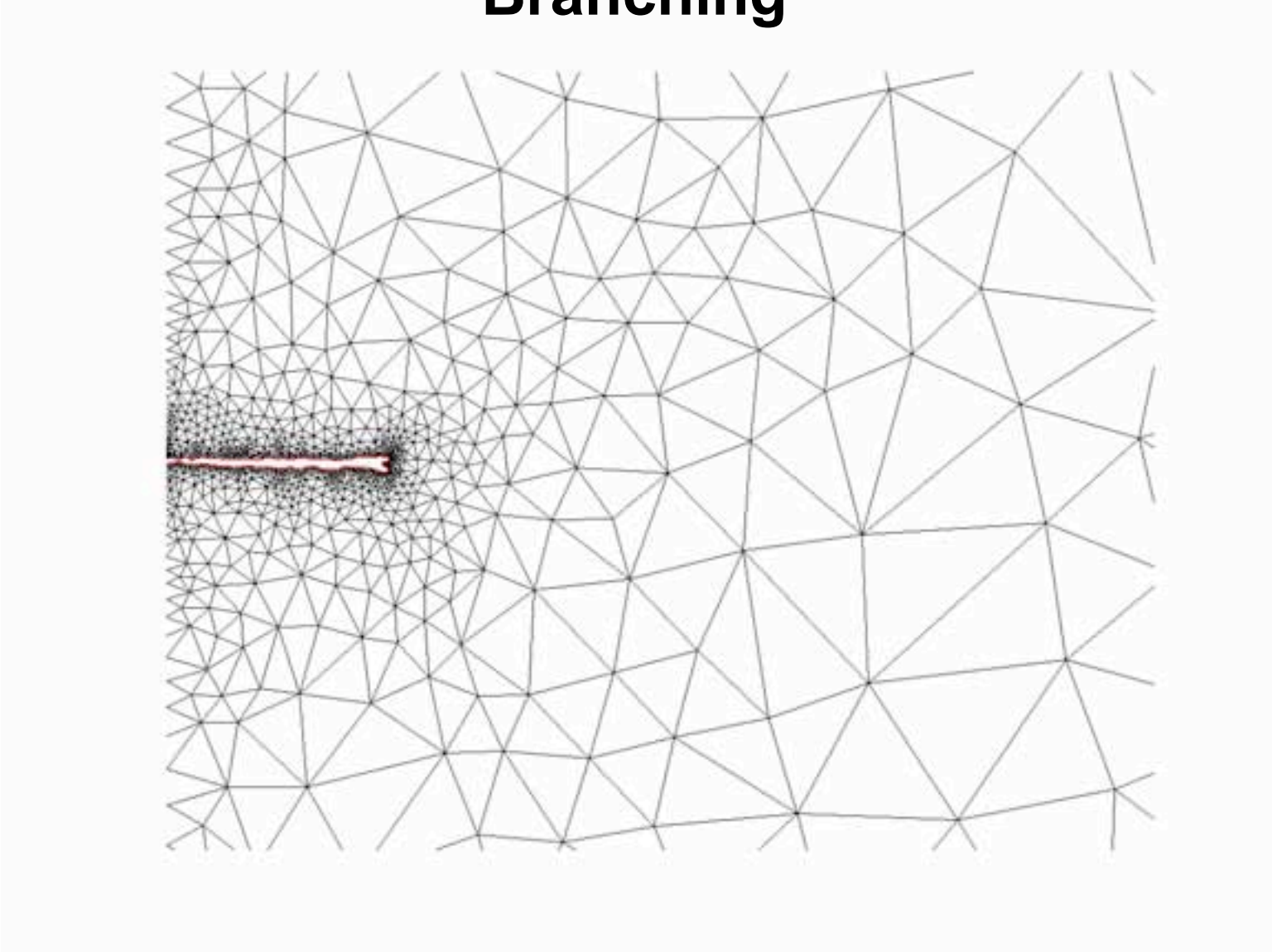
# Branching



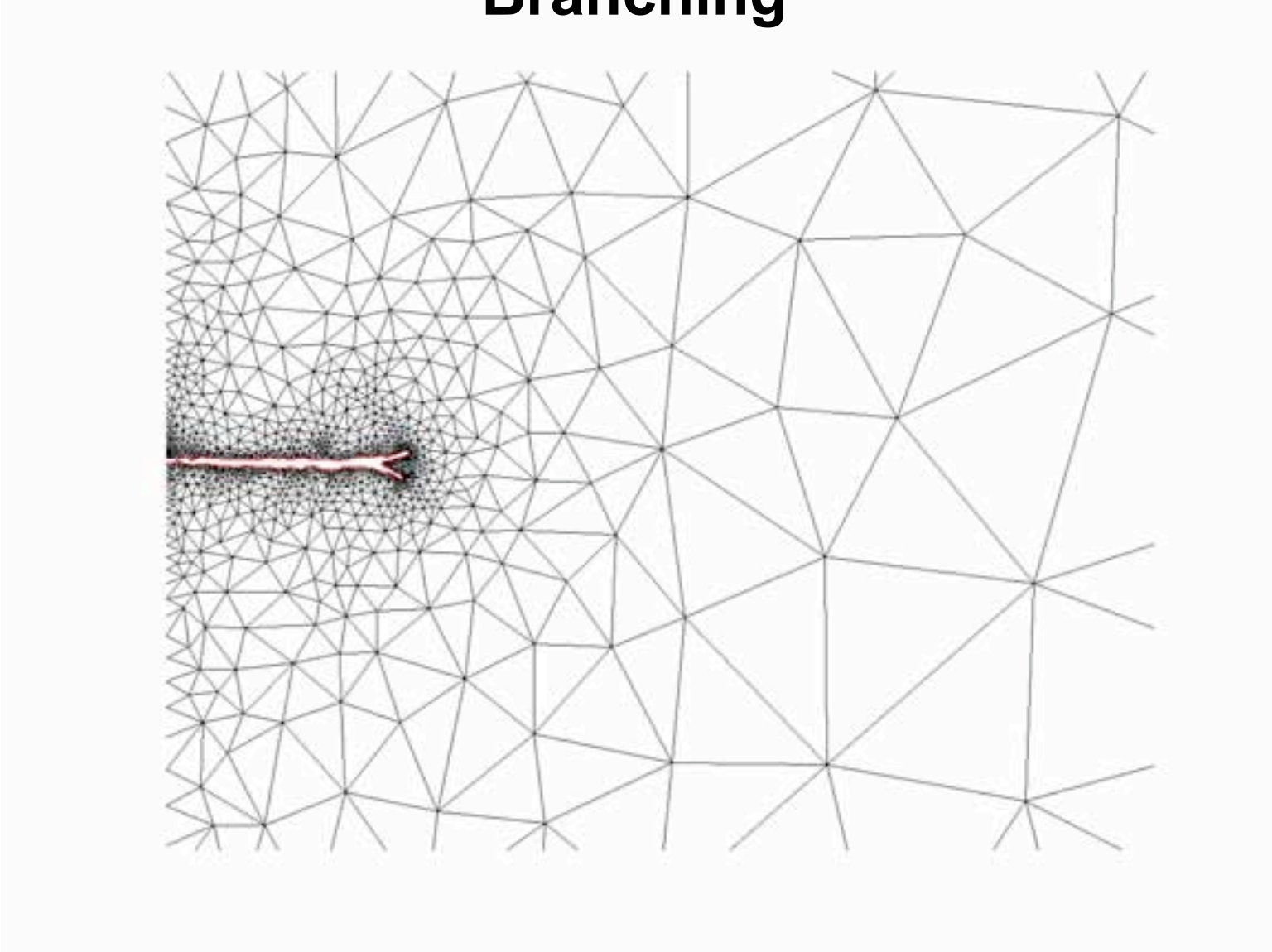
# Branching



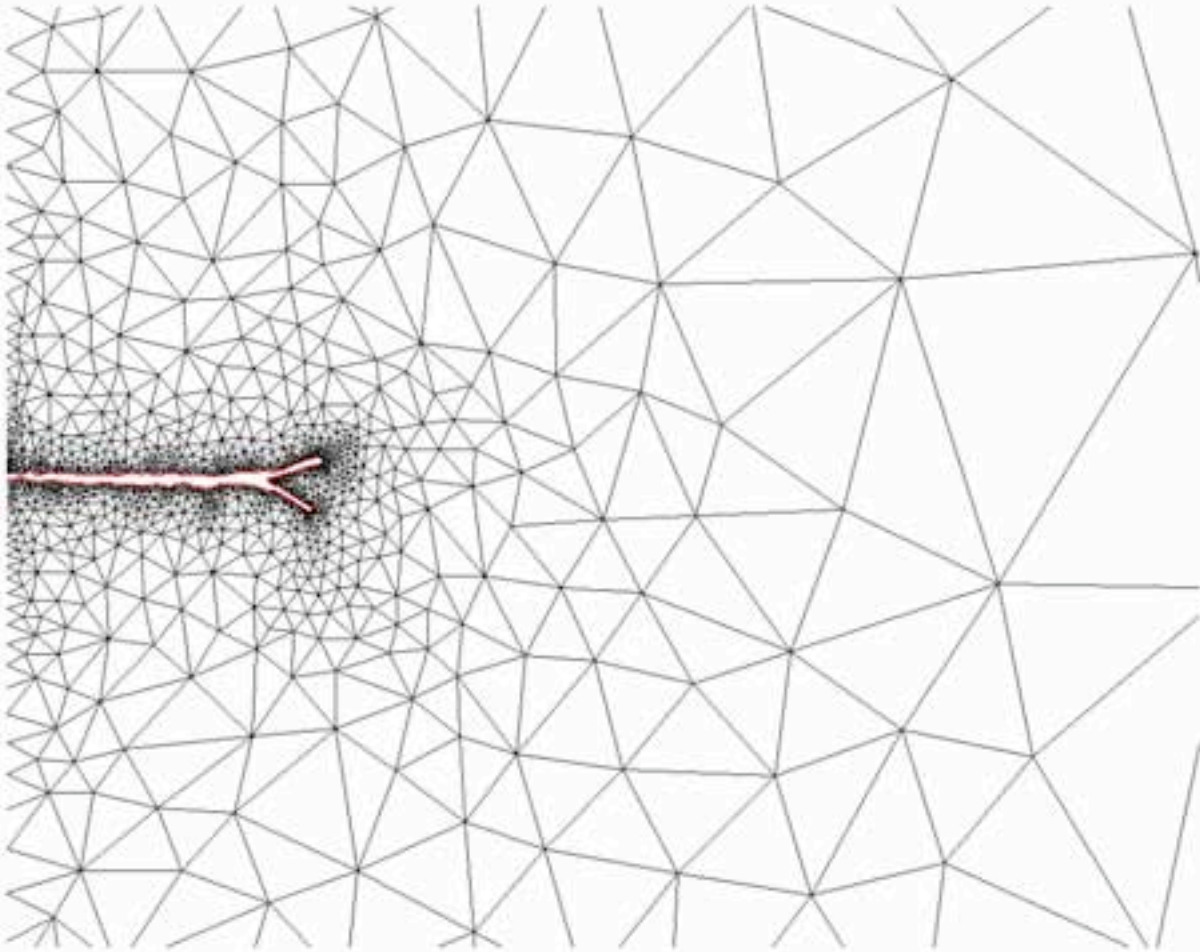
# Branching



# Branching

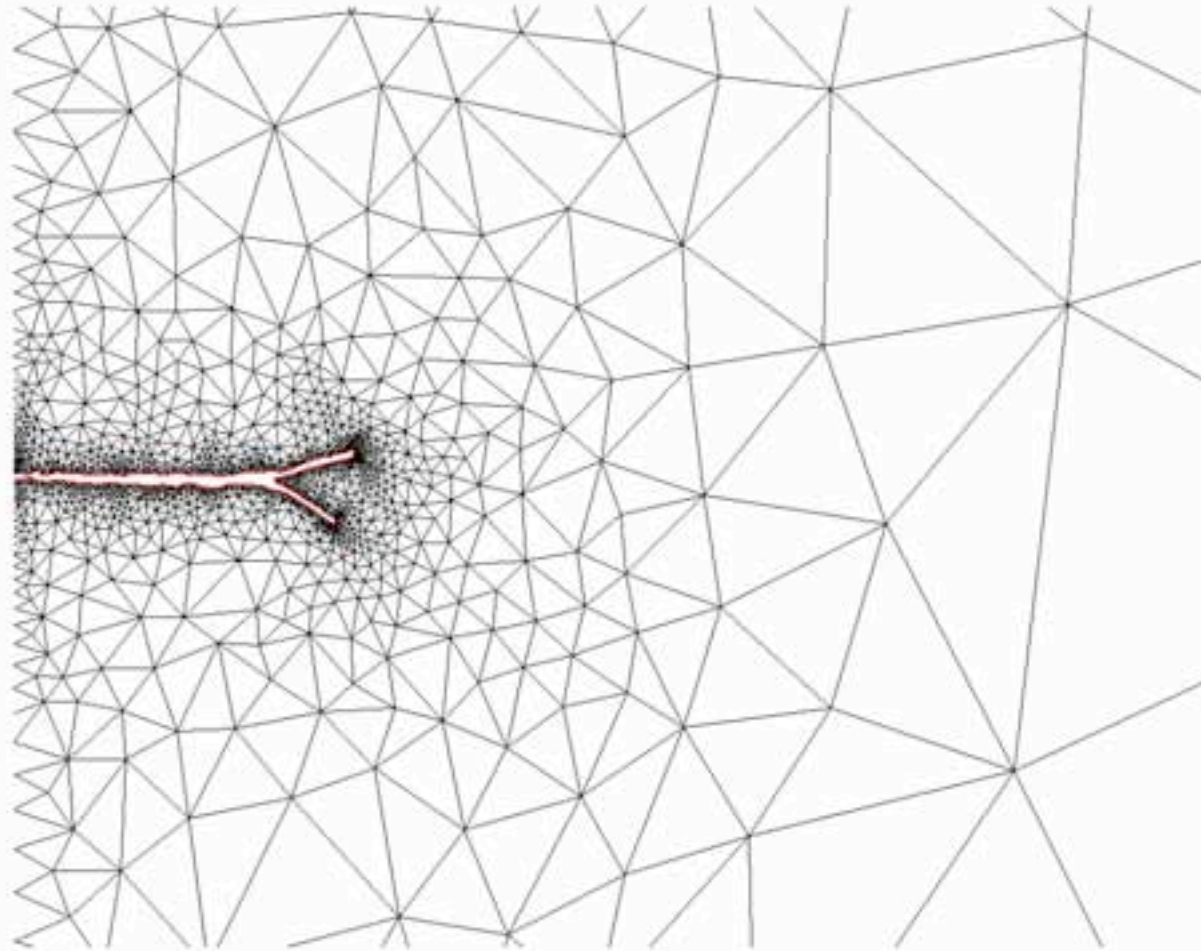


# Branching

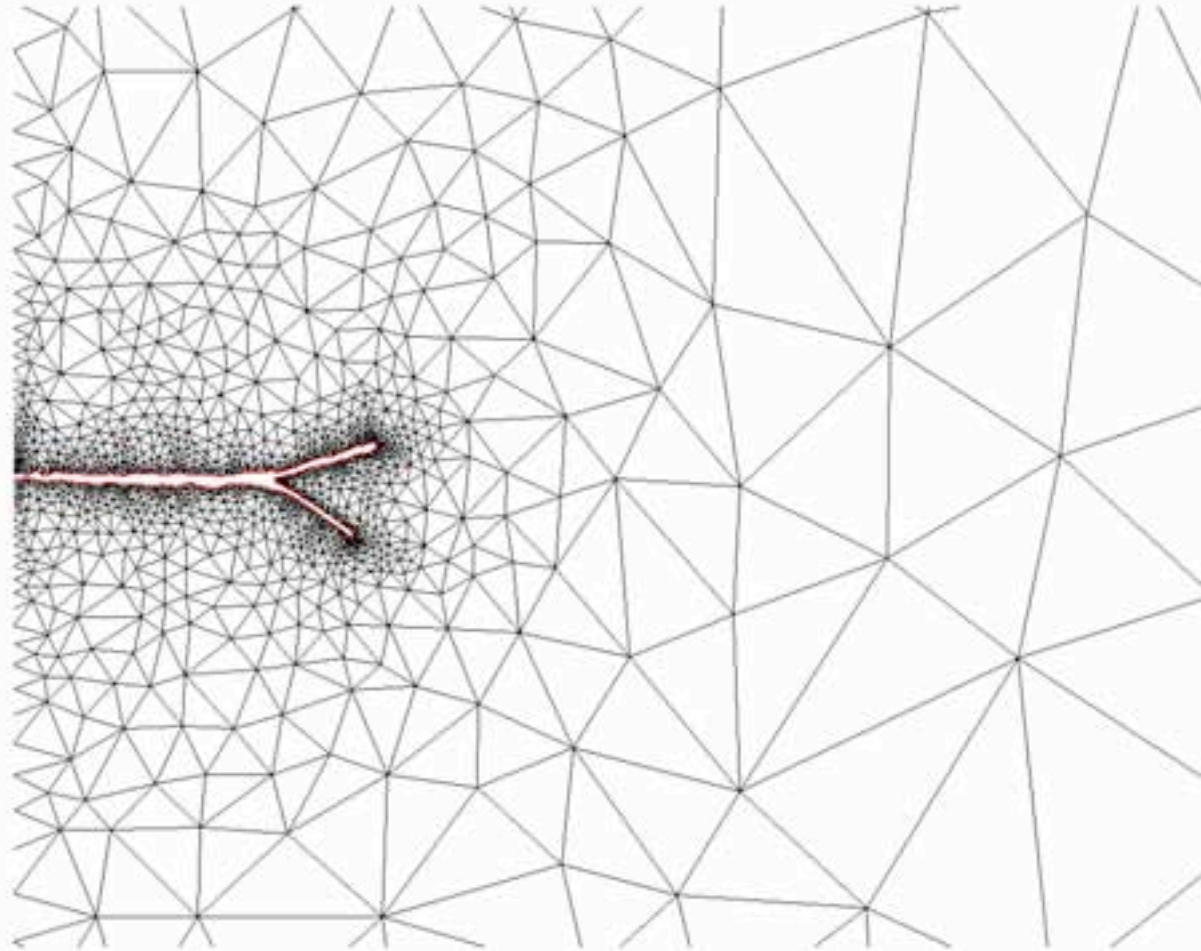




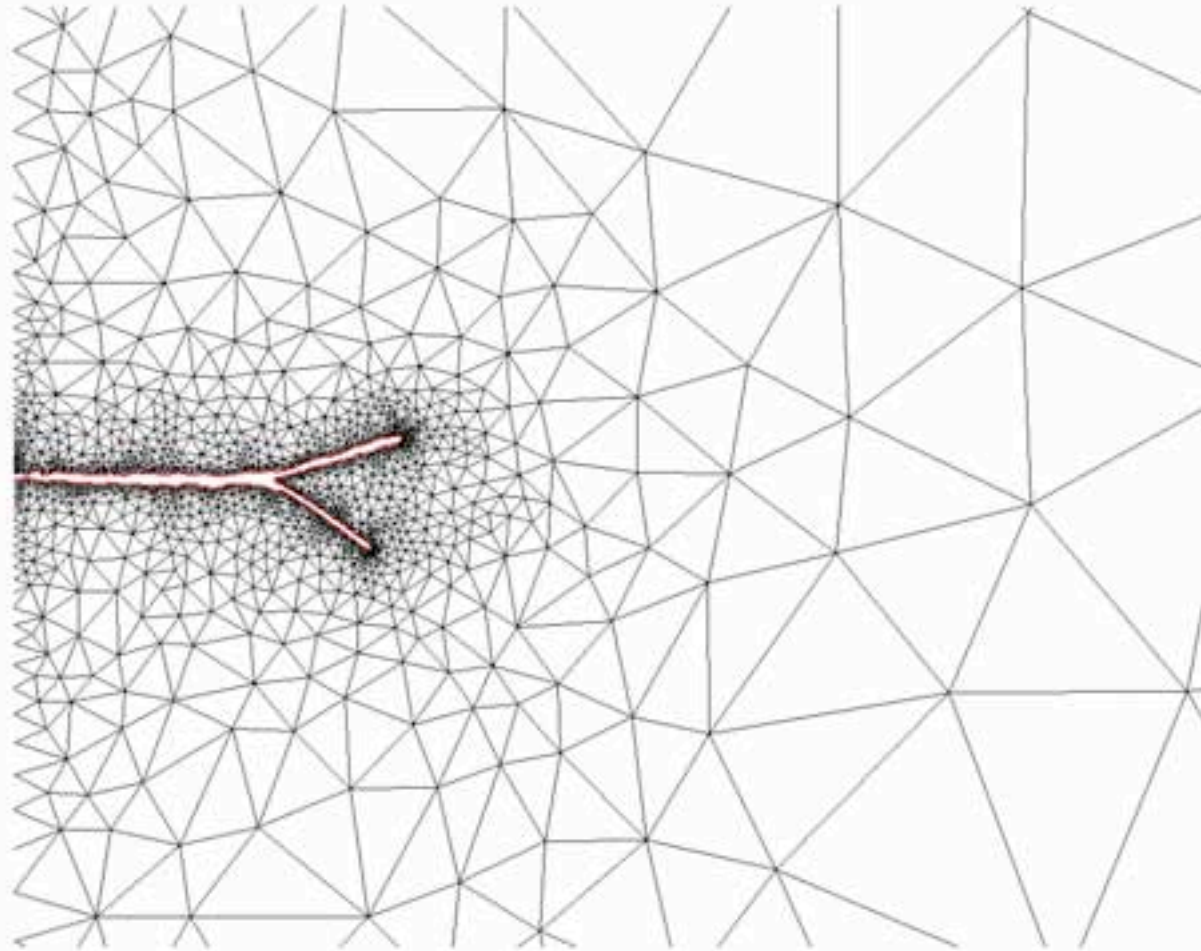
# Branching



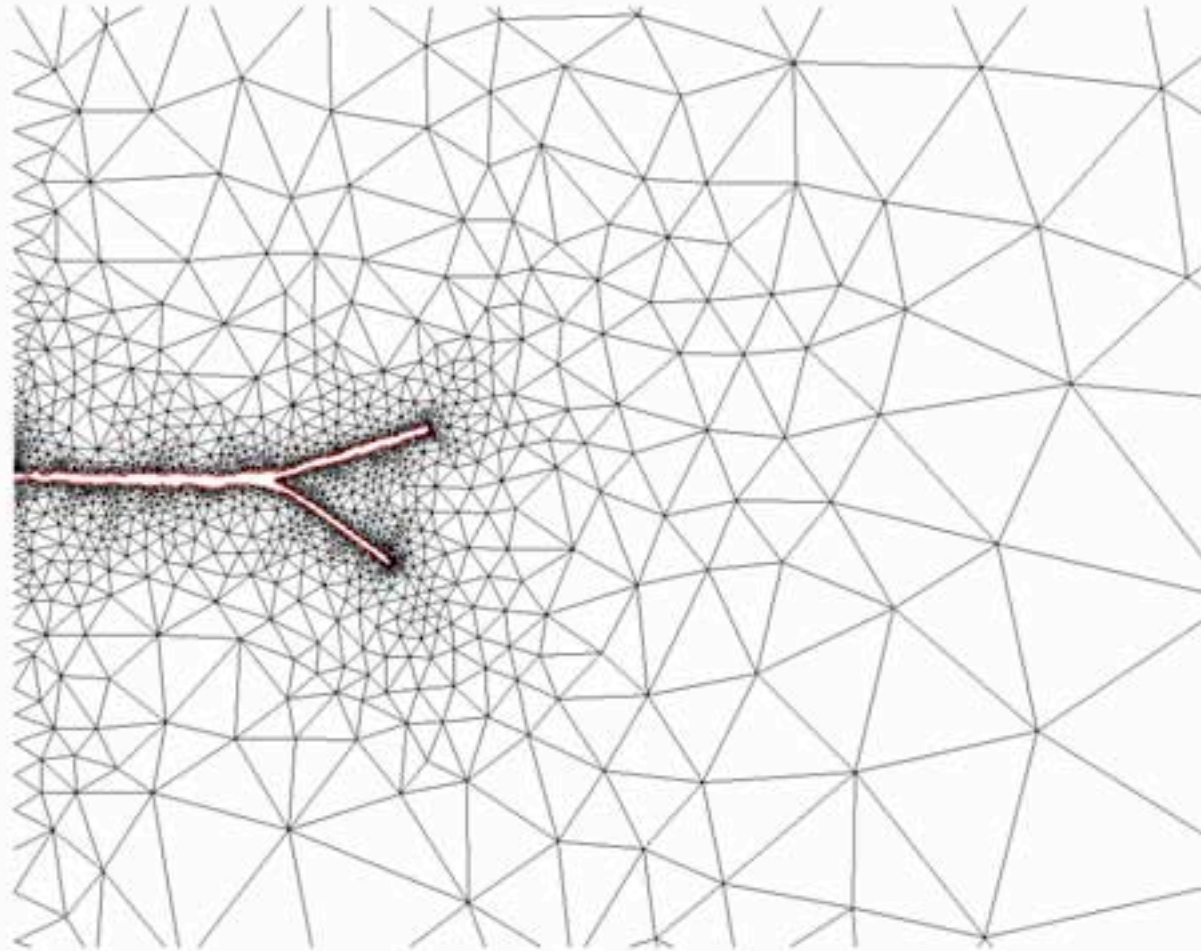
