Fracture as a Moving Boundary Problem

STAMM XVIII, Haifa, Israel, 2012. Orationem Meam.

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LUND UNIVERSITY

STAMM XVIII, Haifa 2012

Fracture as a Moving Boundary Problem

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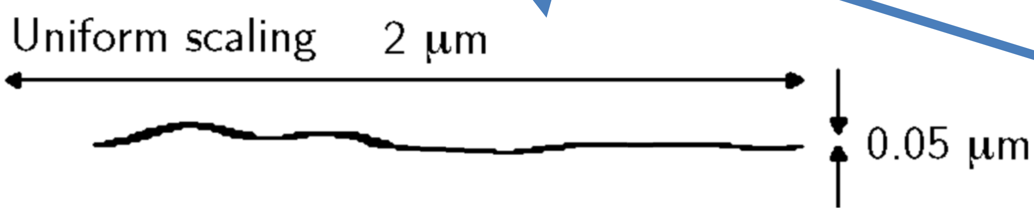
Corroding environment leads to:

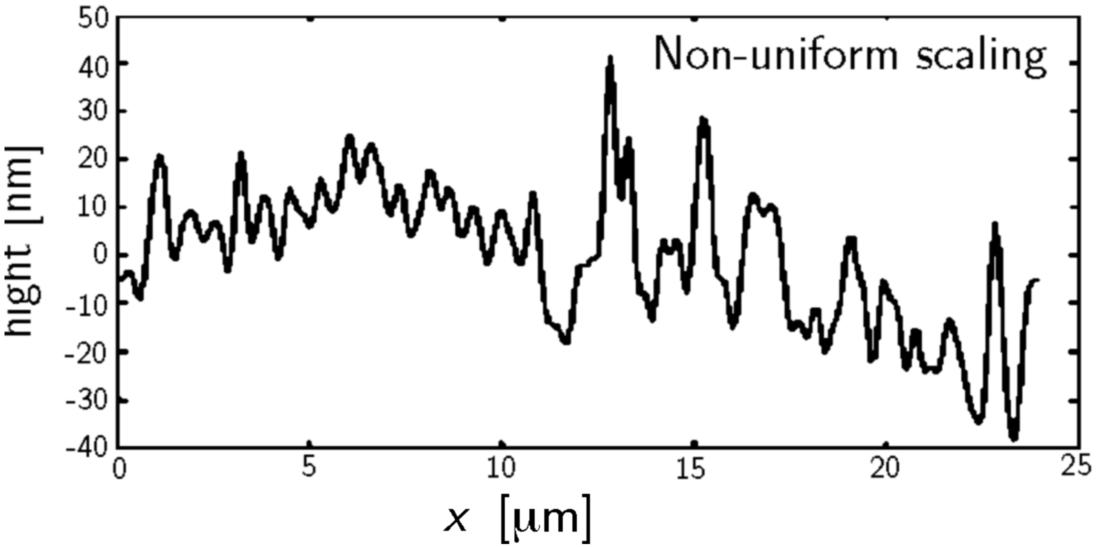
1. Continuous loss of mass
2. Pitting

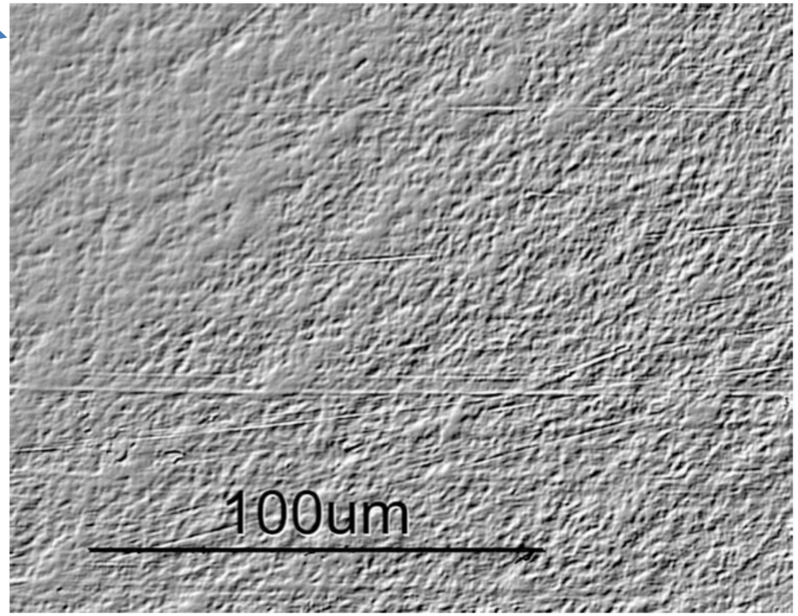
... and with mechanical stress present

3. Surface roughening

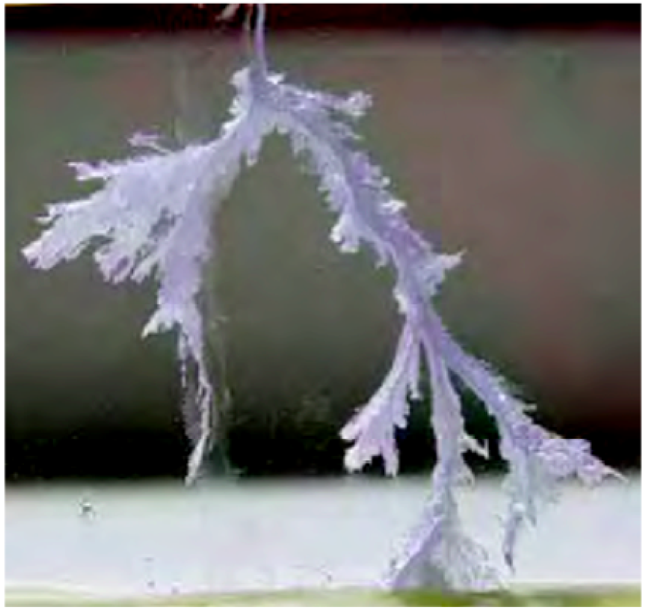
(from Kim *et al*. 2000)



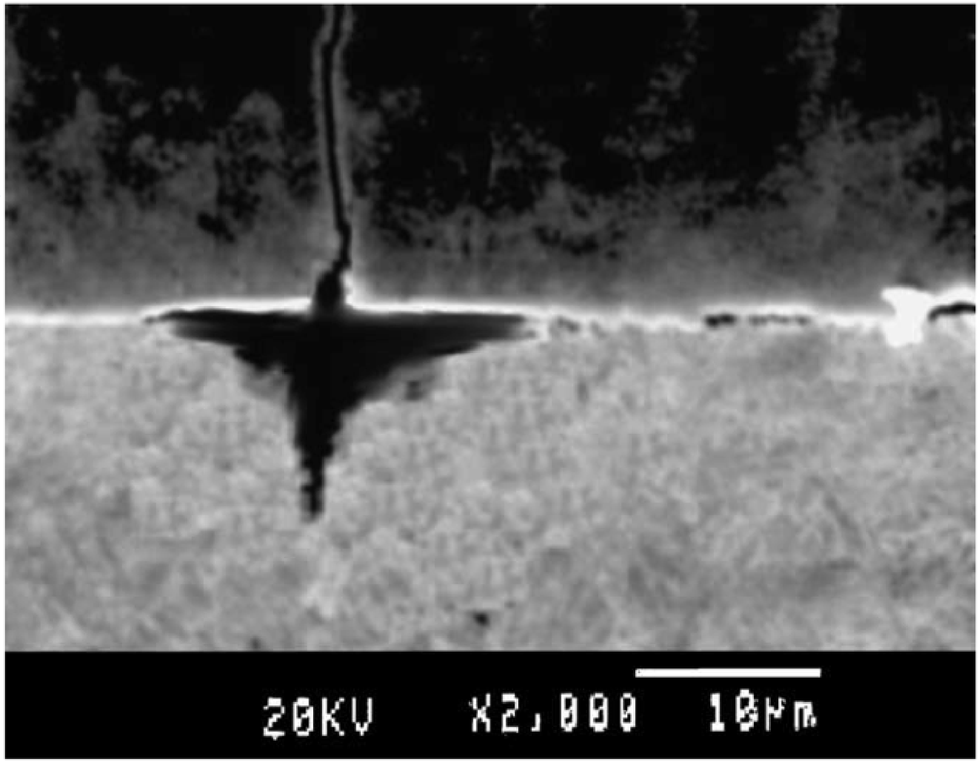




4. Evolving pits
5. Formation of cracks
6. Crack growth
7. Crack branching

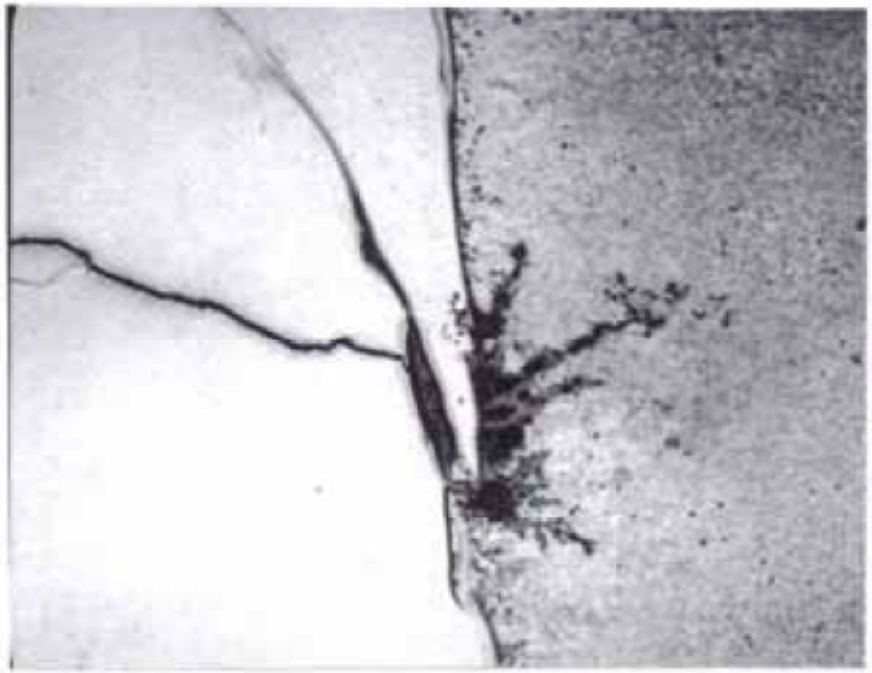


Growing crack in a polycarbonate exposed to acetone (Hejman 2011)

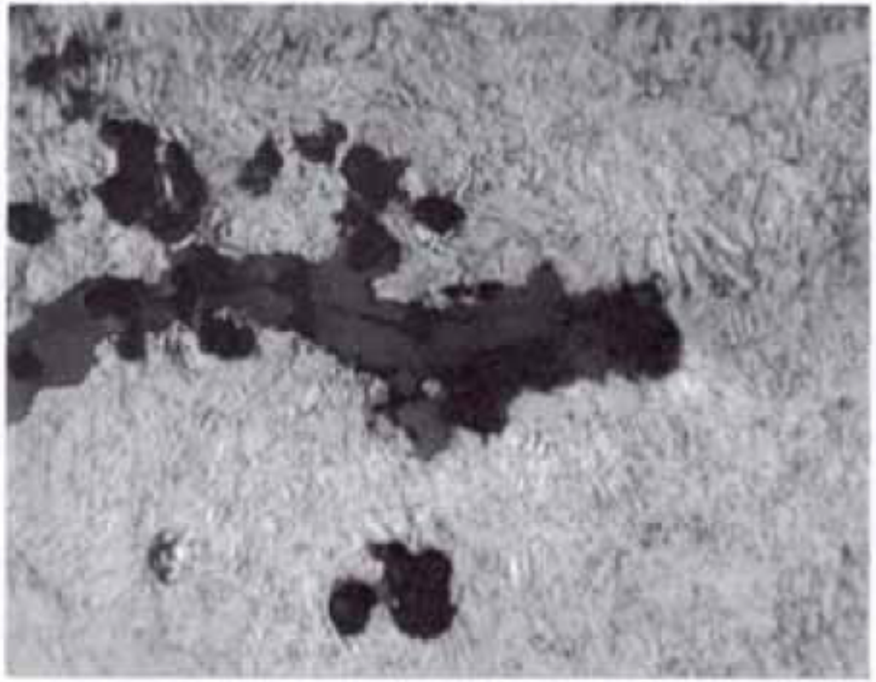


Cr/zone six charge related of land and groove substrate erosion through a micro-crack at the 12:00 bore origin. (Sopok *et al.* 2005)

Corrosion Crack crossing a bi-material interface

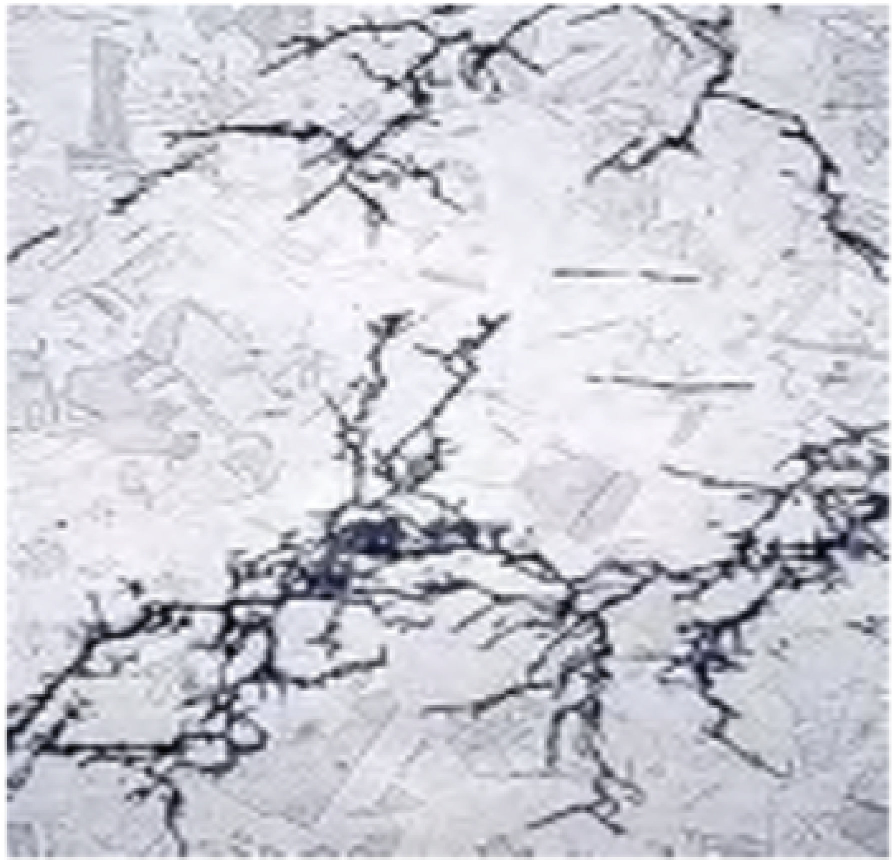


Corrosion crack penetrating a bimaterial interface between austenitic and pressure vessel steel of type SA533C11.



The tip of one of the crack branches. Crack length 7 mm, notch width 10 μm. *Reproduced with permission from Vattenfall AB.*

Biocorrosion



Corrosion in stainless steel in the presence of Gallionella Bacillus

Known to cause SC

Anaerobic Bacteria:
Desulfovibrio, -maculum, -monas

Aerobic Bacteria: *Thiobacillus*

Fungi, Algae, Protozoans

Surface Morphology:

Surface Wave Spectrum

Crack Initiation:

Pit, Cusp and Crack

Crack Growth:

Crack Growth, Blunting and Branching

Evolving Surface Morphology

Asaro-Tiller (1972), Grinfeld (1986, 1993), Srolovitz (1989), Freund (1995), Kim (2000)

Gibb's free energy

$$\Phi = U_c + U_e$$

where

U_c is the free chemical energy and
U_e is the free elastic energy

The free chemical energy

$$U_c = -\gamma \frac{\partial^2 h}{\partial x^2}$$

where

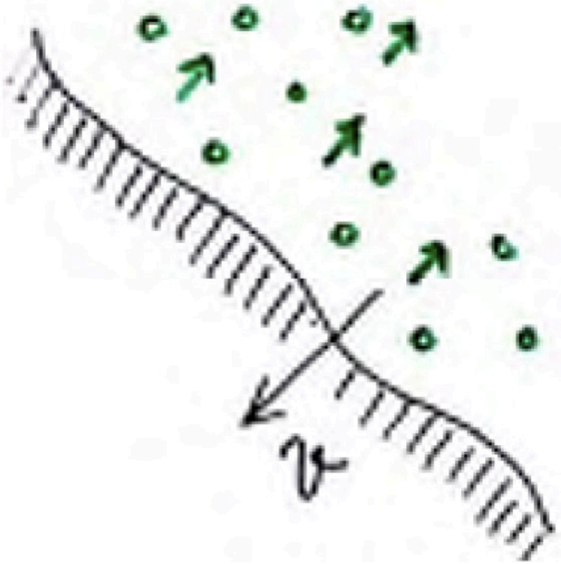
$h(x)$ gives the position of the surface

γ is the surface energy density

The free elastic energy (Cerutti)

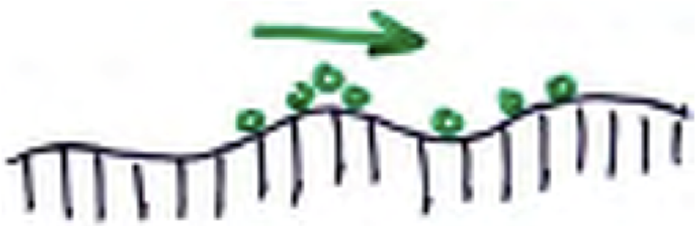
$$U_e = \frac{1}{2}\sigma_{ij}\epsilon_{ij} \sim \frac{1}{2}\mu\frac{\partial h}{\partial x}$$

Evaporation-condensation



$$\frac{\partial h}{\partial t} = -L_1 \Phi$$

Surface diffusion



$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2 \Phi}{\partial x^2}$$

Governing equations:

Evaporation-condensation

$$\frac{\partial h}{\partial t} = L_1 \left(\gamma \frac{\partial^2 h}{\partial x^2} - \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

or surface diffusion

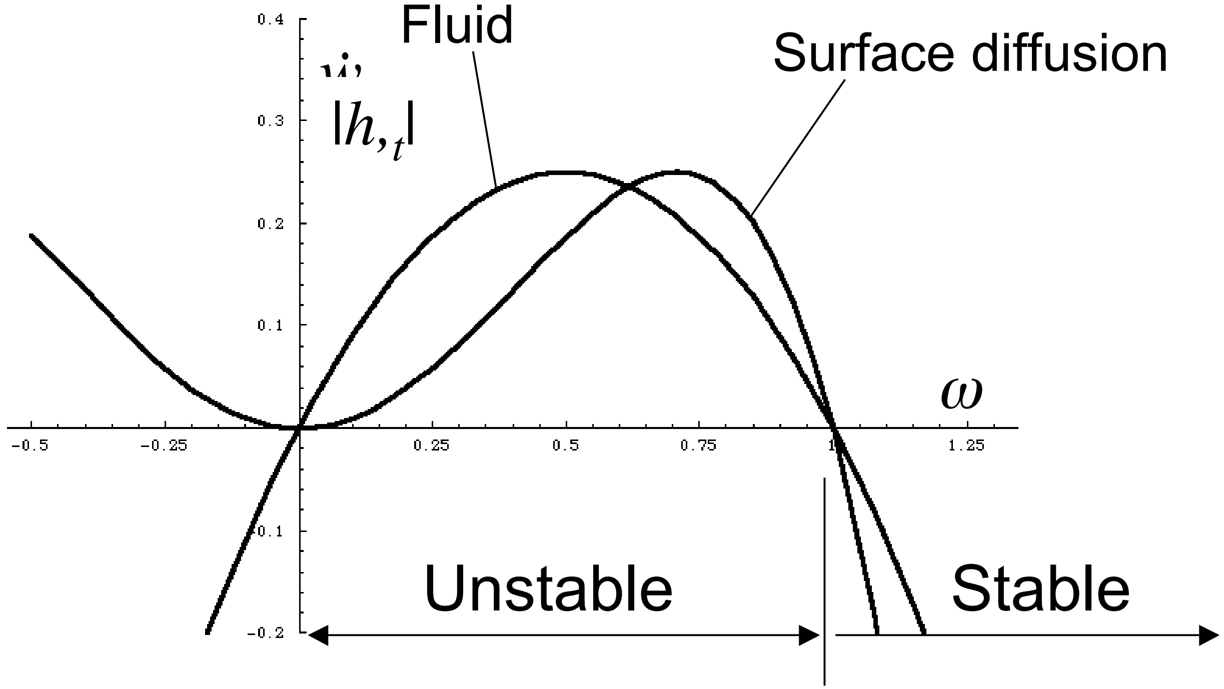
$$\frac{\partial h}{\partial t} = L_2 \frac{\partial^2}{\partial x^2} \left(-\gamma \frac{\partial^2 h}{\partial x^2} + \frac{k}{2} \mu \frac{\partial h}{\partial x} \right)$$

1. Nearly Plane Surface:

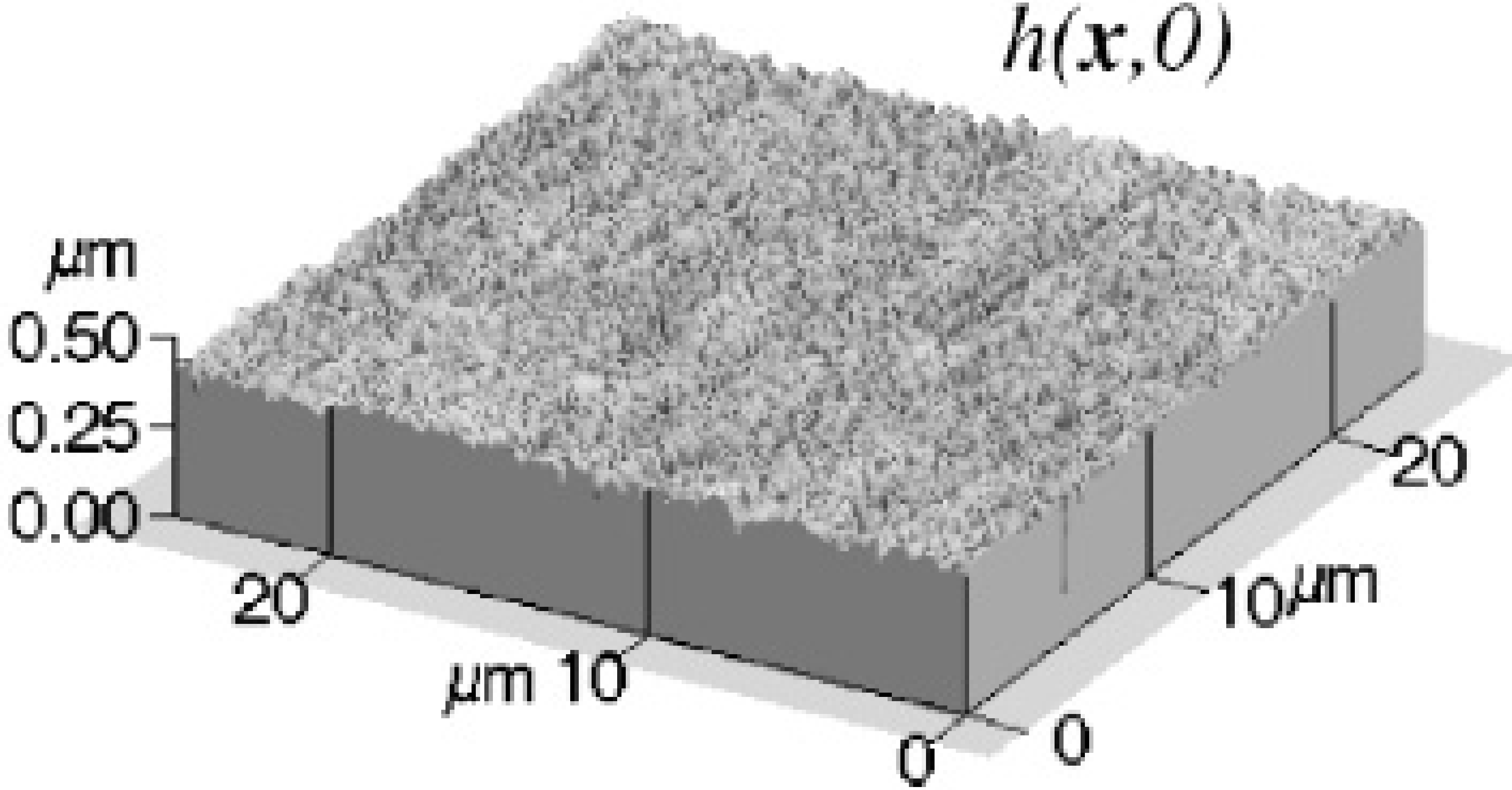
Fouriertransform, put $h = A \sin \omega x \quad |A| << x$

<u>Surface Diffusion</u> <u>Fluid</u>

$h_{,t} = D[(\mu/2)\varepsilon_o \omega^3 - \gamma\omega^4]h \quad h_{,t} = k[(\mu/2)\varepsilon_o \omega - \gamma\omega^2]h$

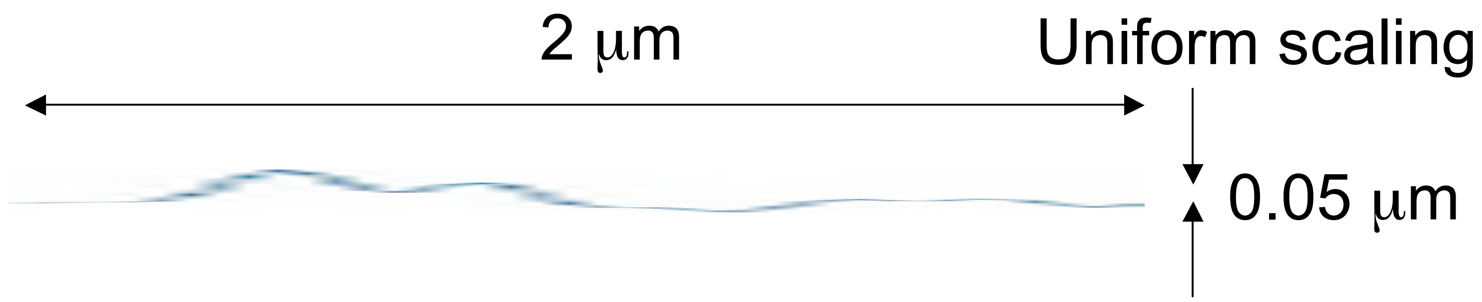


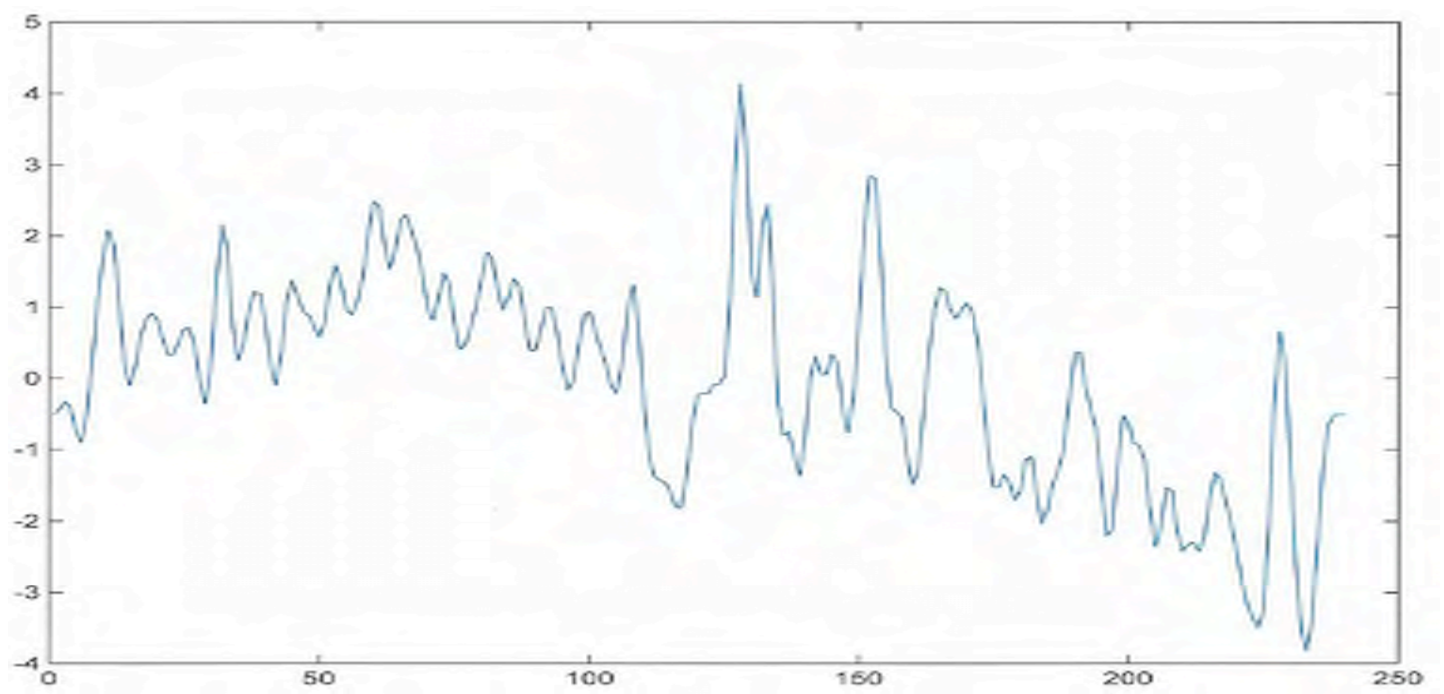
(verified by Kim et al., 2000)



AFM image of shallow etched aluminium (Kim *et al*. 2000)

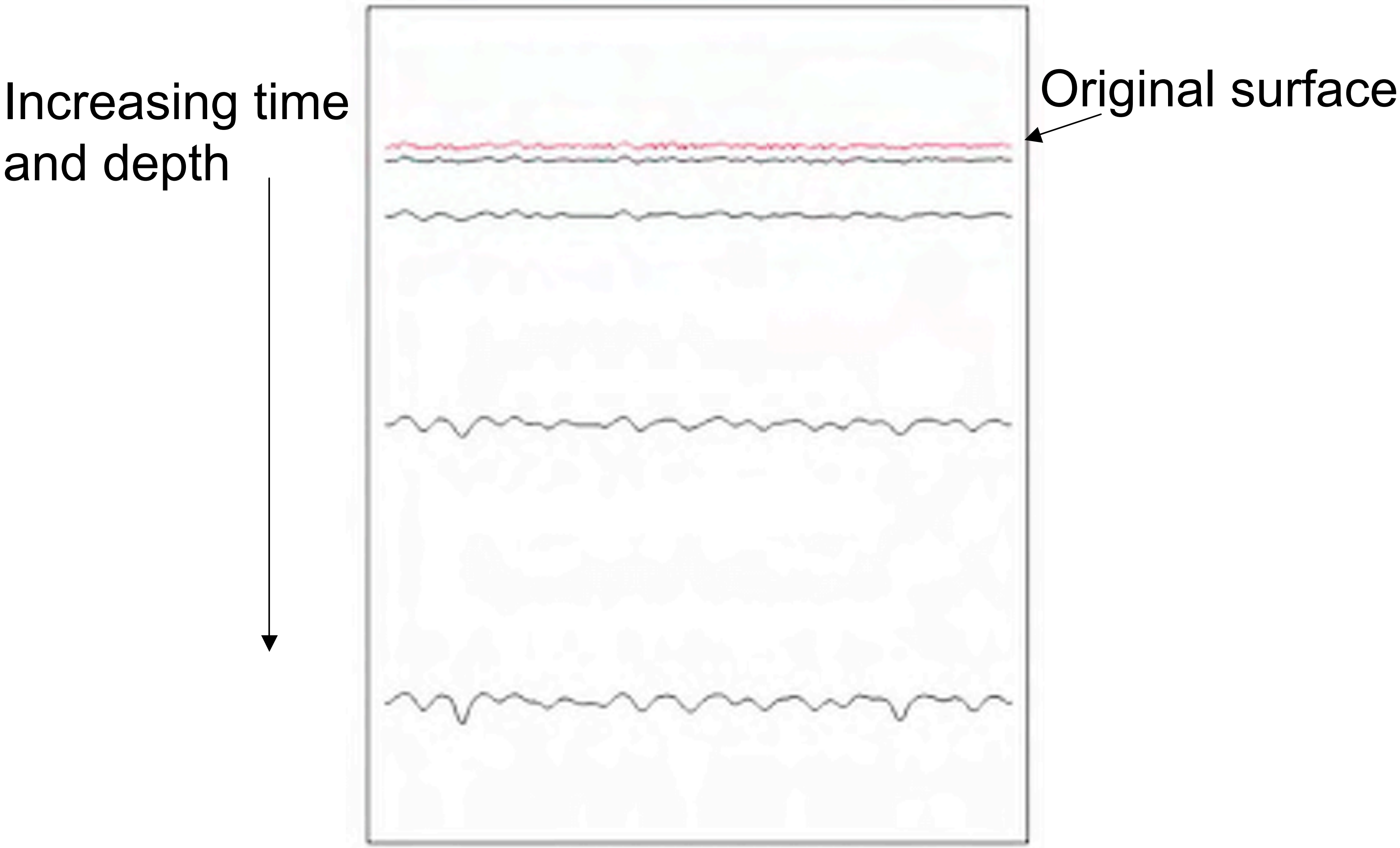
Surface Roughness





(Kim *et al.*, 2000)

FEM calculation of an evolving surface



2. Cusp solutions

Spencer and Meiron(94), Chiu and Gao (95), Yang Xiang and Weinan (02)

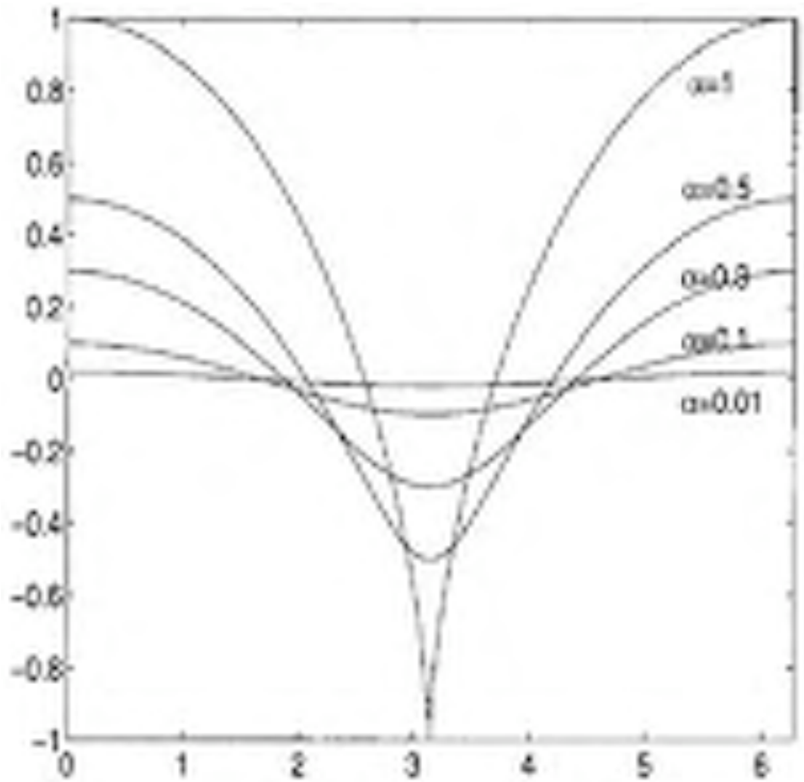
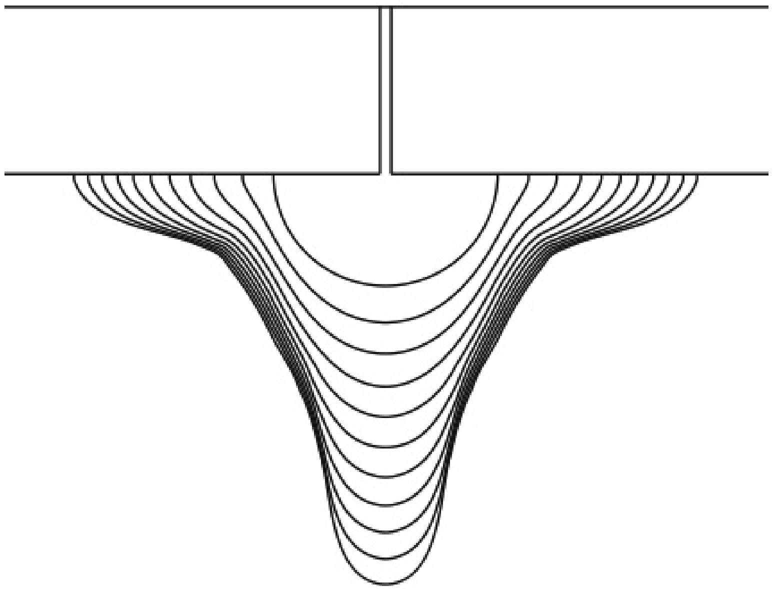
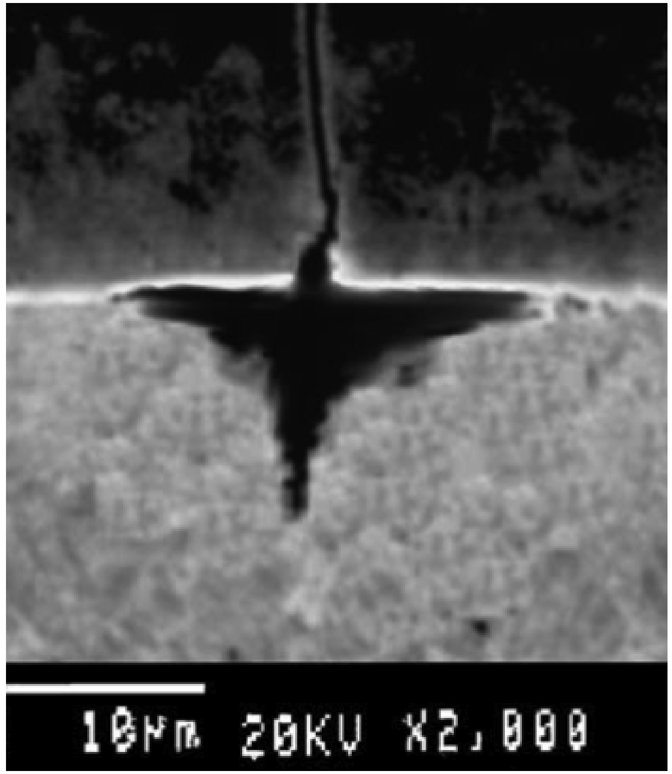


FIG. 1. Cycloid surfaces.

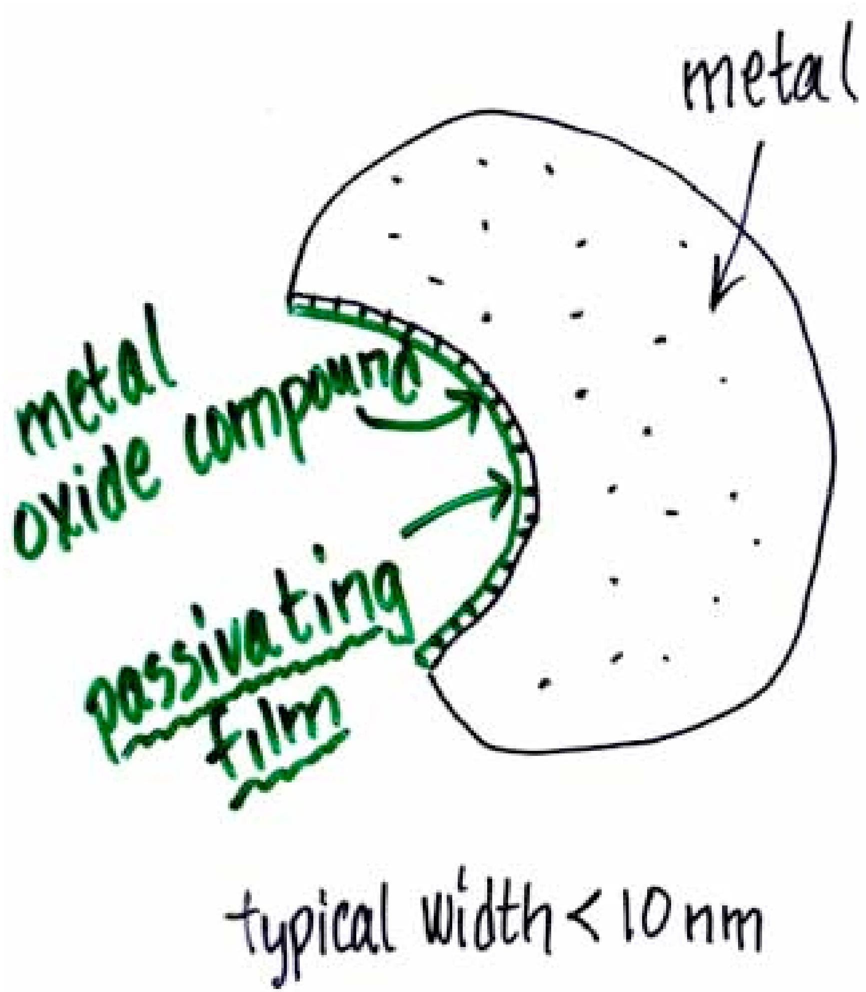




FEM simulation of material dissolution coating (Bjerkén and Ortiz, 2010)

SEM observation of corrosion-erosion crack in a canon bore (Sopok et al. 2005)

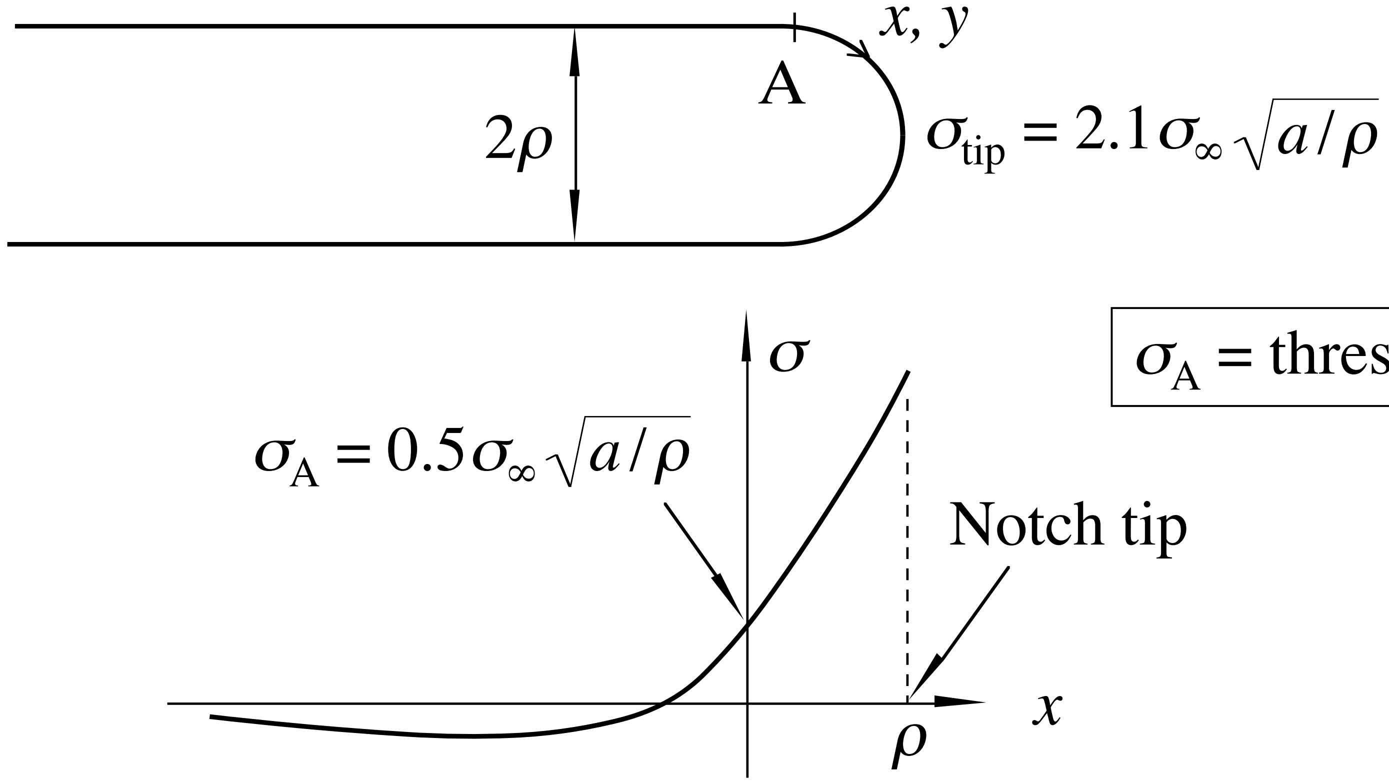
Strain driven material dissolution



Cyclic load damaging the oxide film

$$U = k\,(\sigma - \sigma_f)^n$$

Stress in a half circle notch



σ_A = threshold stress

$K_I = 1.12\sigma_\infty\sqrt{\pi a}$, $\sigma_{th} = \sigma_A$ => $\rho = 0.625(K_I/\sigma_{th})^2$

2. Cusp solutions

Spencer and Meiron(94), Chiu and Gao (95), Yang Xiang and Weinan (02)

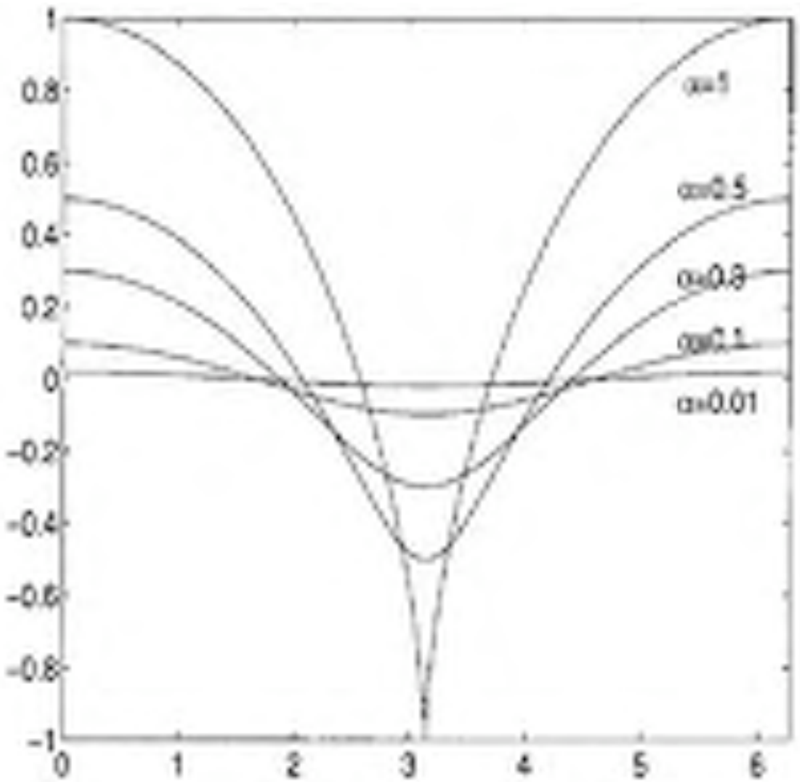
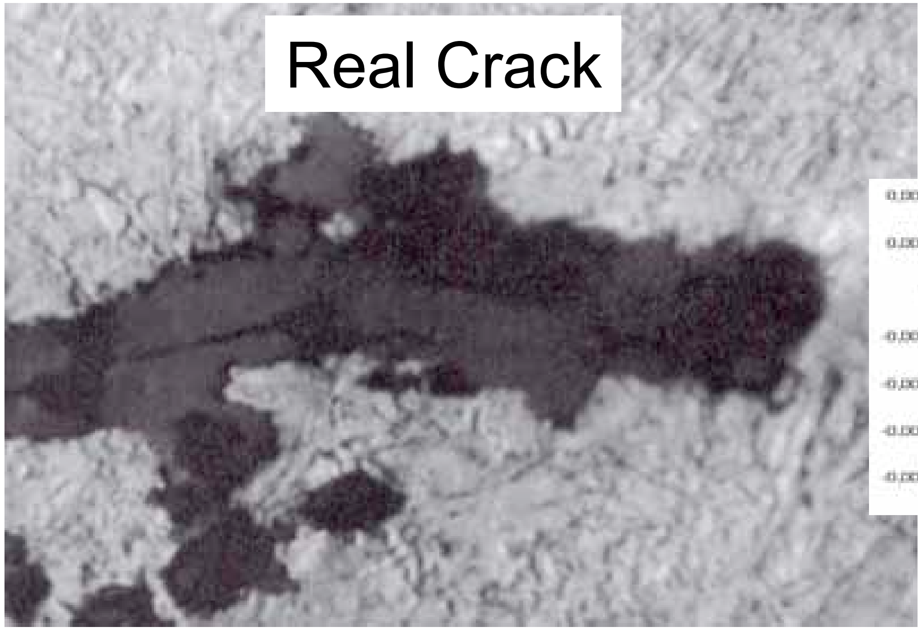
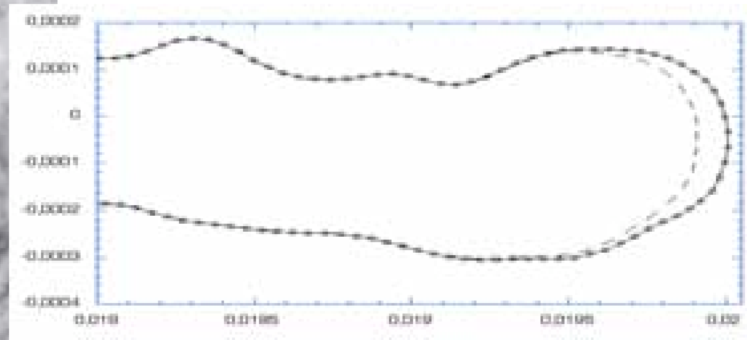


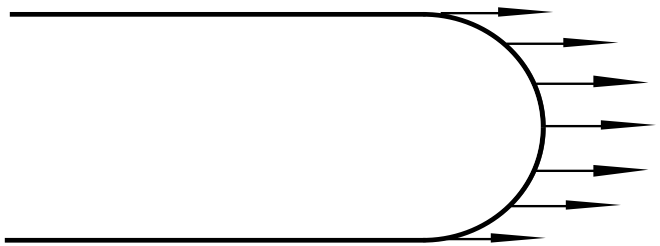
FIG. 1. Cycloid surfaces.