# Branching



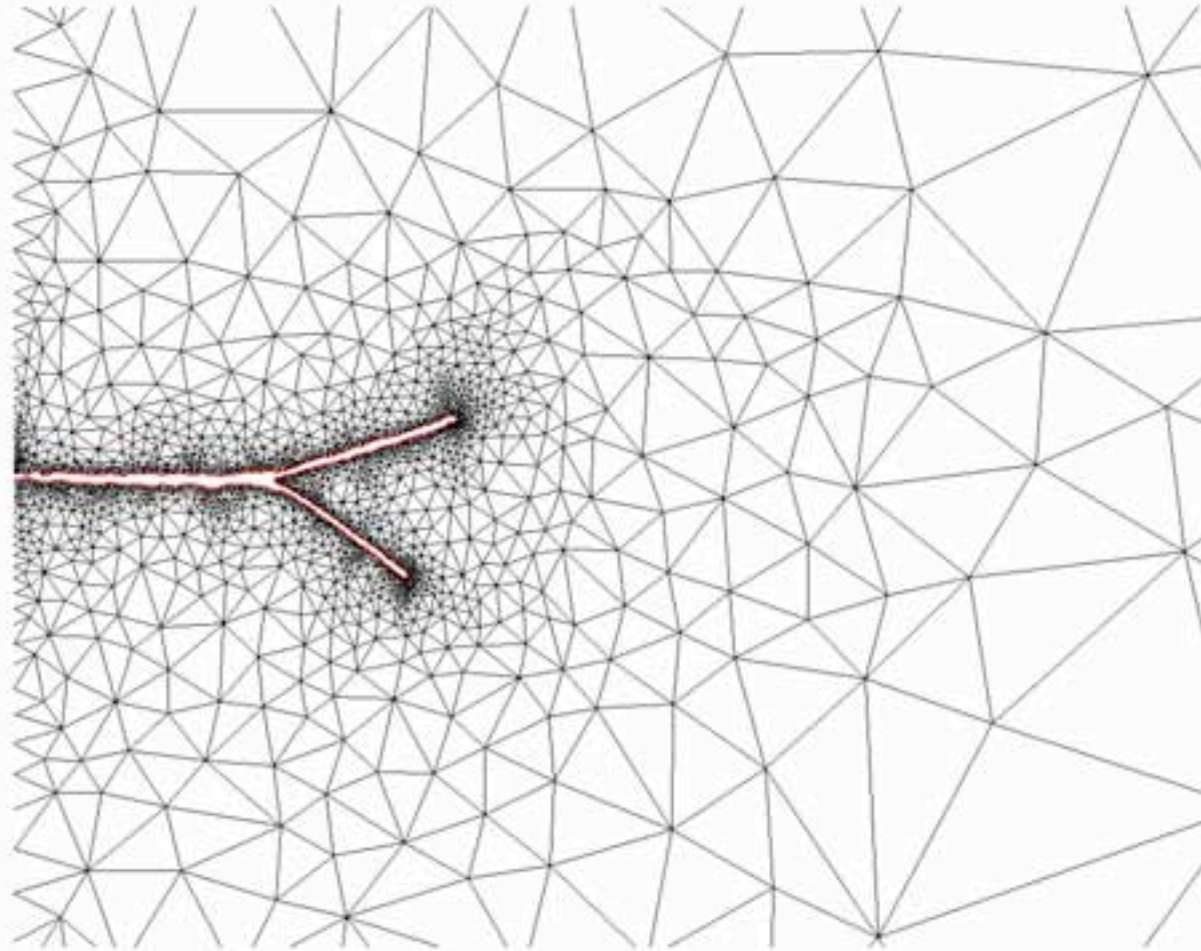
# Branching



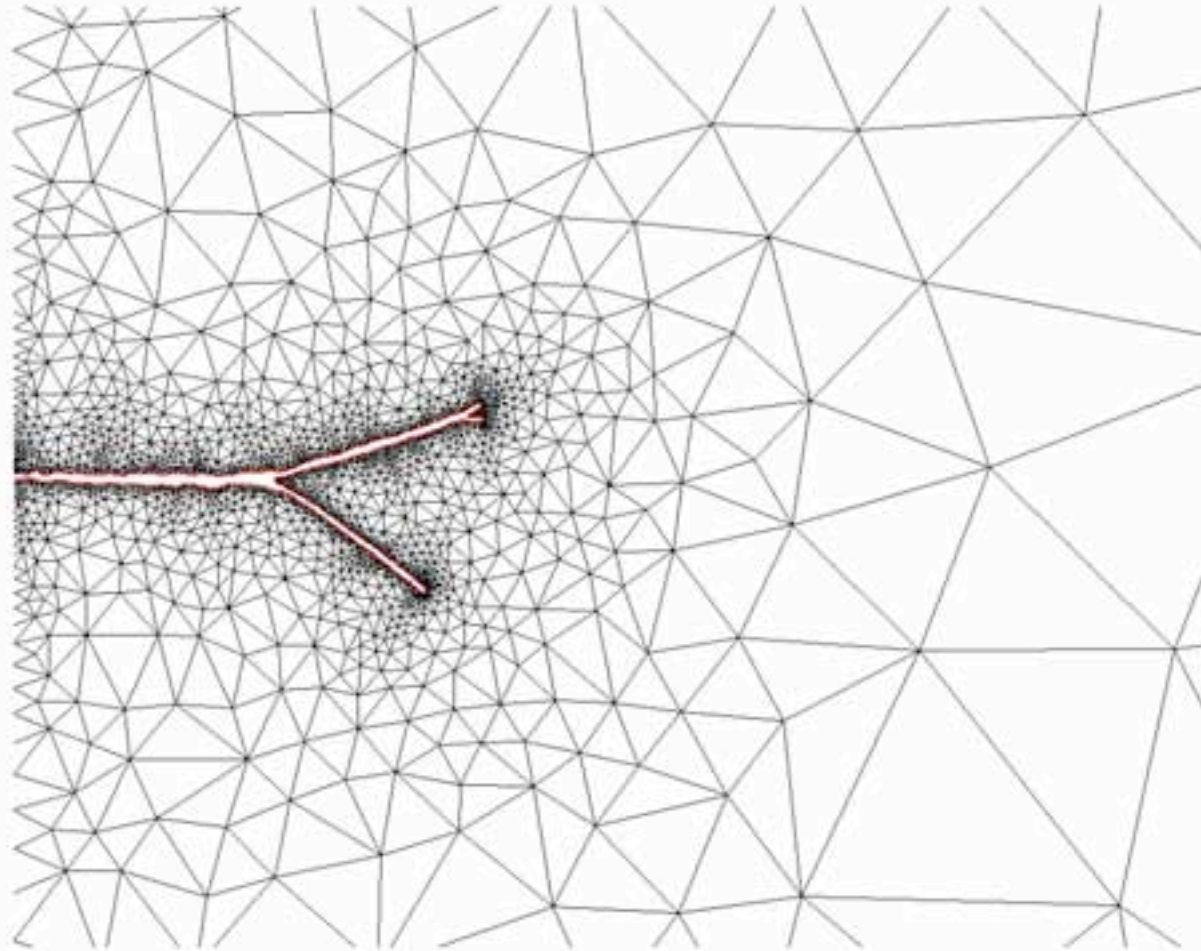
# Branching



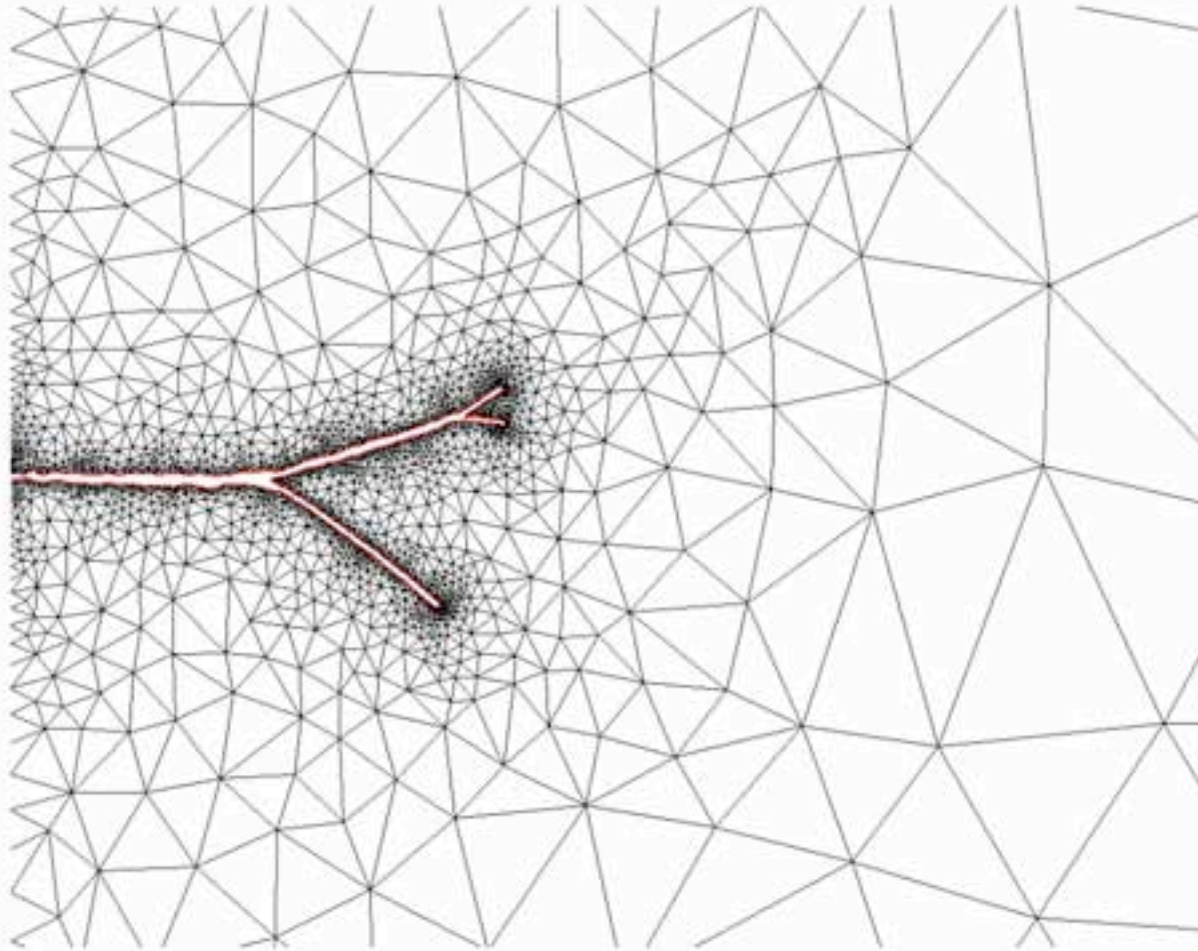
# Branching



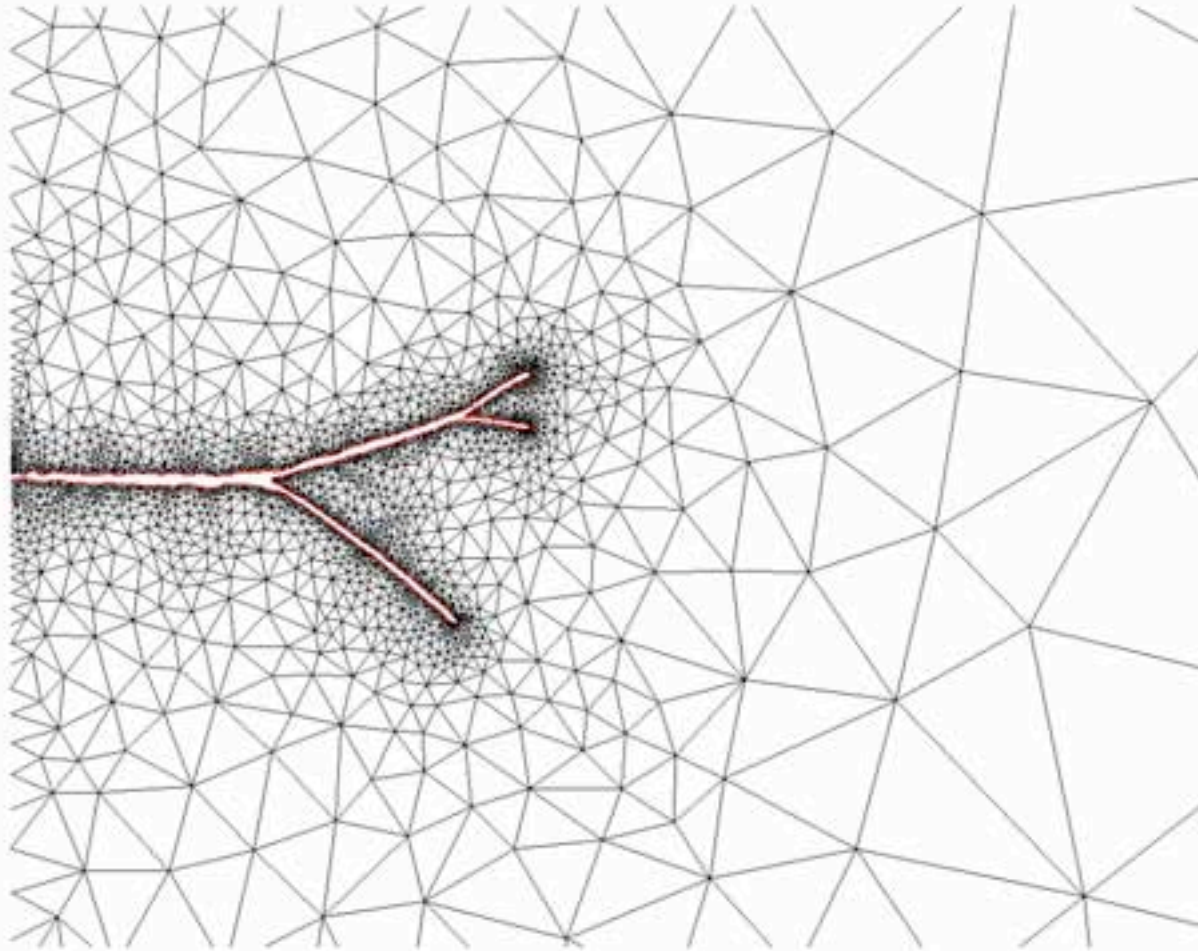
# Branching



# Branching

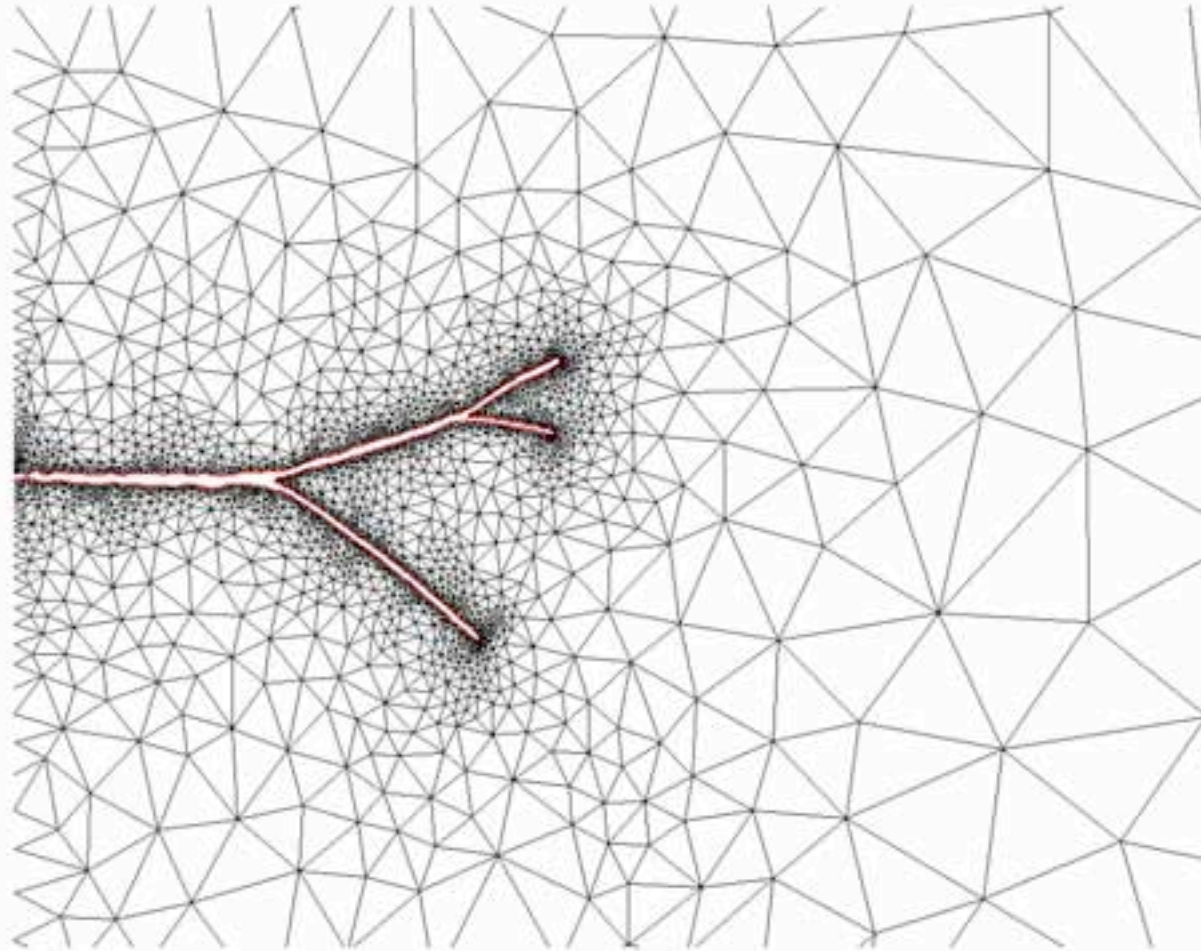


# Branching

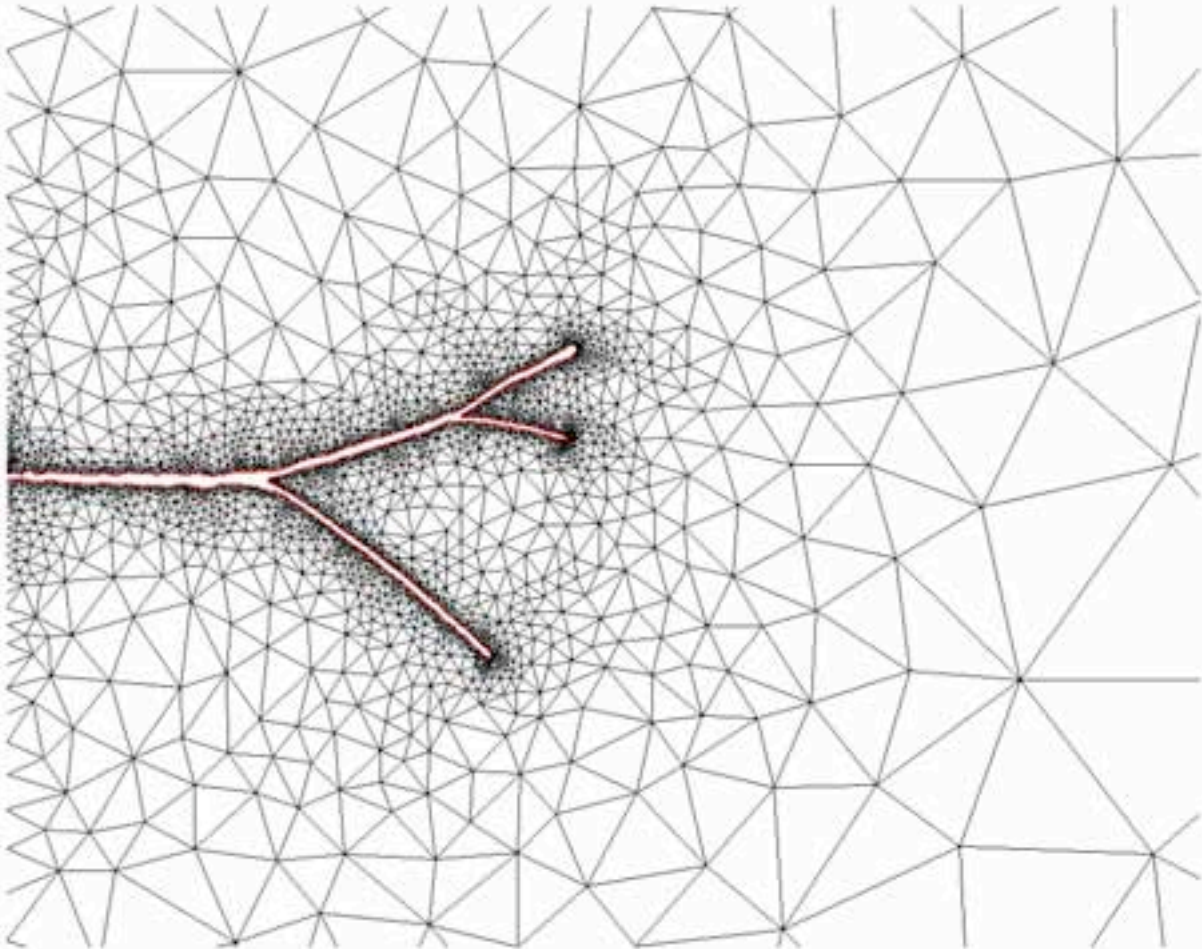




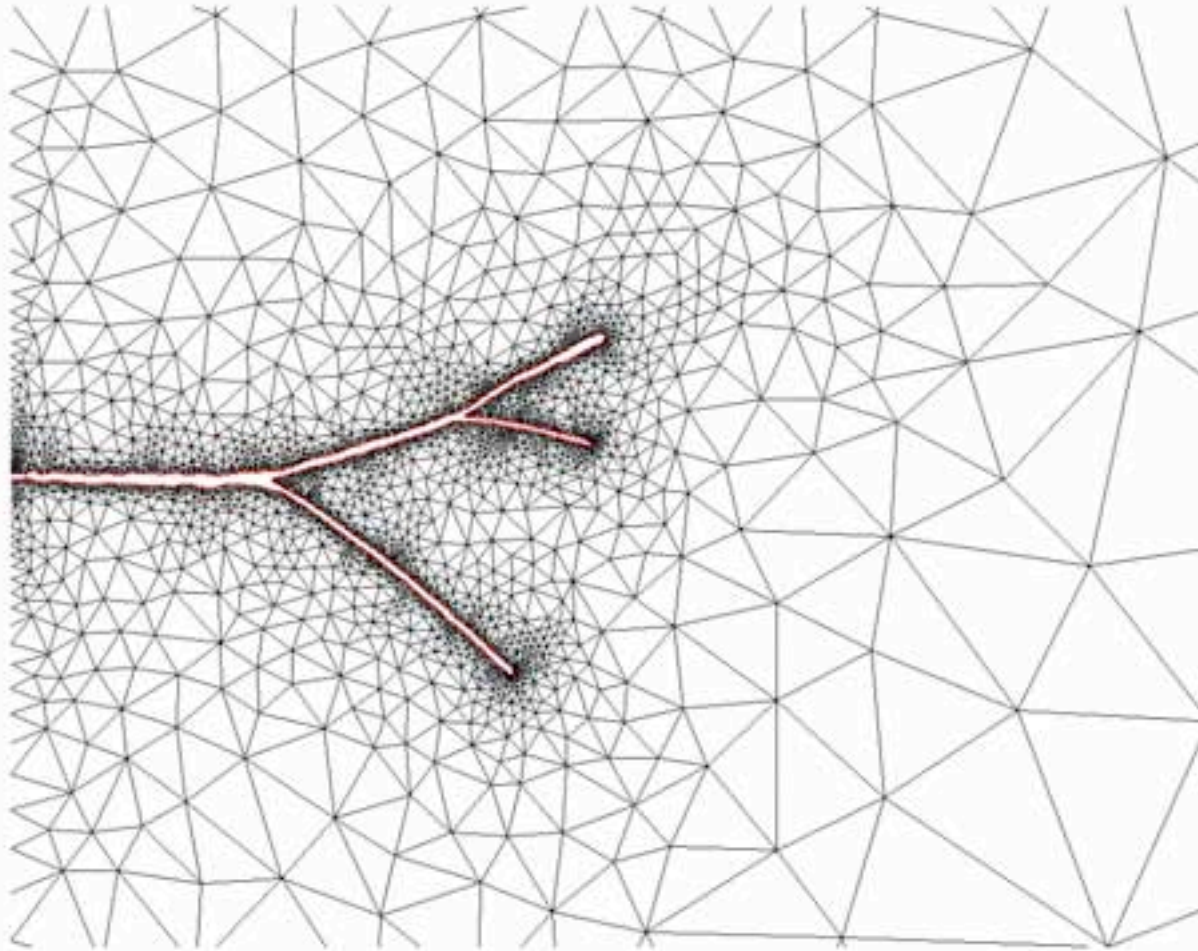