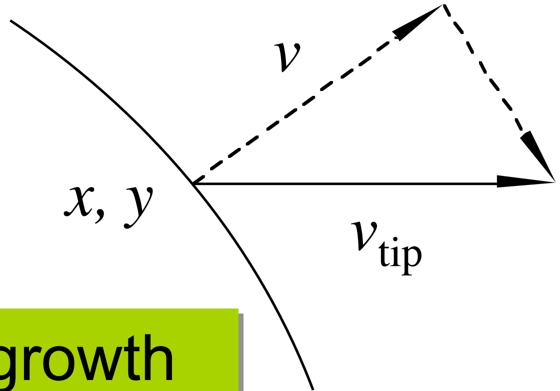
Steady State Crack-tip Shape



FEM result

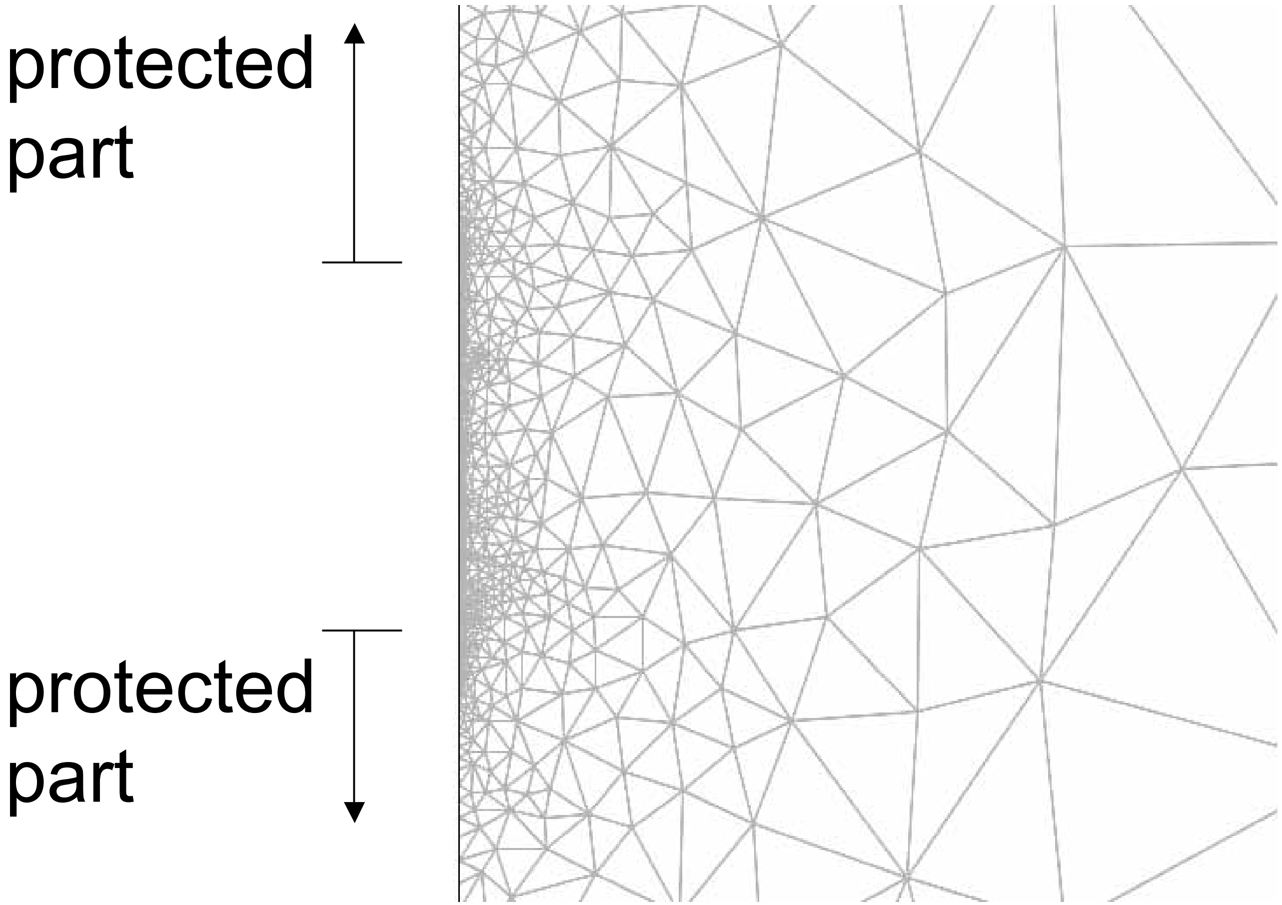




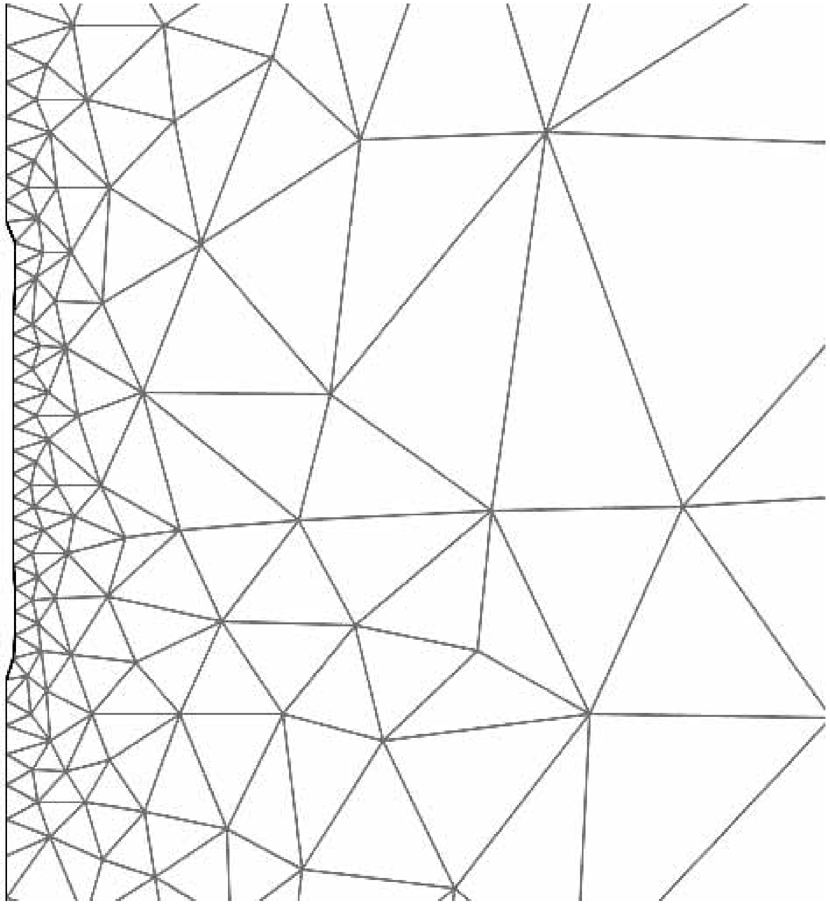


Constant Component in the growth direction: $v = (x/\rho)\ v_{tip}$

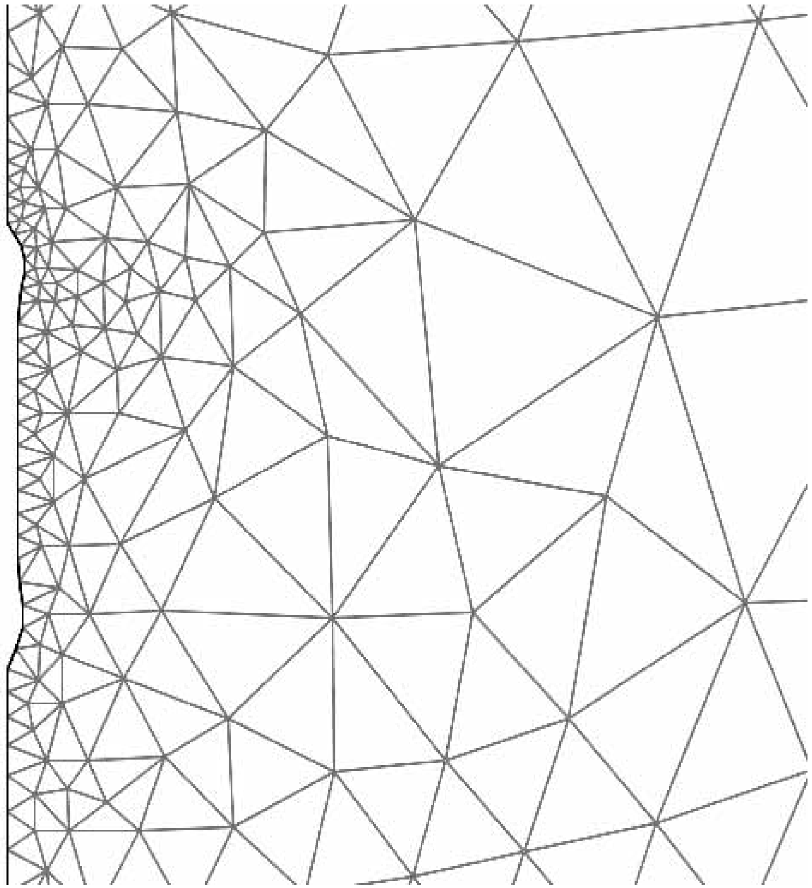
Corroding Surface



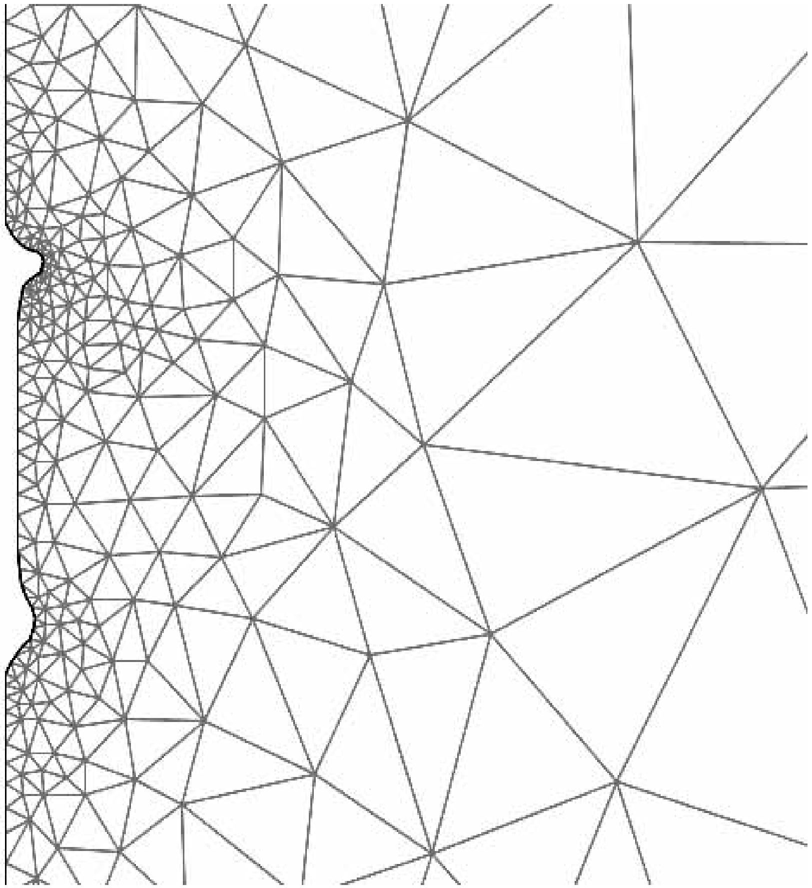
Corroding Surface



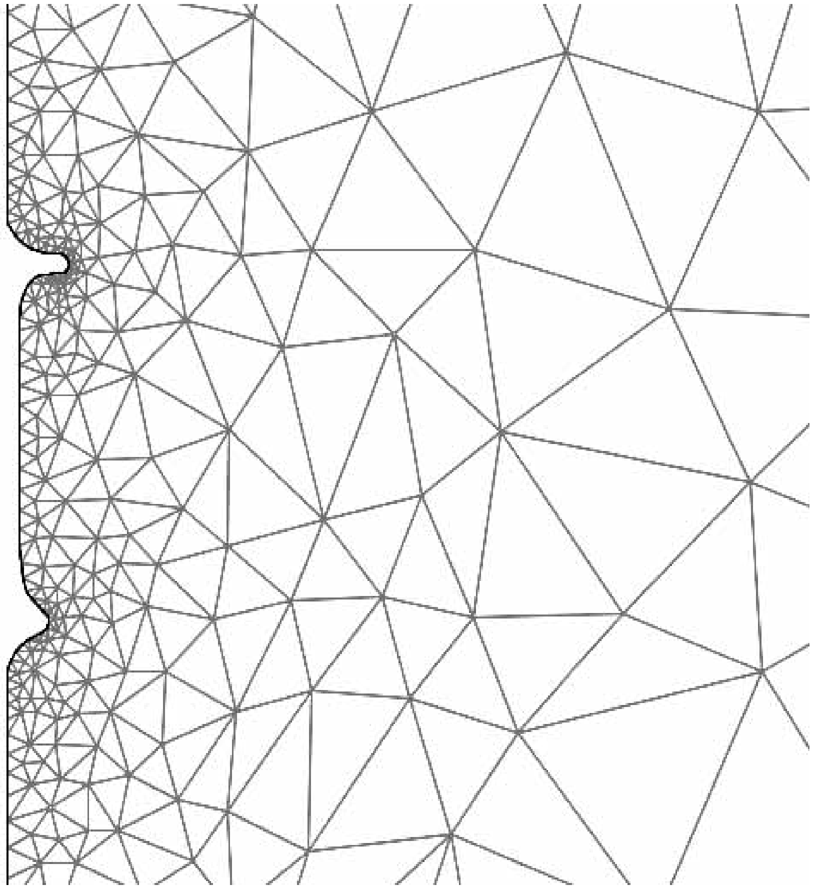
Corroding Surface



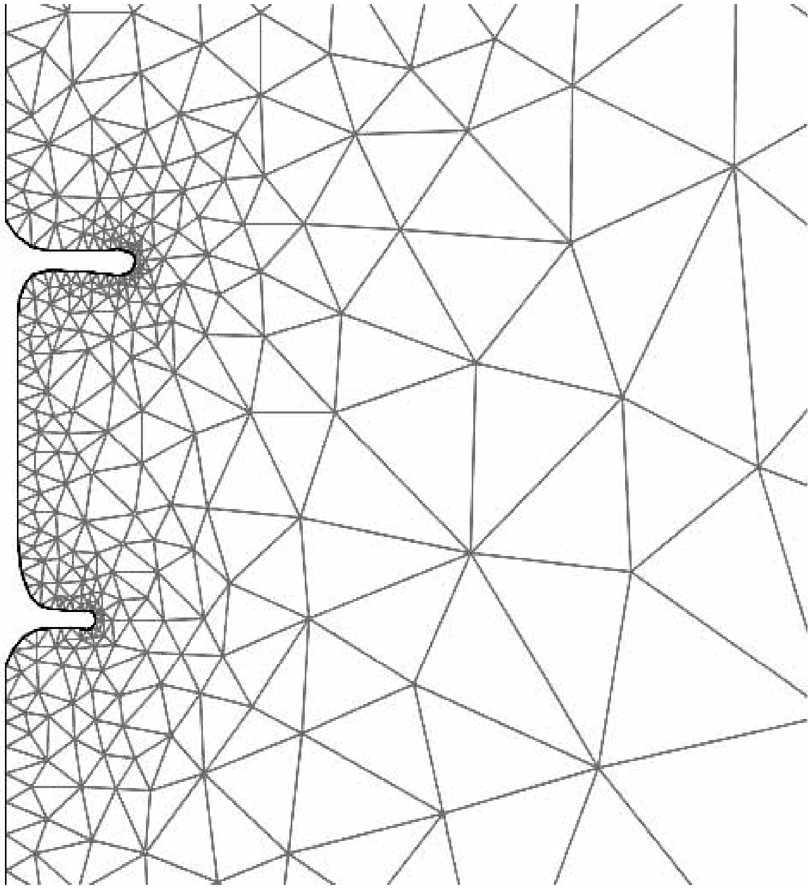
Corroding Surface



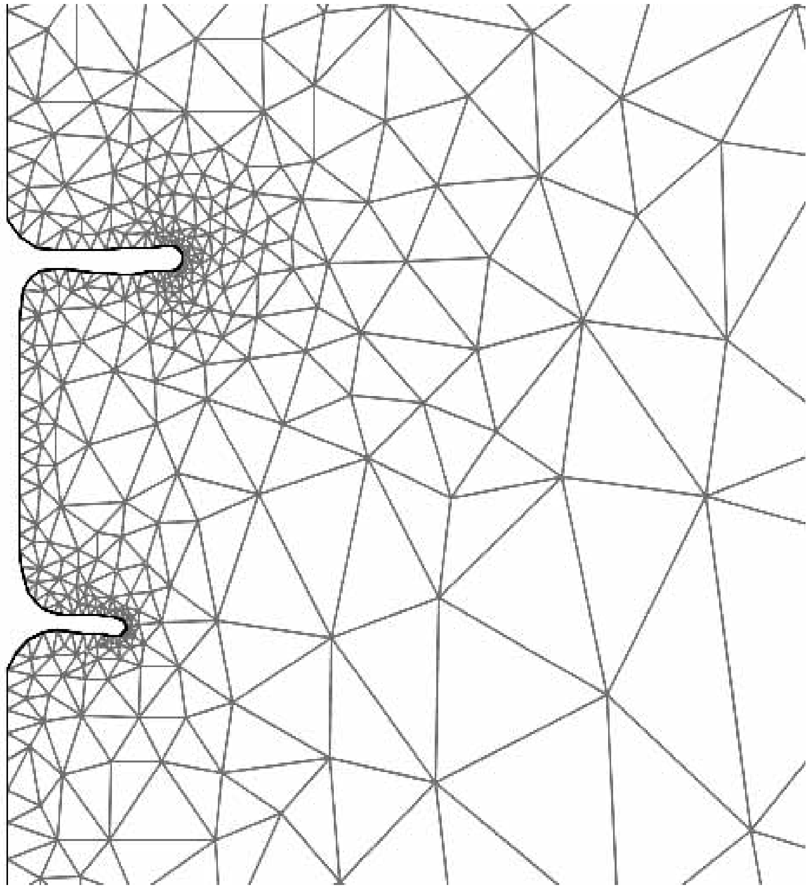
Corroding Surface



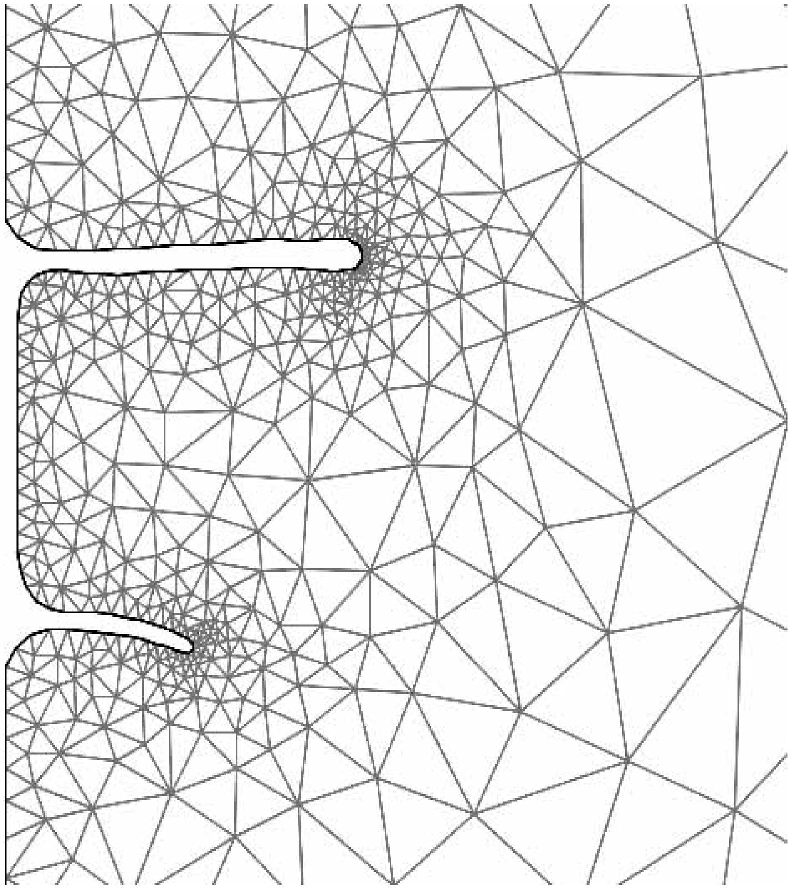
Corroding Surface



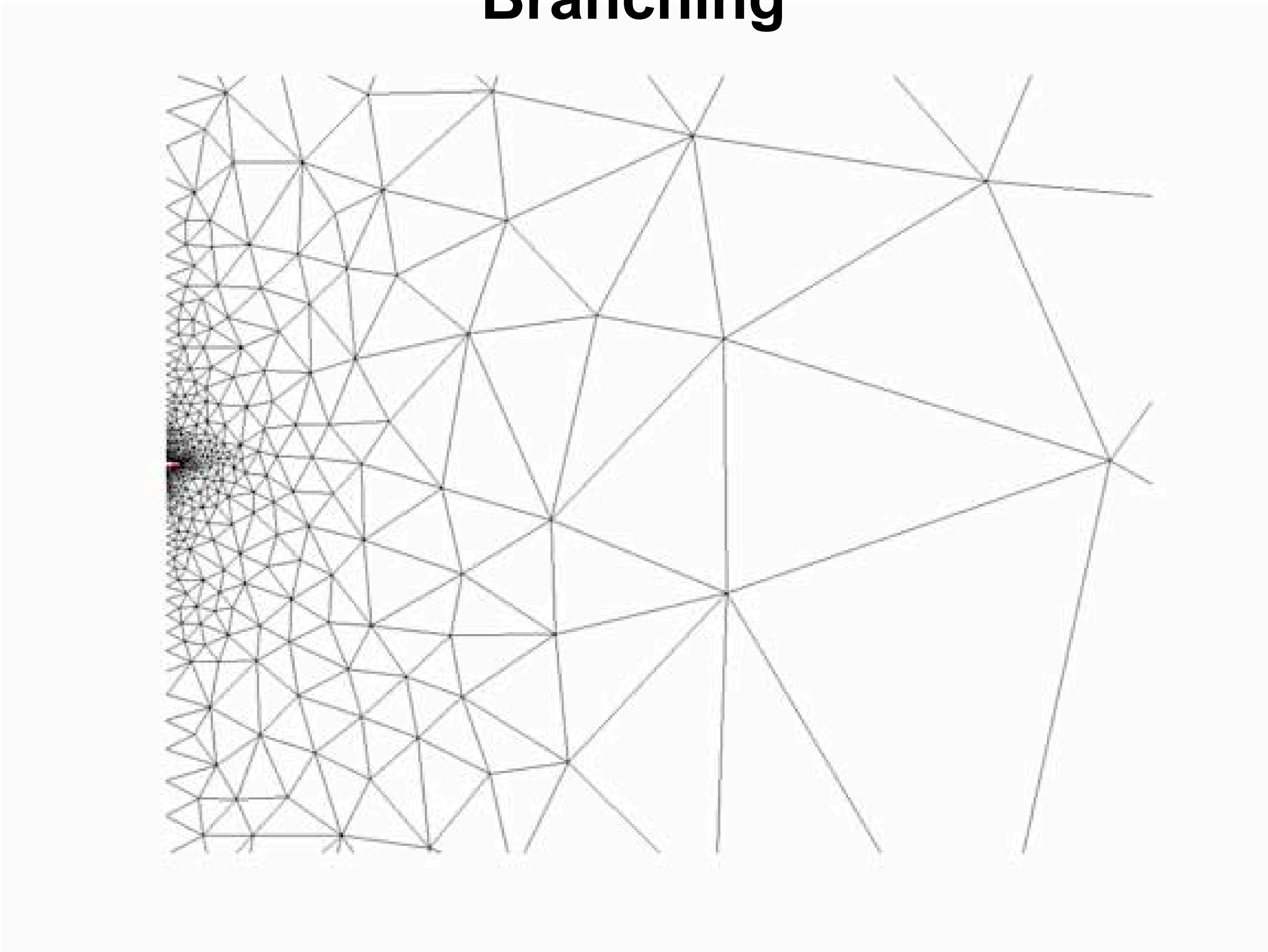
Corroding Surface



Corroding Surface



Branching



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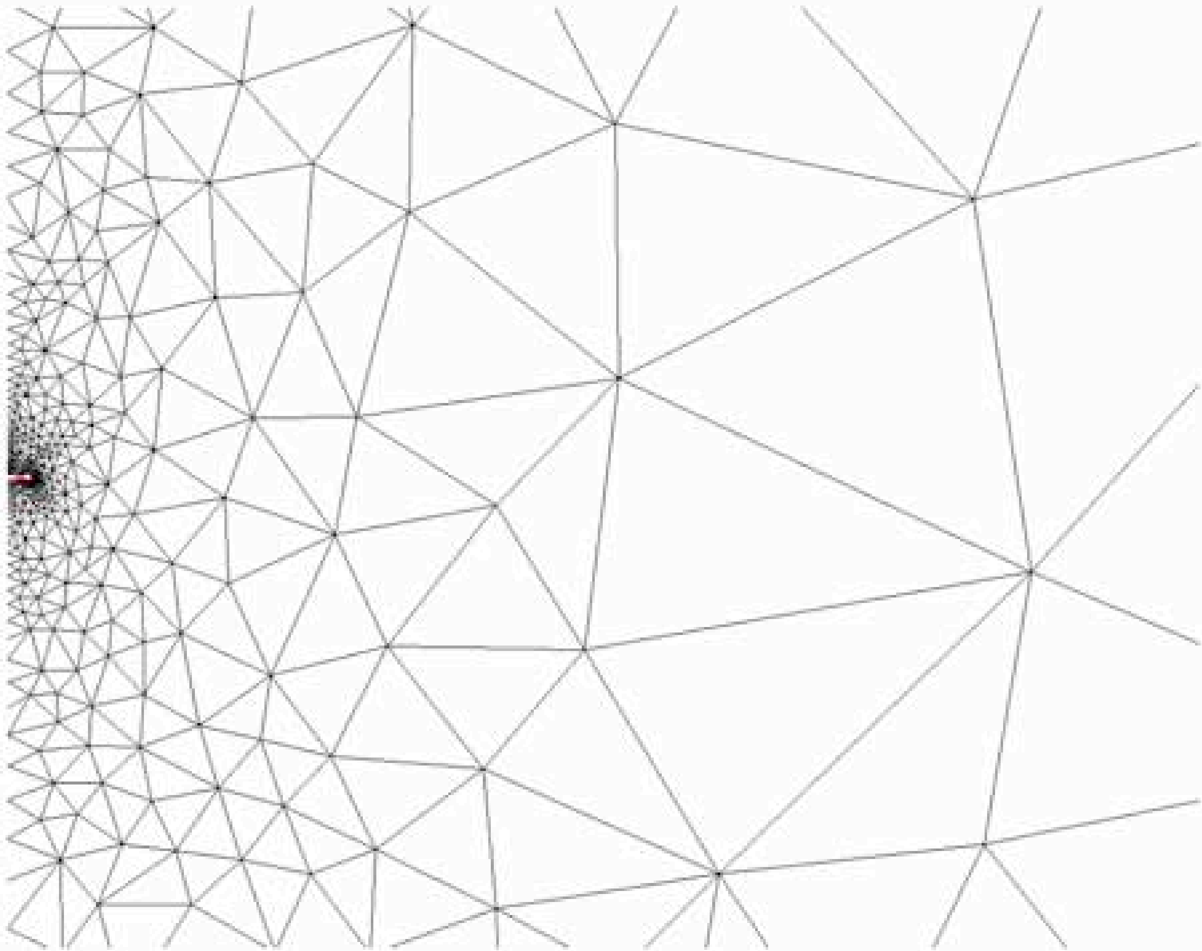

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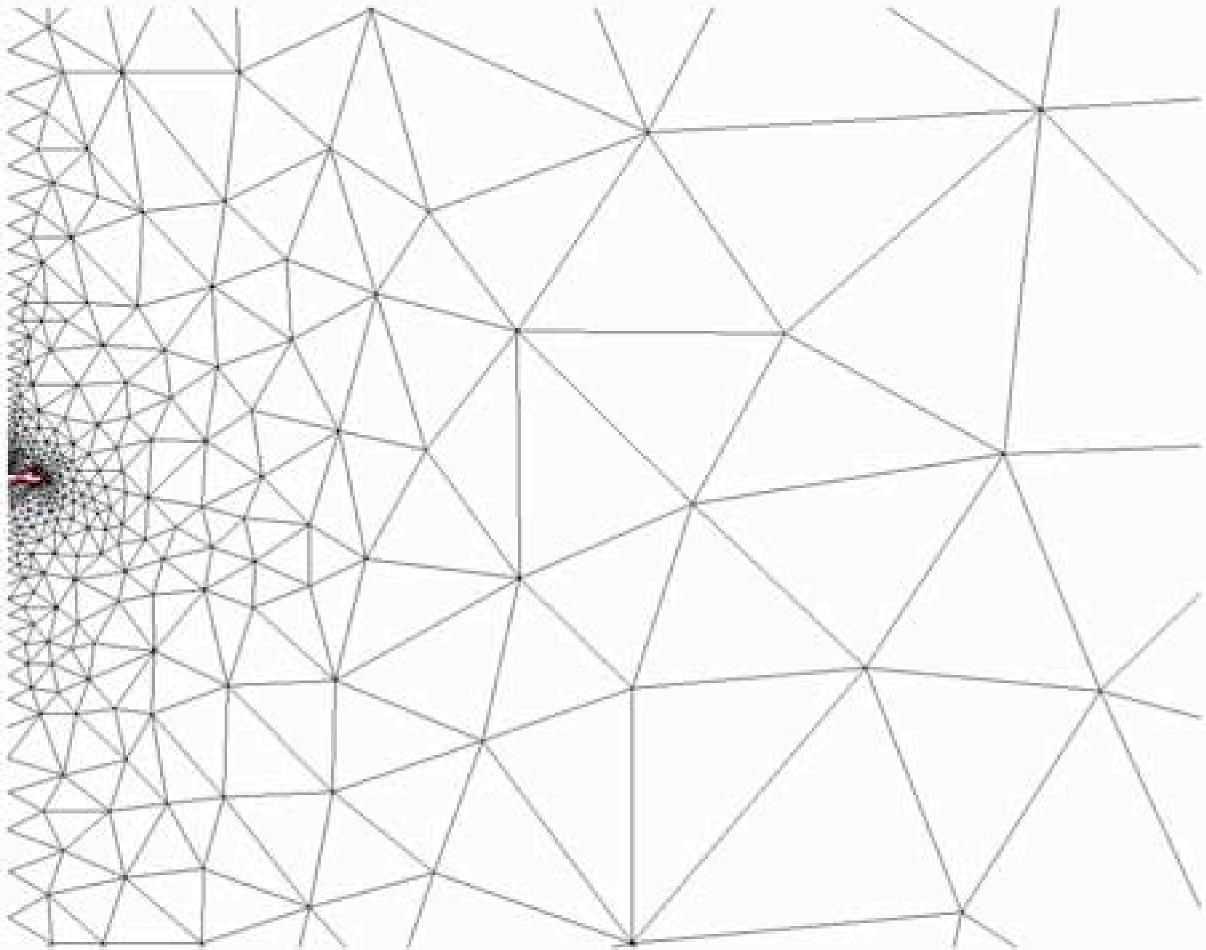

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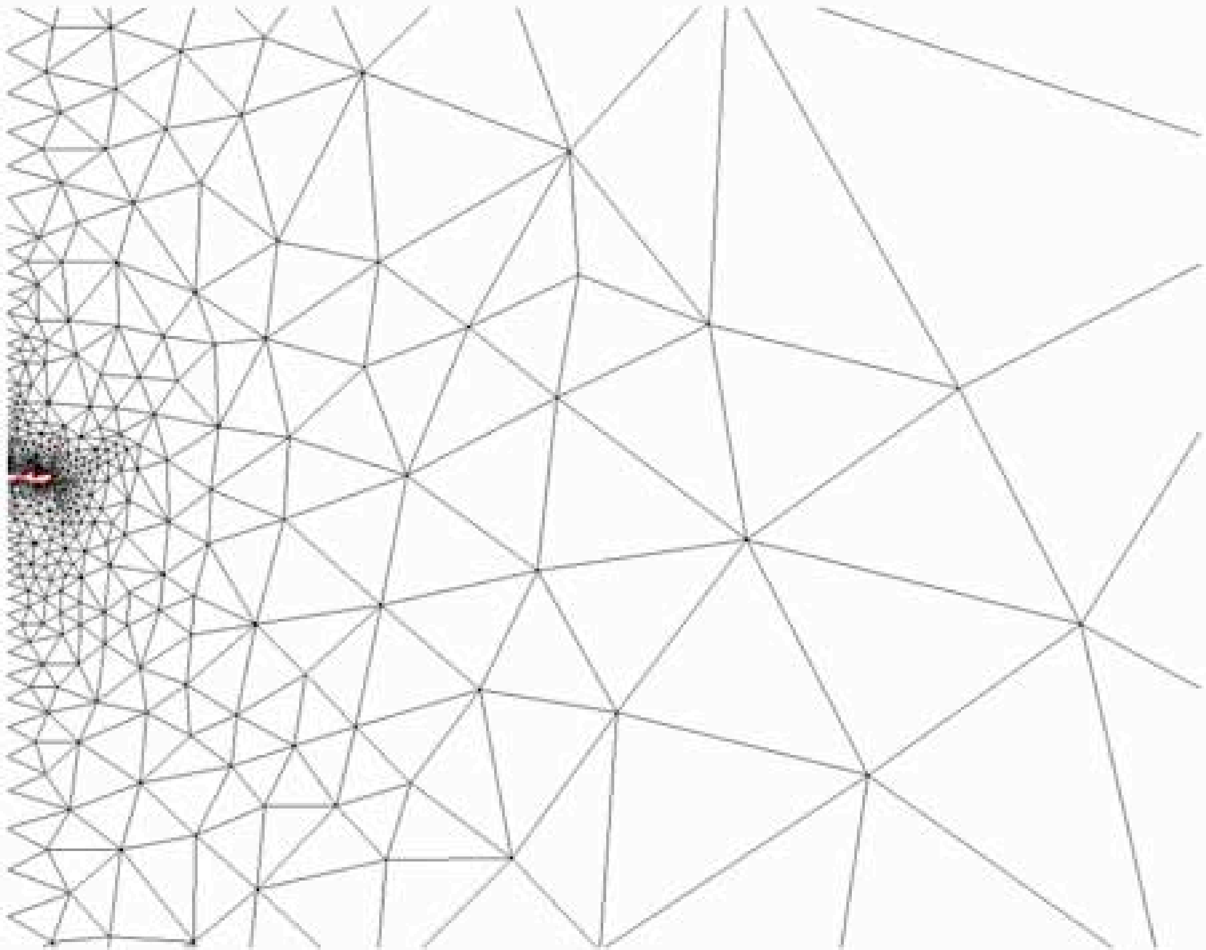

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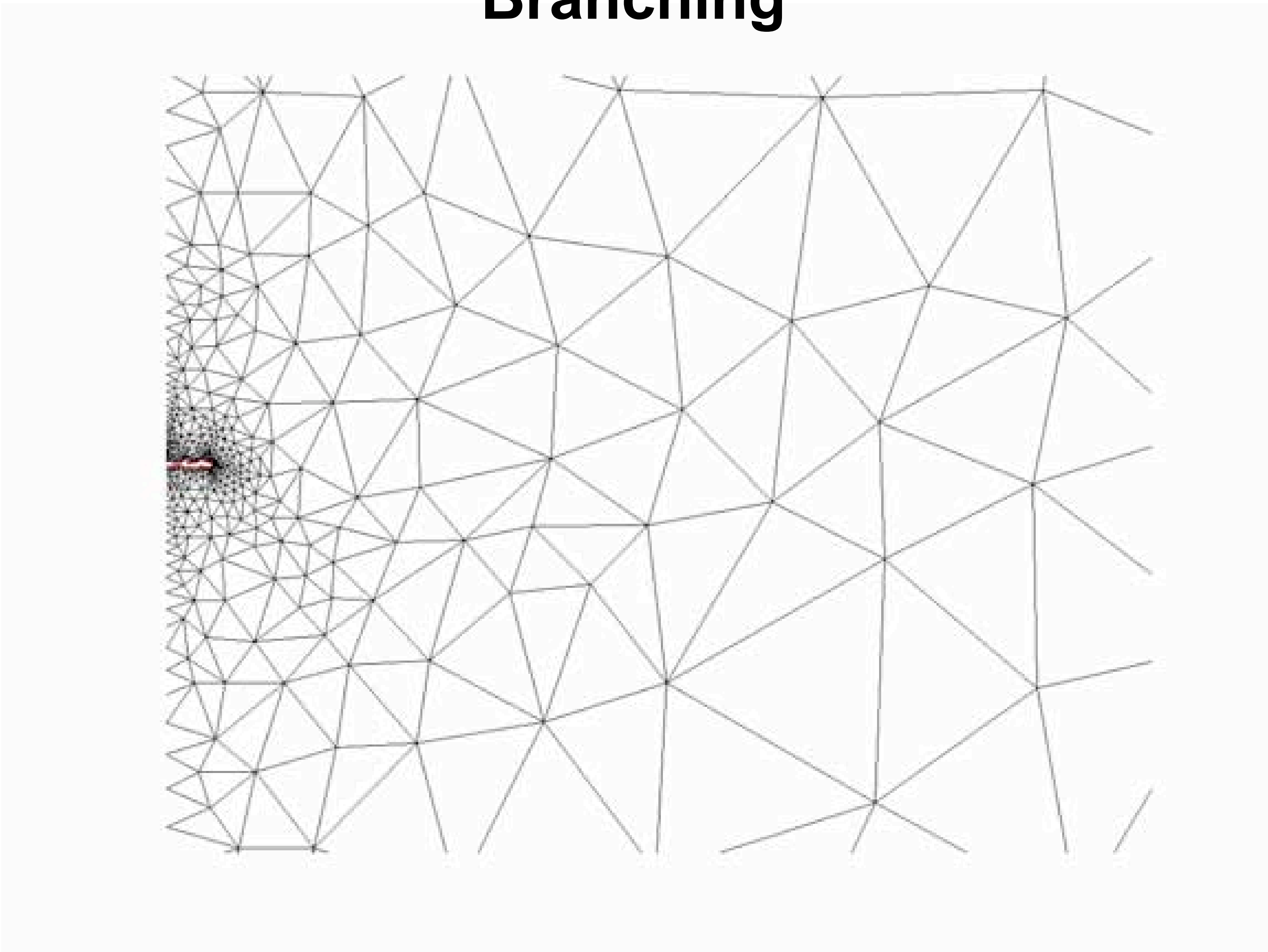

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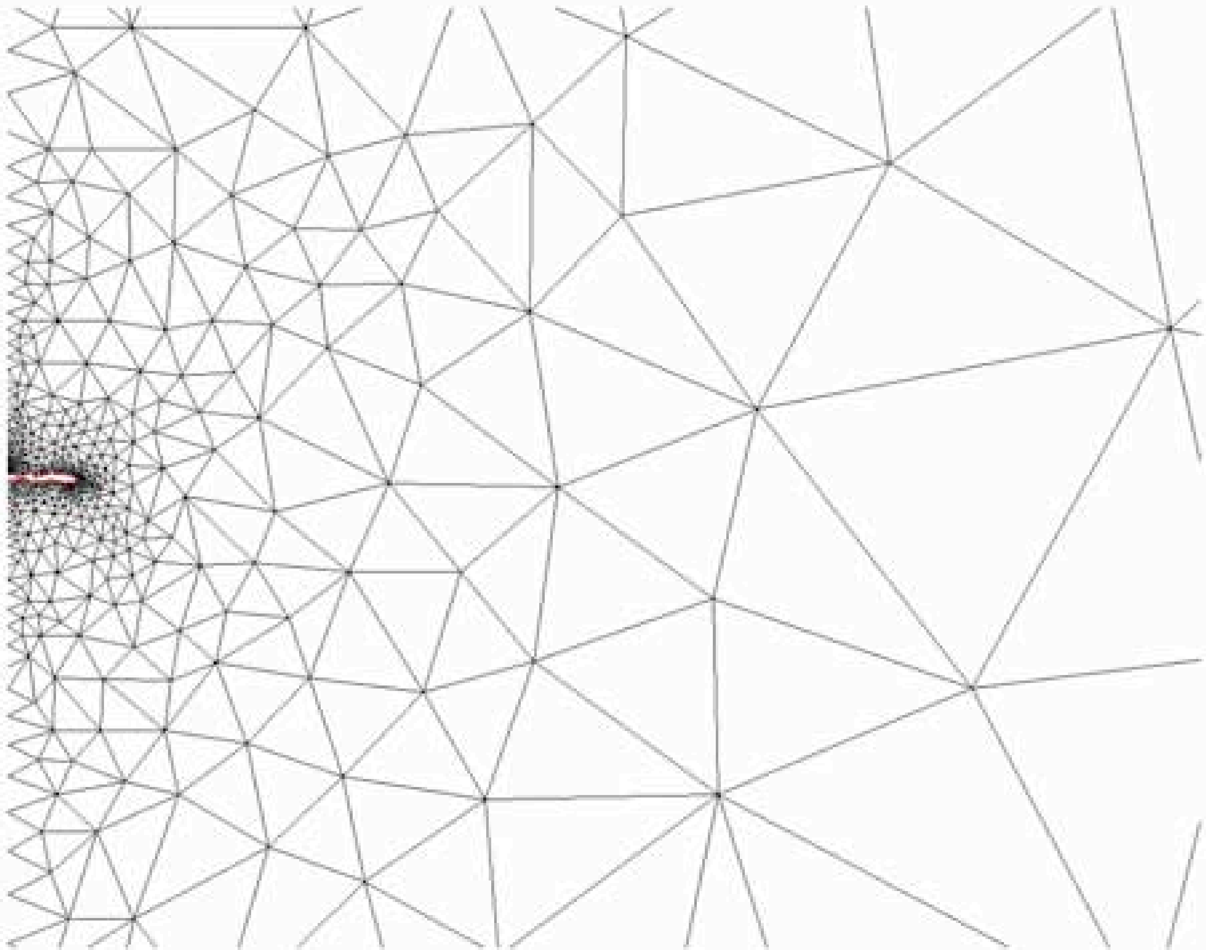

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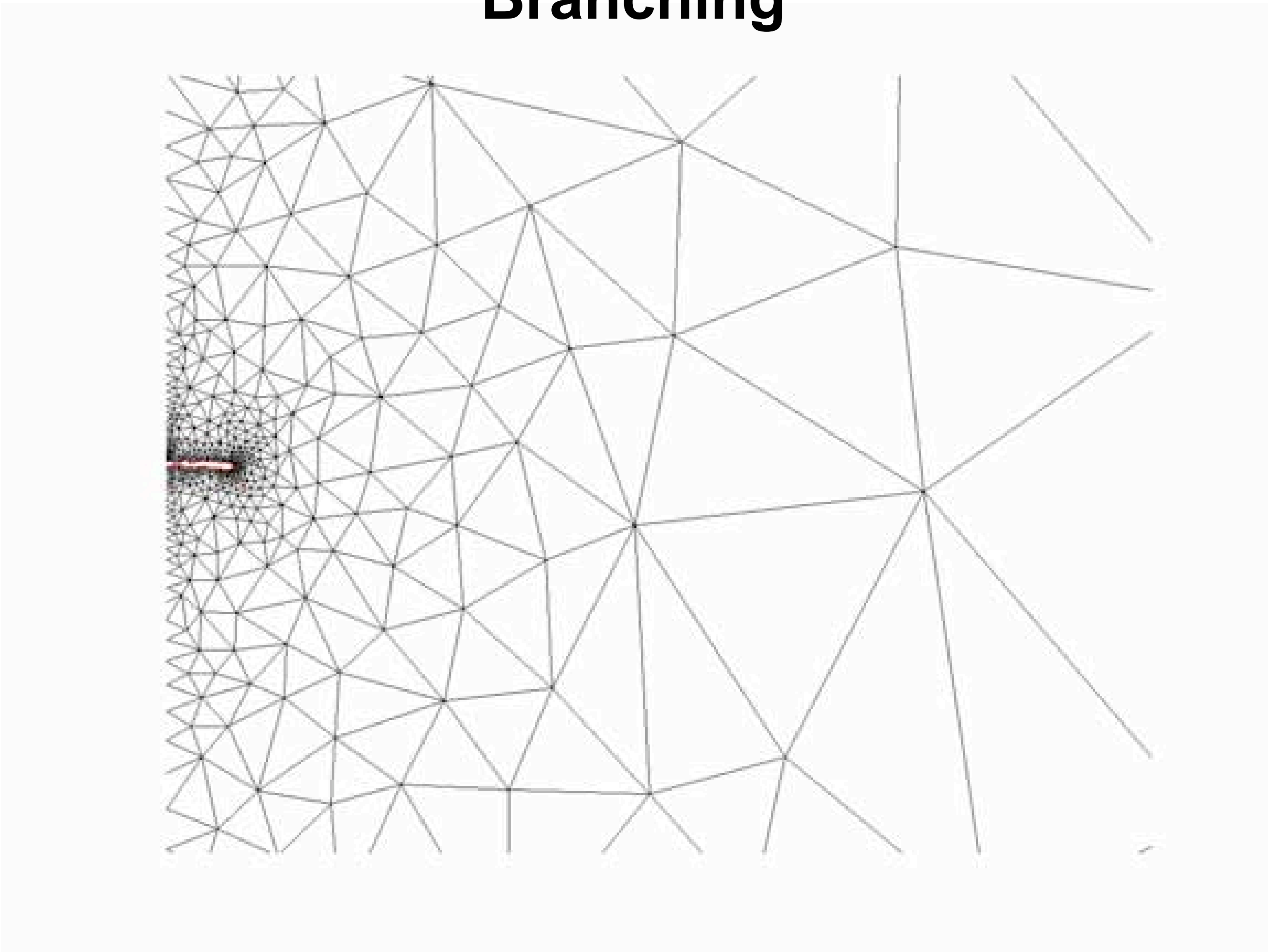

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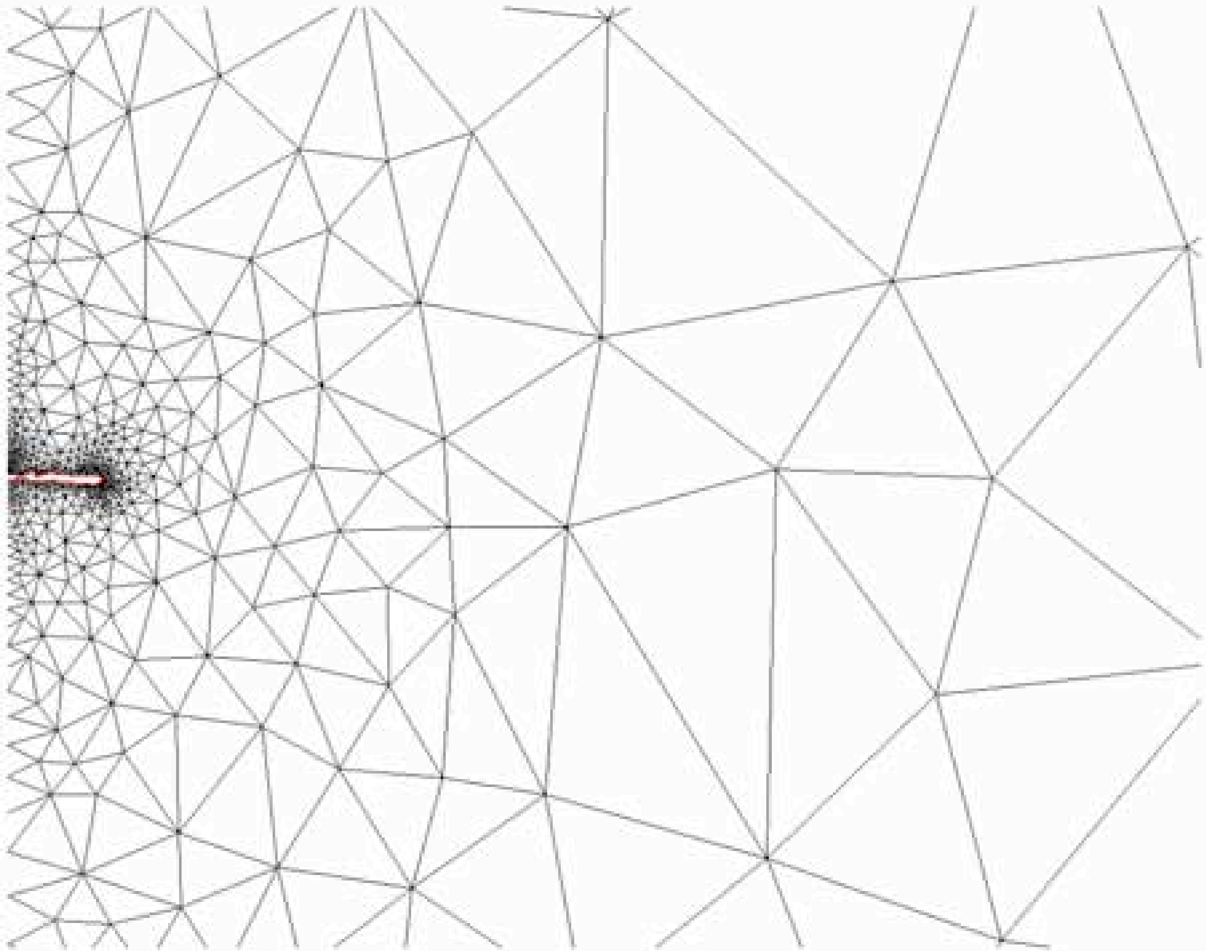

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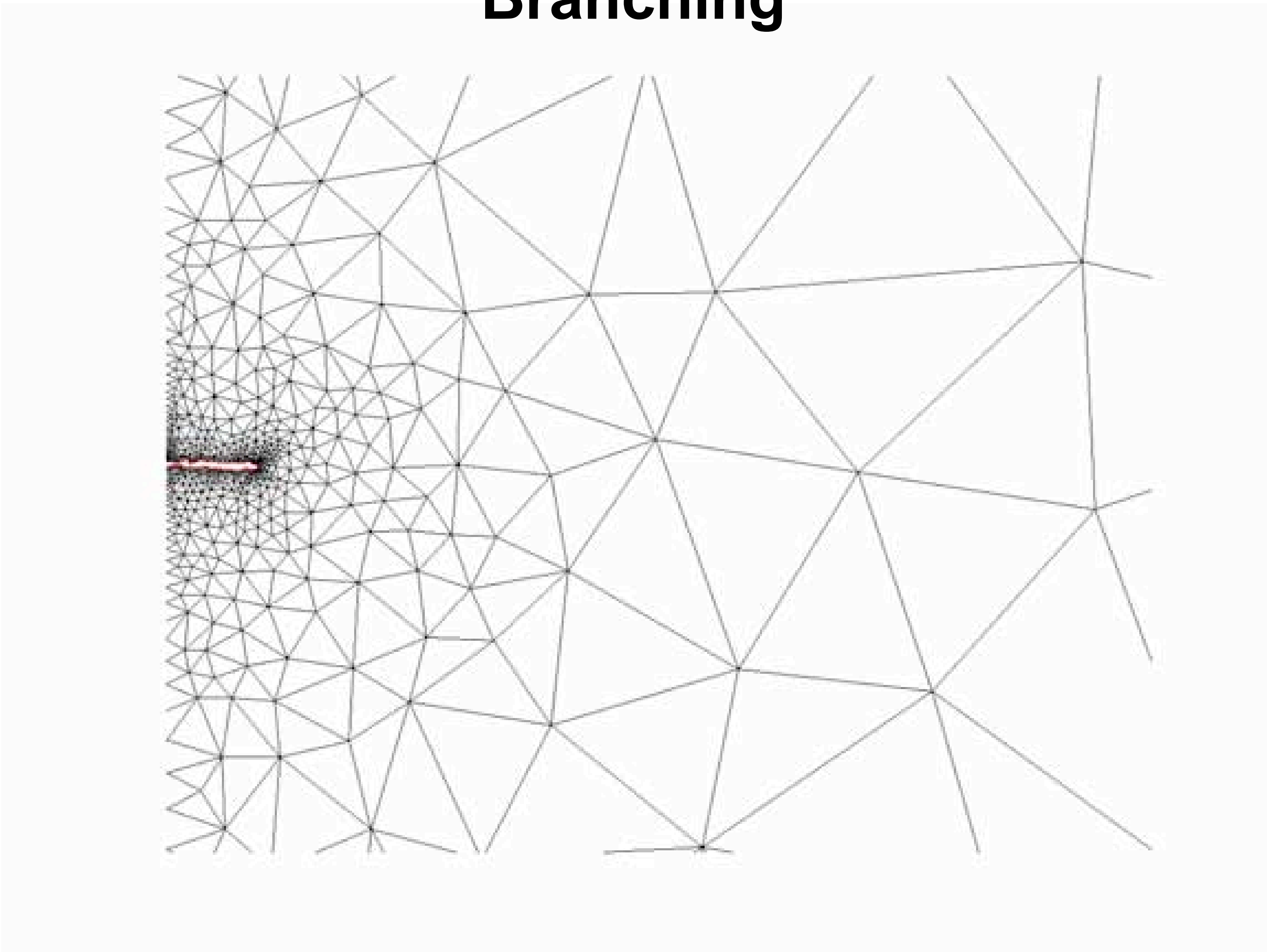

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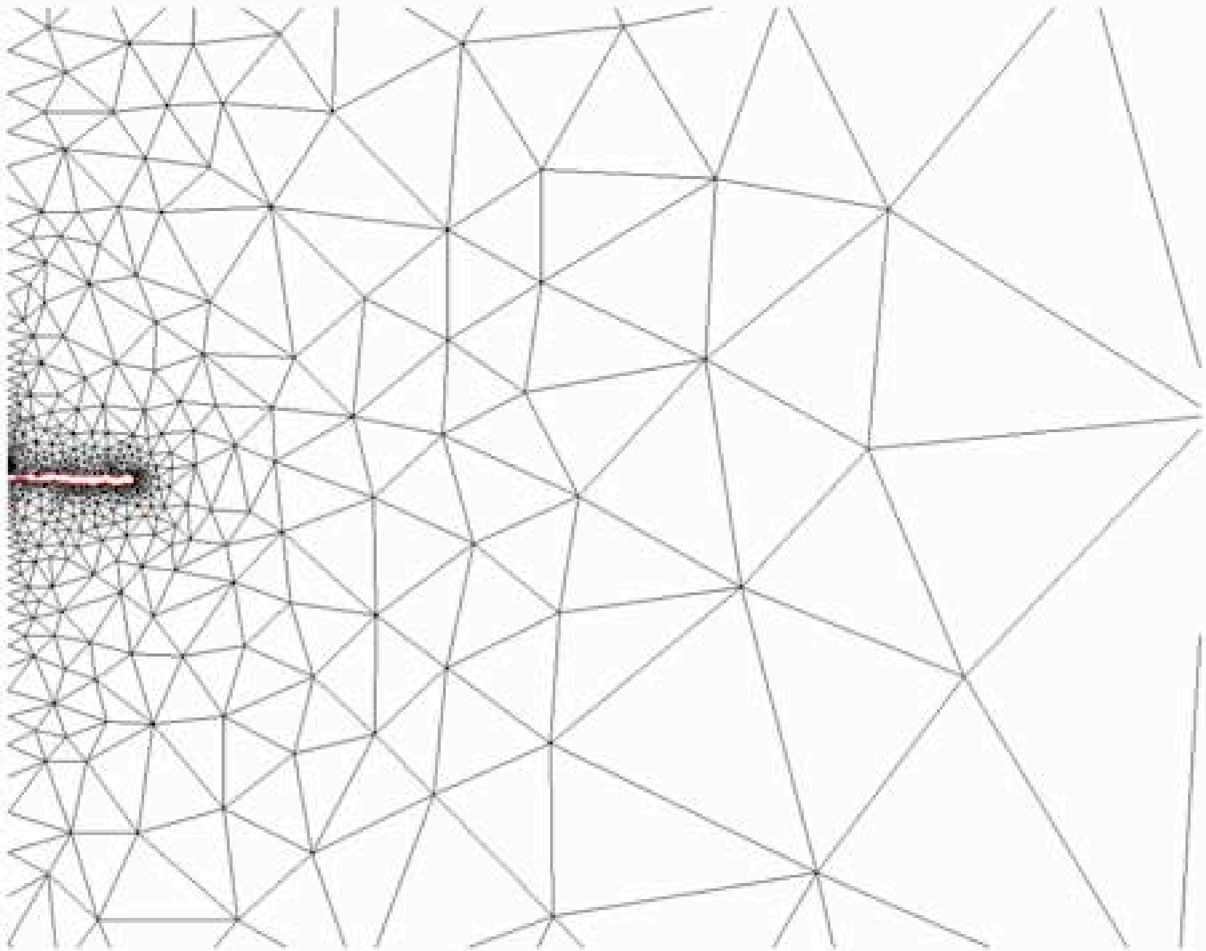

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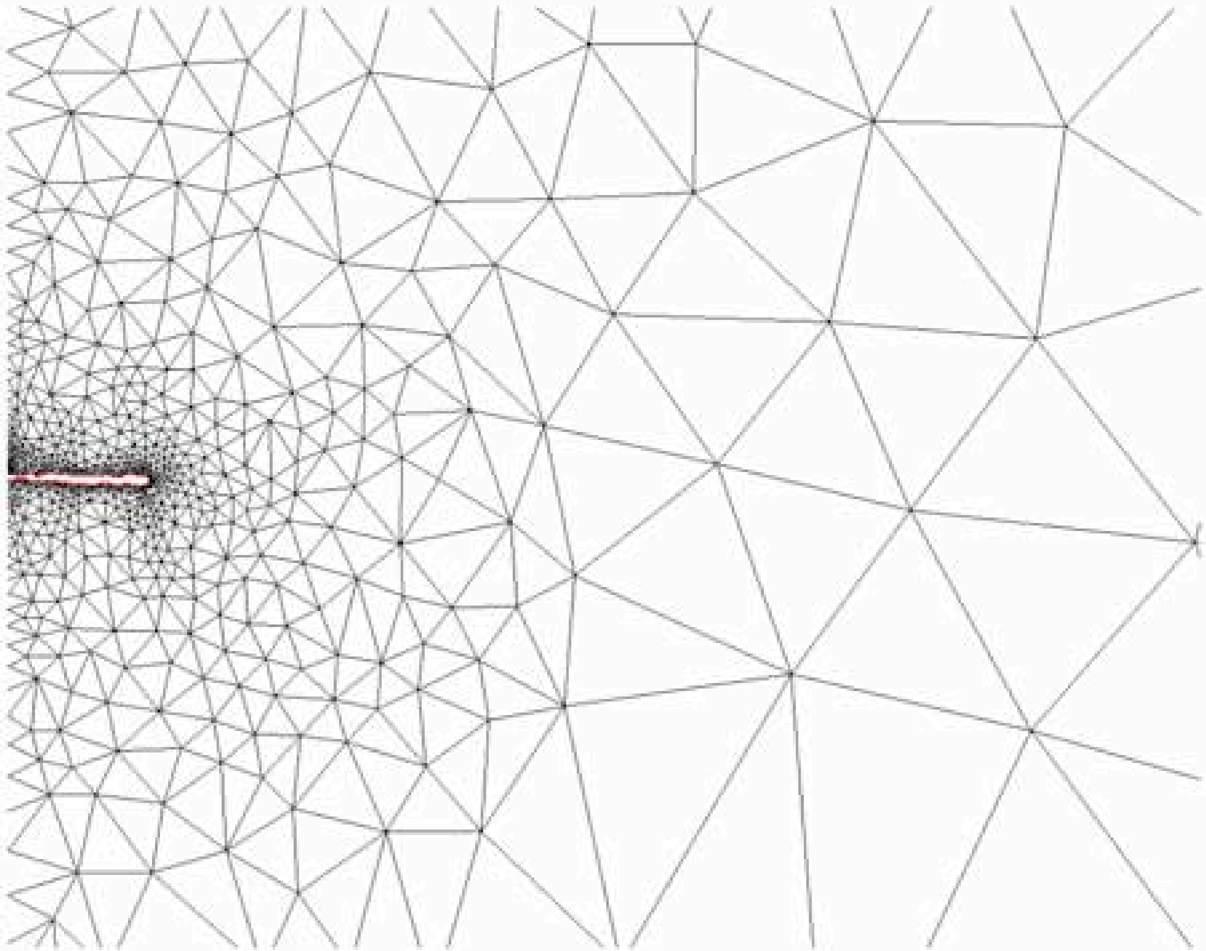

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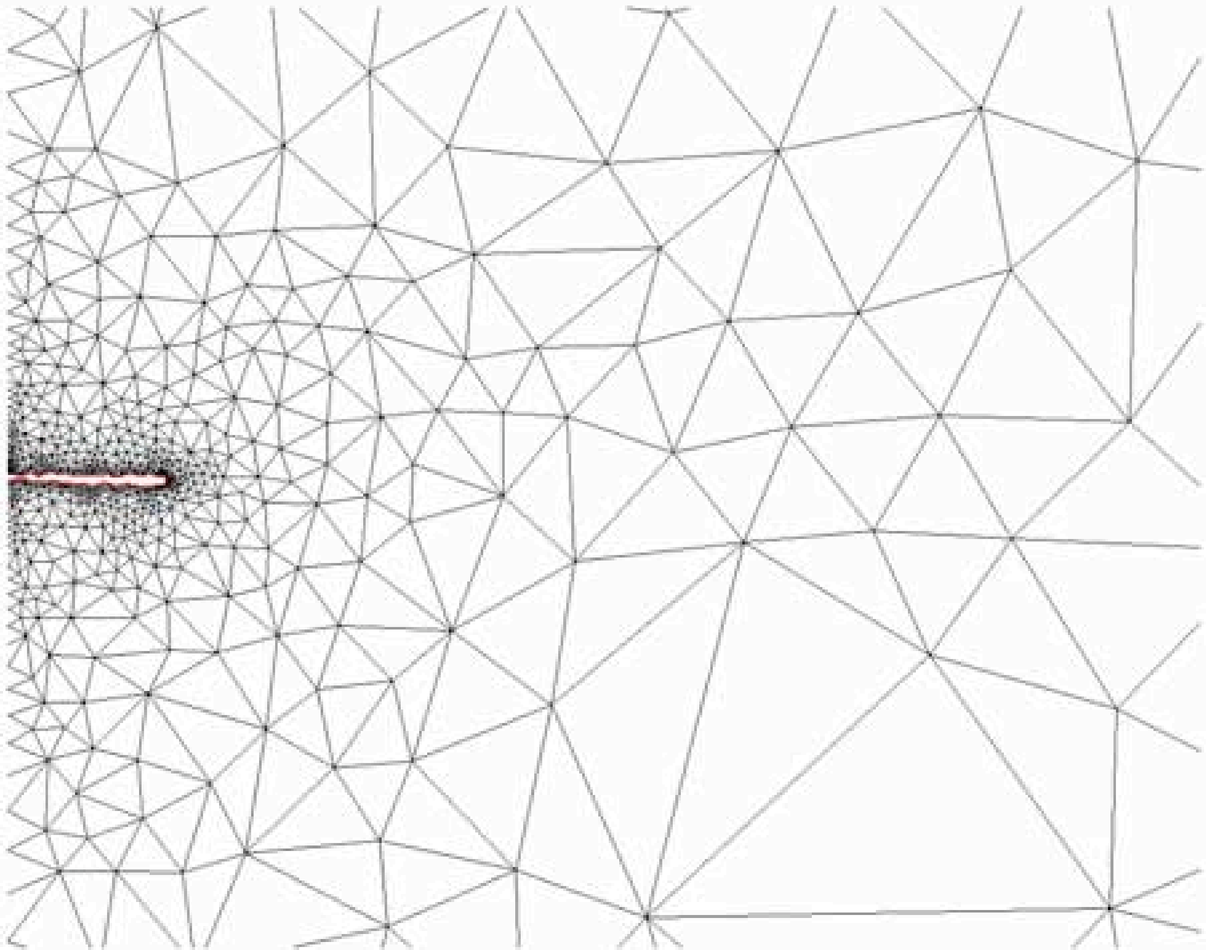

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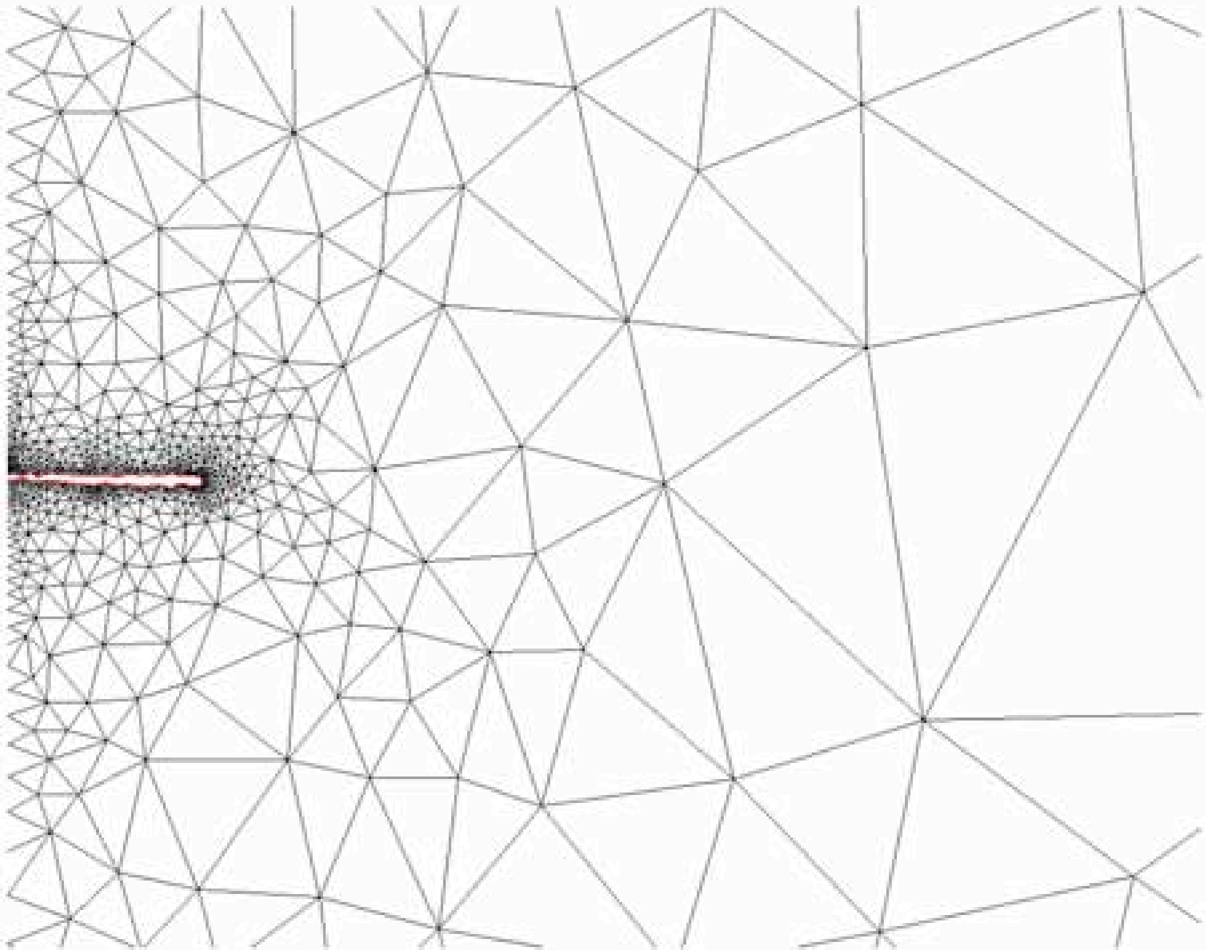

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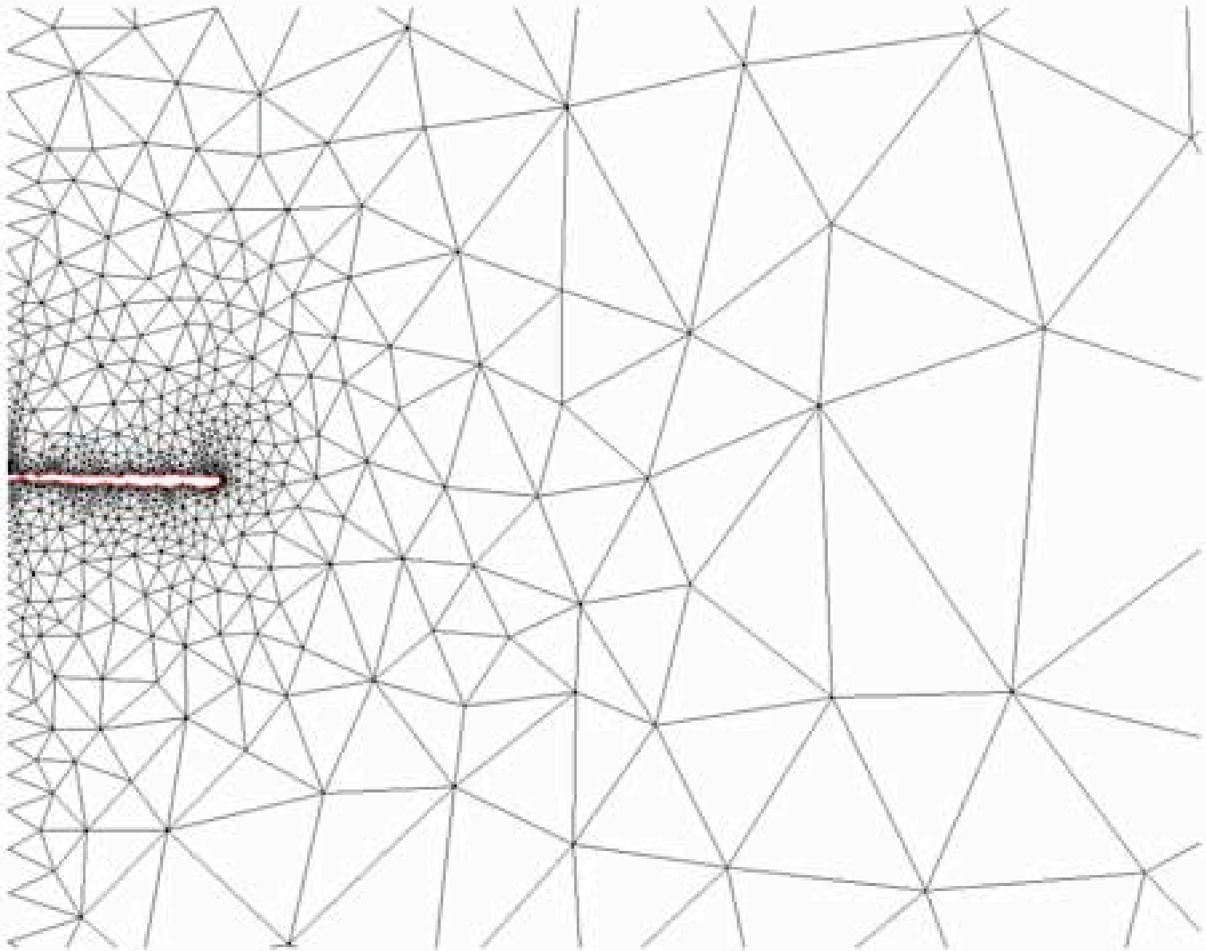

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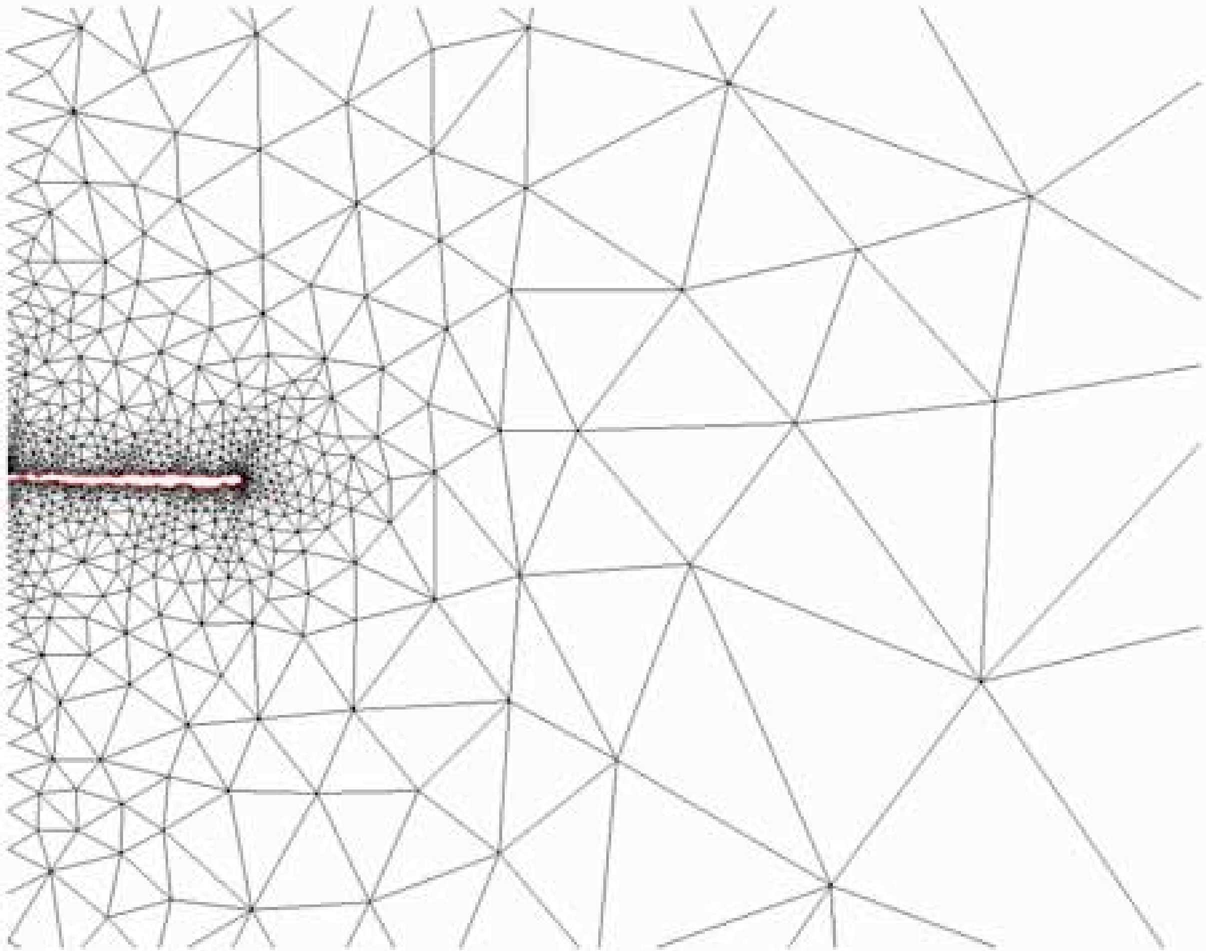

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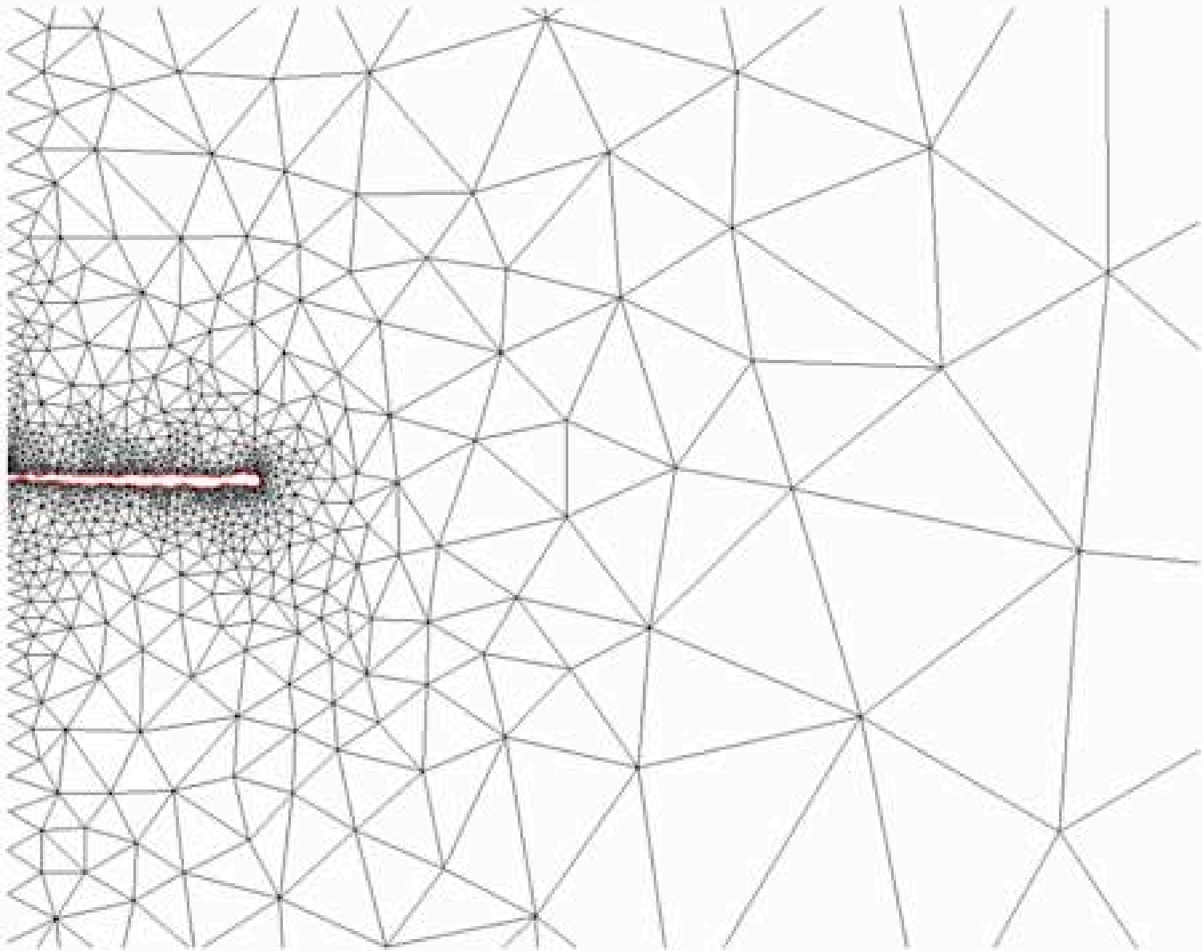

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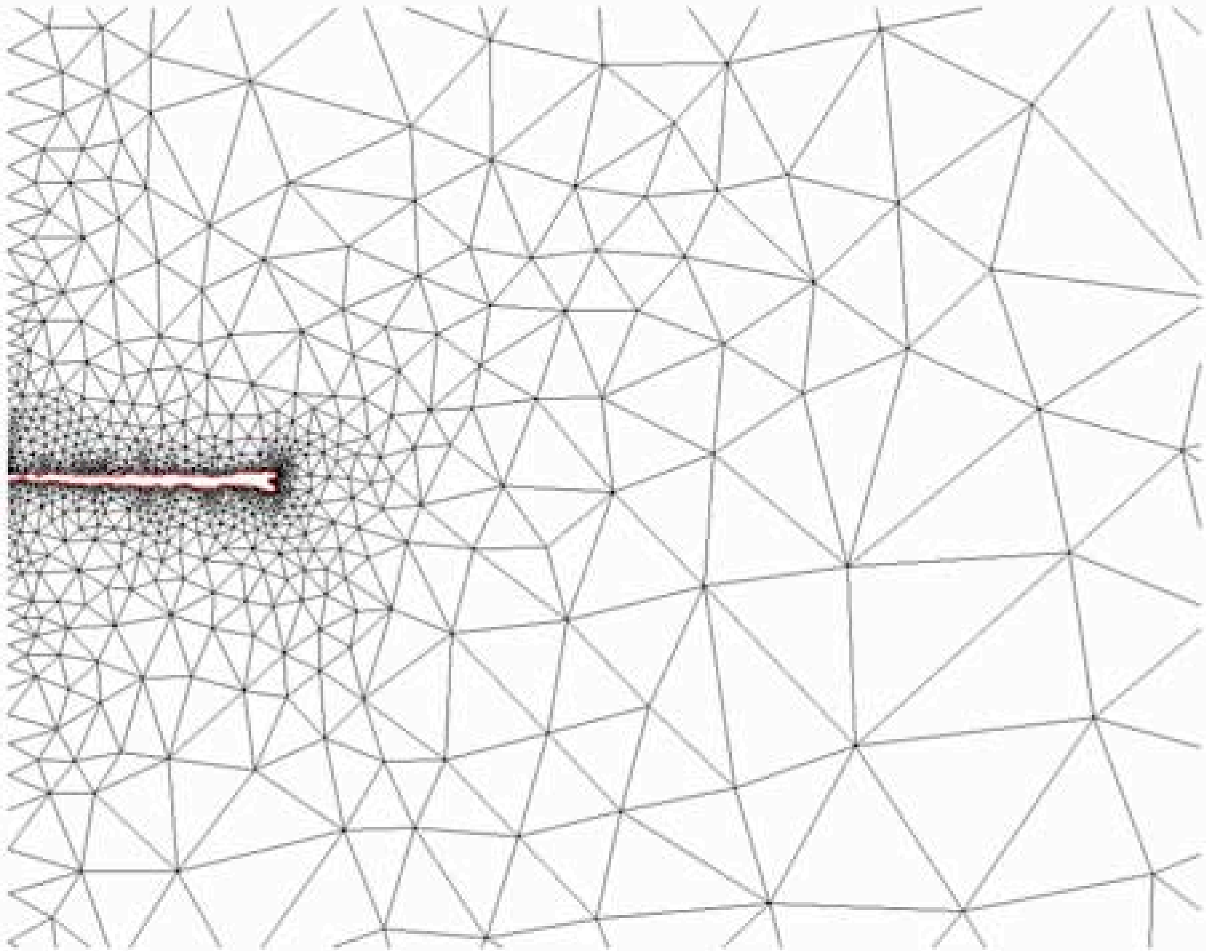