# Branching



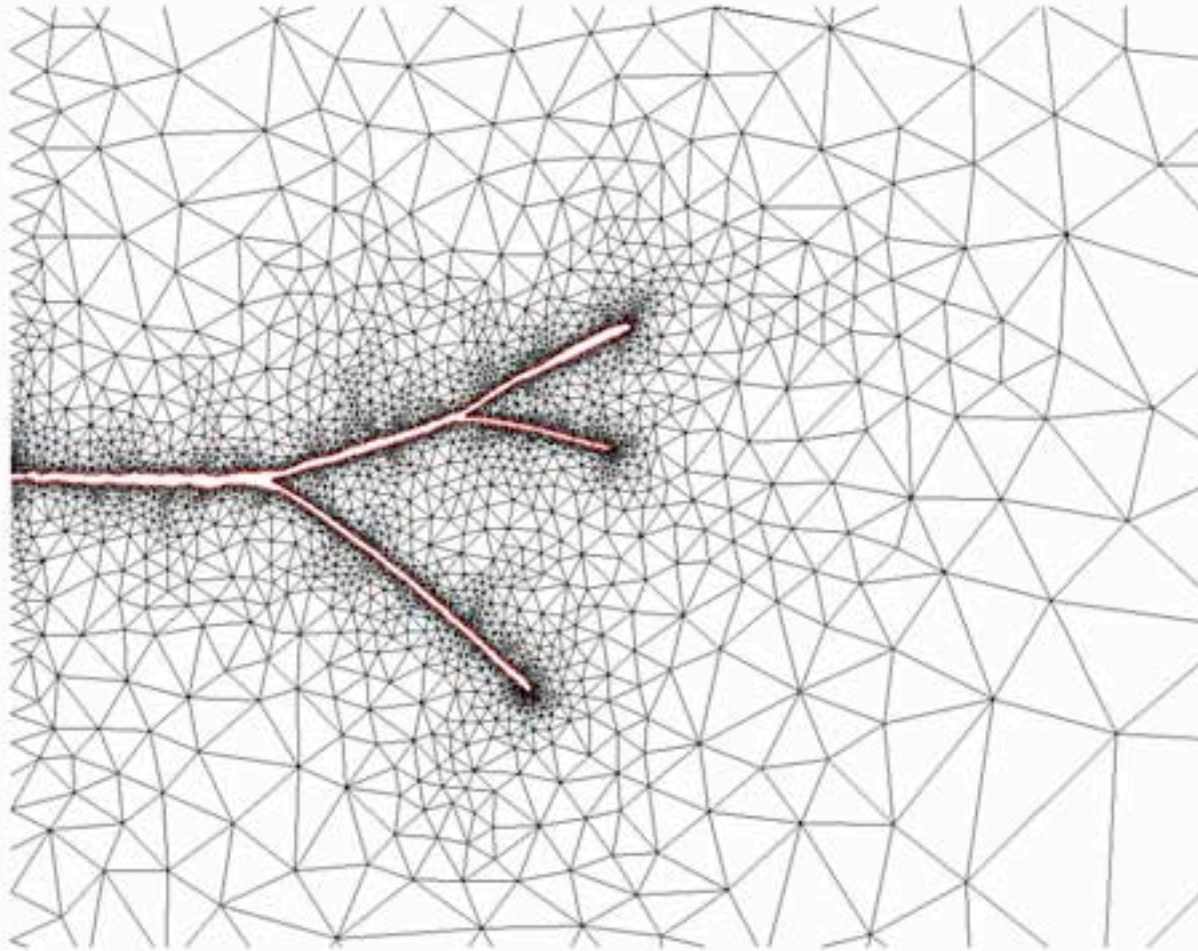
# Branching



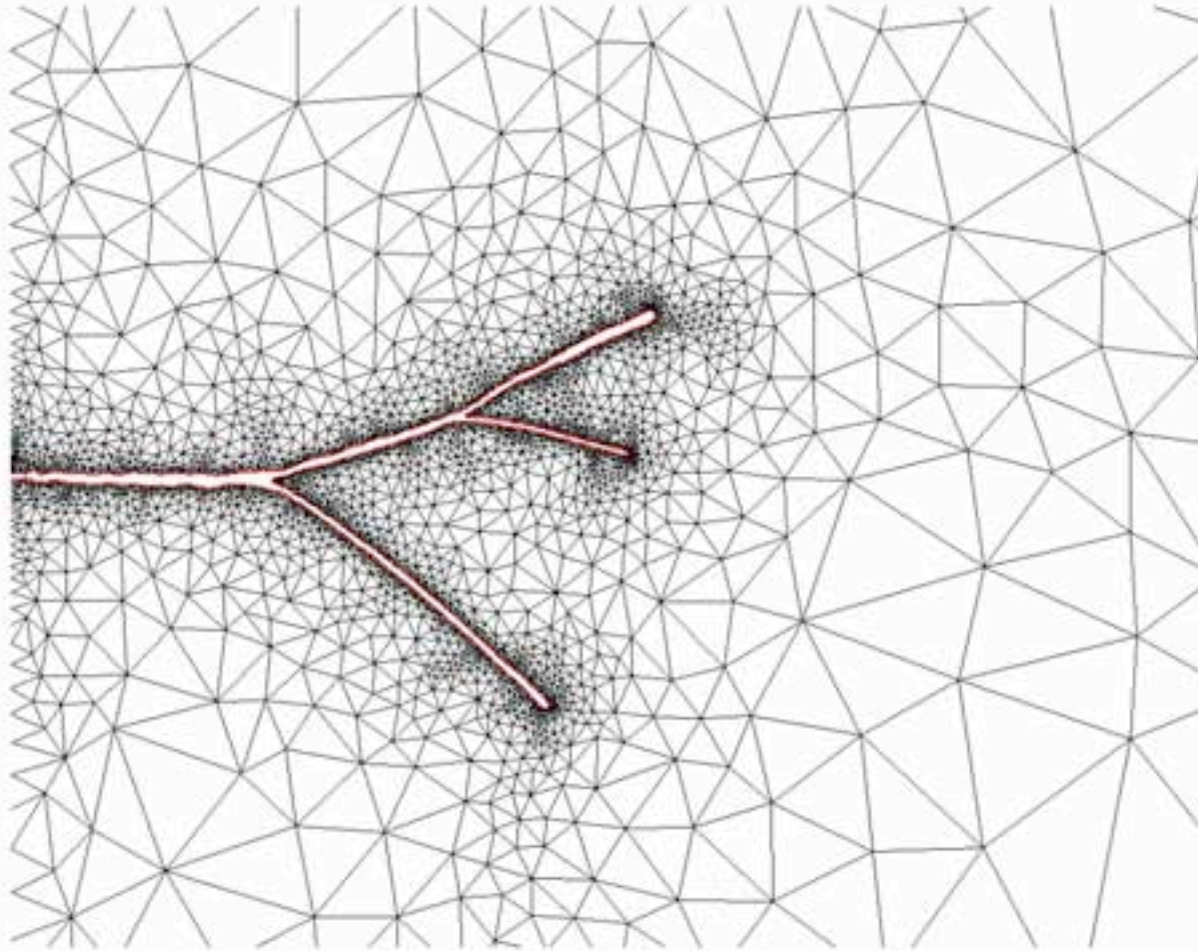
# Branching



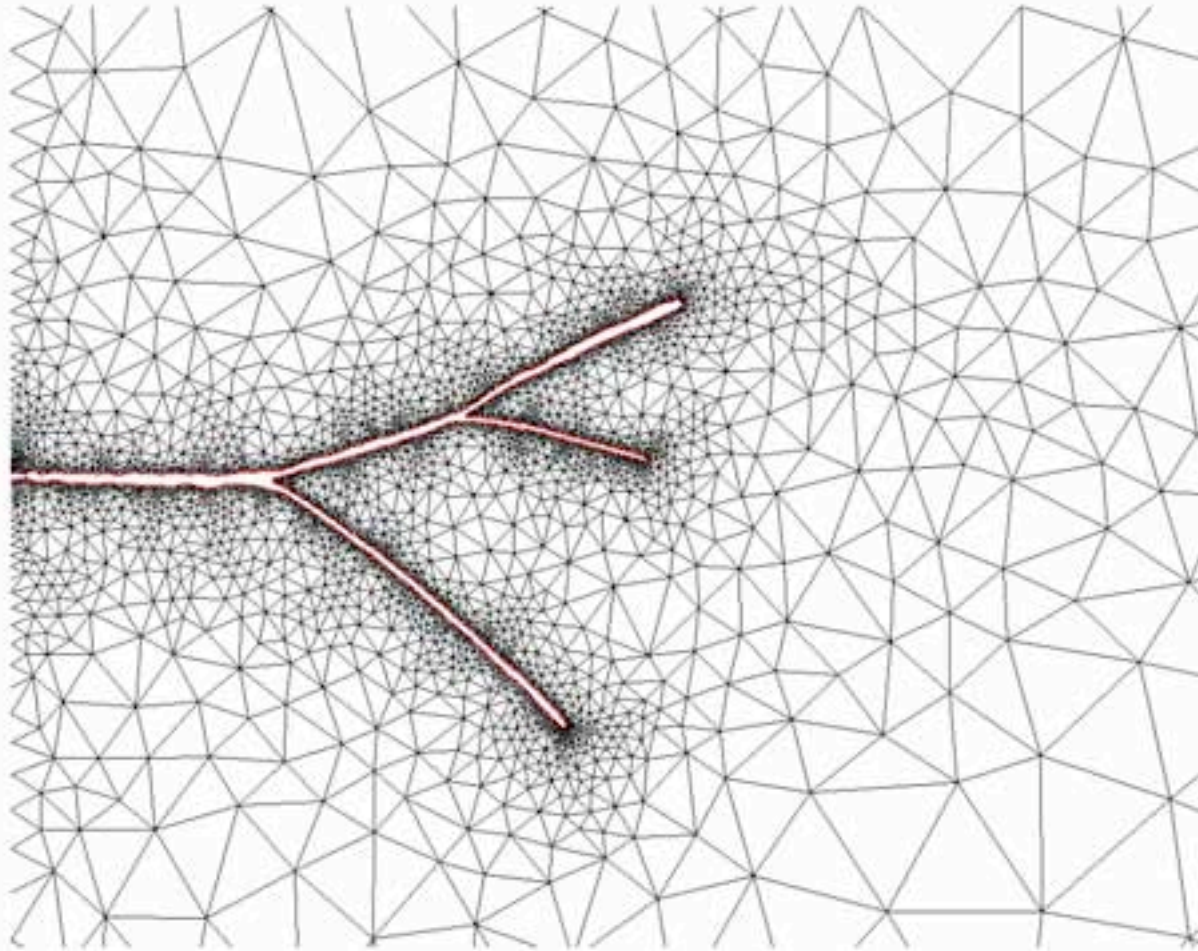
# Branching



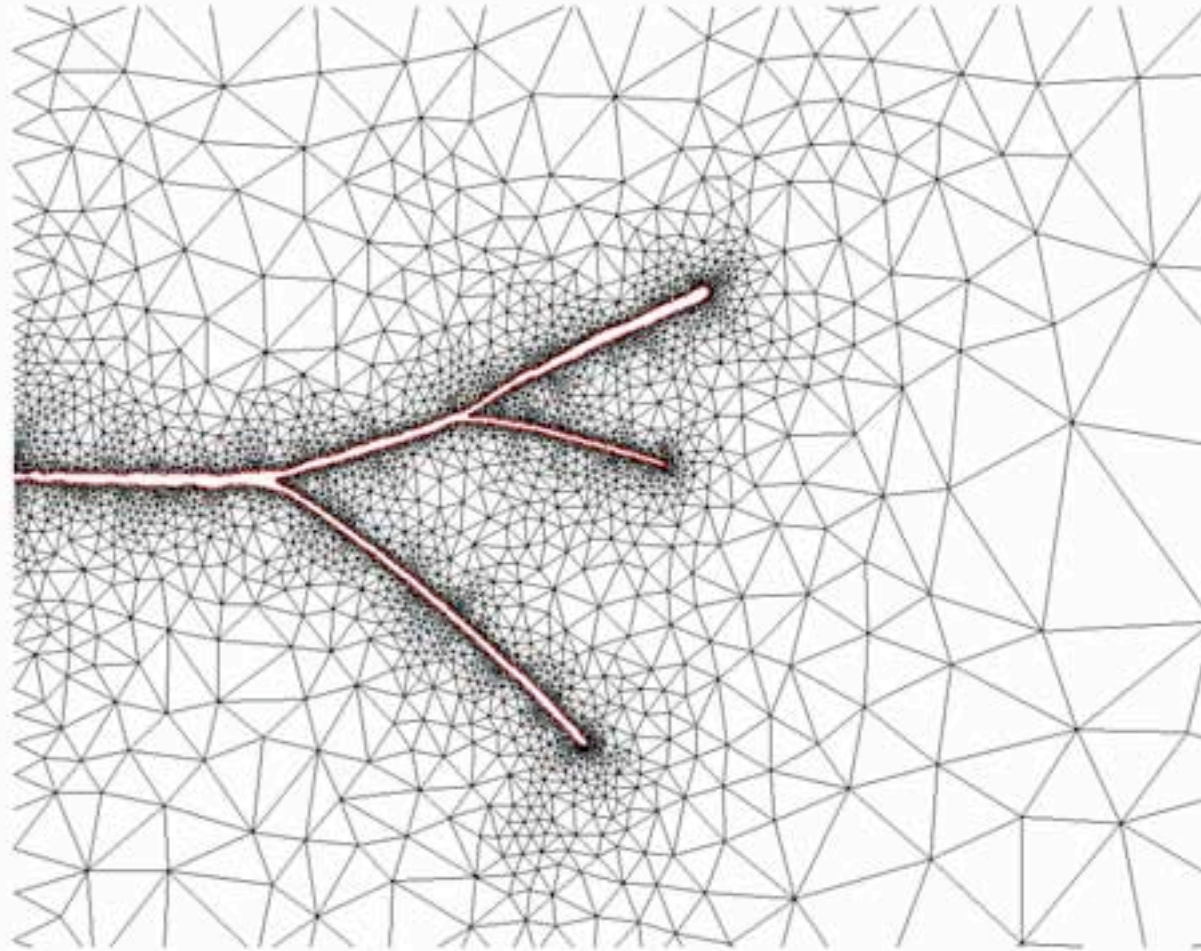
# Branching



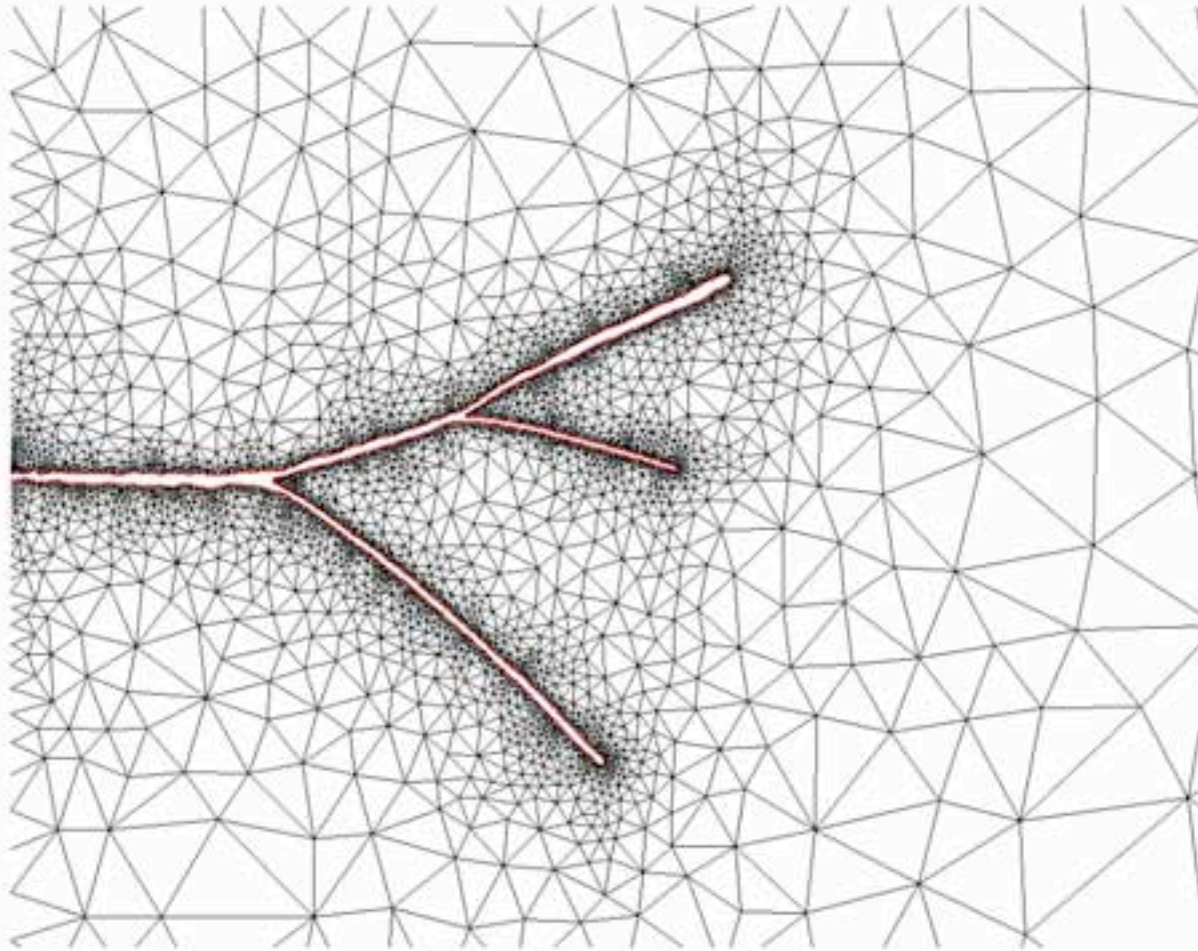
# Branching



# Branching

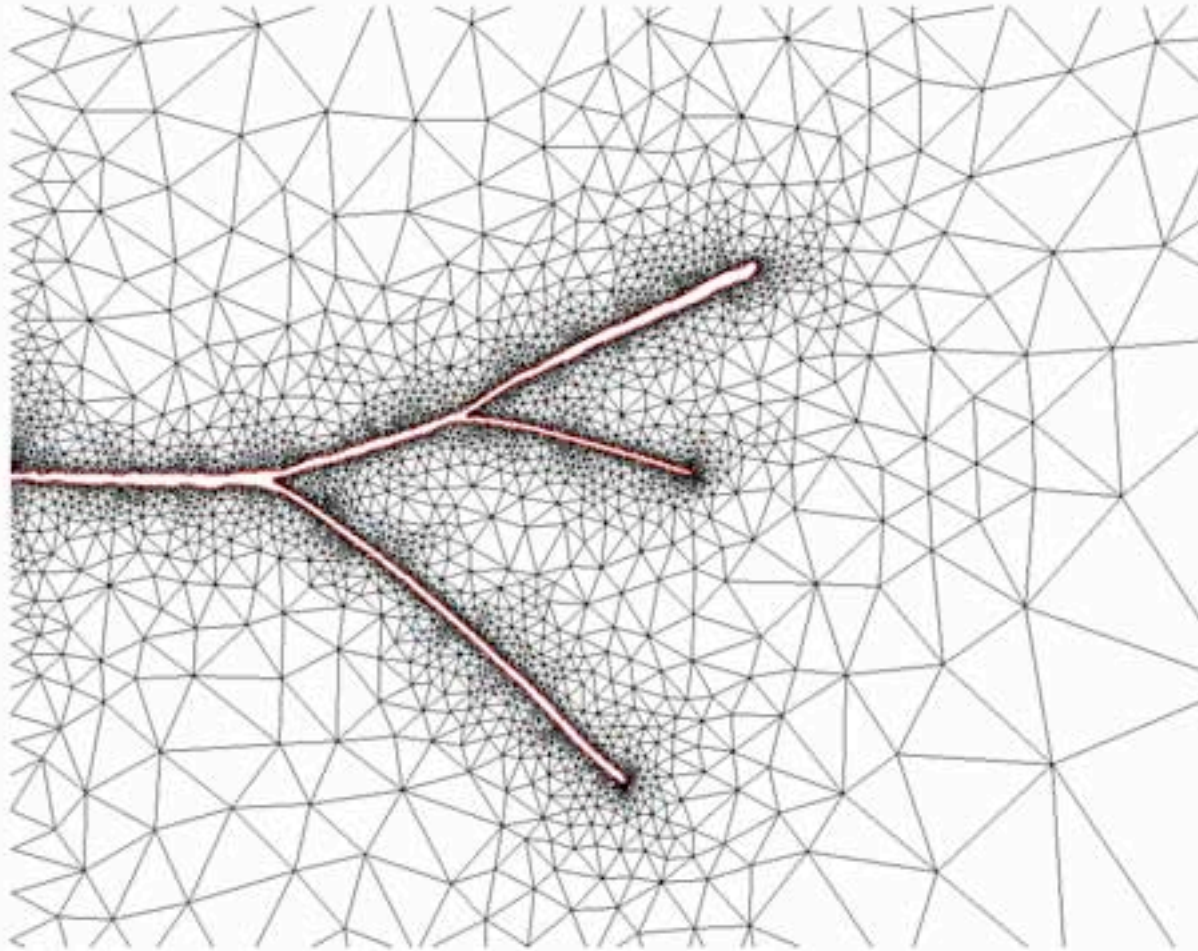


# Branching

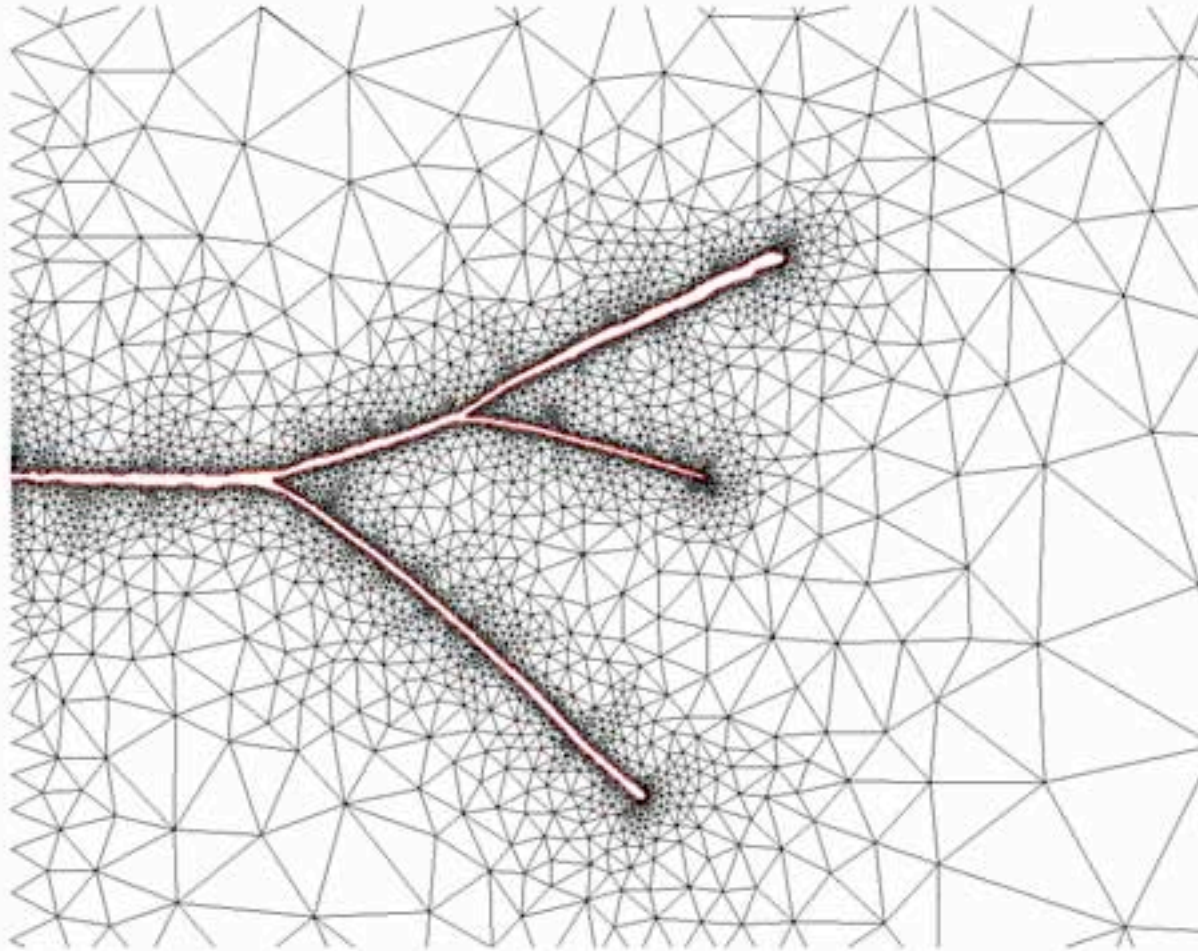