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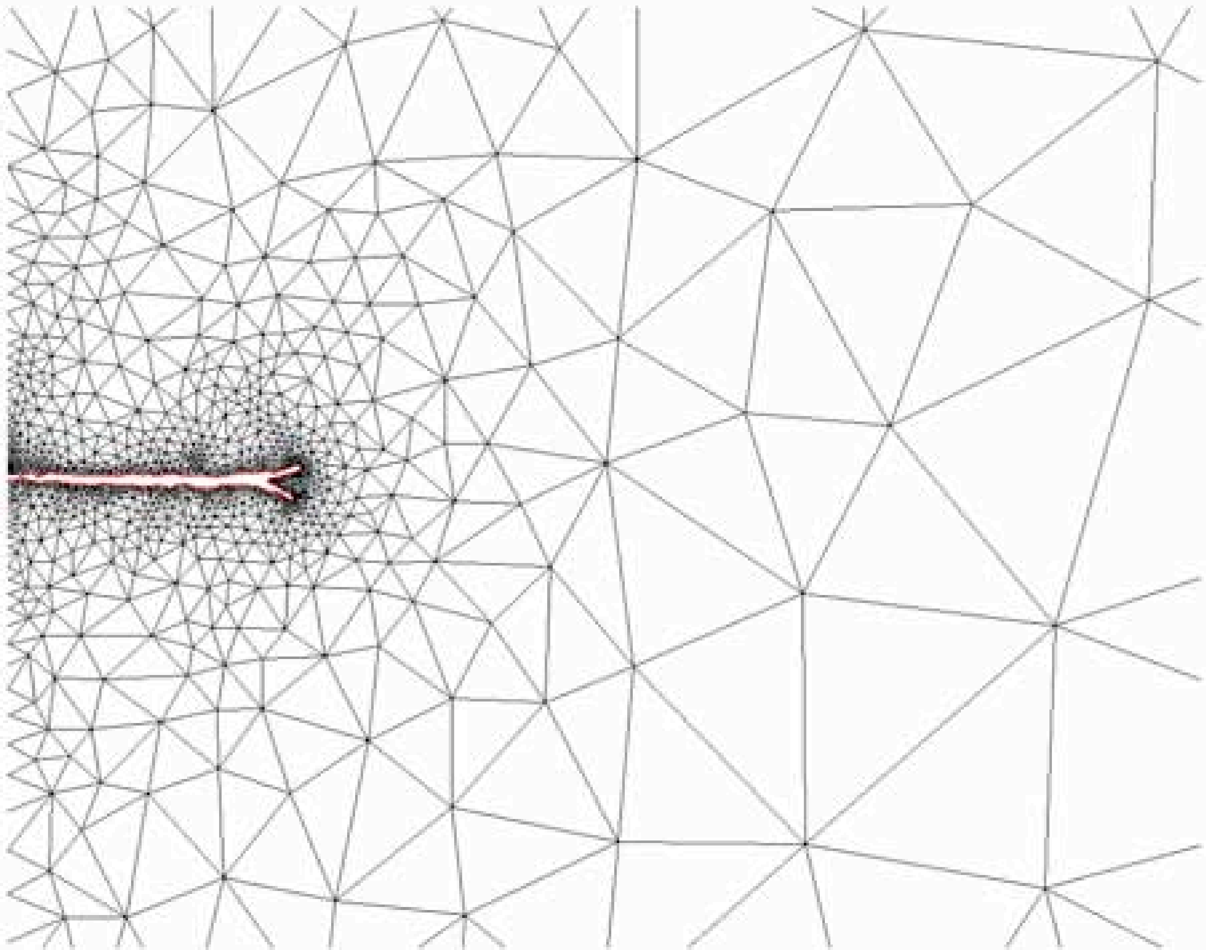

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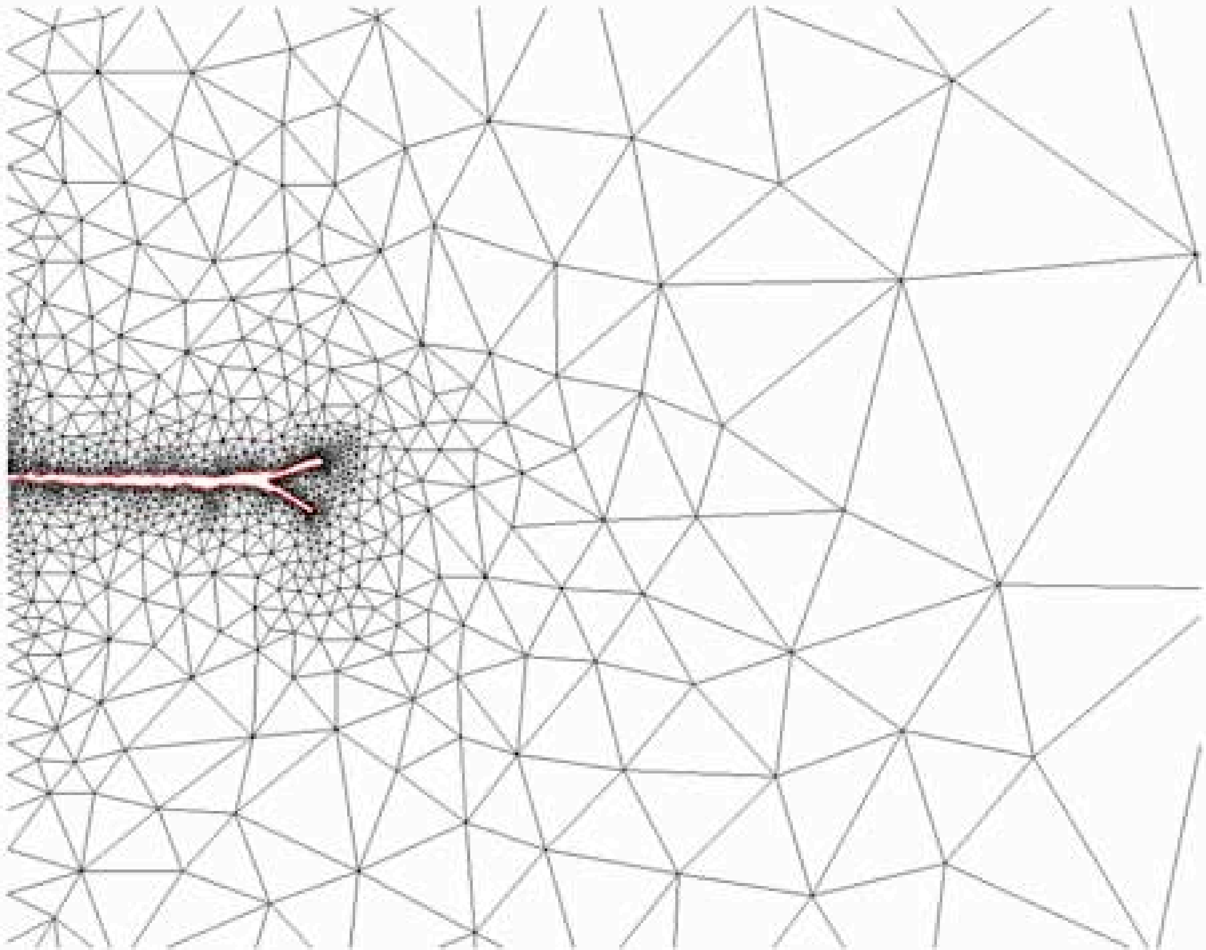

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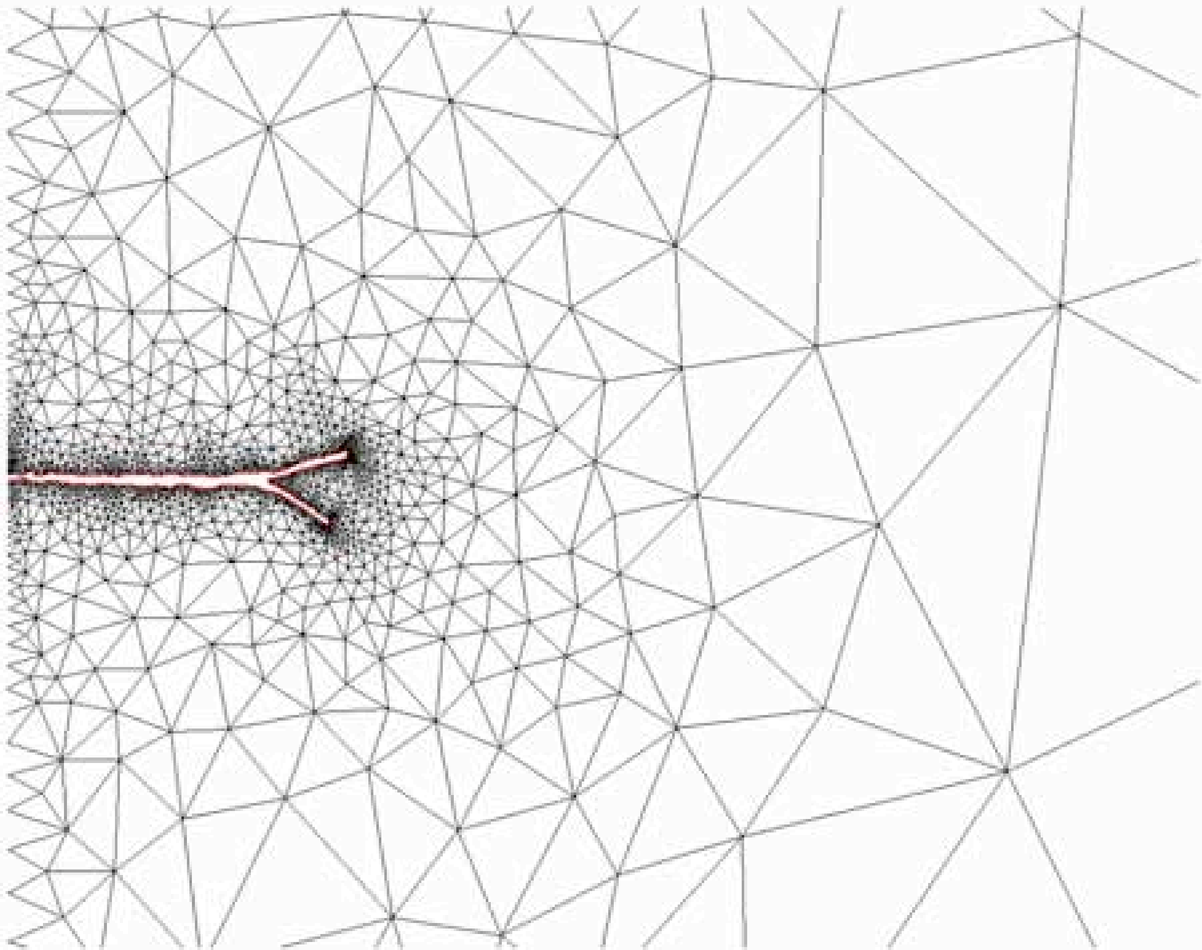

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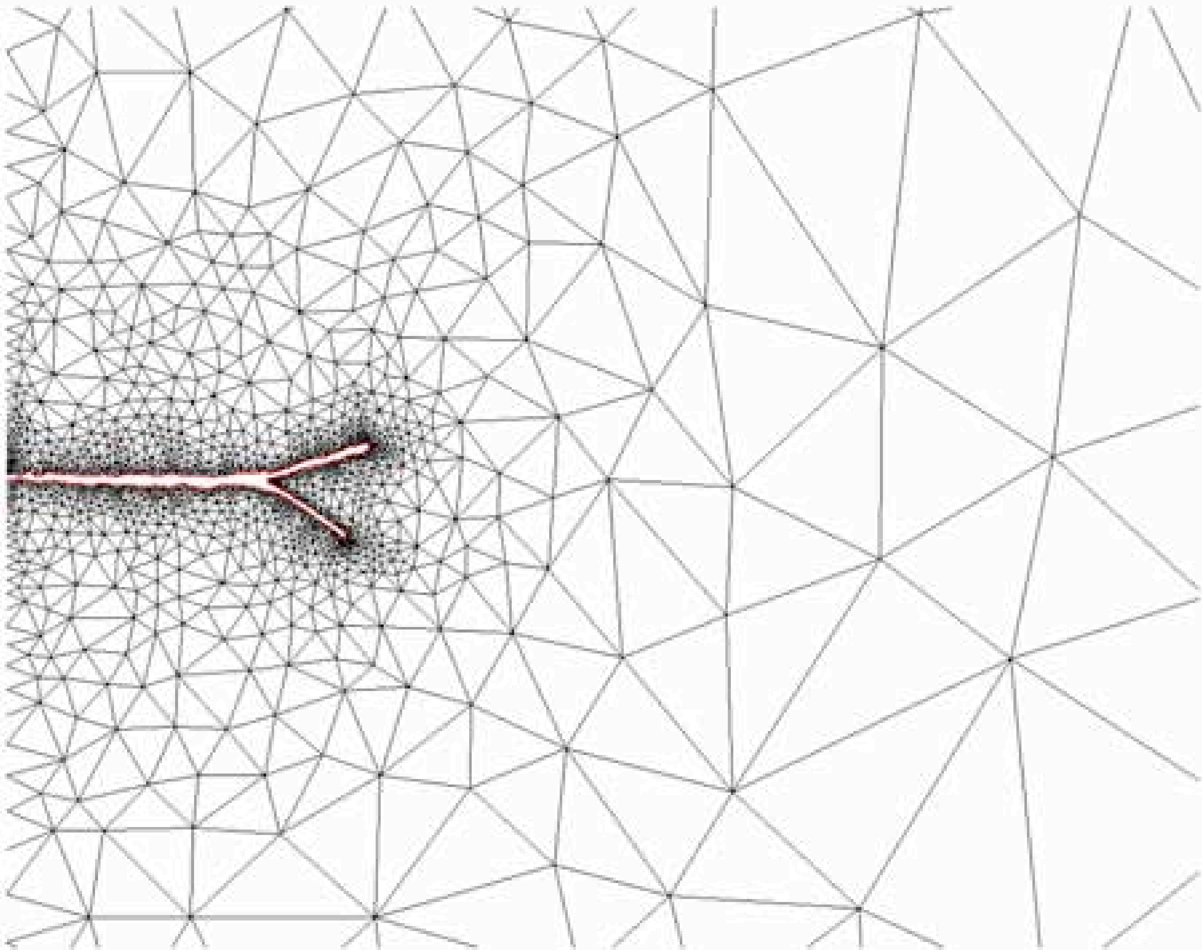

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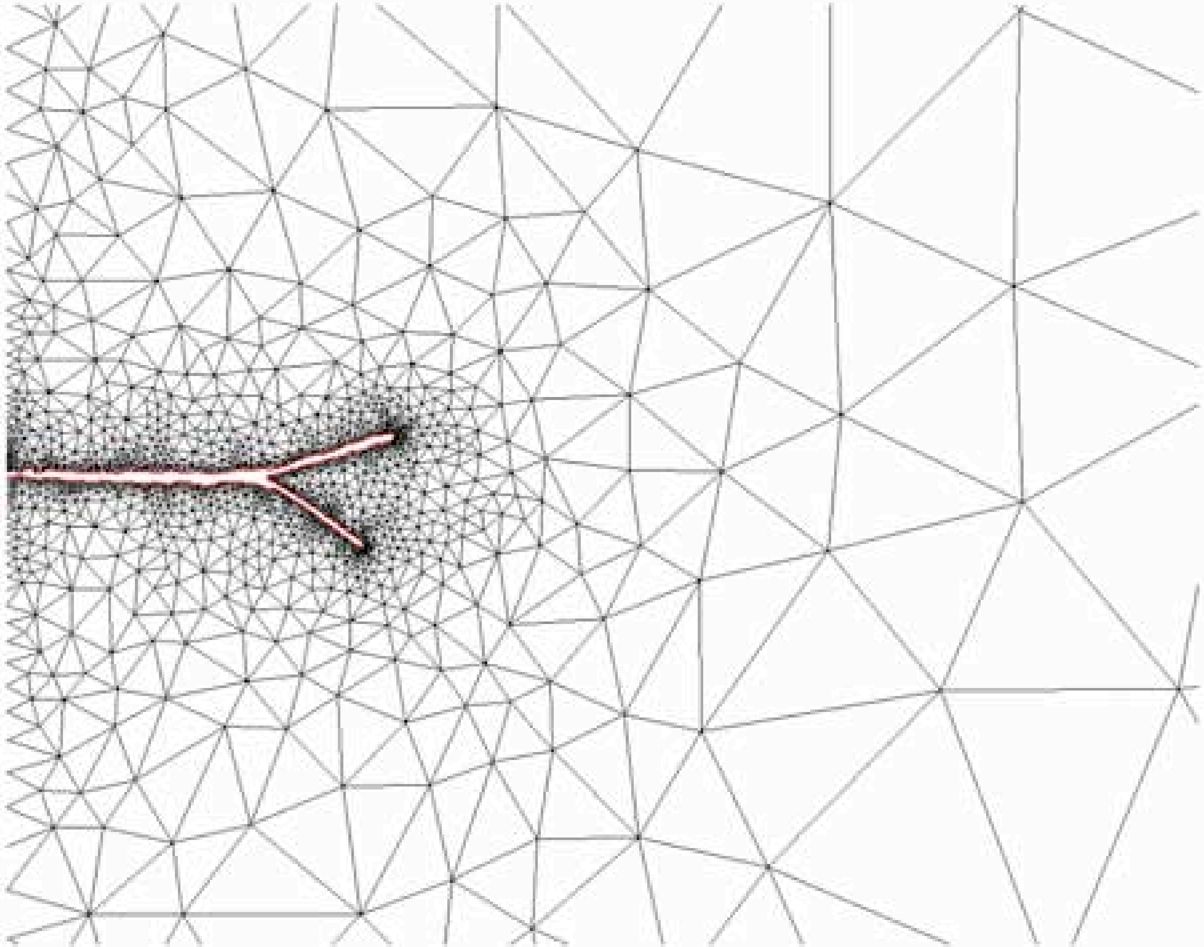

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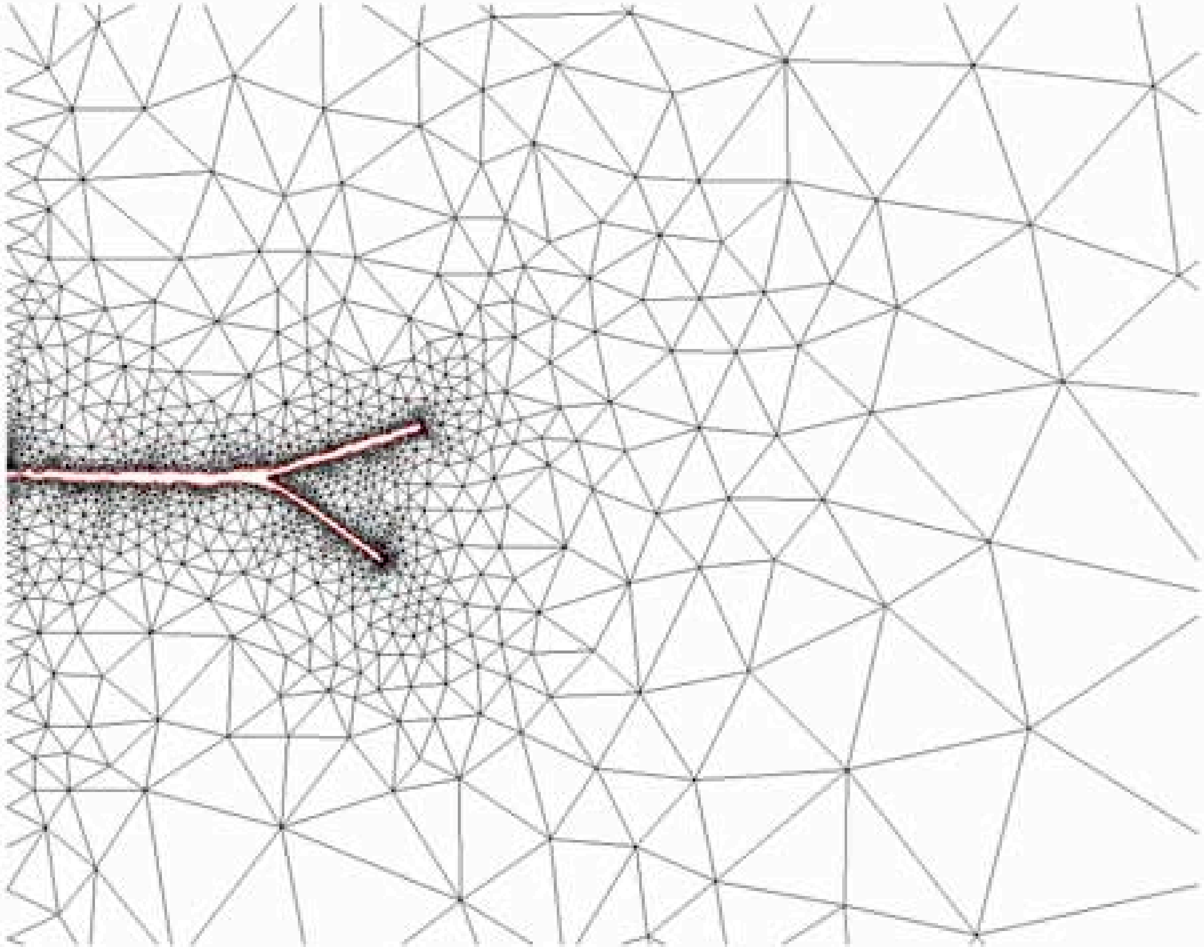

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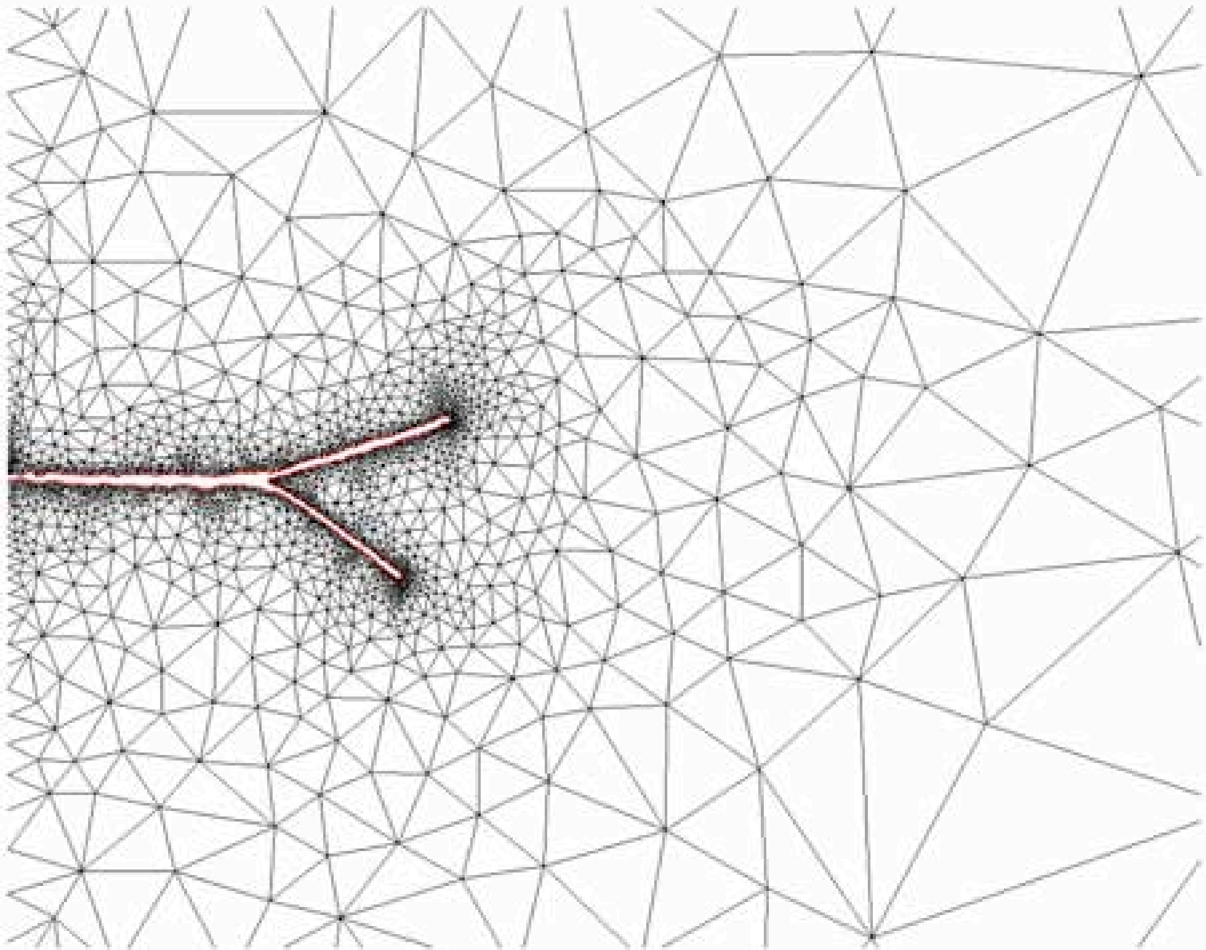

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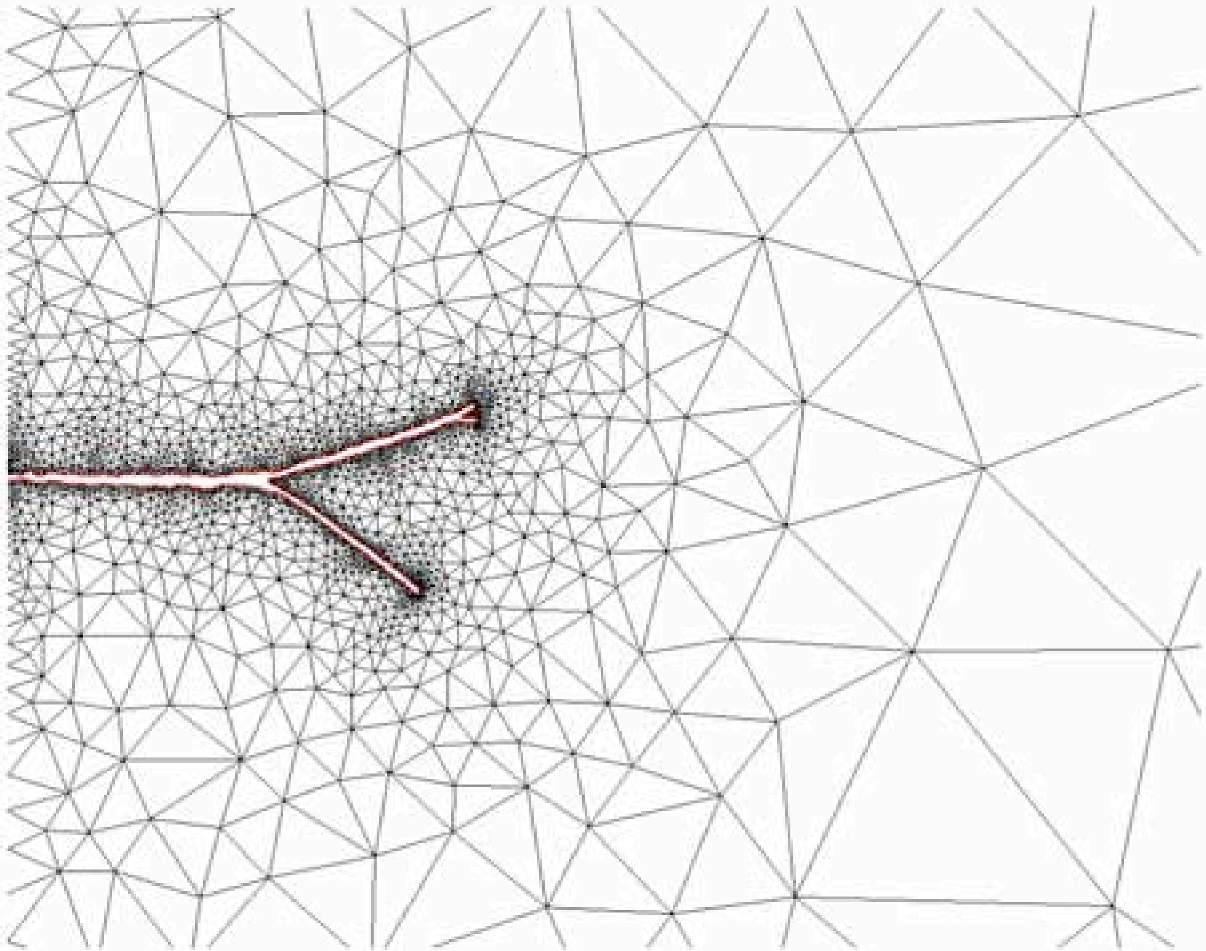

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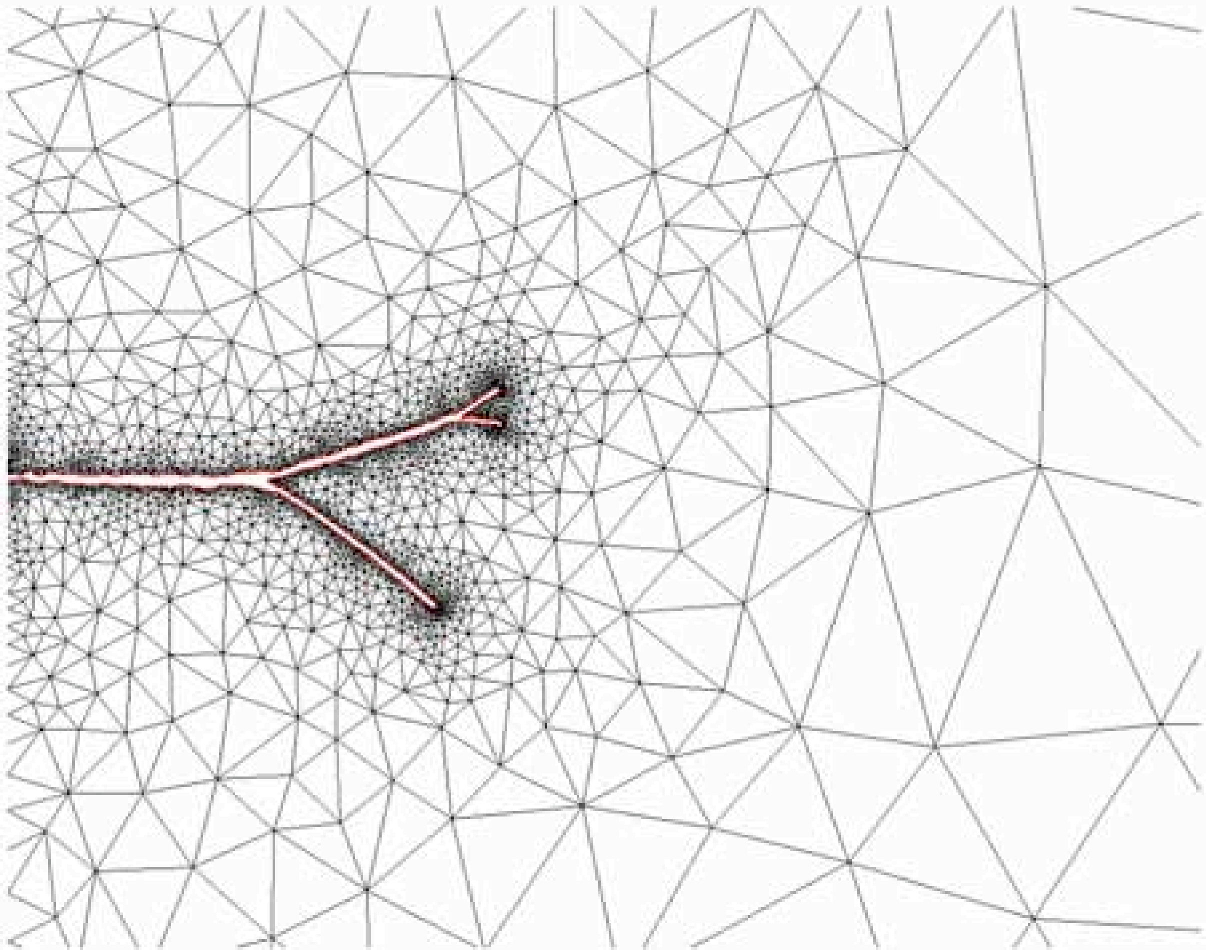

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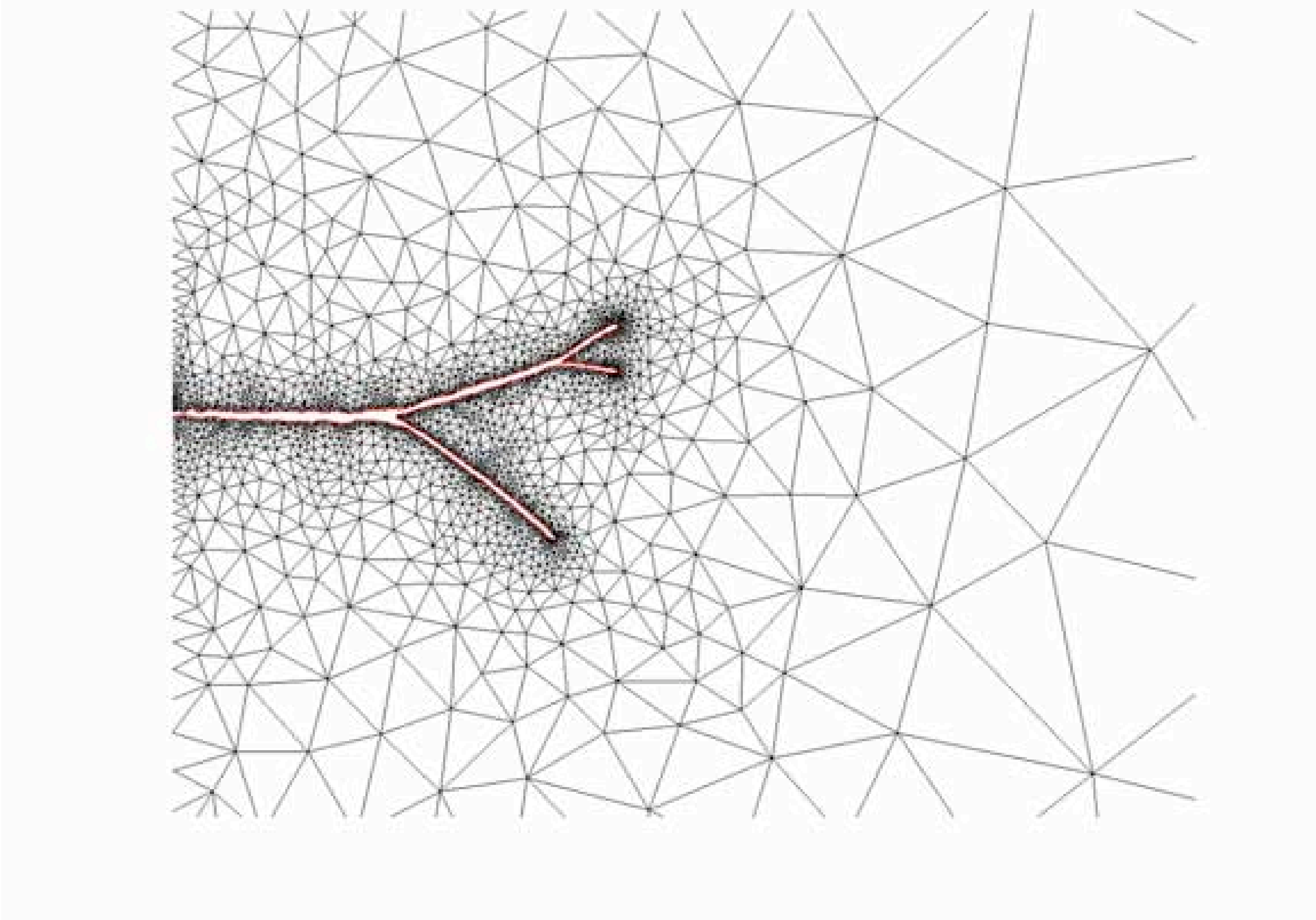

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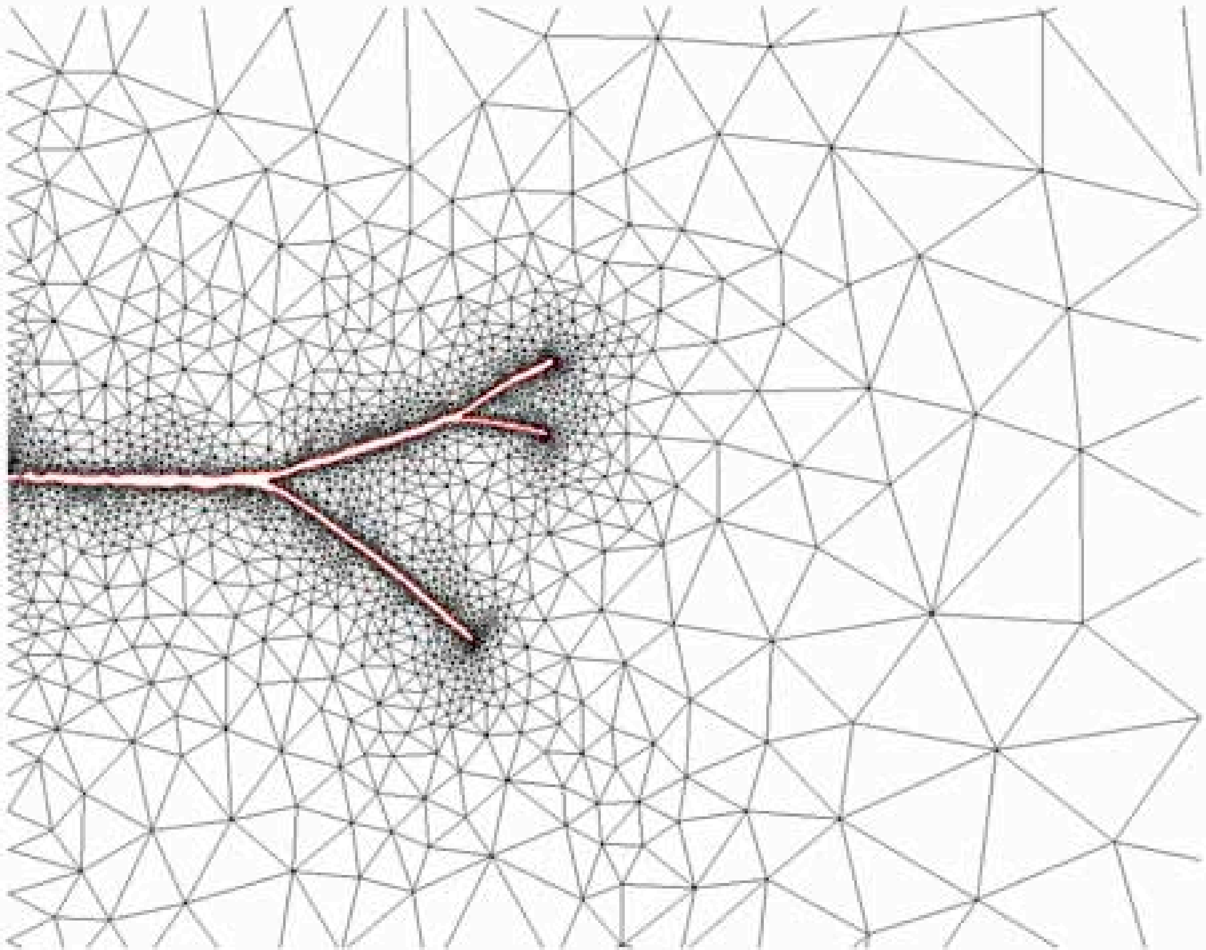

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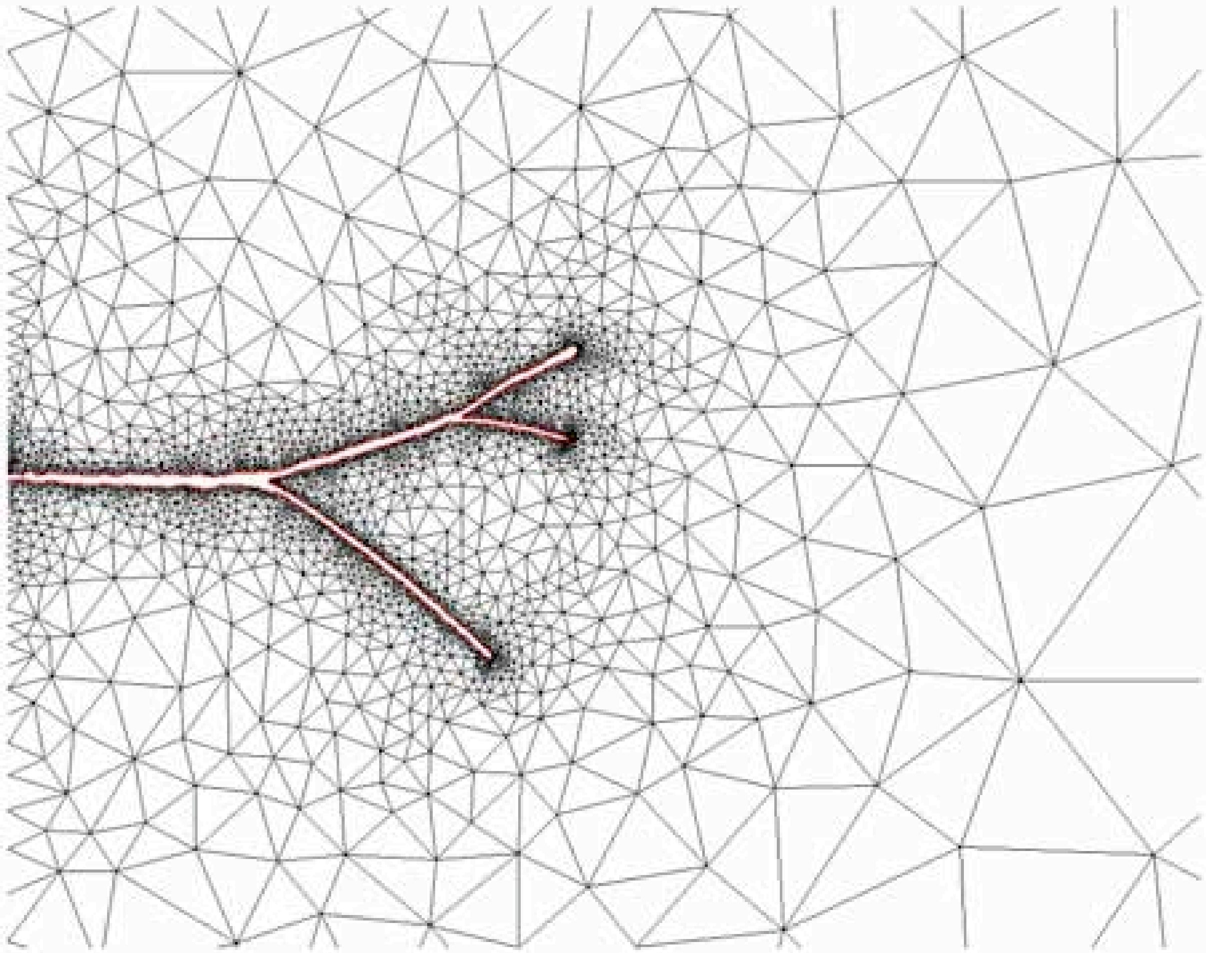

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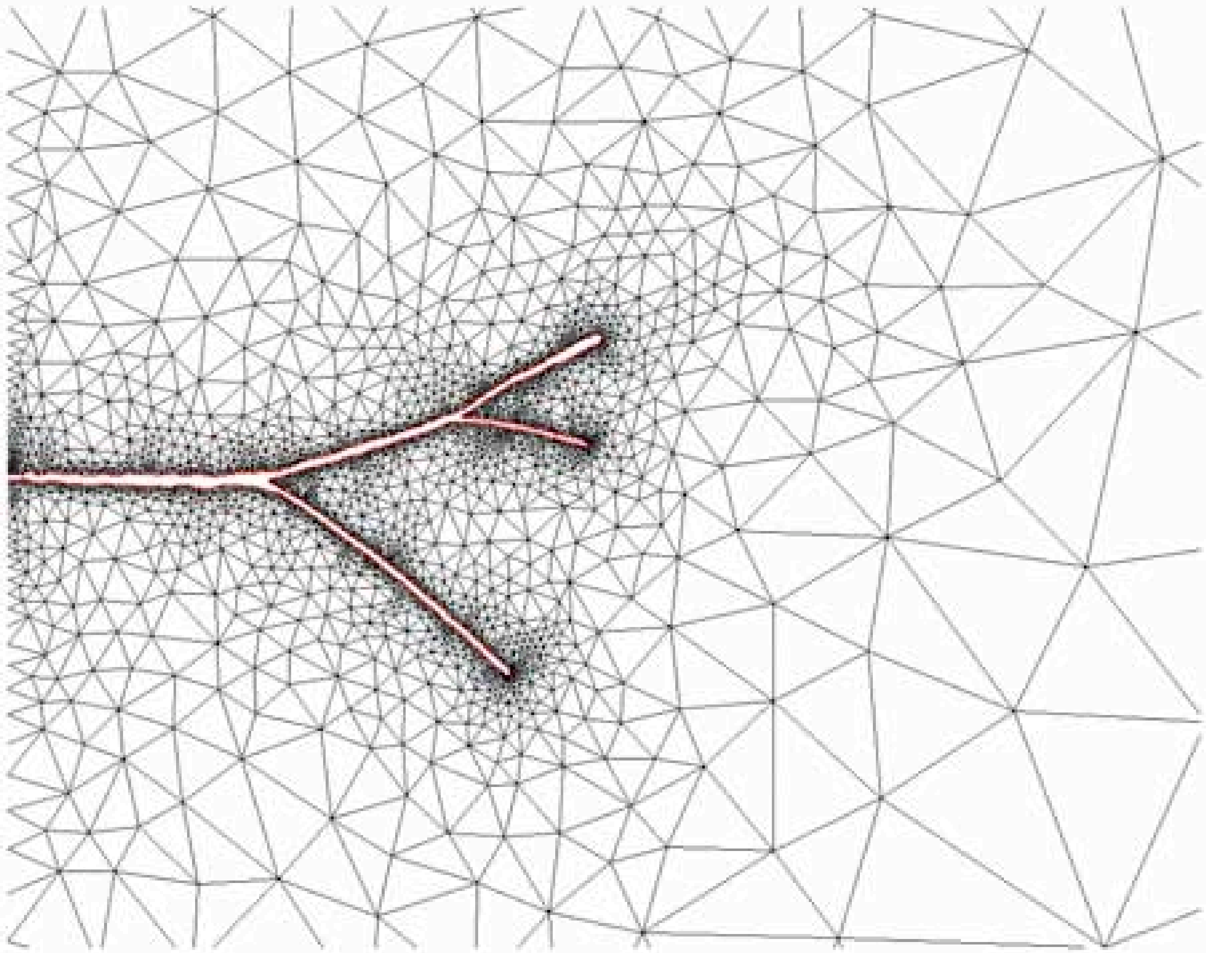

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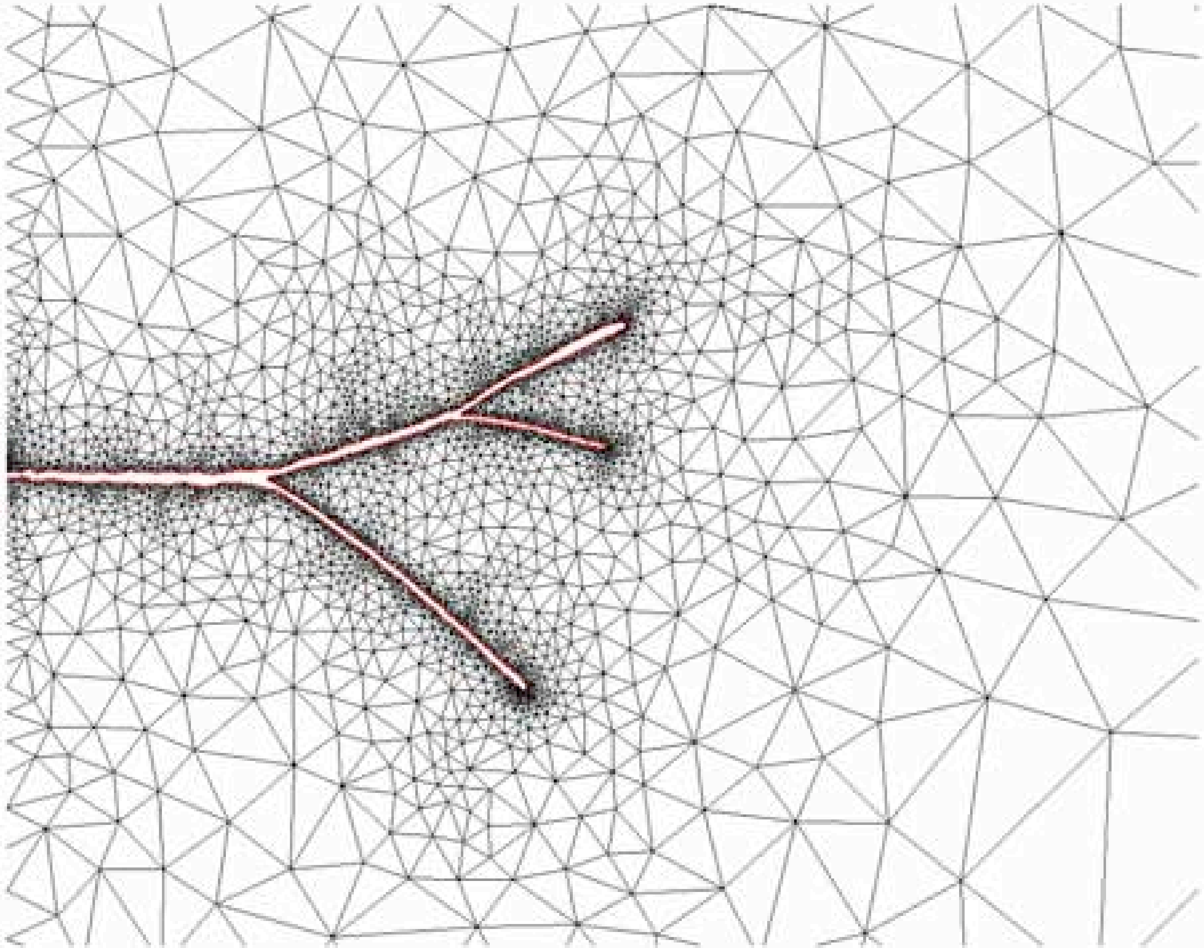
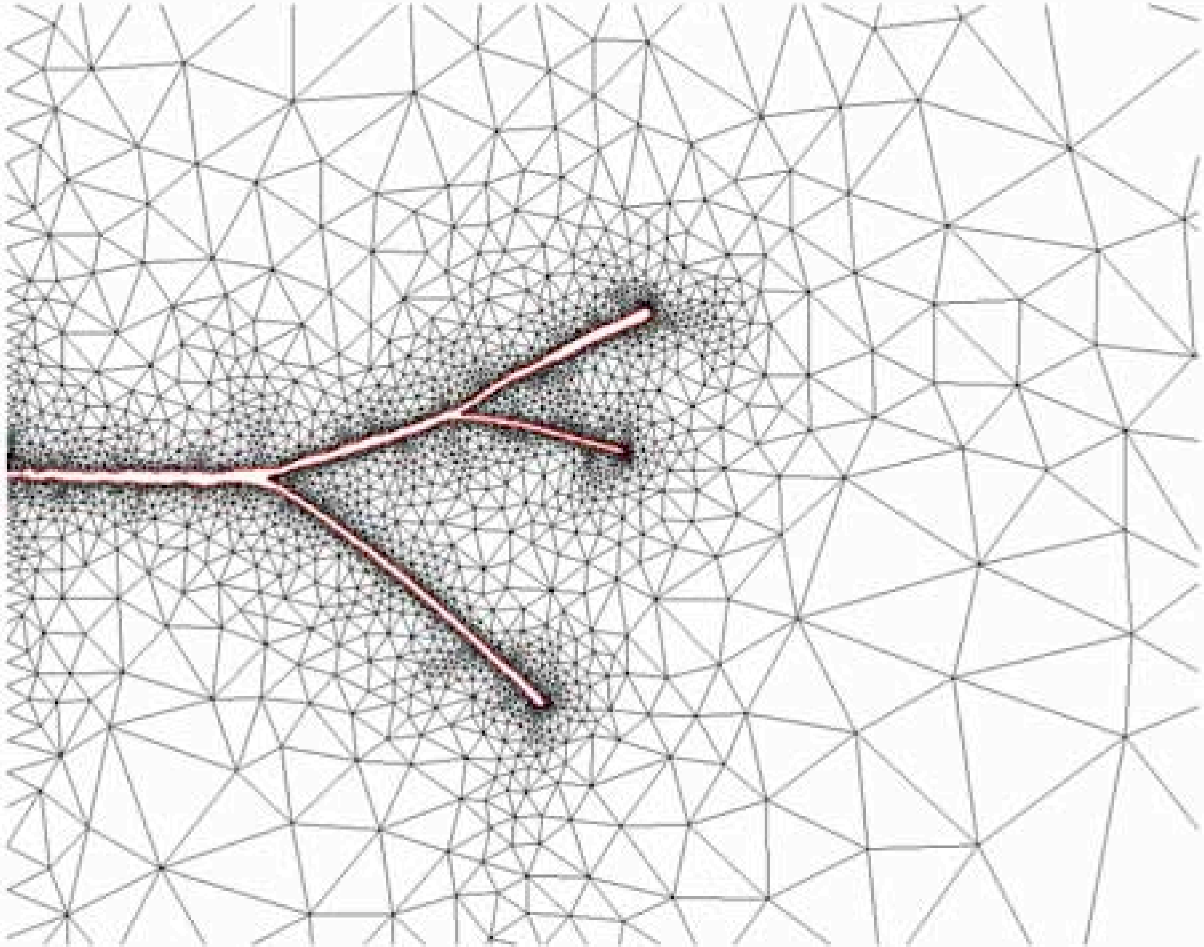


Branching



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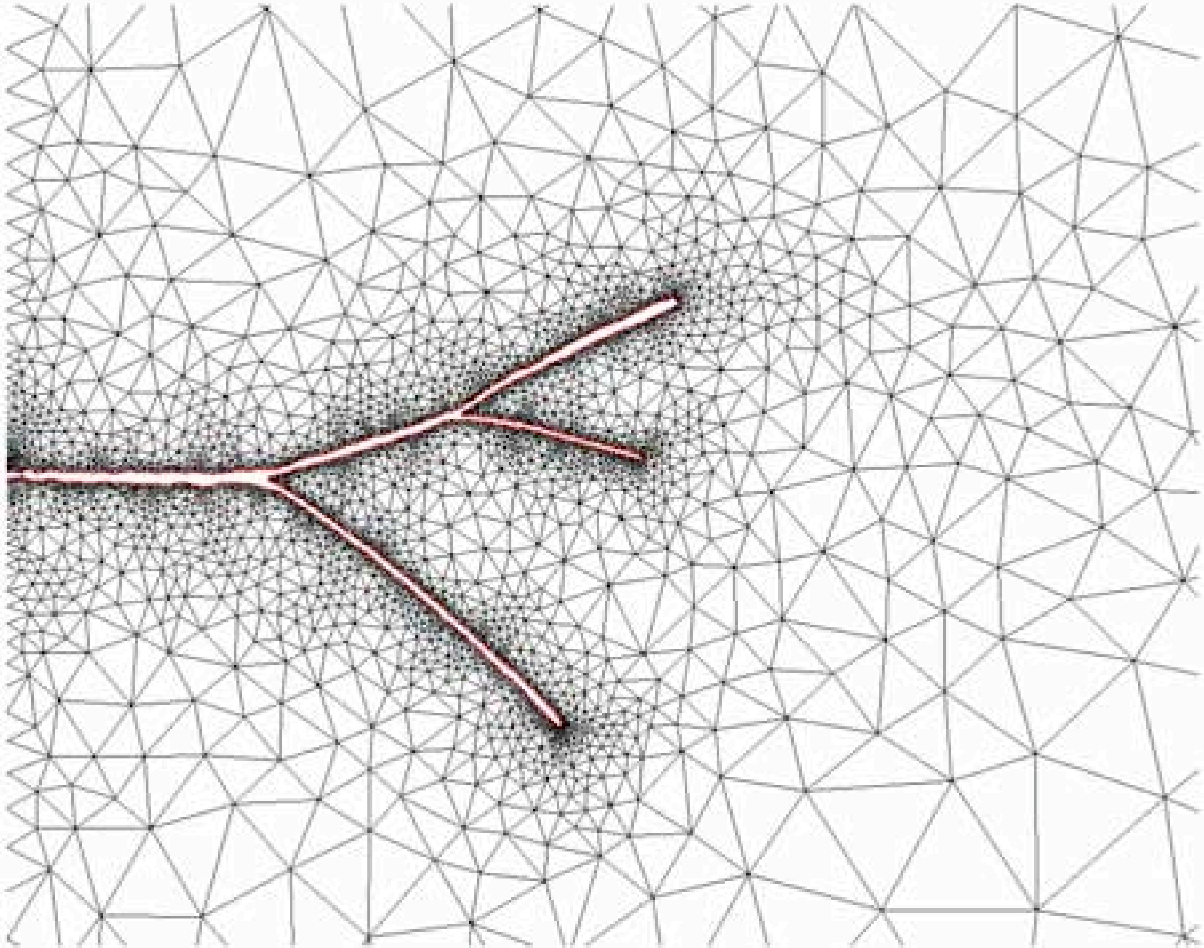

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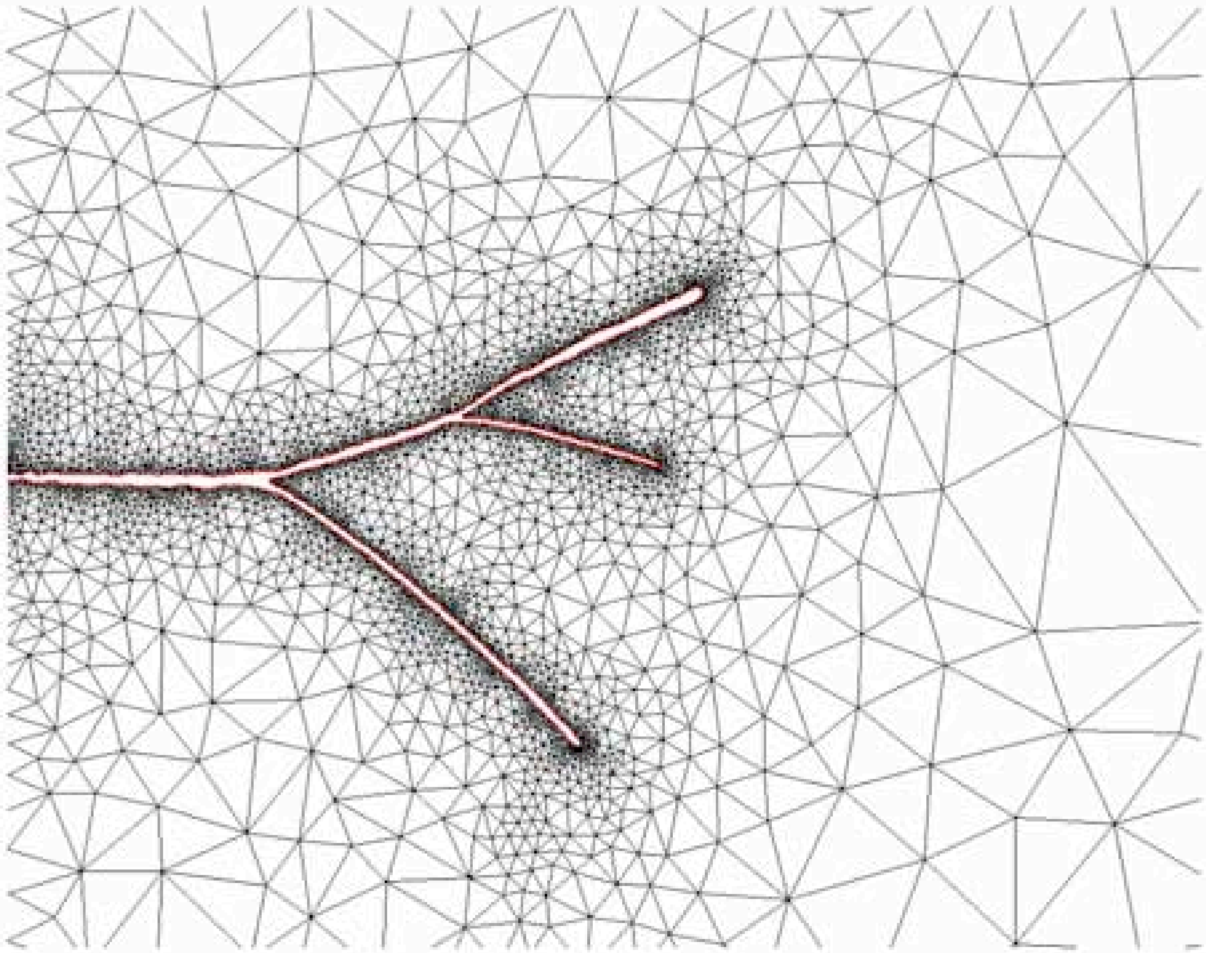

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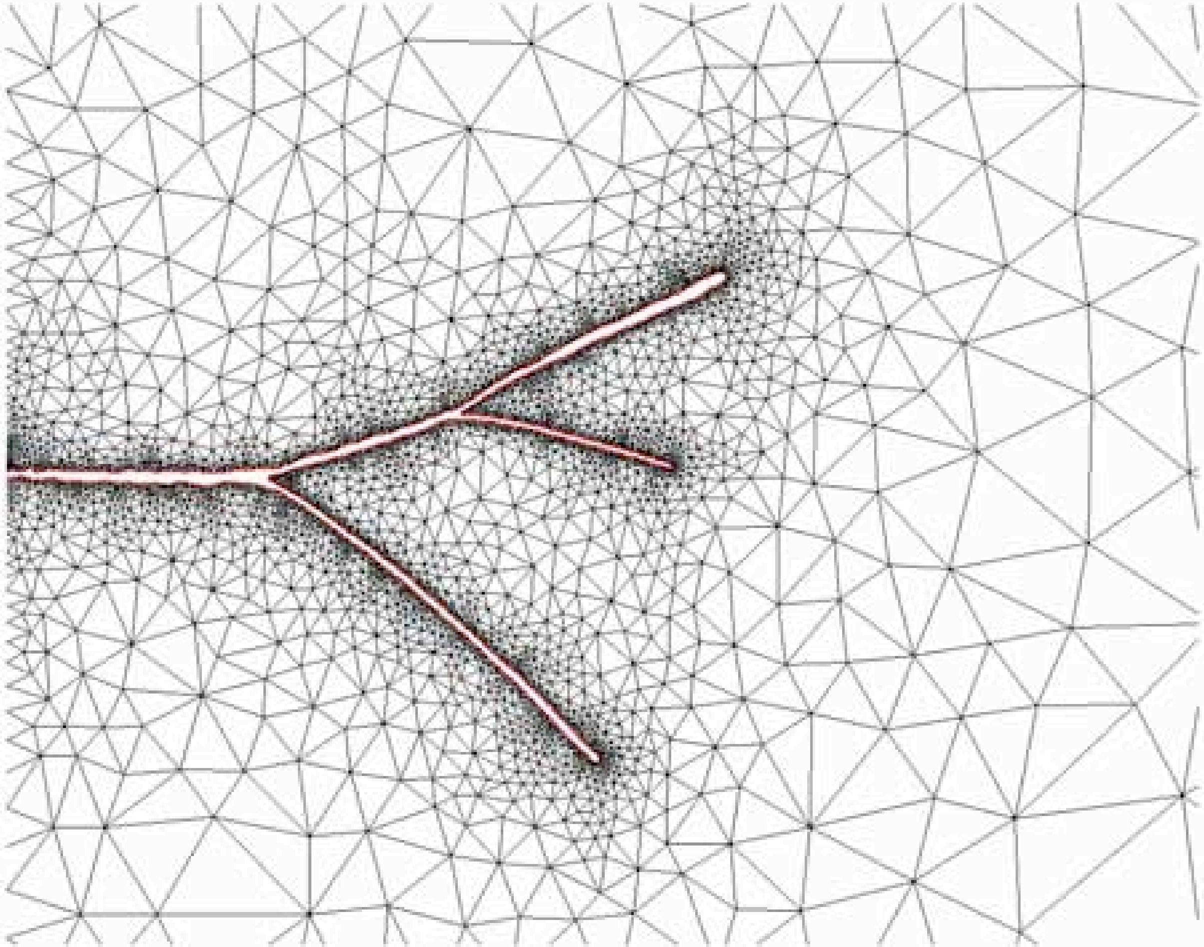

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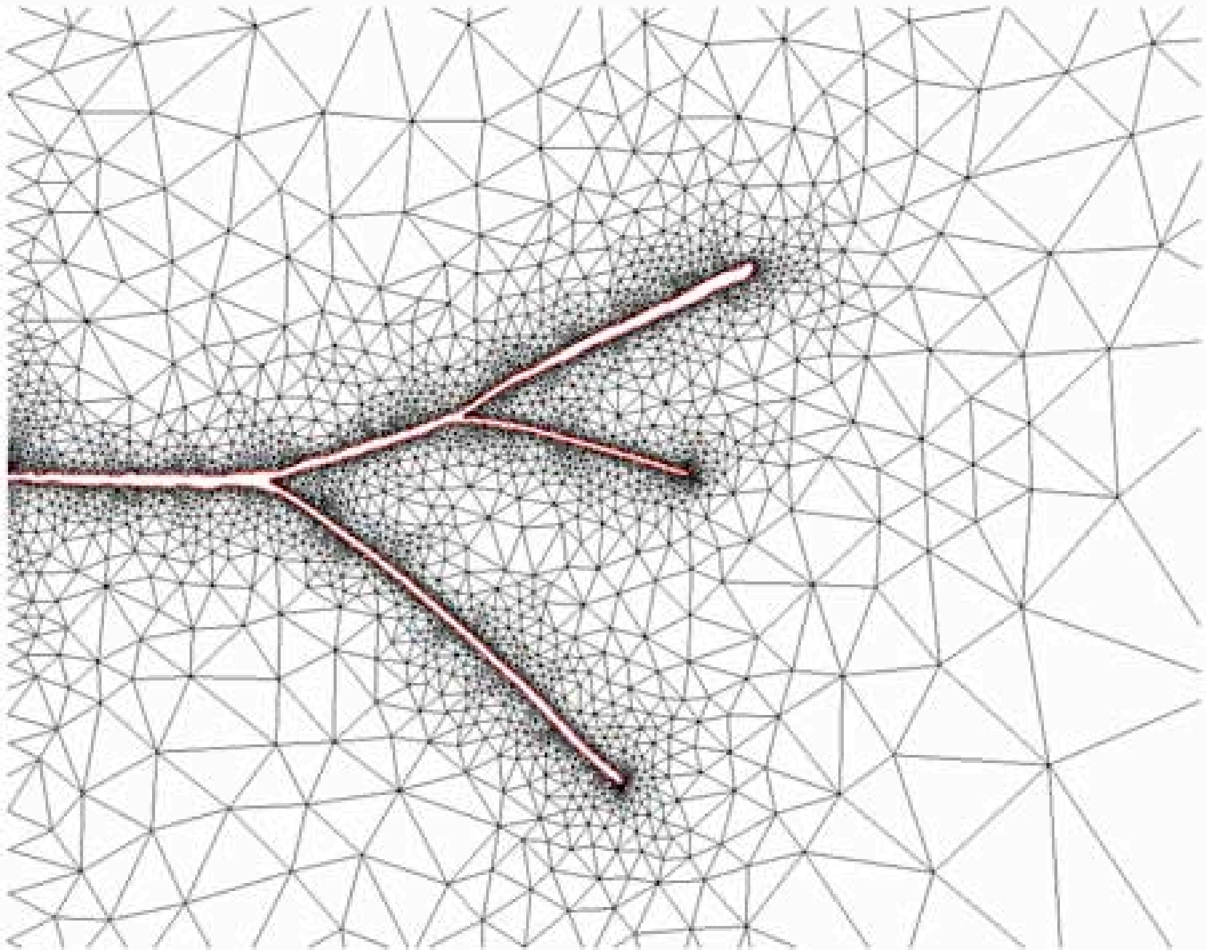