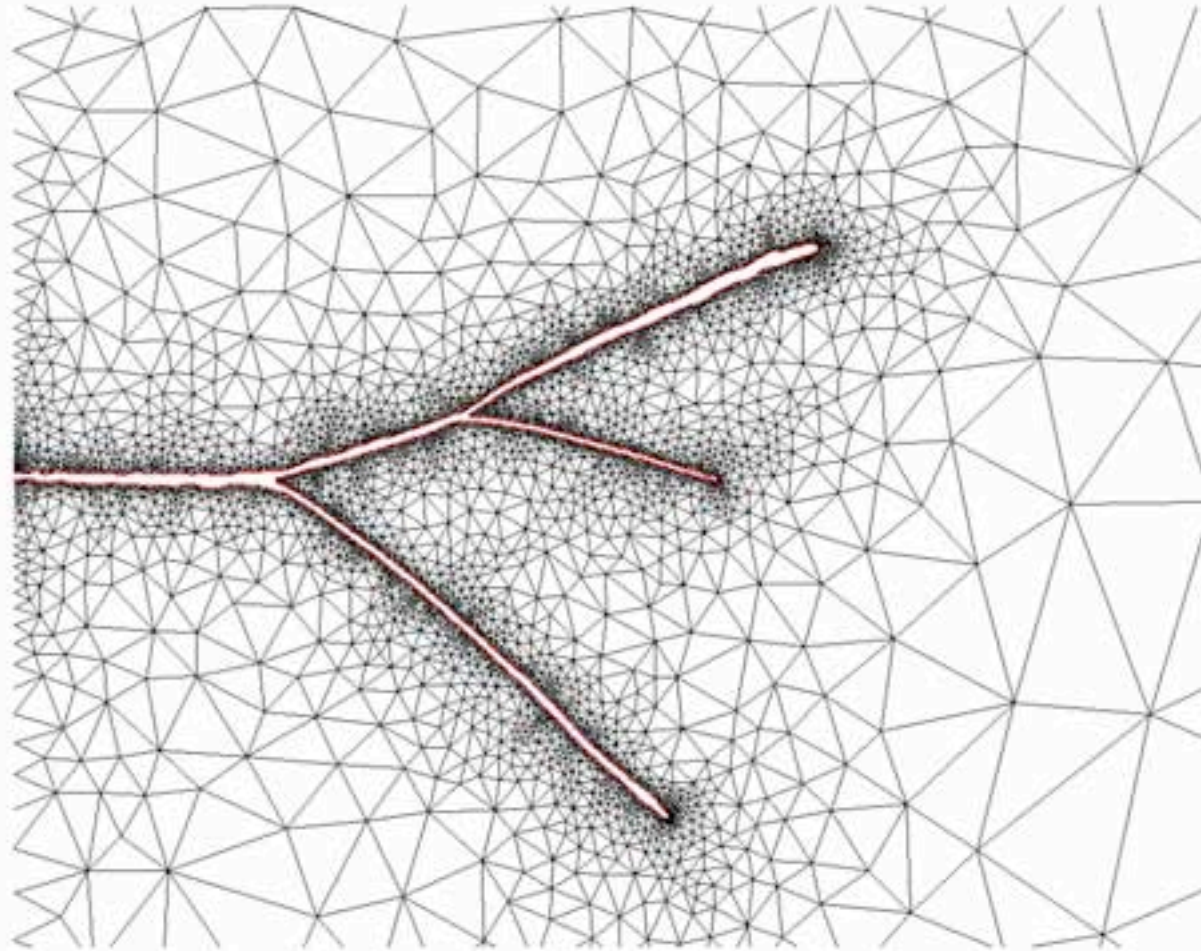
# Branching



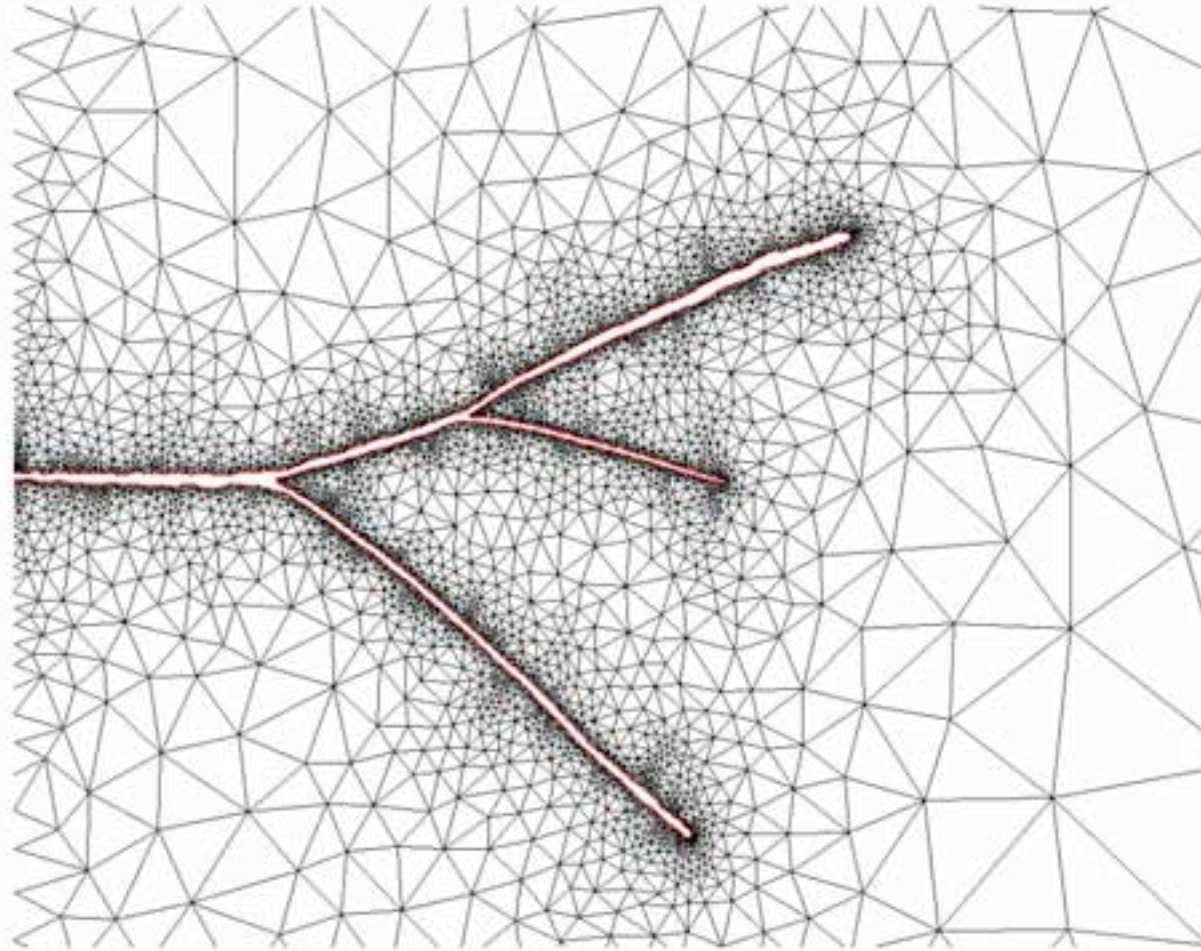
# Branching



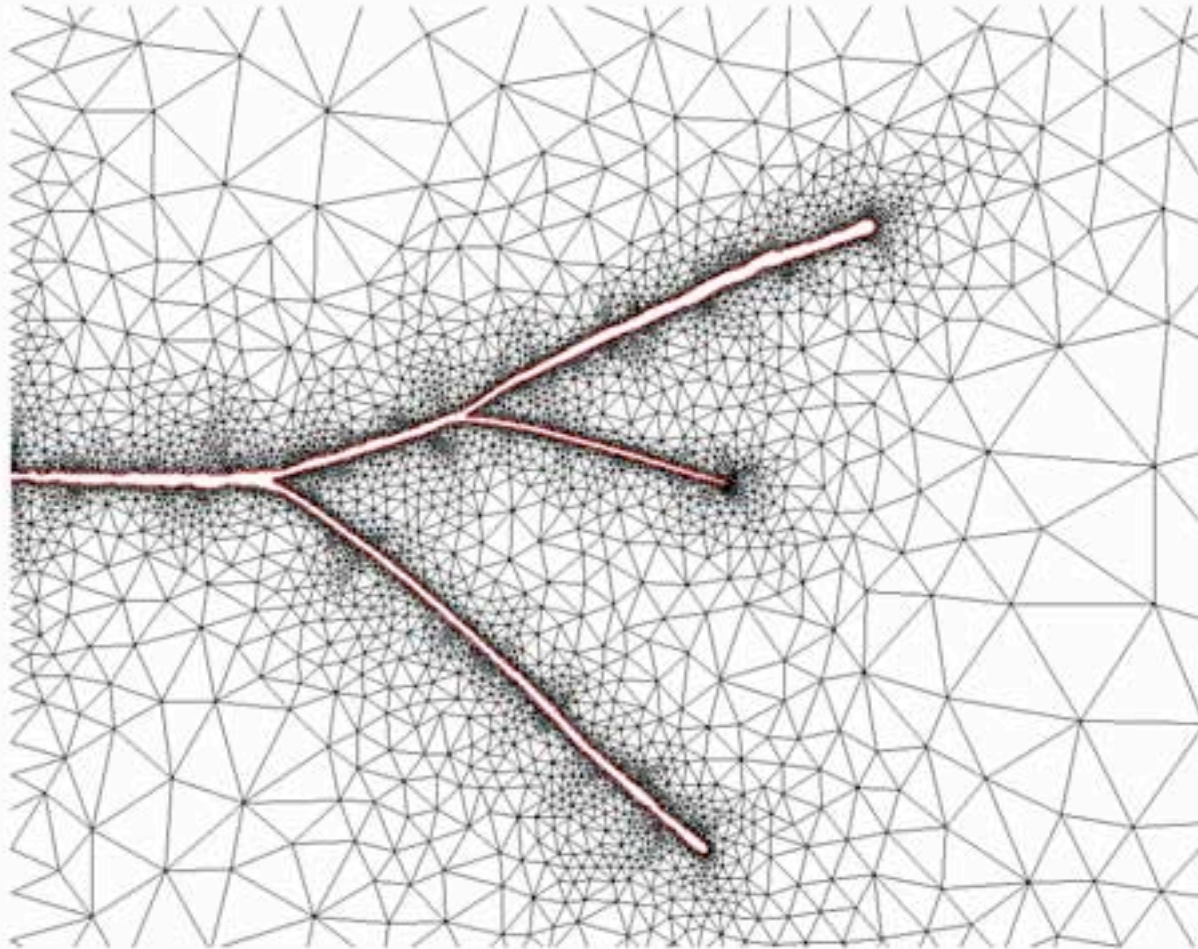
# Branching



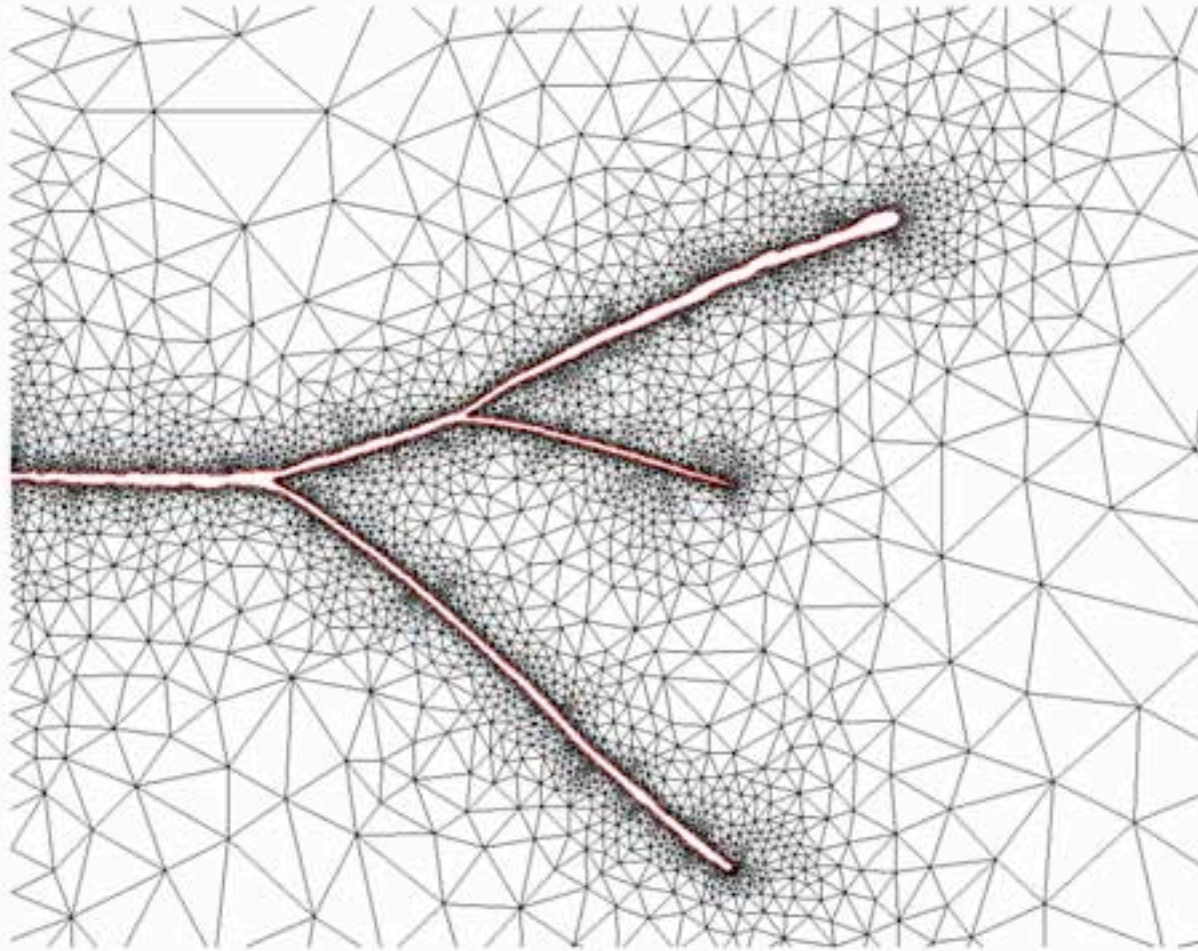
# Branching



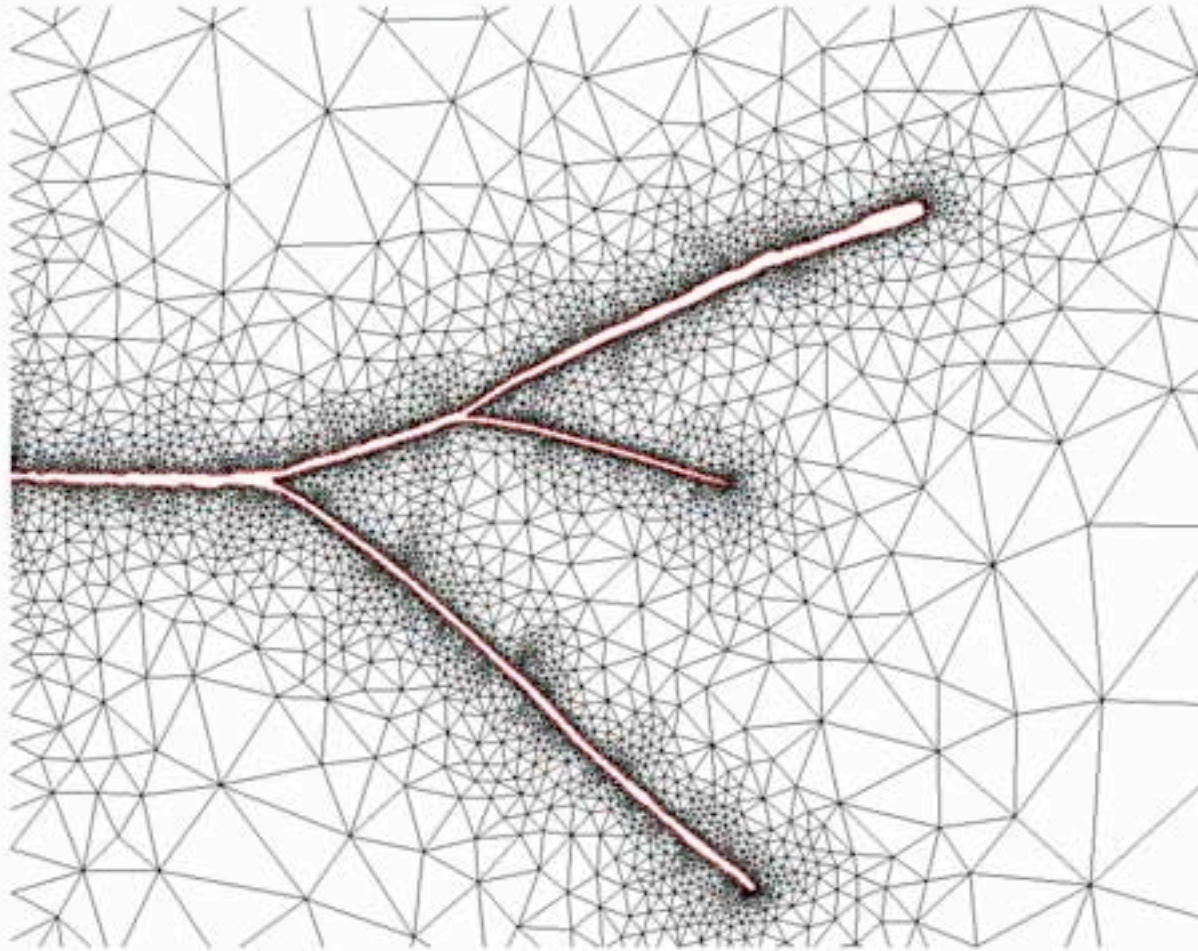
# Branching



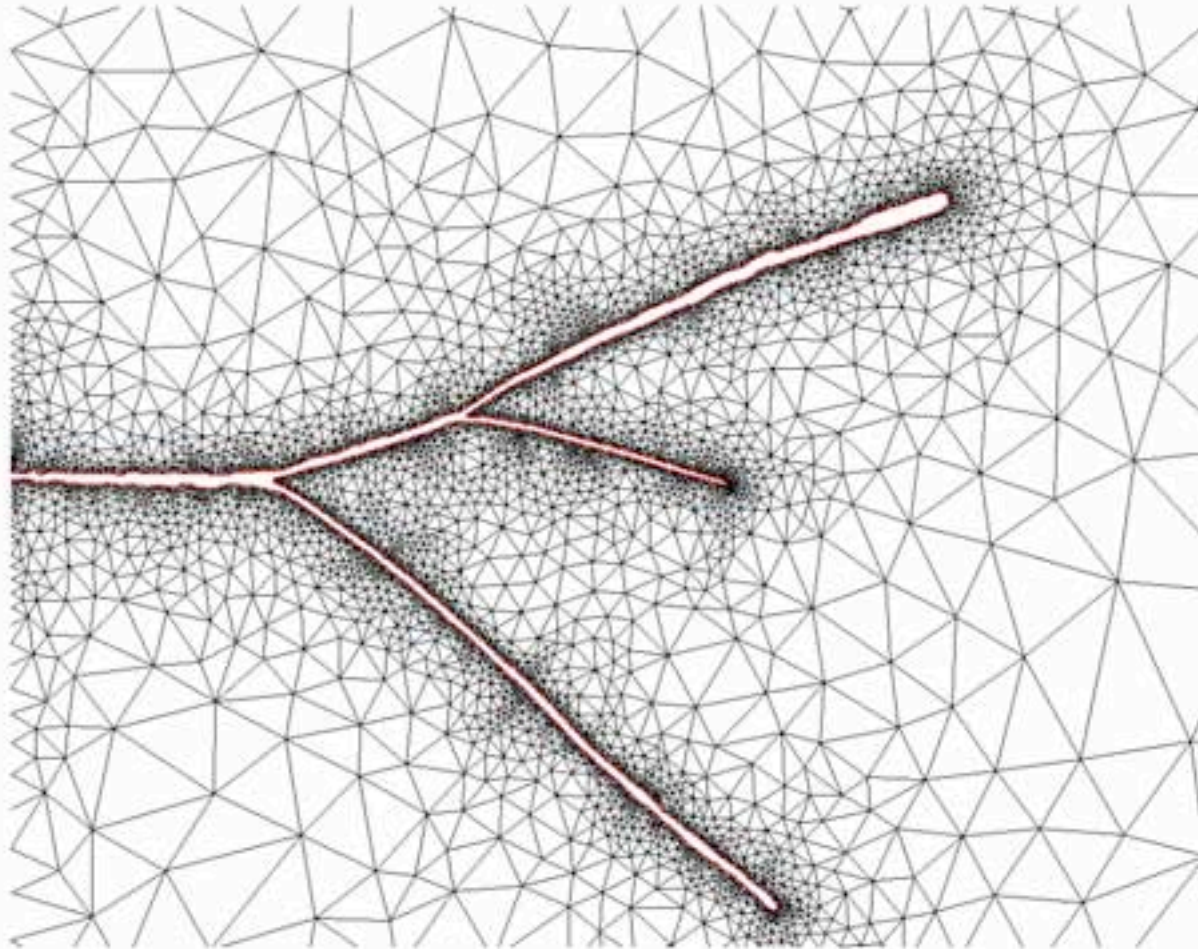
# Branching



# Branching

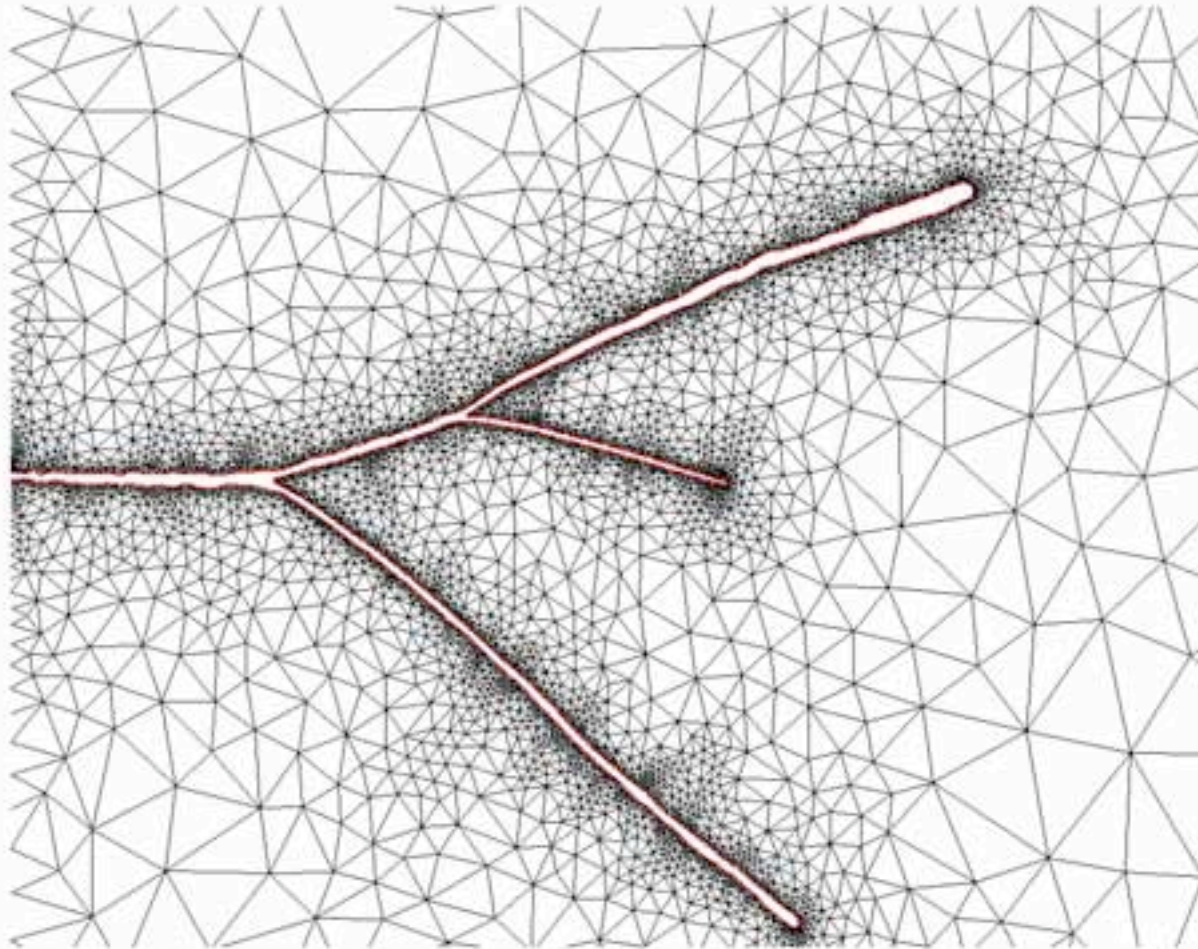


# Branching

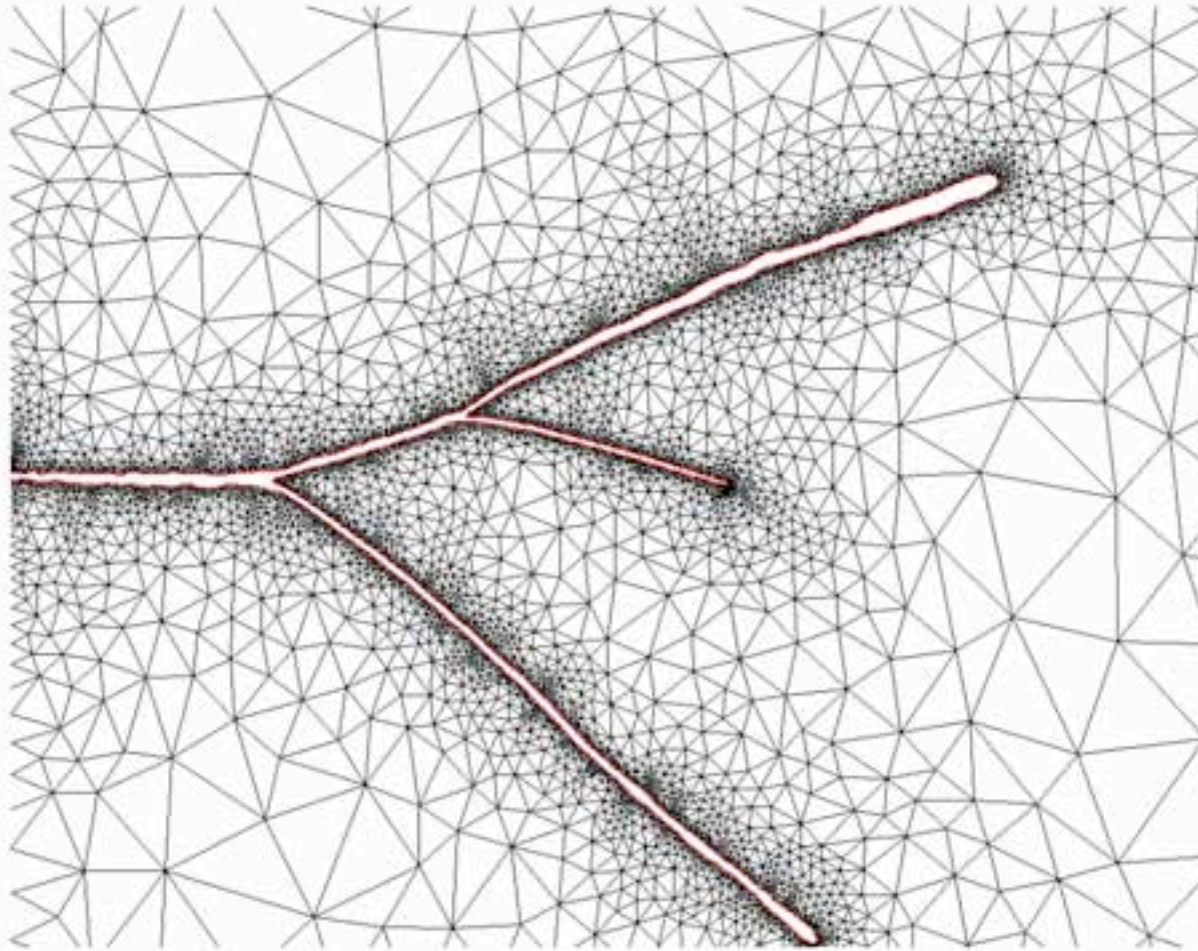




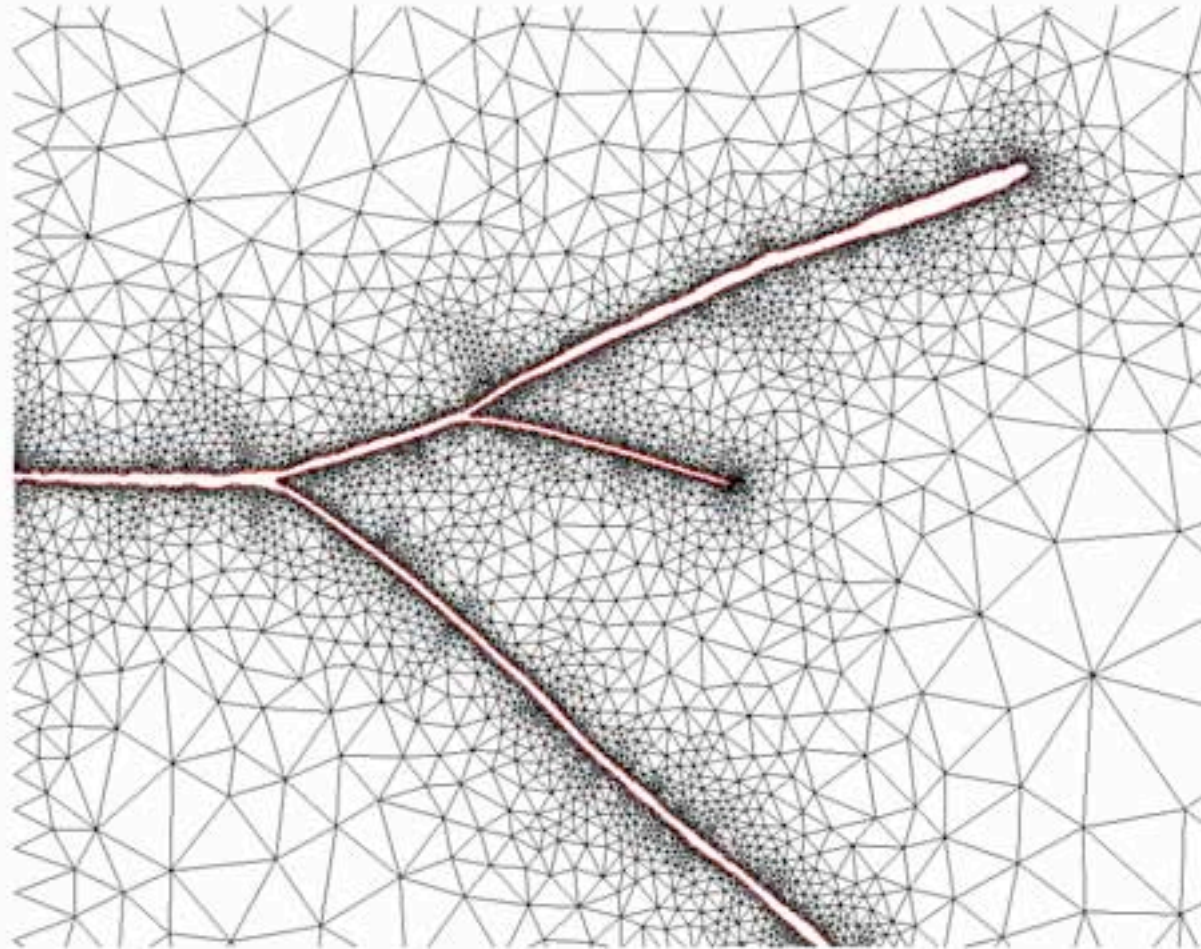
# Branching



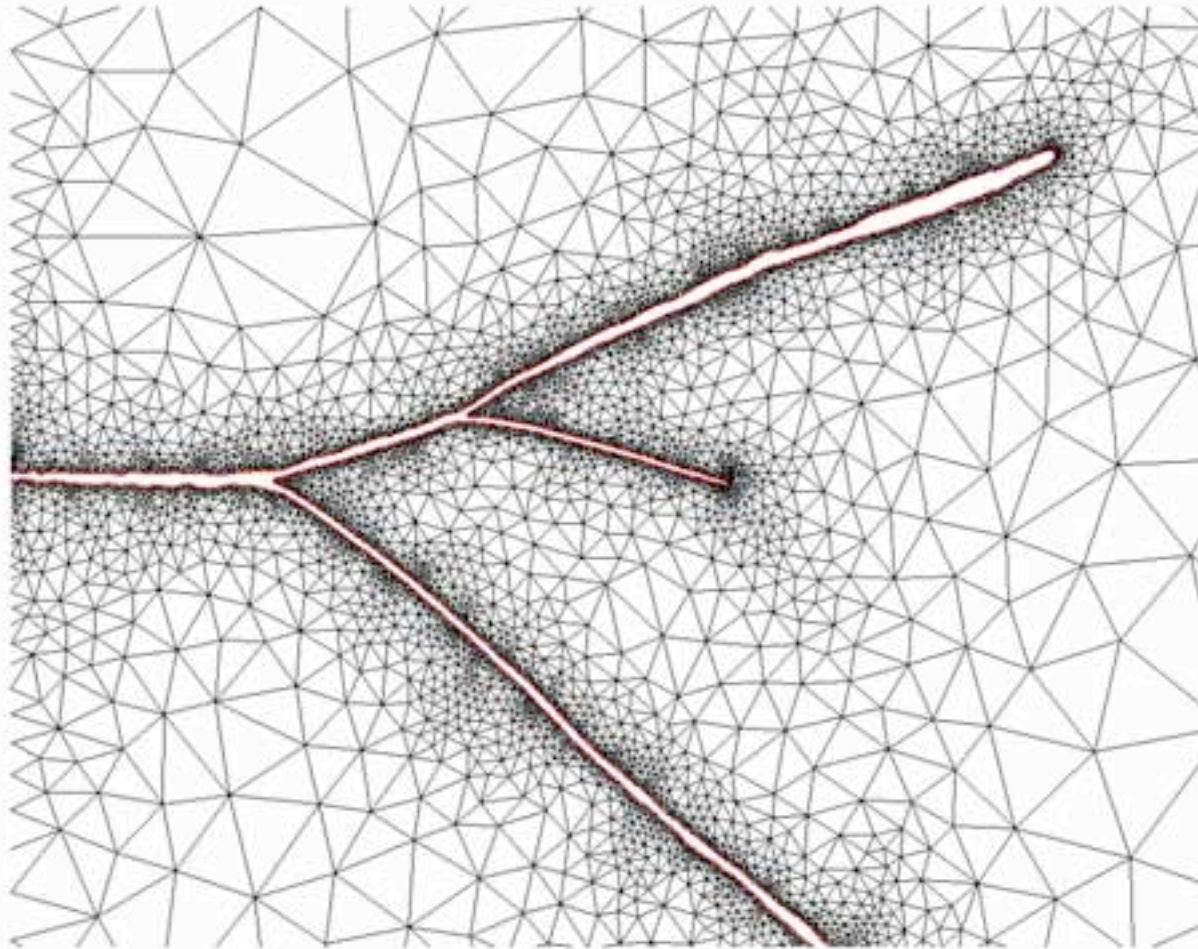
# Branching



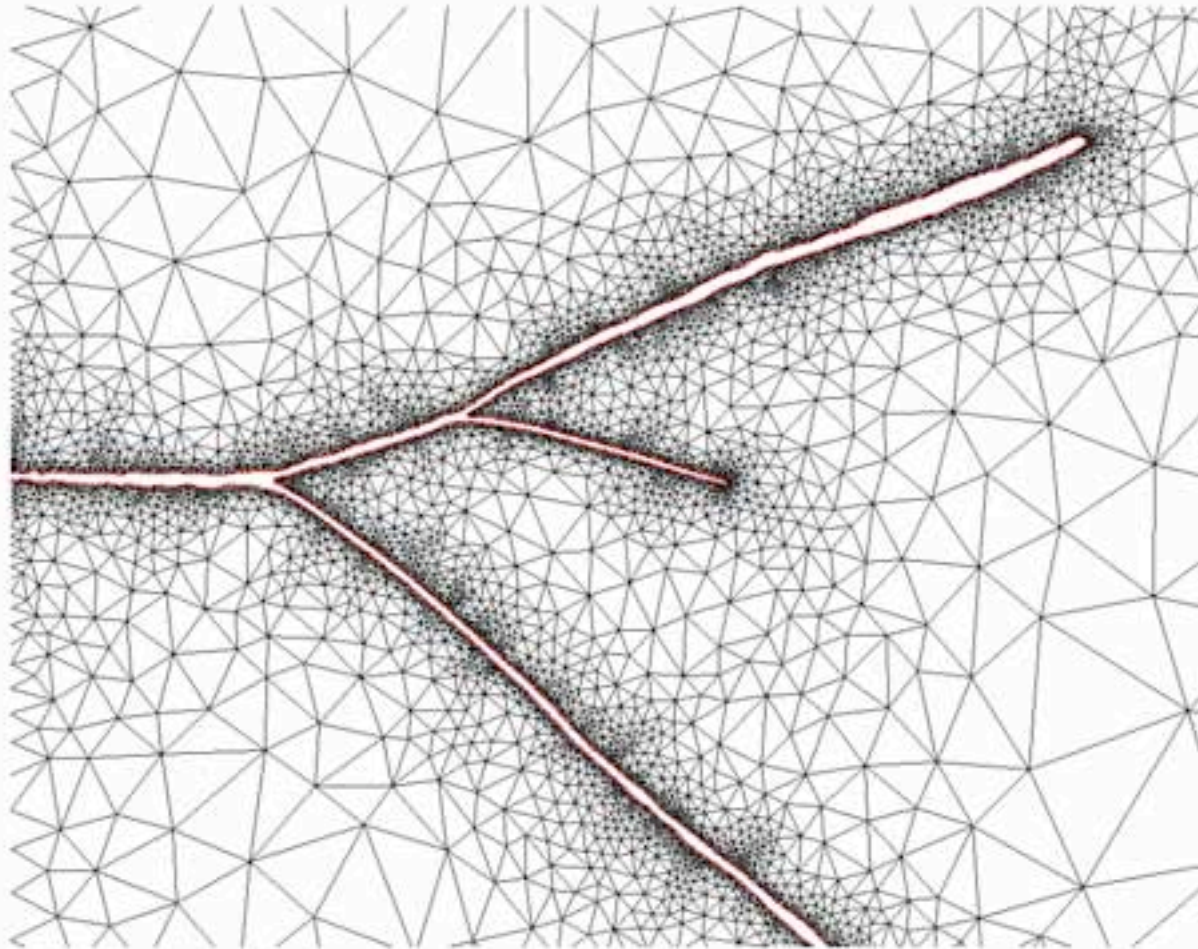
# Branching



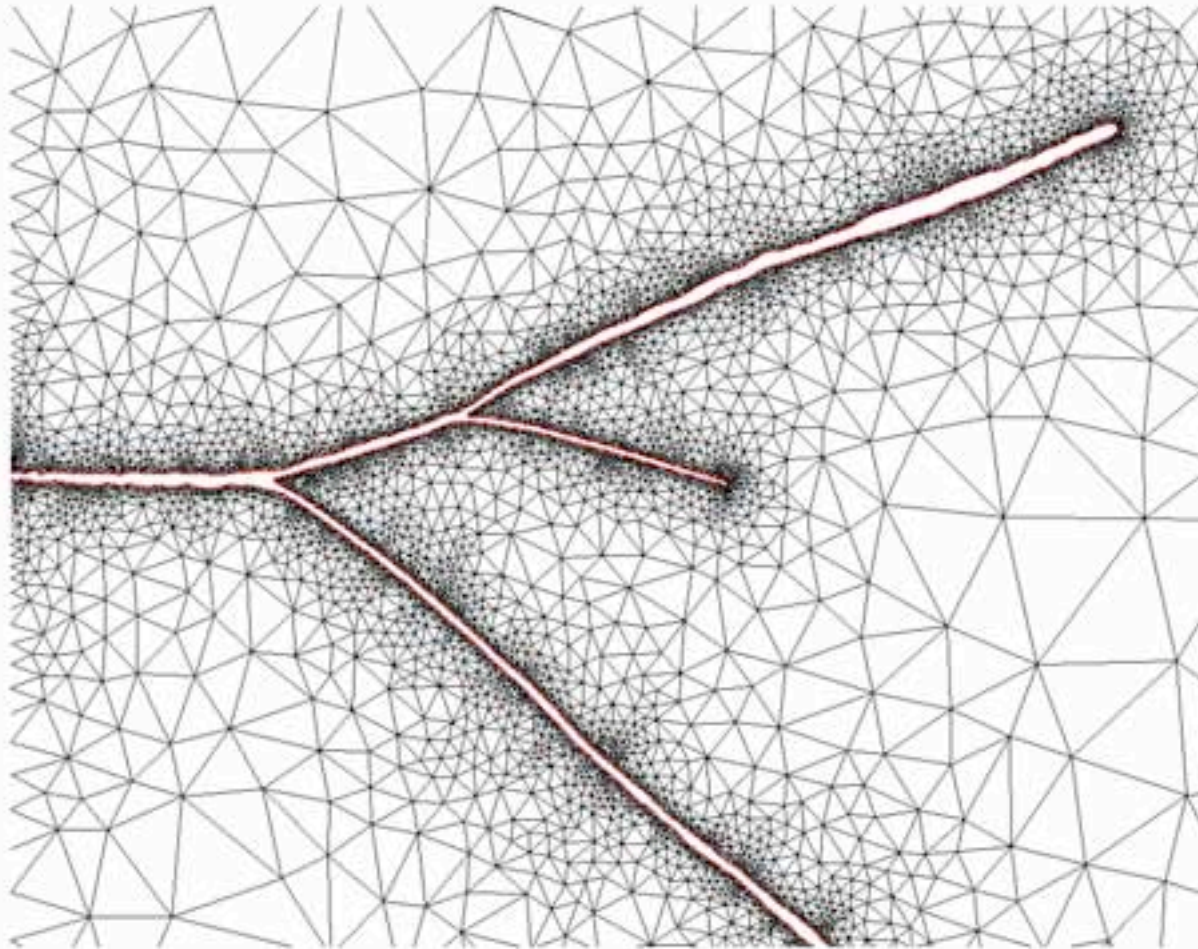
# Branching



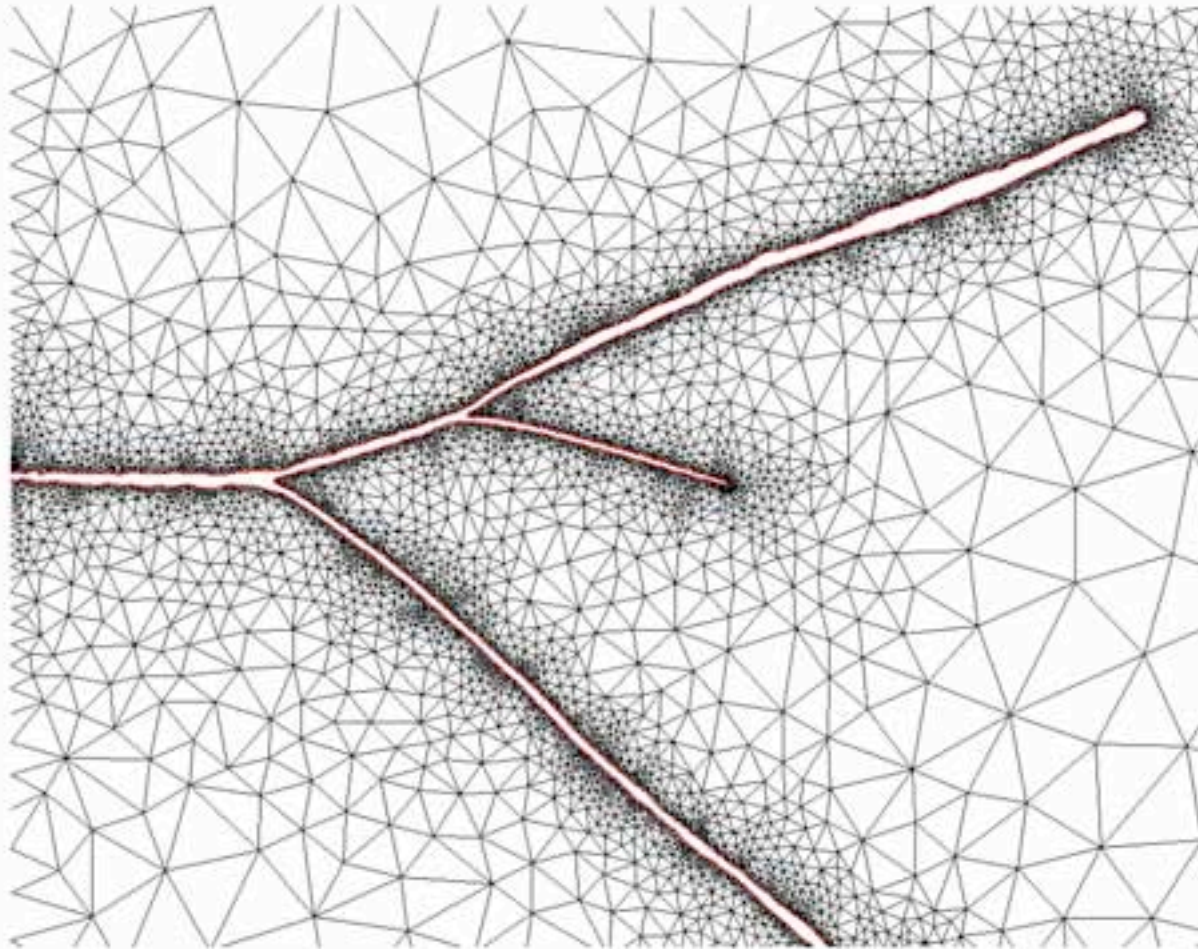