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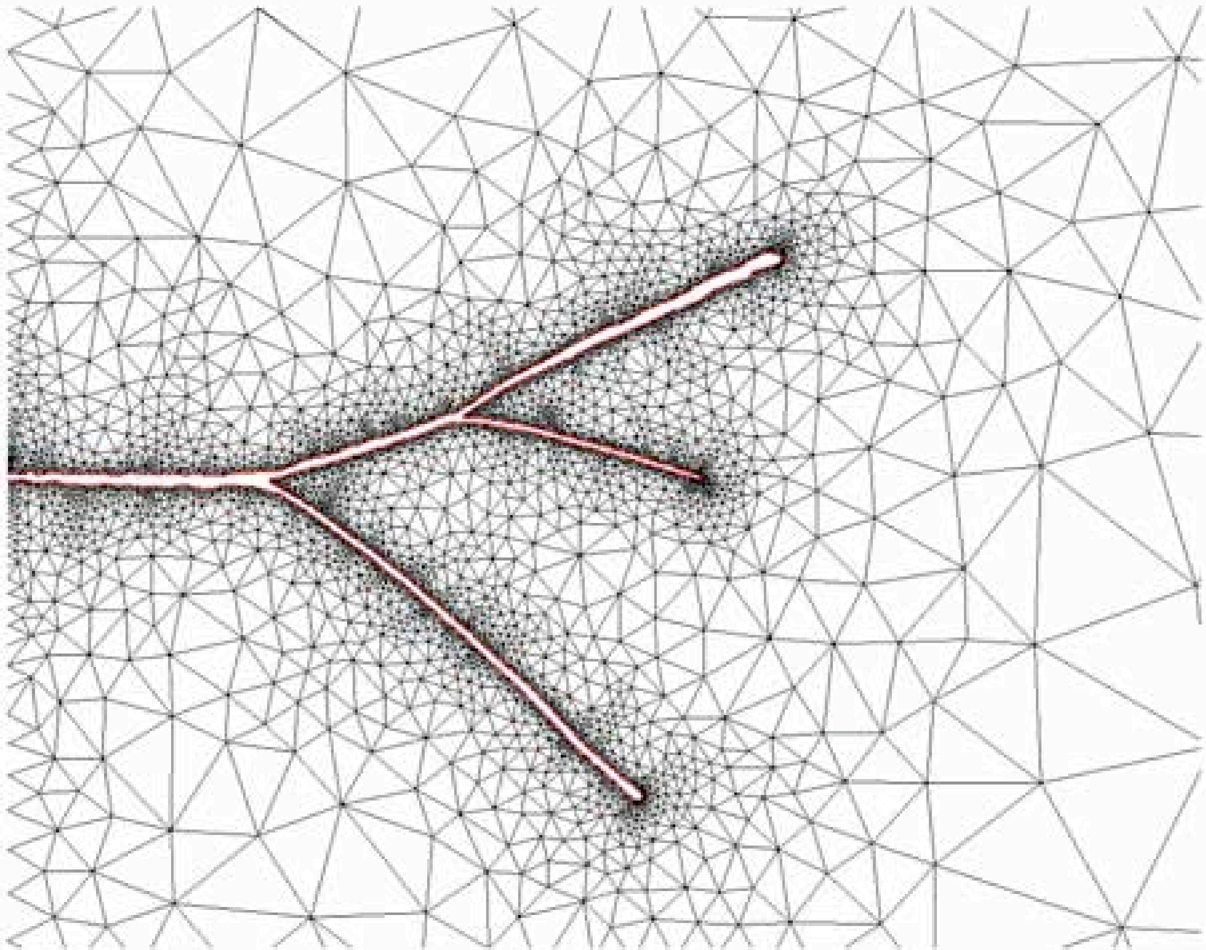

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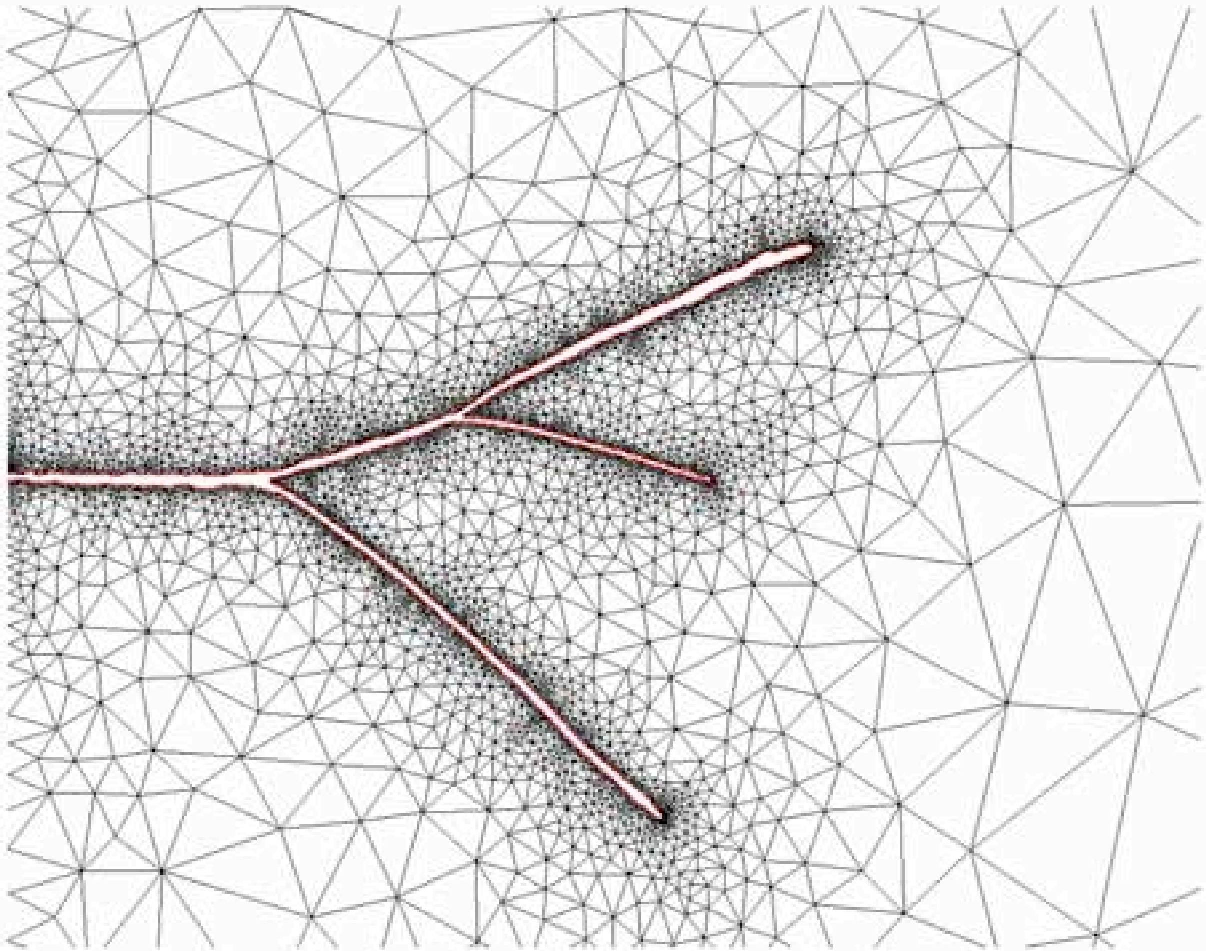

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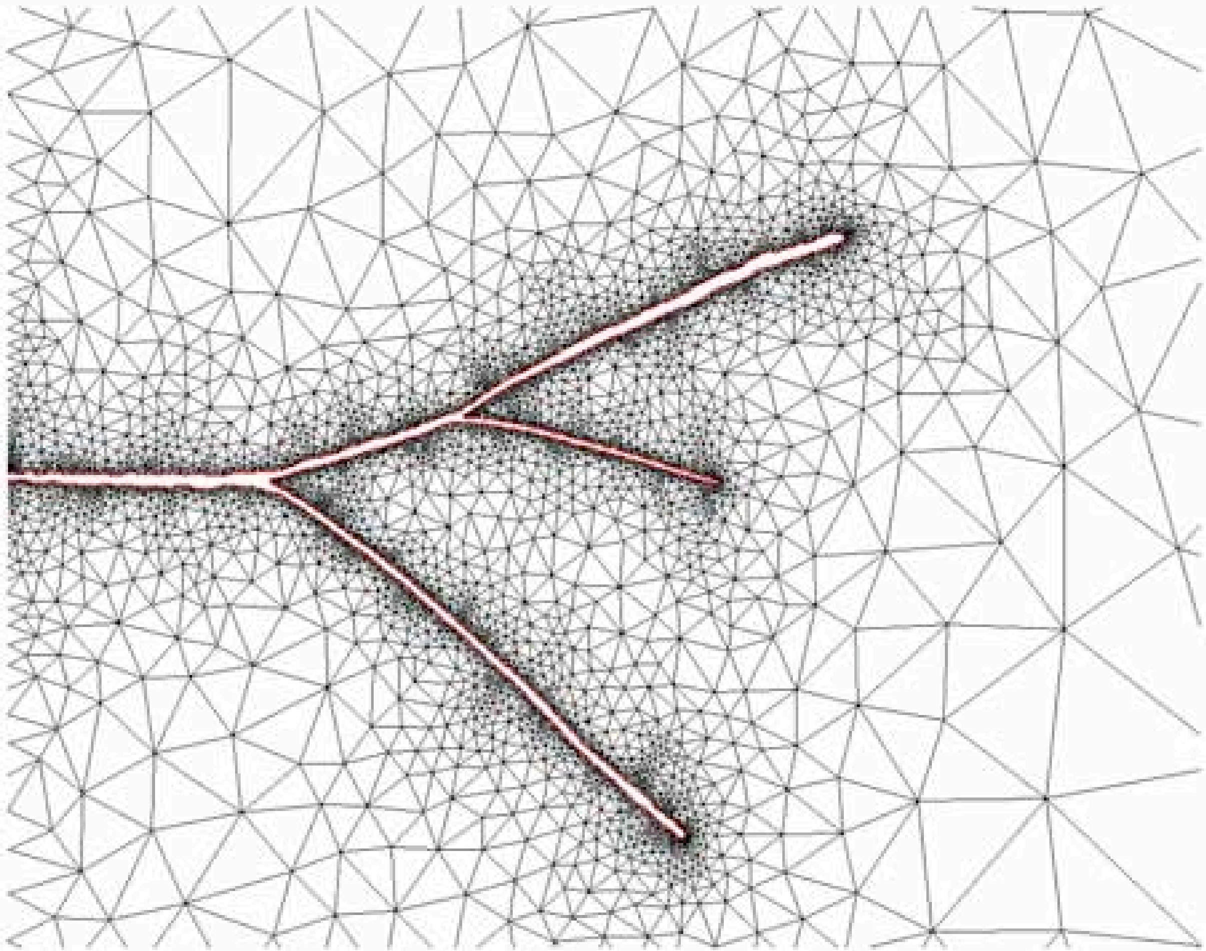

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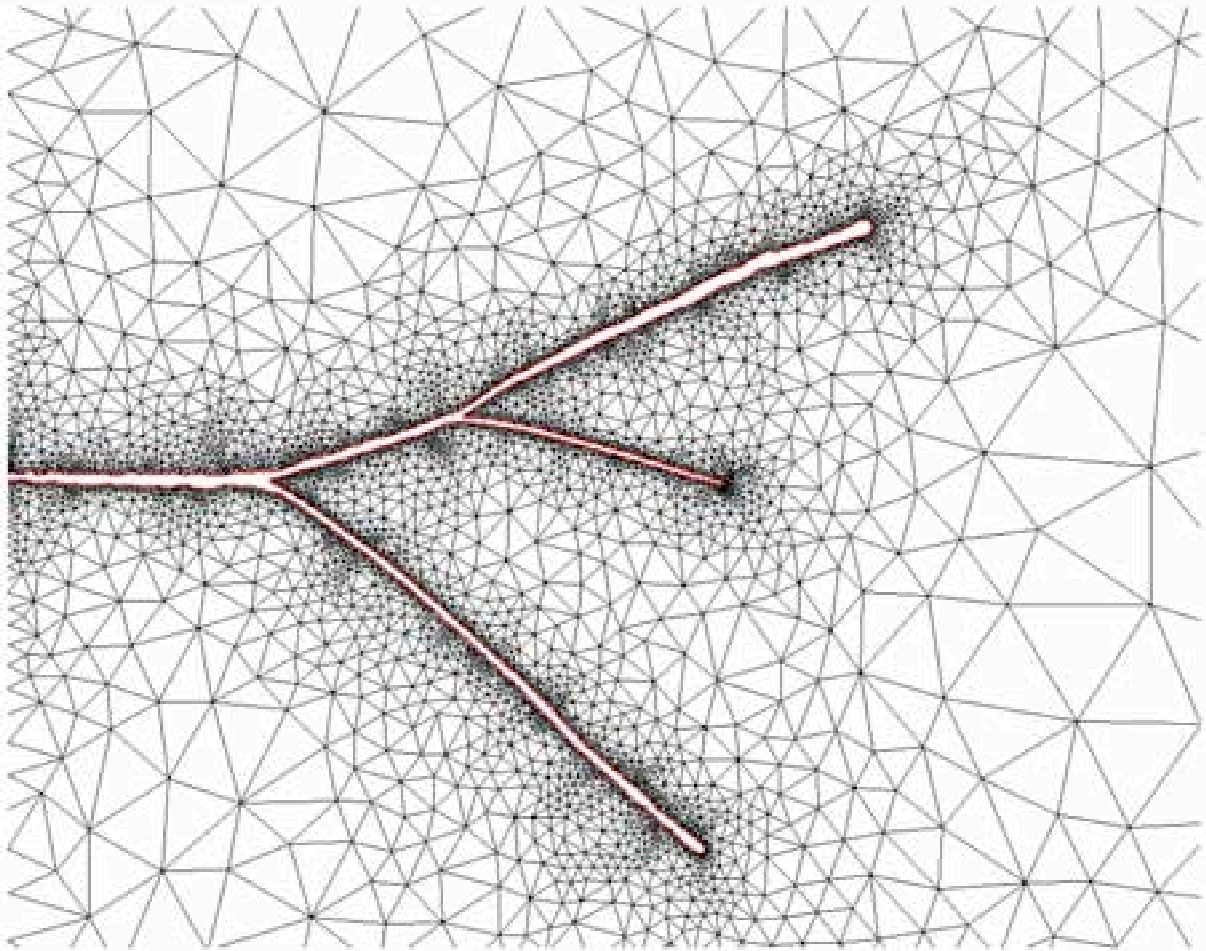

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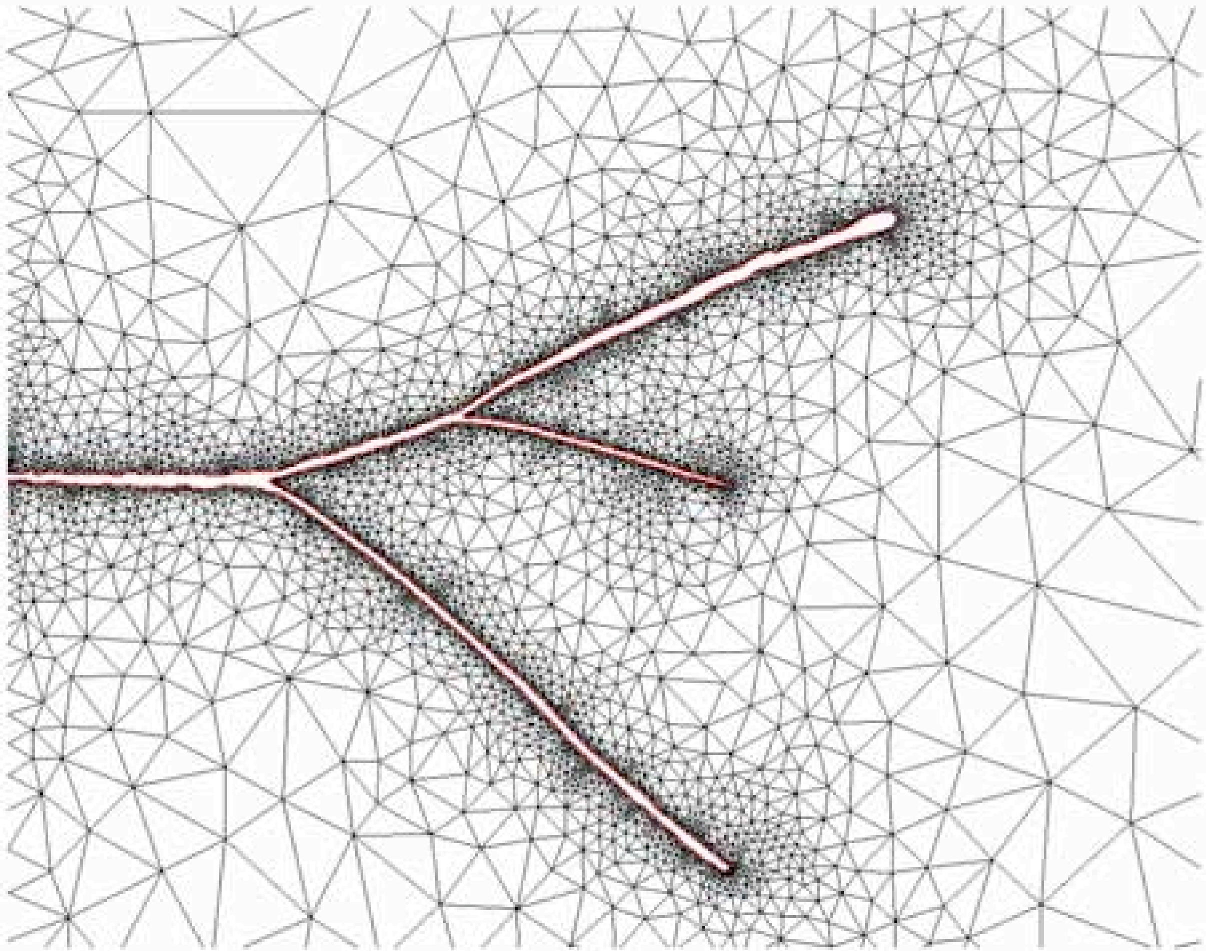

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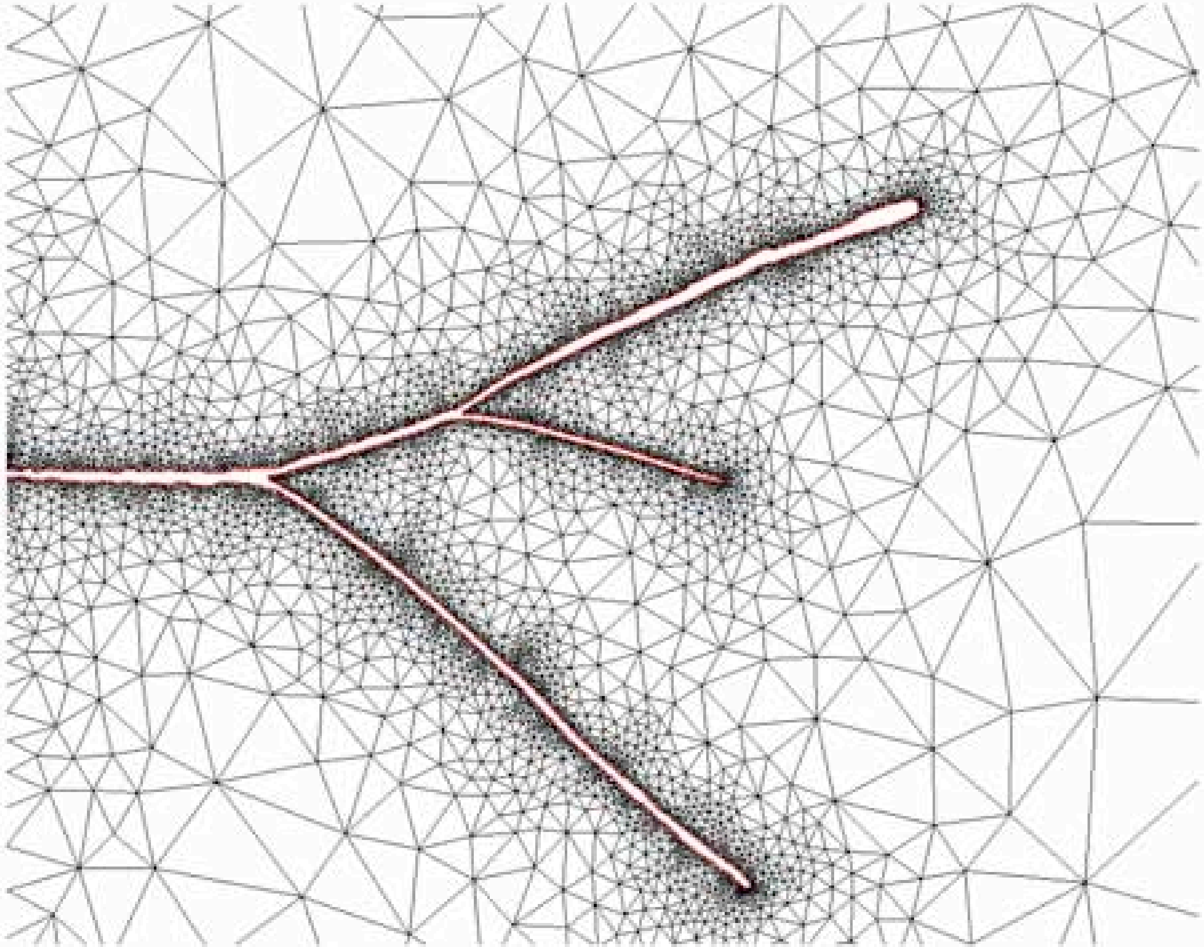

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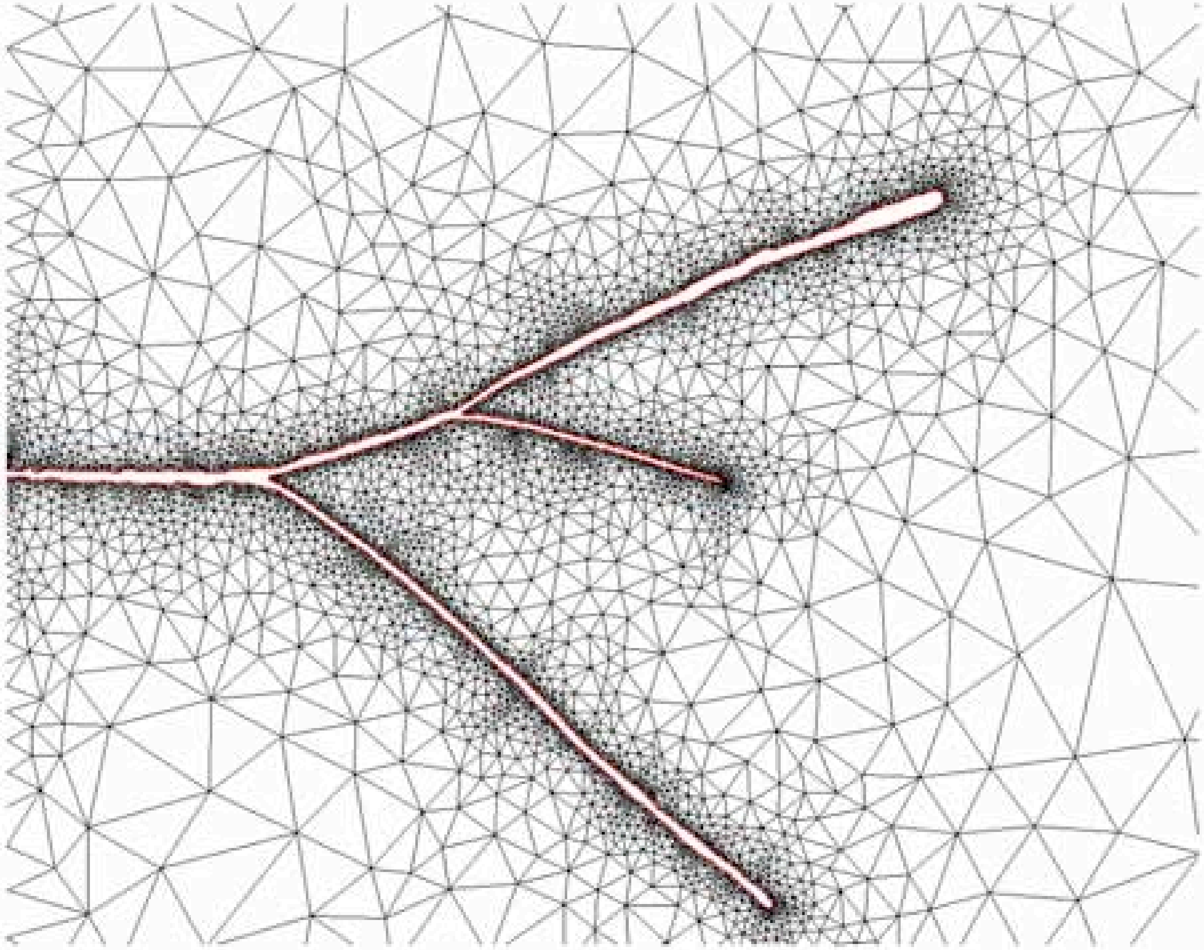

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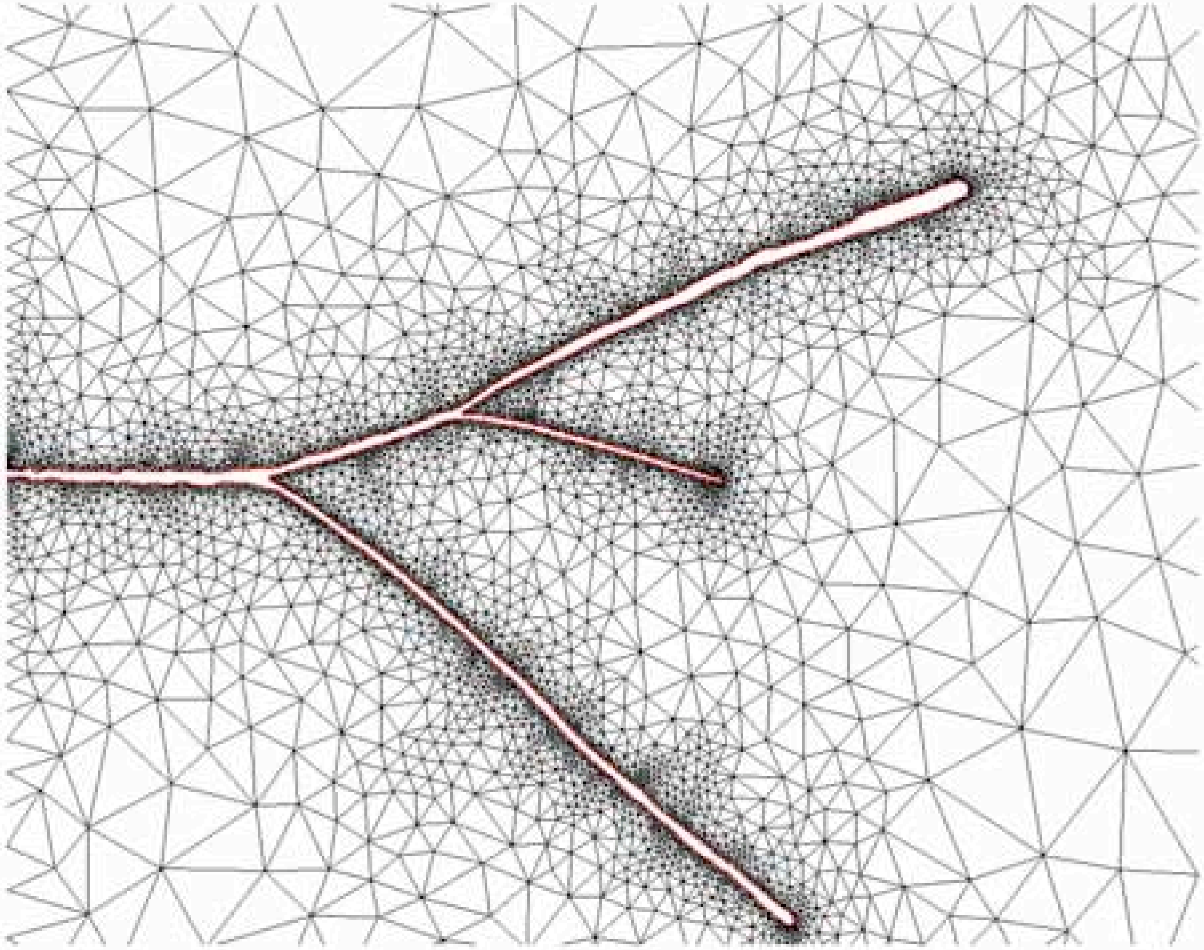

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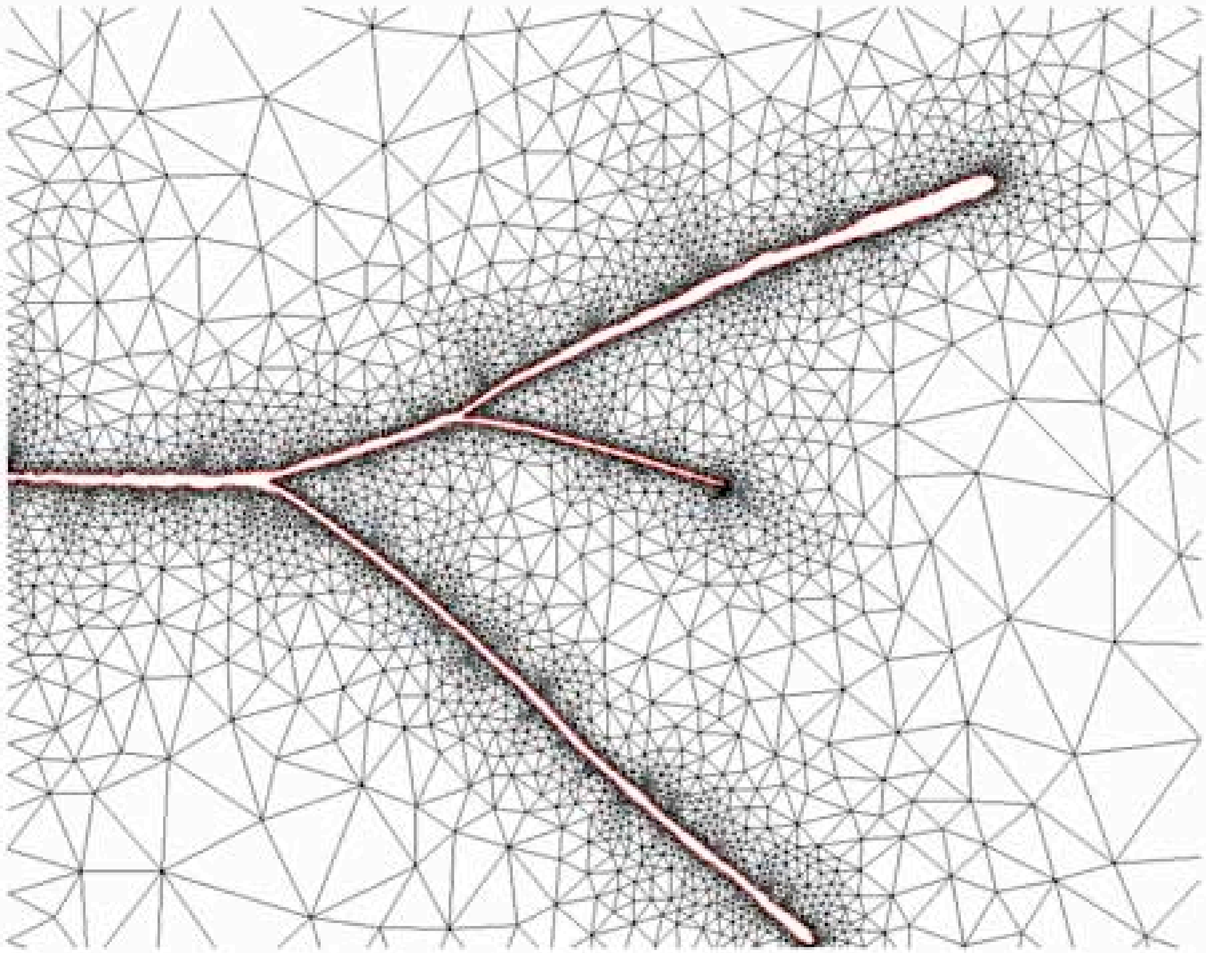

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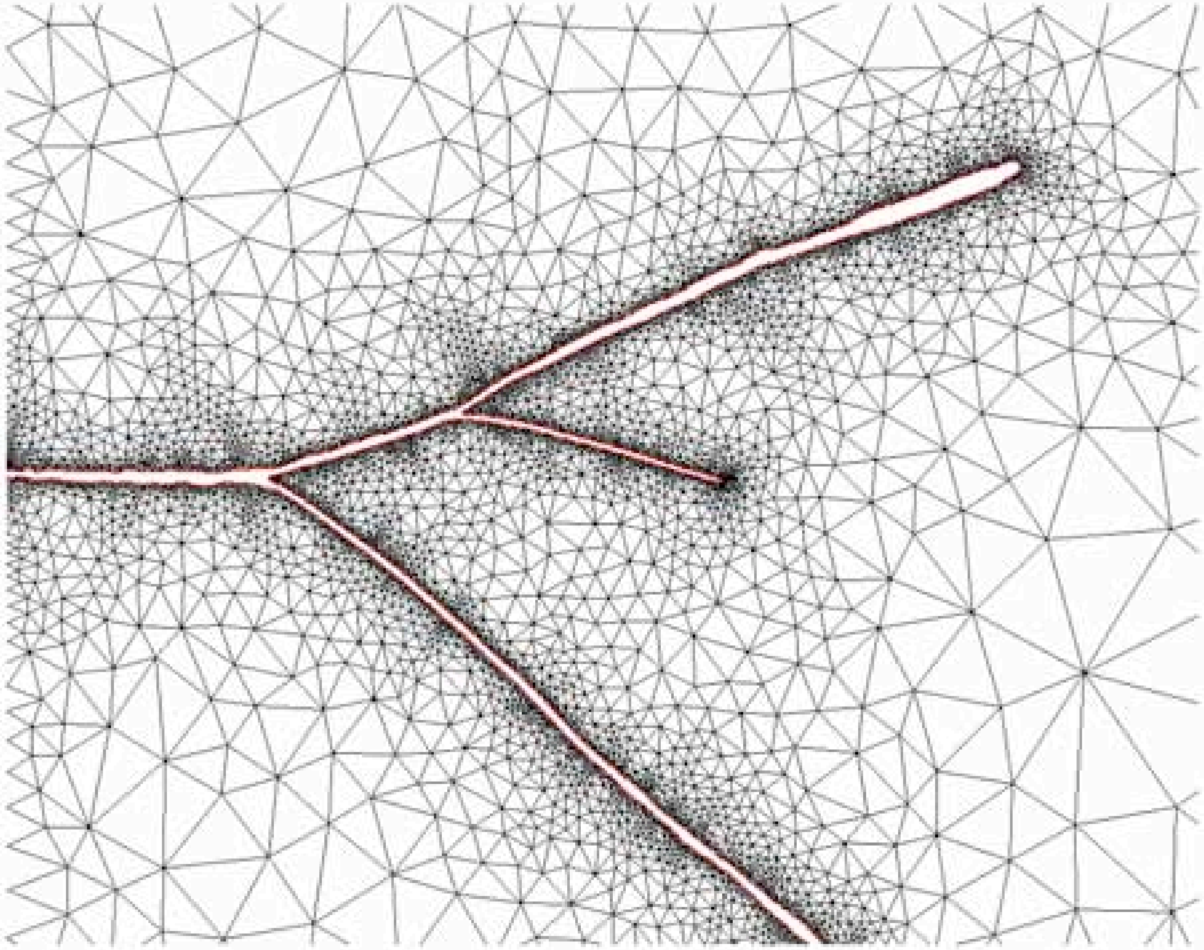