# Branching



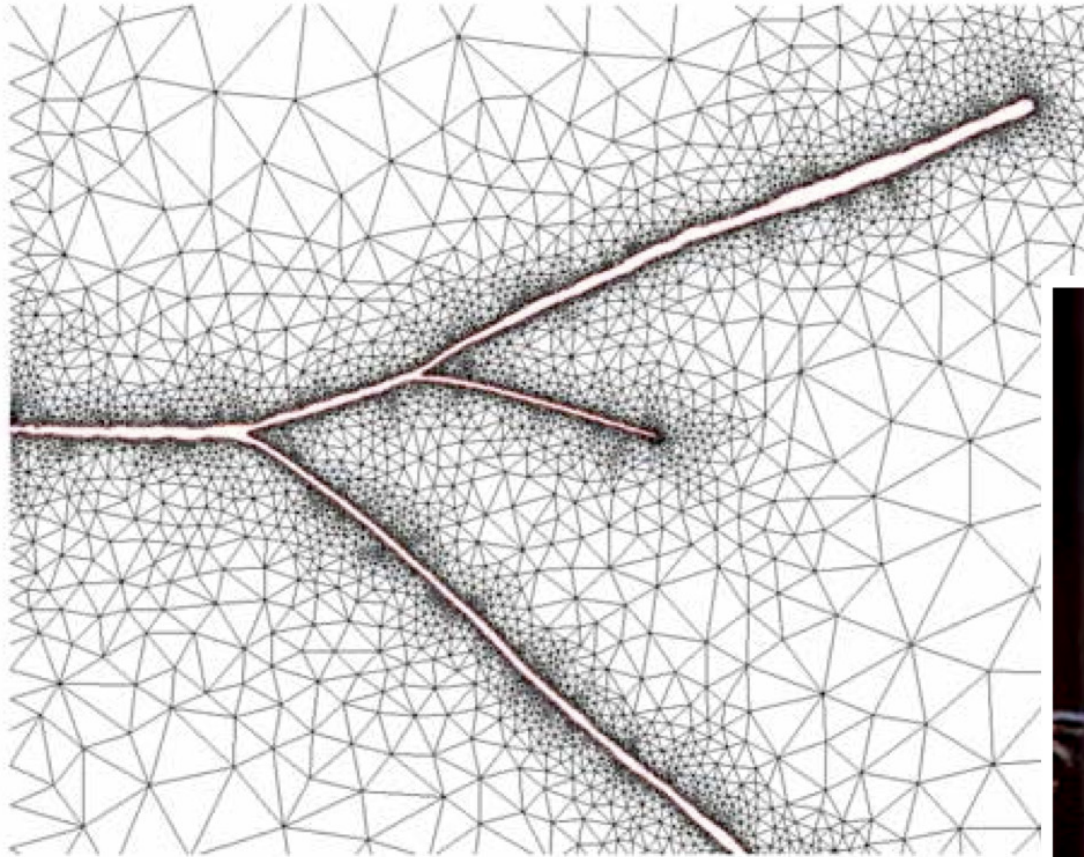
# Branching



# Branching

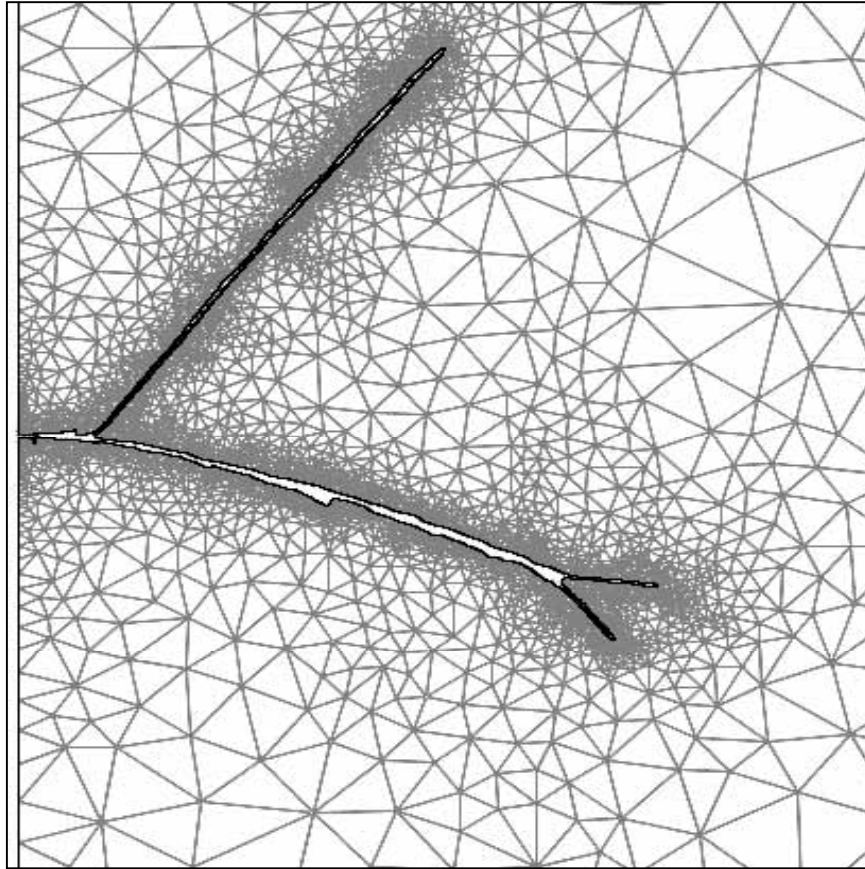


# Branching





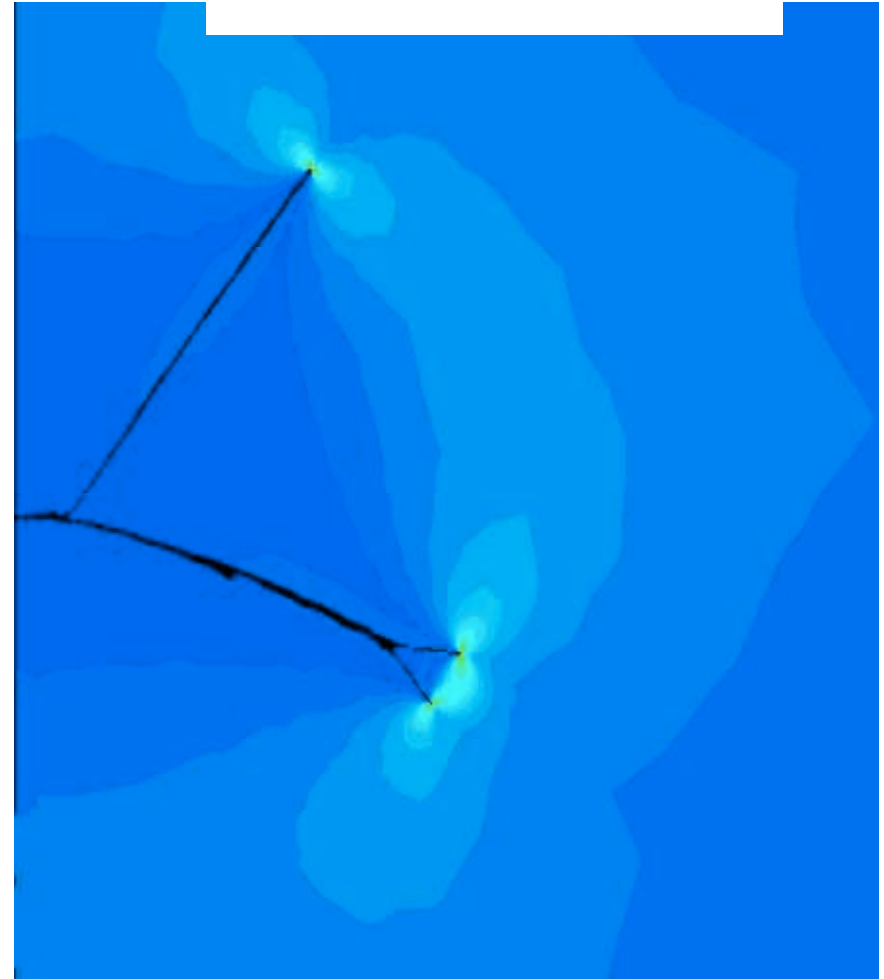
# Crack-tip load



Before branching

$$K_I = K_I', \rho = \rho'$$

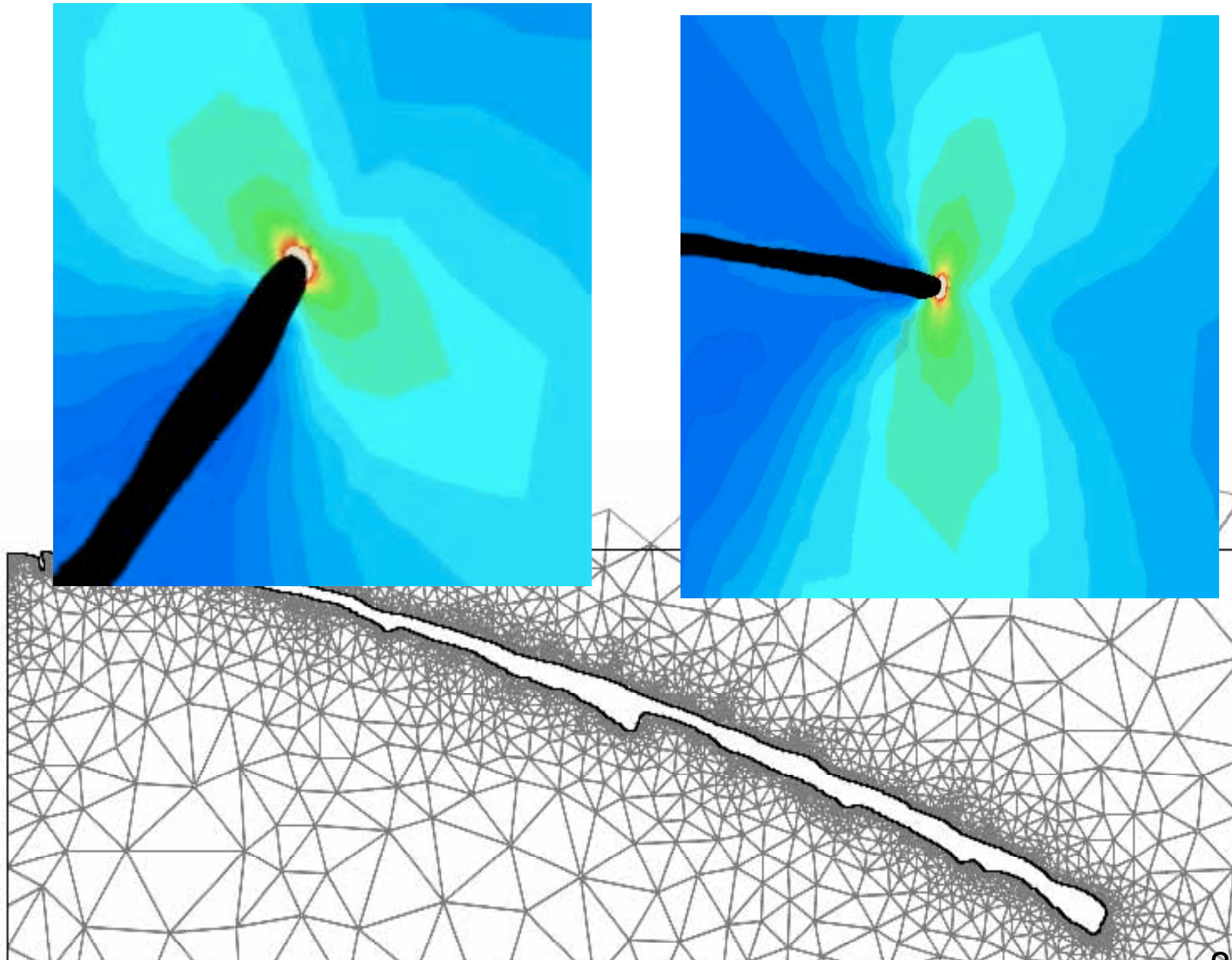
effective stress



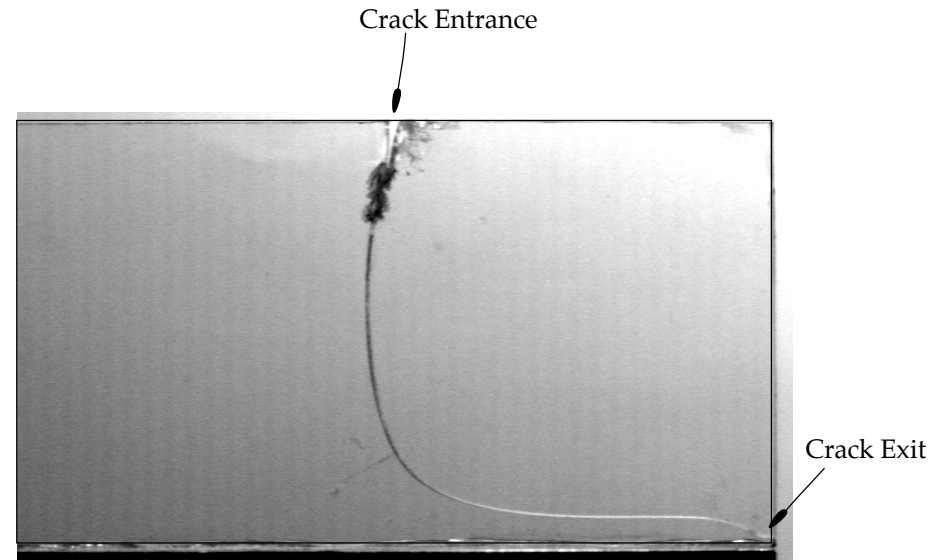
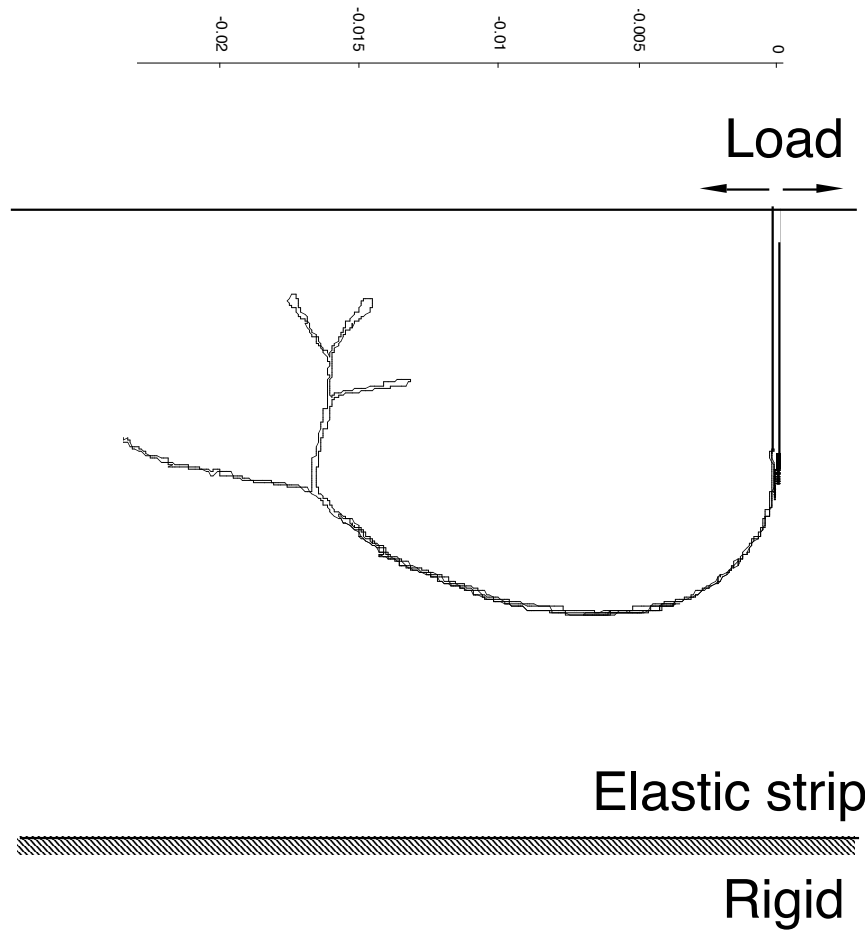
After branching

$$K_I \approx 0.7 K_I', \rho = \rho' / 2$$

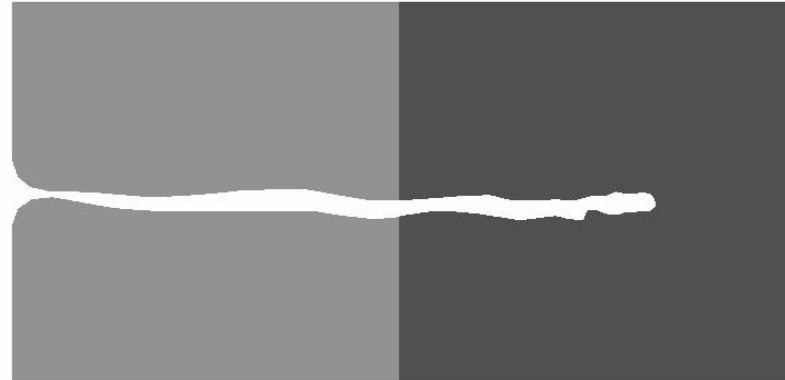
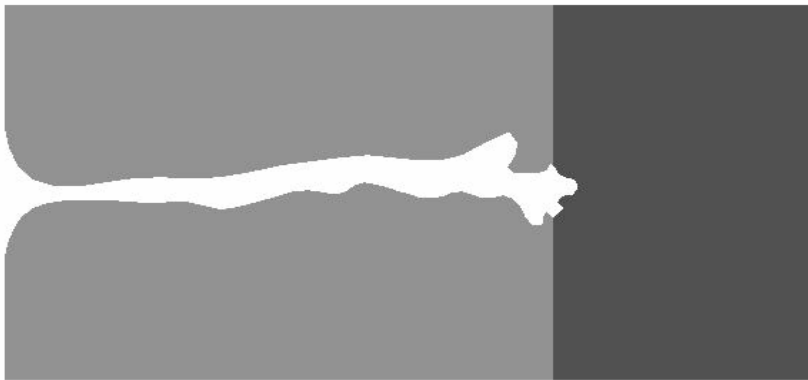
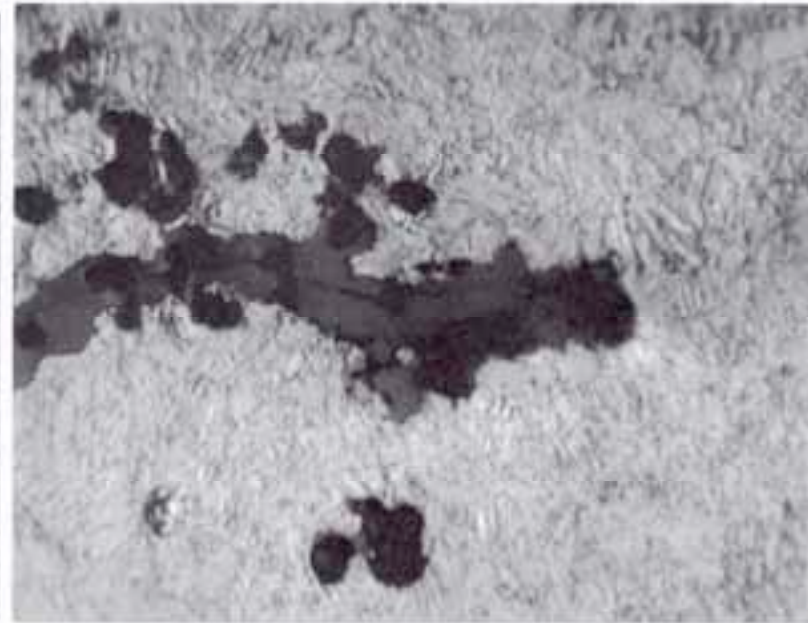
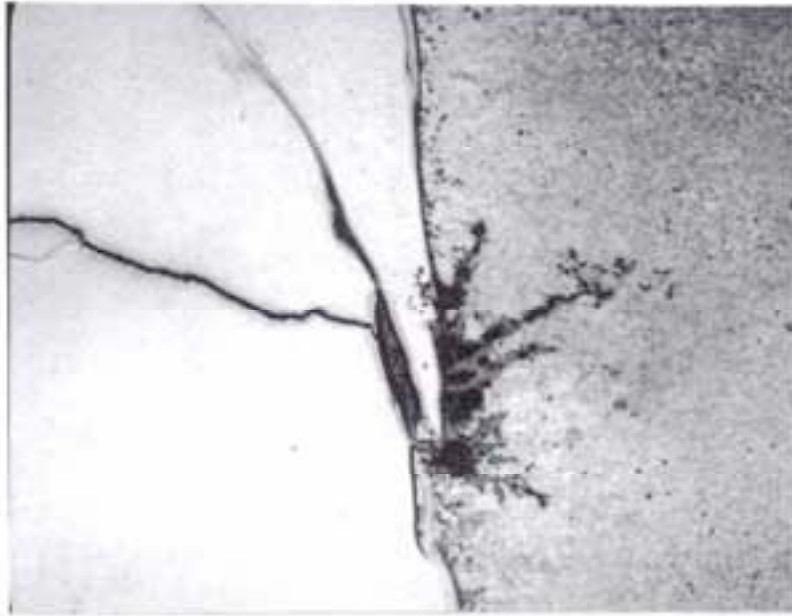
# Crack-tip Load



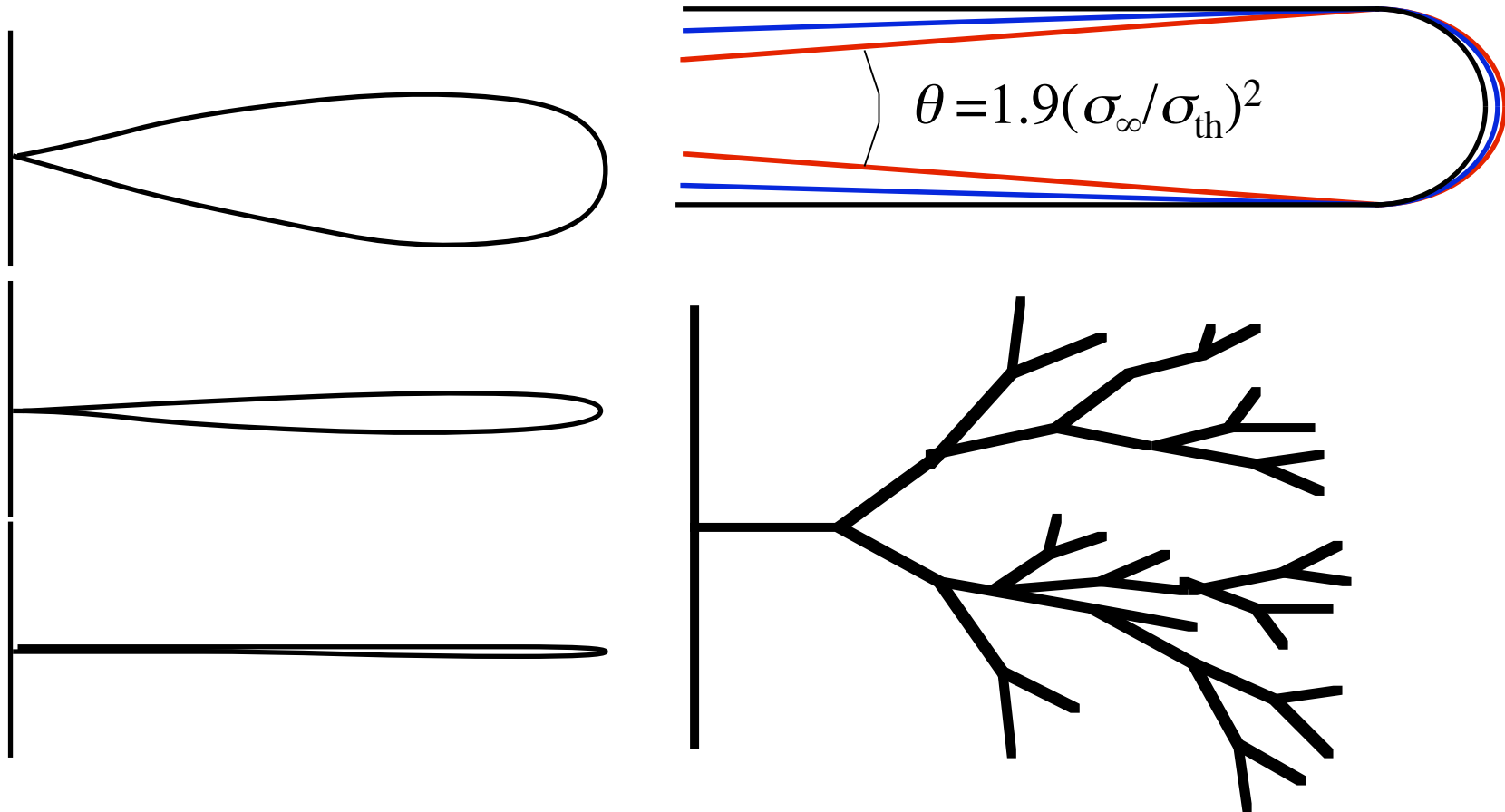
# Crack seem to follow a mode I path



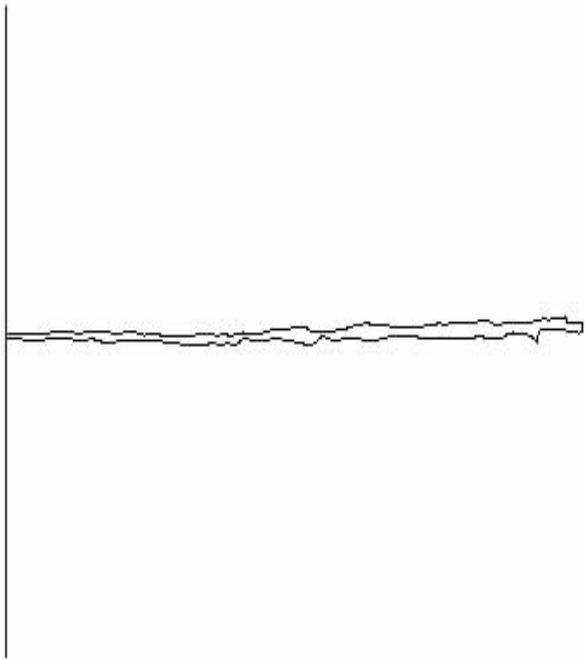
# From Cladding to Grey Material



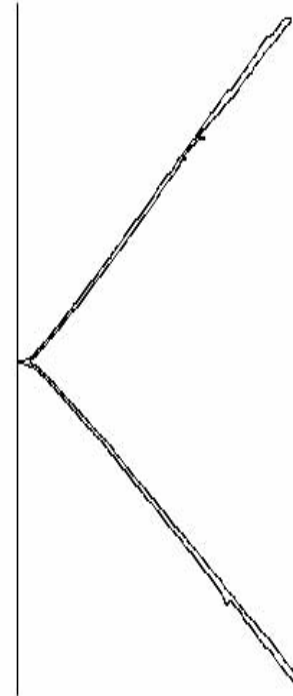
# Missing lengthparameter => Selfsimilar growth



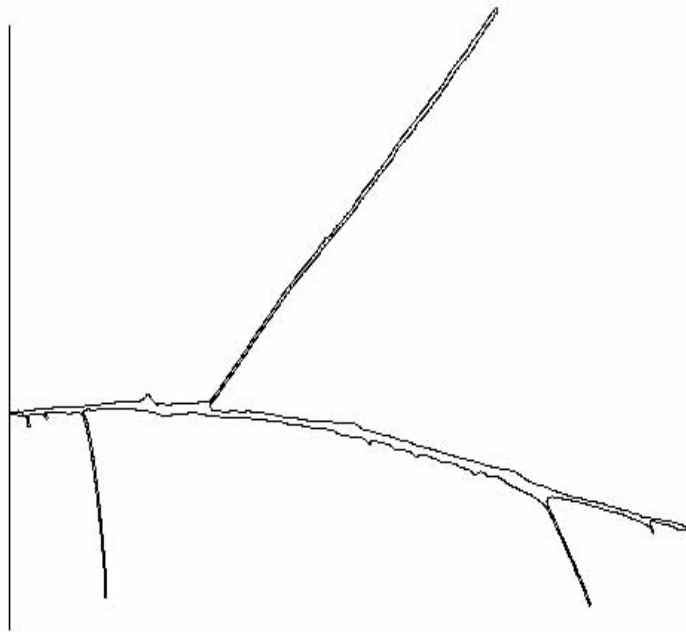
$$\varepsilon_f/\varepsilon_\infty = 0.3, \sigma_x/\sigma_y = 0$$



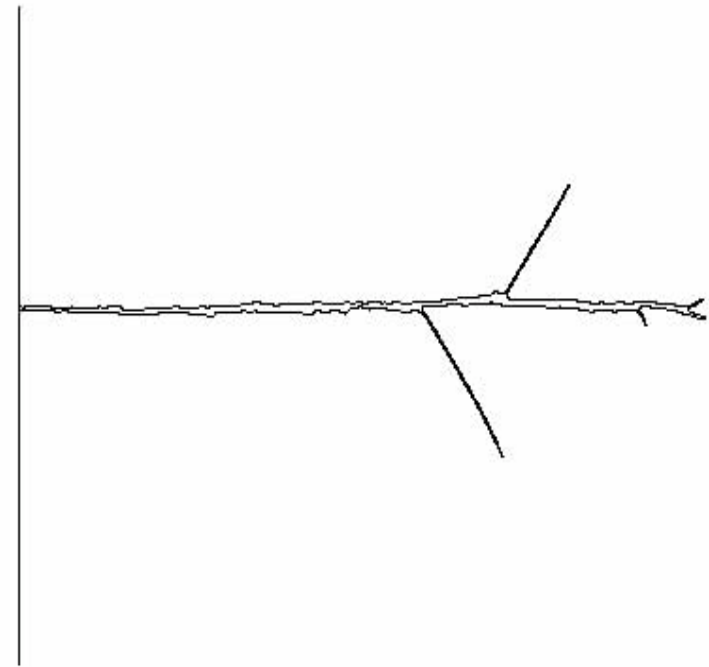
$$\varepsilon_f/\varepsilon_\infty = 0.3, \sigma_x/\sigma_y = 1.1$$



























$$\varepsilon_f/\varepsilon_\infty = 0.6, \sigma_x/\sigma_y = 0.9$$



$$\varepsilon_f/\varepsilon_\infty = 1.5, \sigma_x/\sigma_y = 0.7$$



	$\varepsilon_f/\varepsilon_\infty = 0$	0.3	0.5	0.75	1	1.2
$\sigma_x/\sigma_y = 0$						
0.5						
0.7						
0.9						



Contributions to the free energy (Ginzburg & Landau, 1950)

$$\mathcal{F} = \mathcal{F}_{el} + \mathcal{F}_{ch} + \mathcal{F}_{gr}$$

Volume totals:

Elastic energy  $\mathcal{F}_{el} = \int \frac{G(\psi)}{2} (\nabla w)^2 dV$

Chemical energy  $\mathcal{F}_{ch} = \int U(\psi) dV$

Gradient energy  $\mathcal{F}_{gr} = \int \frac{g_b}{2} (\nabla \psi)^2 dV$

Antiplane deformation  $\Rightarrow$  Two free variables

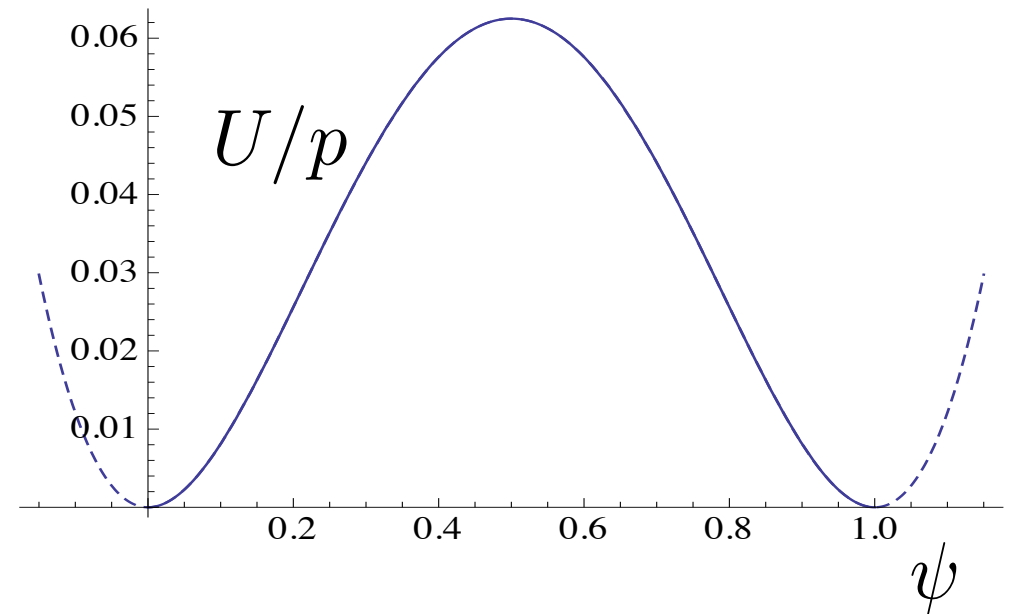
Displacements  $w$  and phase (density)  $\psi$

$$\frac{\partial \psi}{\partial t} = -L_{\psi} \frac{\delta \mathcal{F}}{\delta \psi} , \quad \frac{\partial w}{\partial t} = -L_w \frac{\delta \mathcal{F}}{\delta w}$$

Ginzburg, Landau (50)

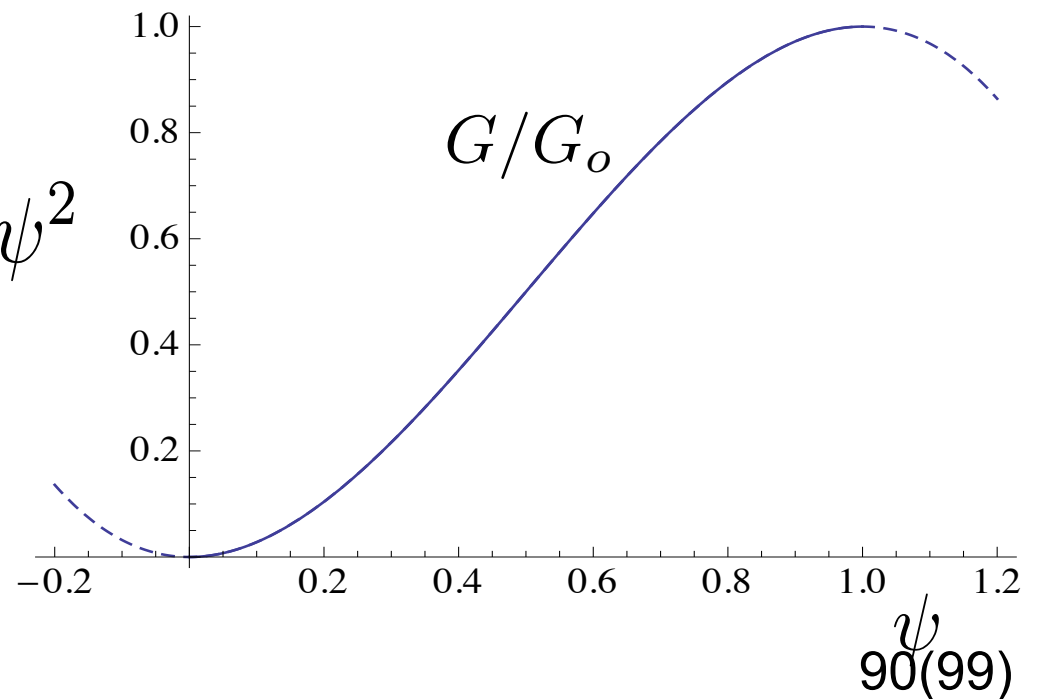
Double-well  
chemical potential

$$U(\psi) = p \psi^2 (1 - \psi)^2$$



Shear modulus

$$G(\psi) = G_o(\psi)(-2\psi + 3)\psi^2$$



## Evolution of the phase

$$\frac{\partial \psi}{\partial t} = -L_\psi \left[ \frac{1}{2} G'(\psi) (\nabla w)^2 + p\psi(\psi^2 - 1) - g_b \Delta \psi \right]$$

## Evolution of the displacements

$$\frac{\partial w}{\partial t} = L_w \nabla \cdot [G(\psi) \nabla w]$$

At equilibrium:  $\nabla \cdot [G(\psi) \nabla w] = 0$

One dimension (Ginzburg, Landau)

$$g_b \psi_o'' - p \psi_o (\psi_o^2 - 1) = 0$$

solution

$$\psi_o = \tanh(x / \sqrt{2g_b})$$

With mechanical loading

$$g_b \psi'' - p \psi (\psi^2 - 1) + \kappa p (\psi^2 - 1) - \frac{c}{L_\psi} \psi' = 0$$

Seek perturbation solution:

$$\psi(x) = \psi_o(x) + \omega f(x) \quad \text{as} \quad \omega = c / L_\psi G_o (\nabla w)^2 \sqrt{p g_b} \rightarrow 0$$

The result

$$\psi'_o = \psi_o^2 - 1 = \frac{1}{\sqrt{2g_b}} \operatorname{sech}^2(x / \sqrt{2g_b}) \quad ,$$

gives

$$\left(\kappa - \frac{c}{L_\psi}\right) \frac{p}{\sqrt{2g_b}} \operatorname{sech}^2(x / \sqrt{2g_b}) = 0 \quad ,$$

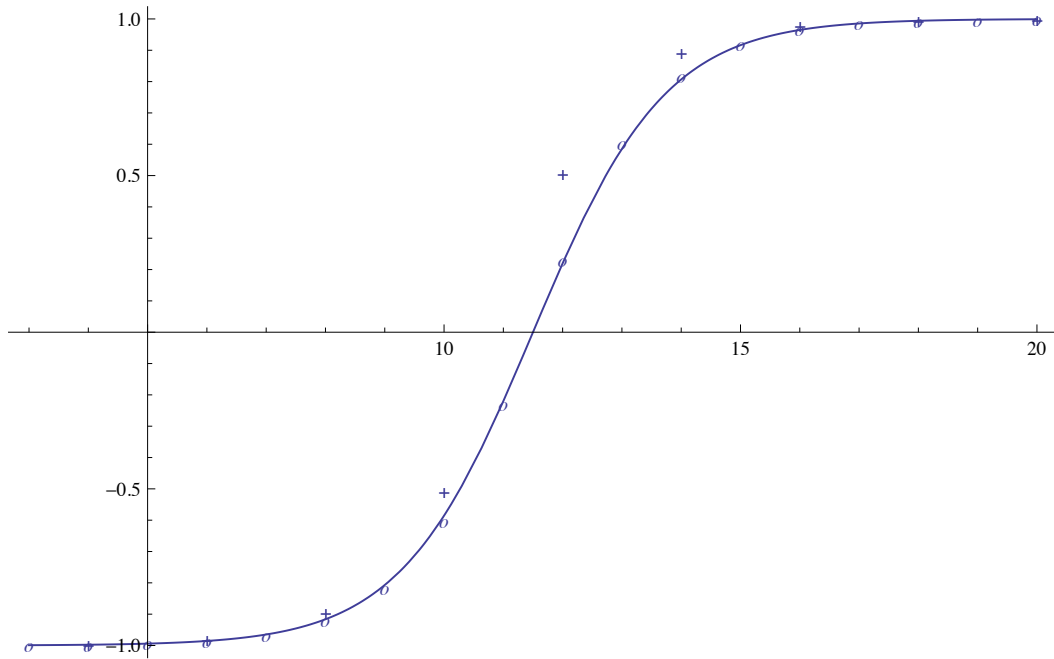
i.e. the speed of the corroding edge

$$c = L_\psi \kappa = \frac{3}{4} p L_\psi G_o (\nabla w)^2 \sqrt{\frac{2g_b}{p}} \quad .$$

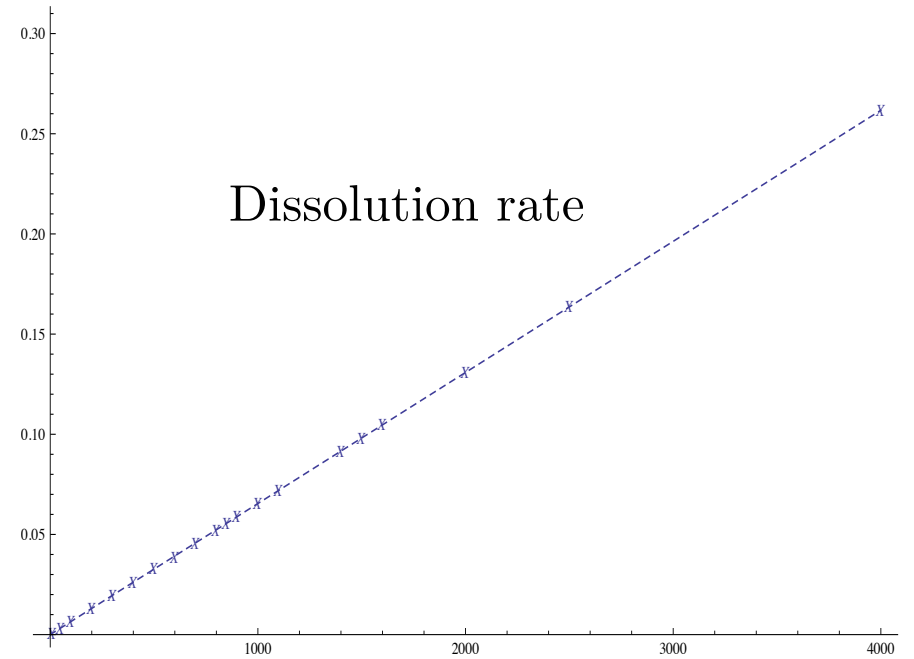
Steady state solution

$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2 \sqrt{\frac{2g_b}{p}}t\right)$$

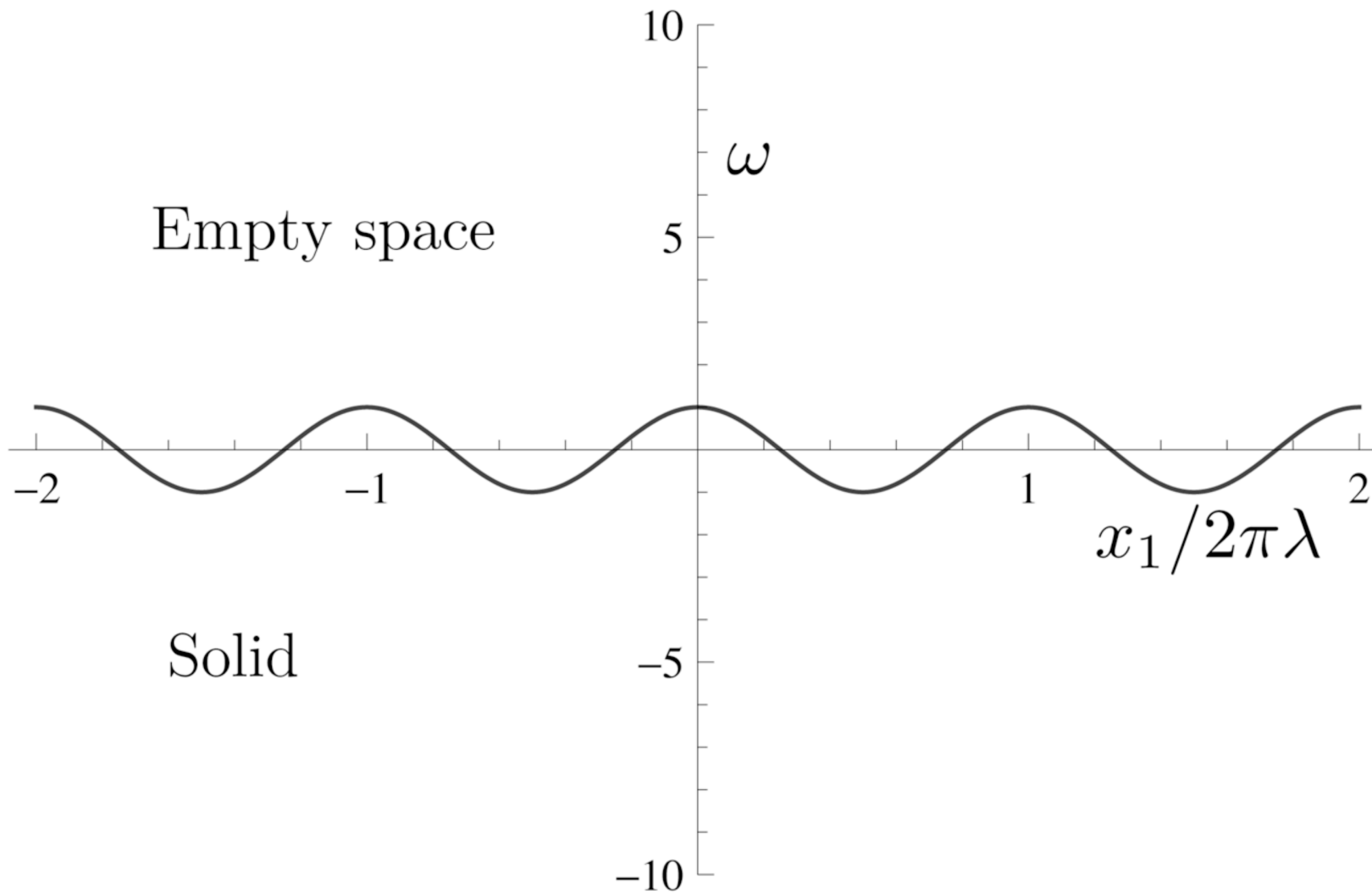
Transition Rate vs. Distance



Dissolution Rate vs. Tensile Stress



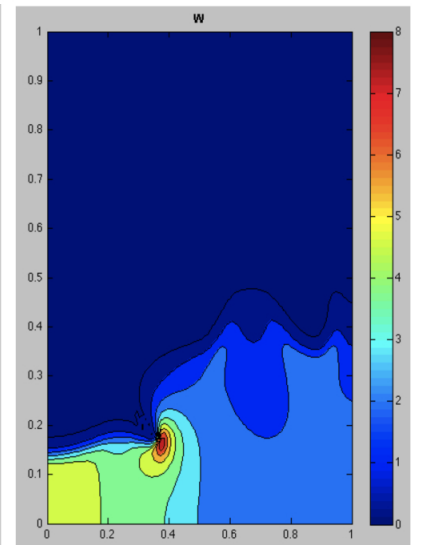
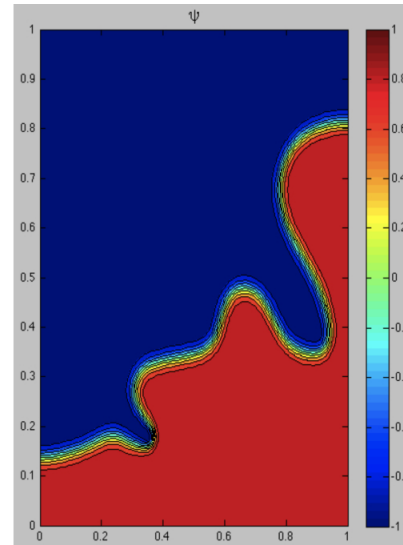
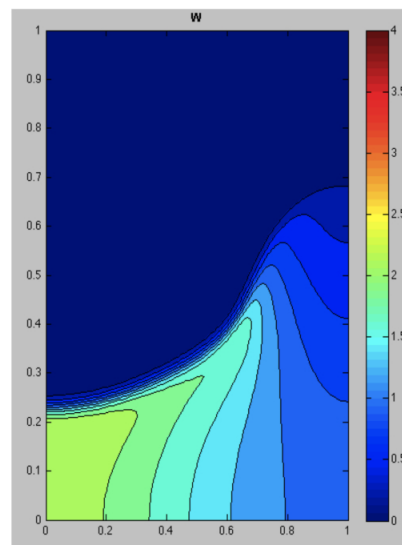
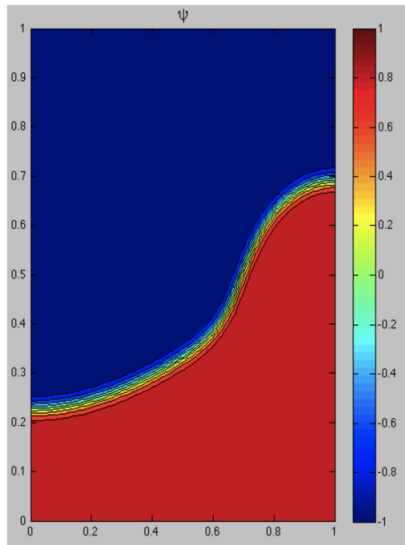
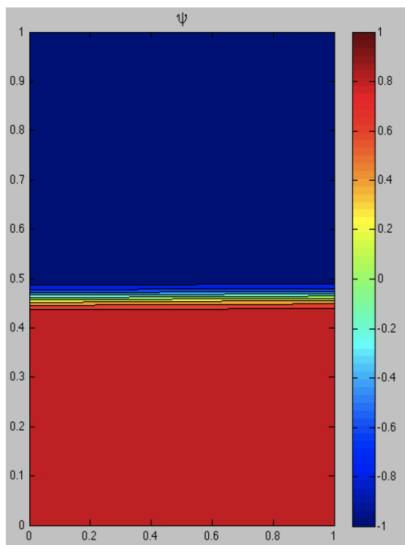
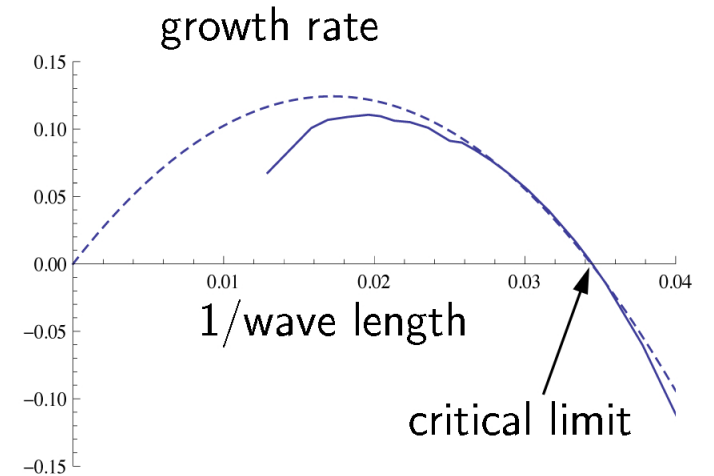
Tensile stress





$$\left(\frac{d}{dx_2} - 2\beta\right)(f' + f^2 - 1) = 0$$

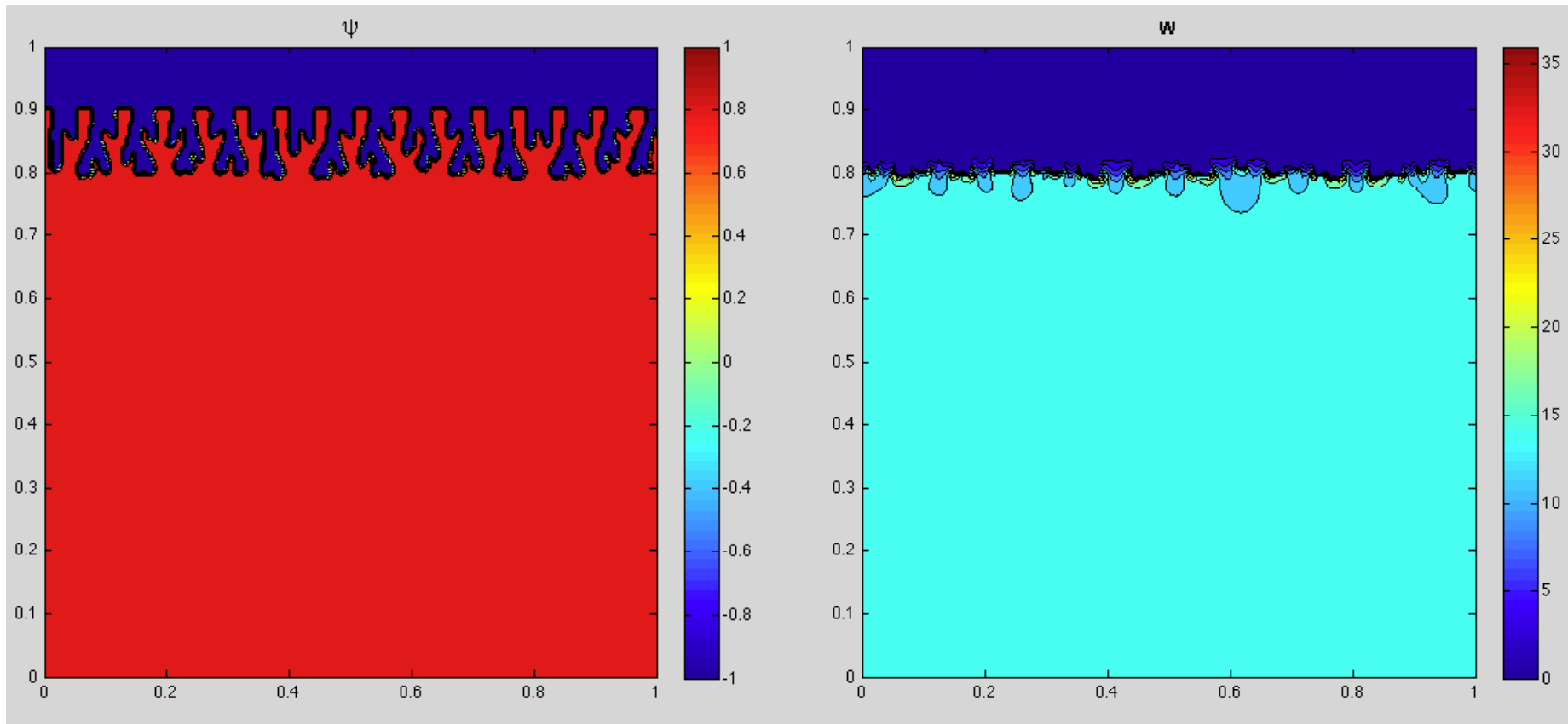
$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$



Red is remaining material

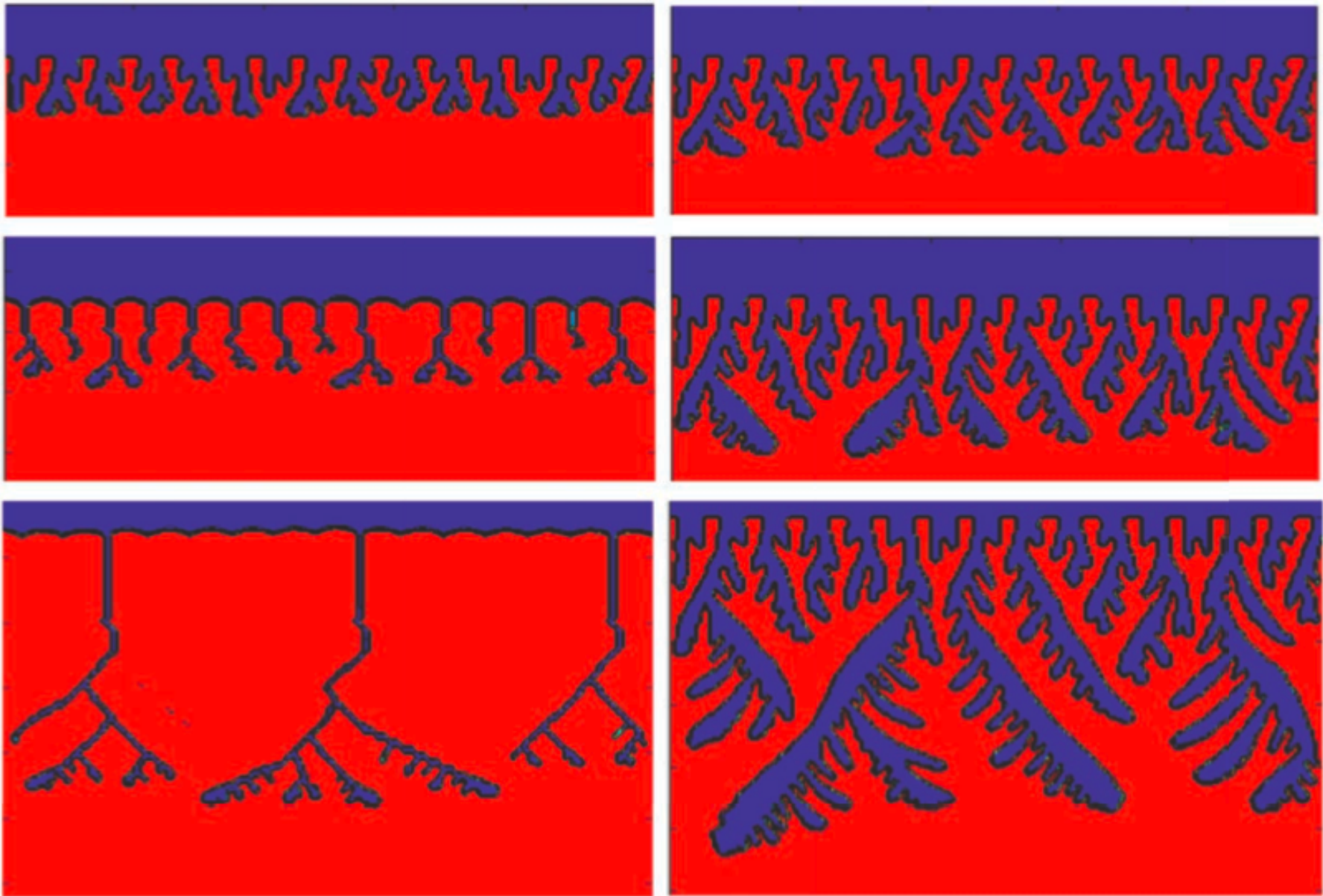
Effective Stress

Effective Stress



Red is remaining material

Effective stress



Without general corrosion

with general corrosion

# Summary

Stress corrosion can be modelled as a moving boundary problem

Surface instability, formation cracks and crack growth are captured

Branching occurs as the blunted crack front become unstable

Phase field modelling simplifies the analysis

Time dependent solution to the Ginzburg-Landau equation found