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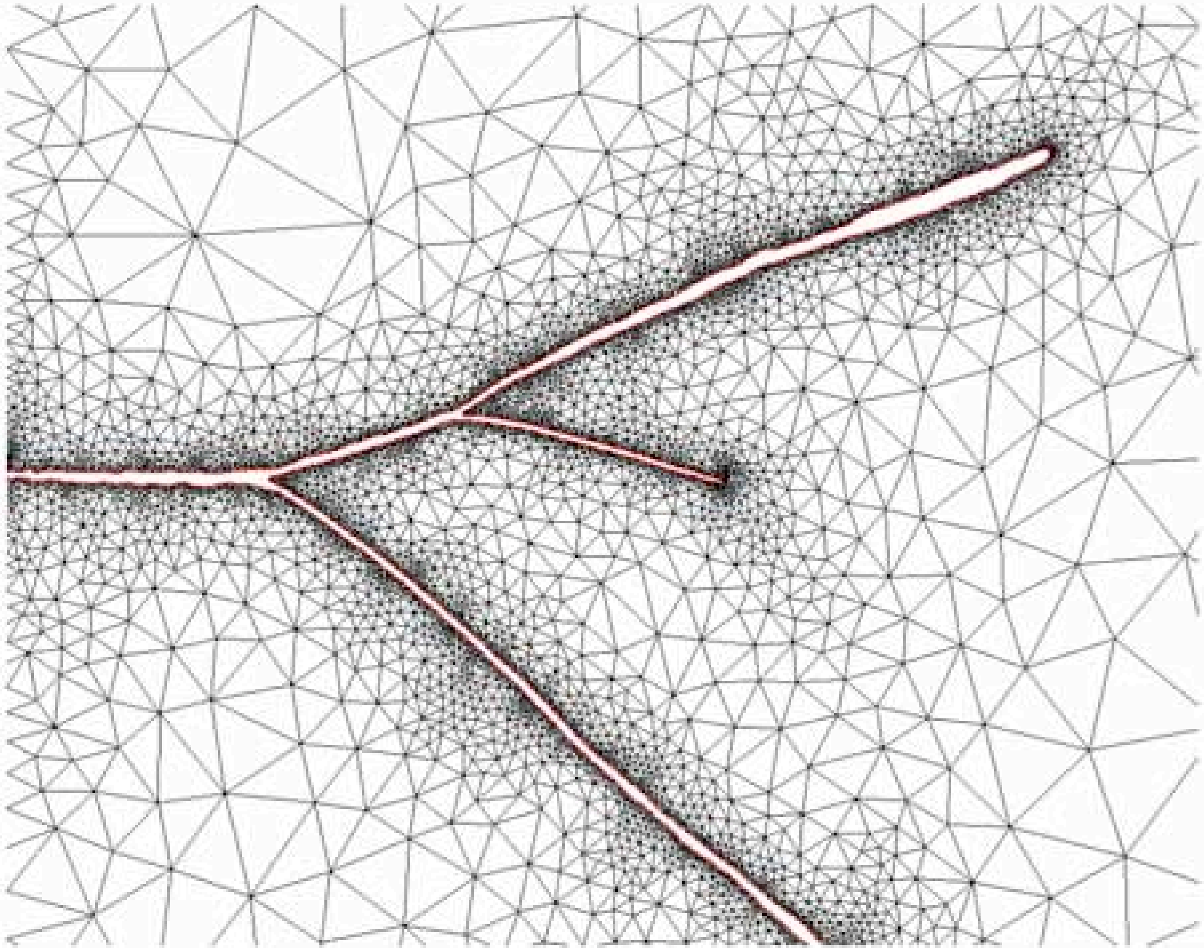

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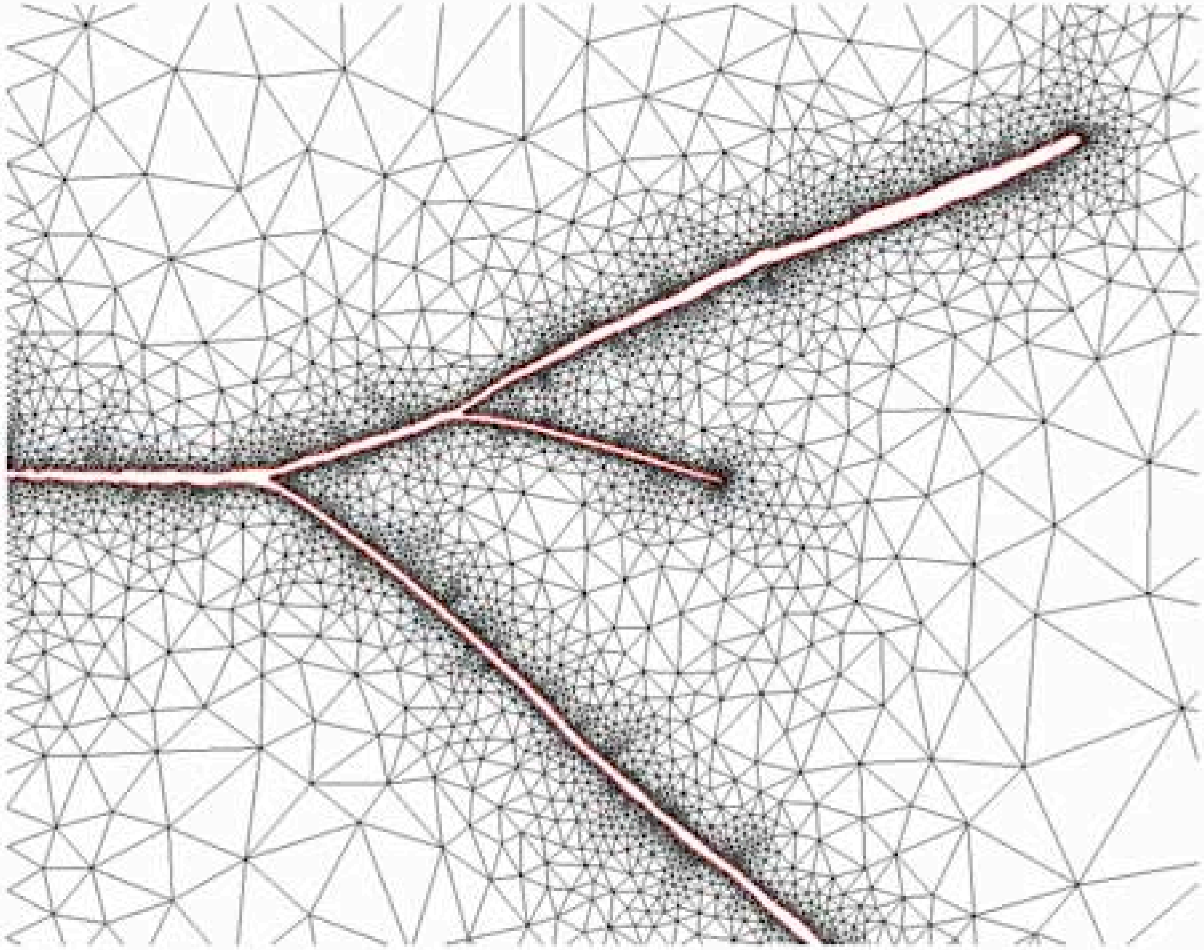

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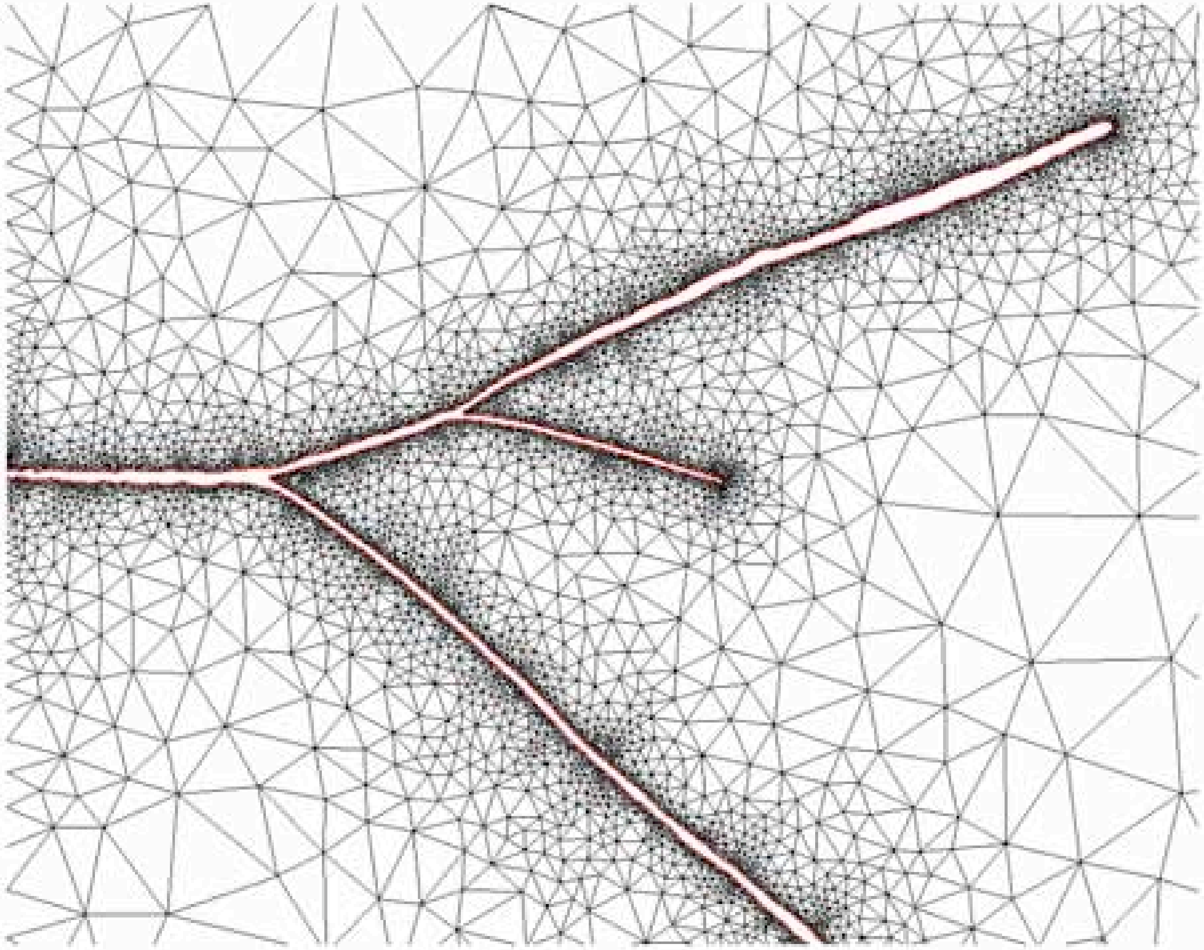

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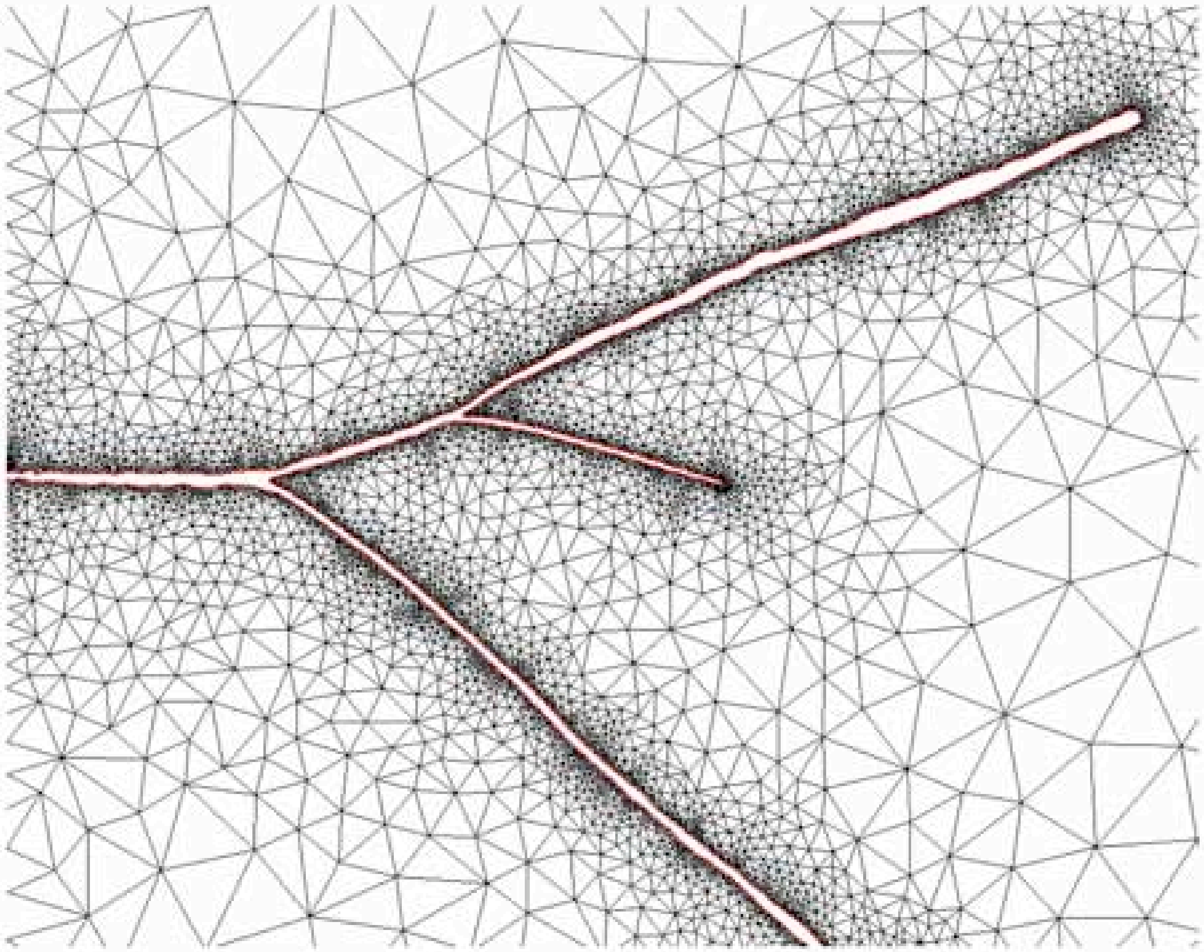

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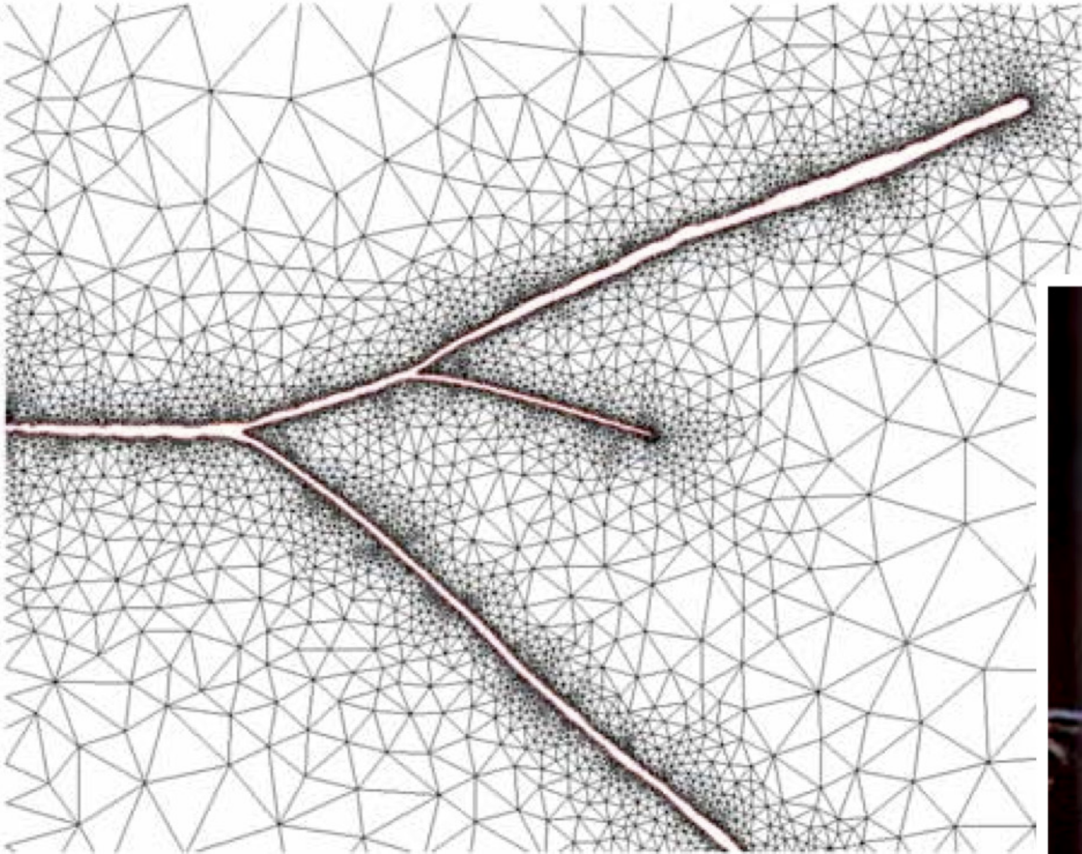
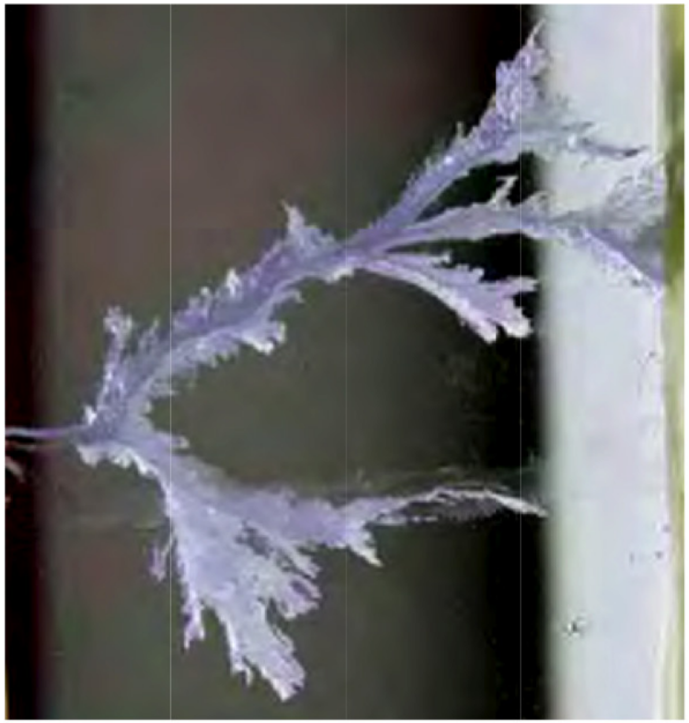
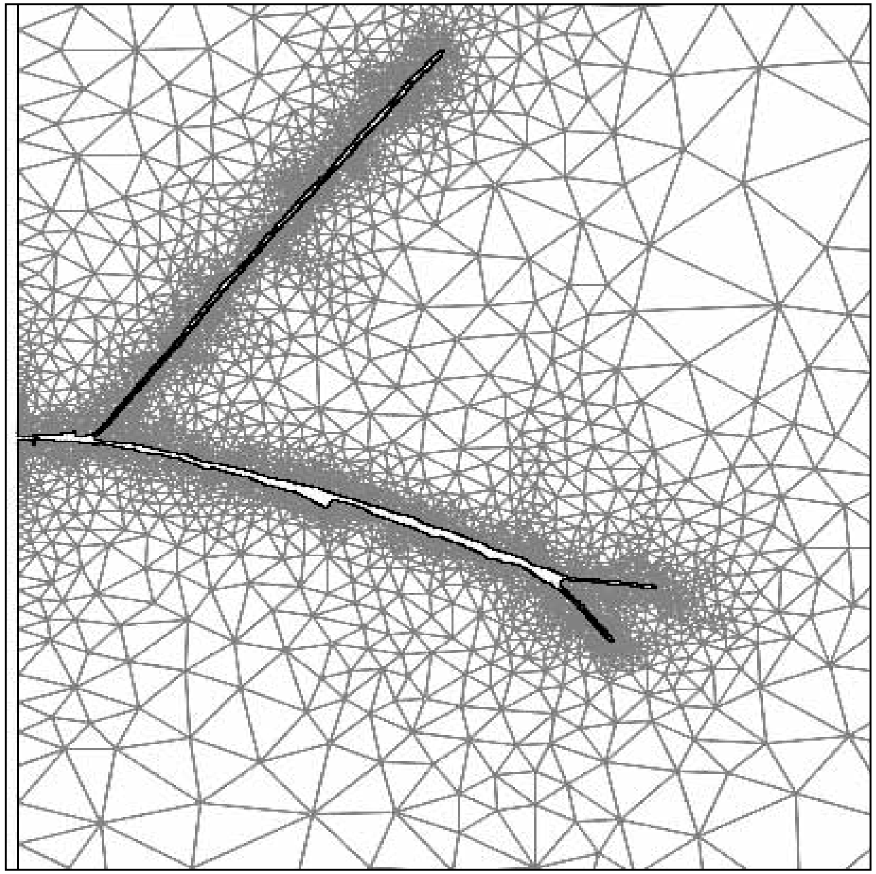
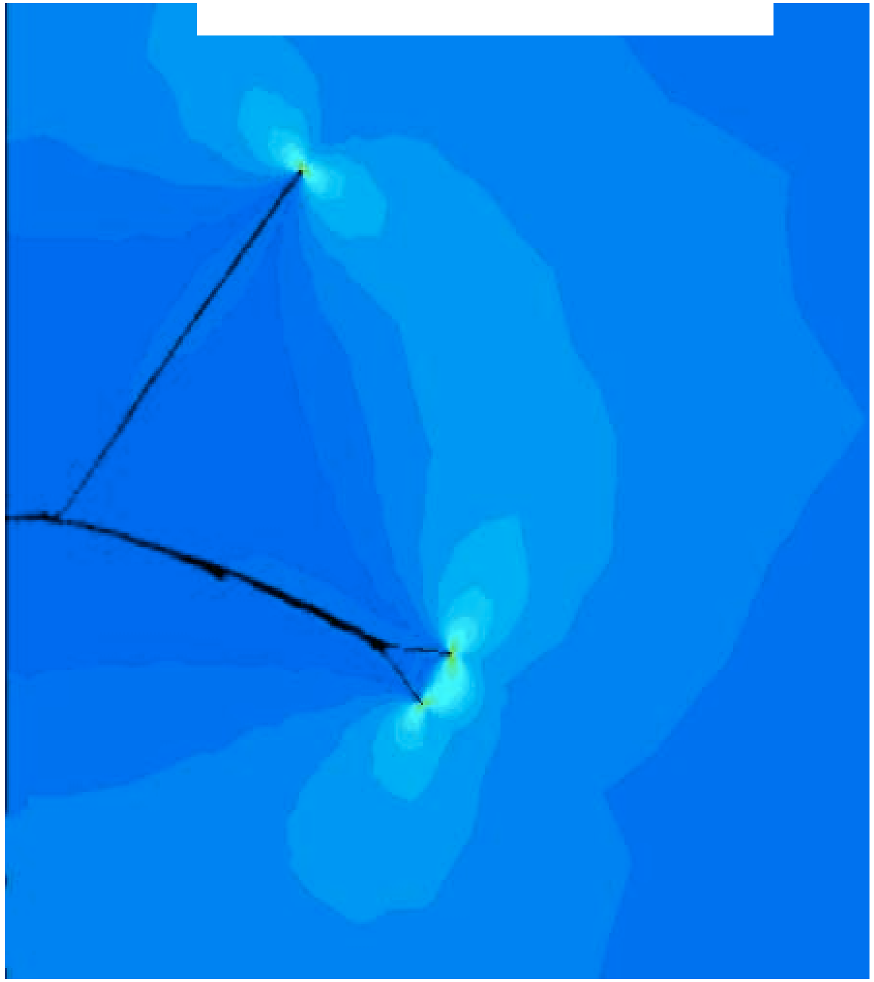




Crack-tip load

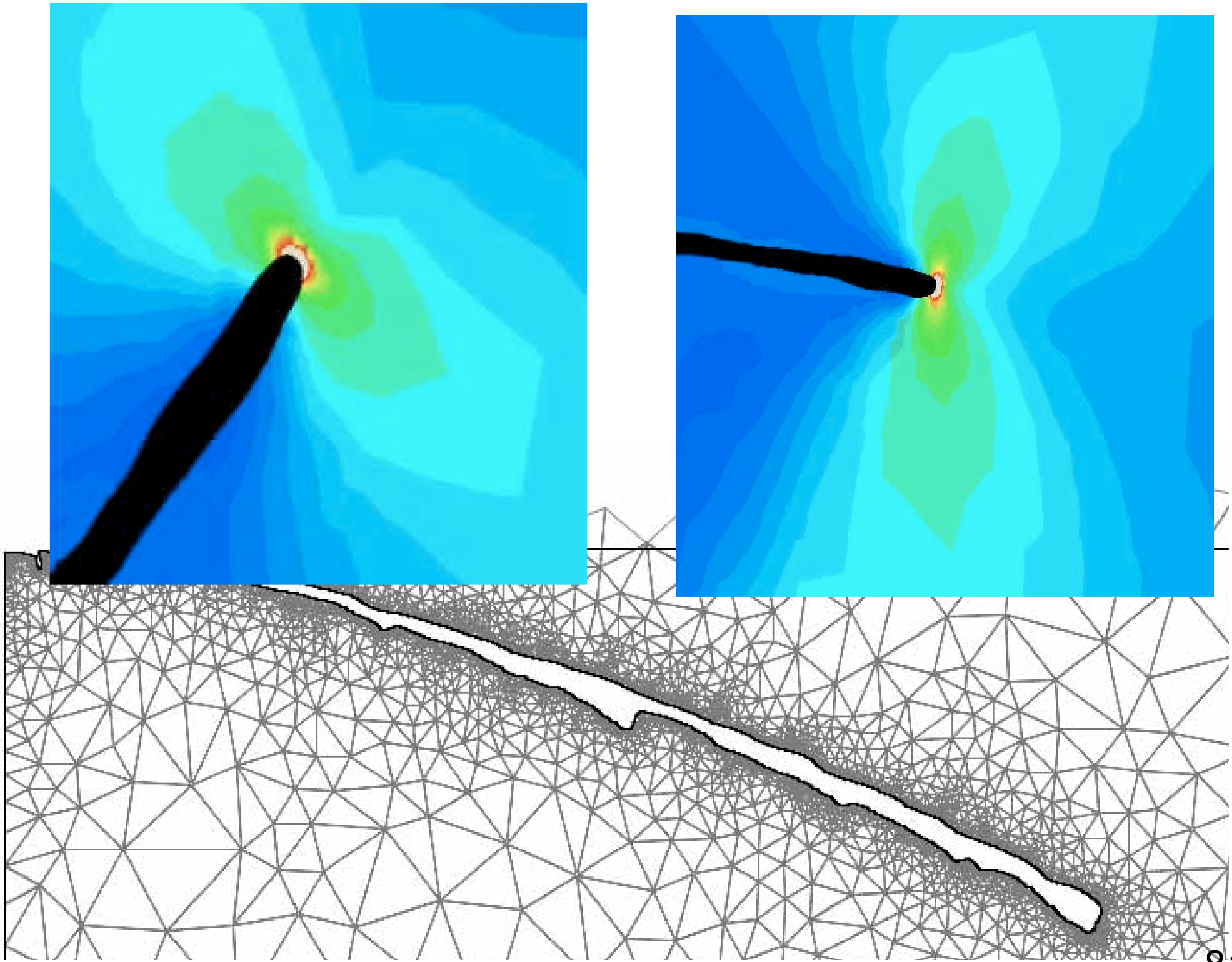


Before branching
$K_I = K_I', \rho = \rho'$

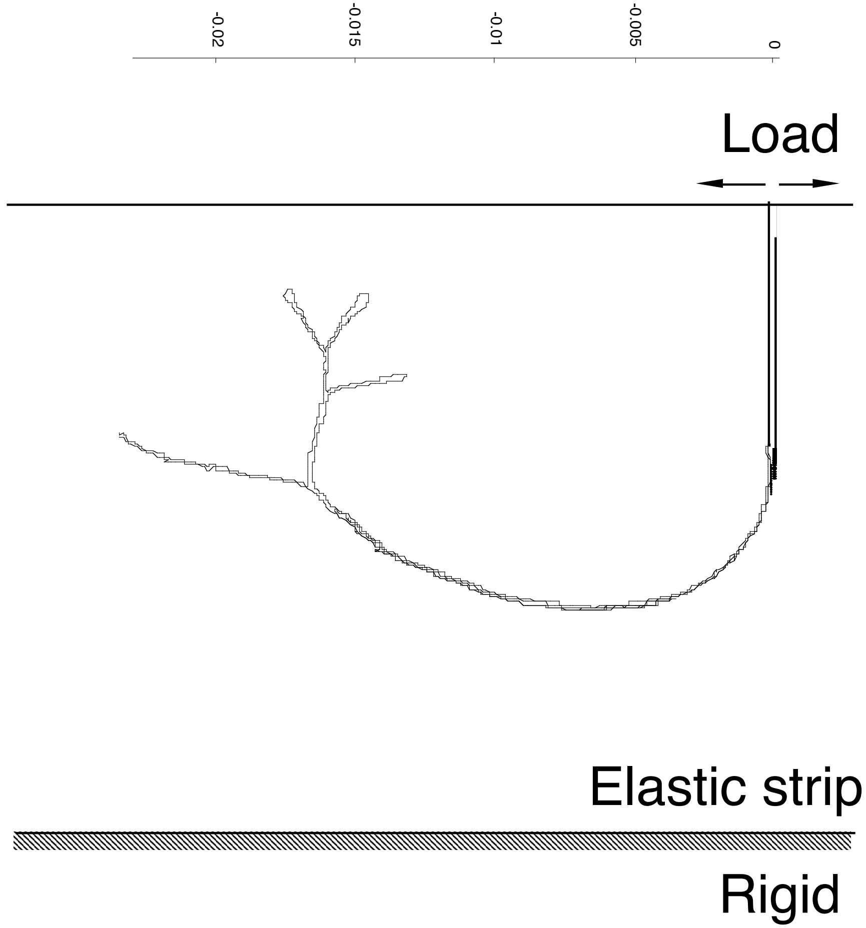


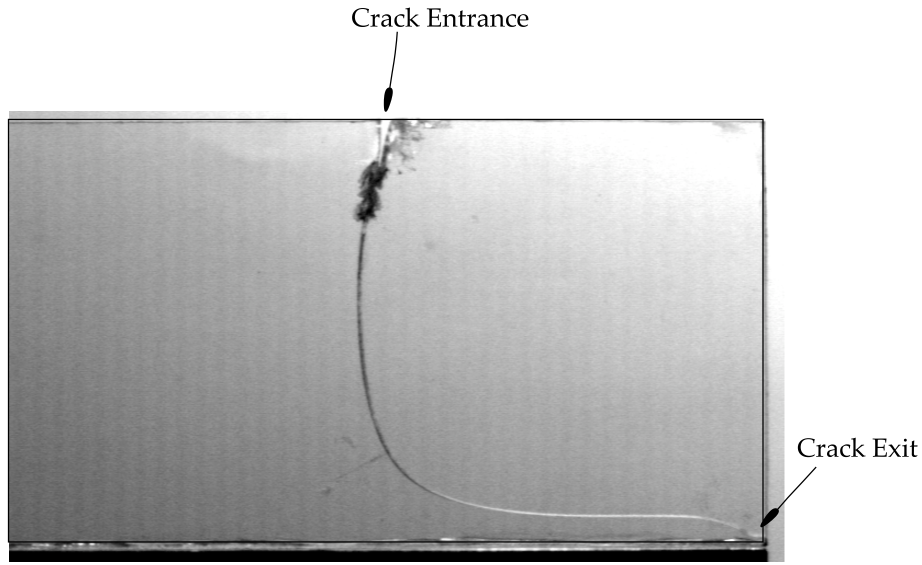
After branching
$K_I \approx 0.7K_I', \rho = \rho'/2$

Crack-tip Load

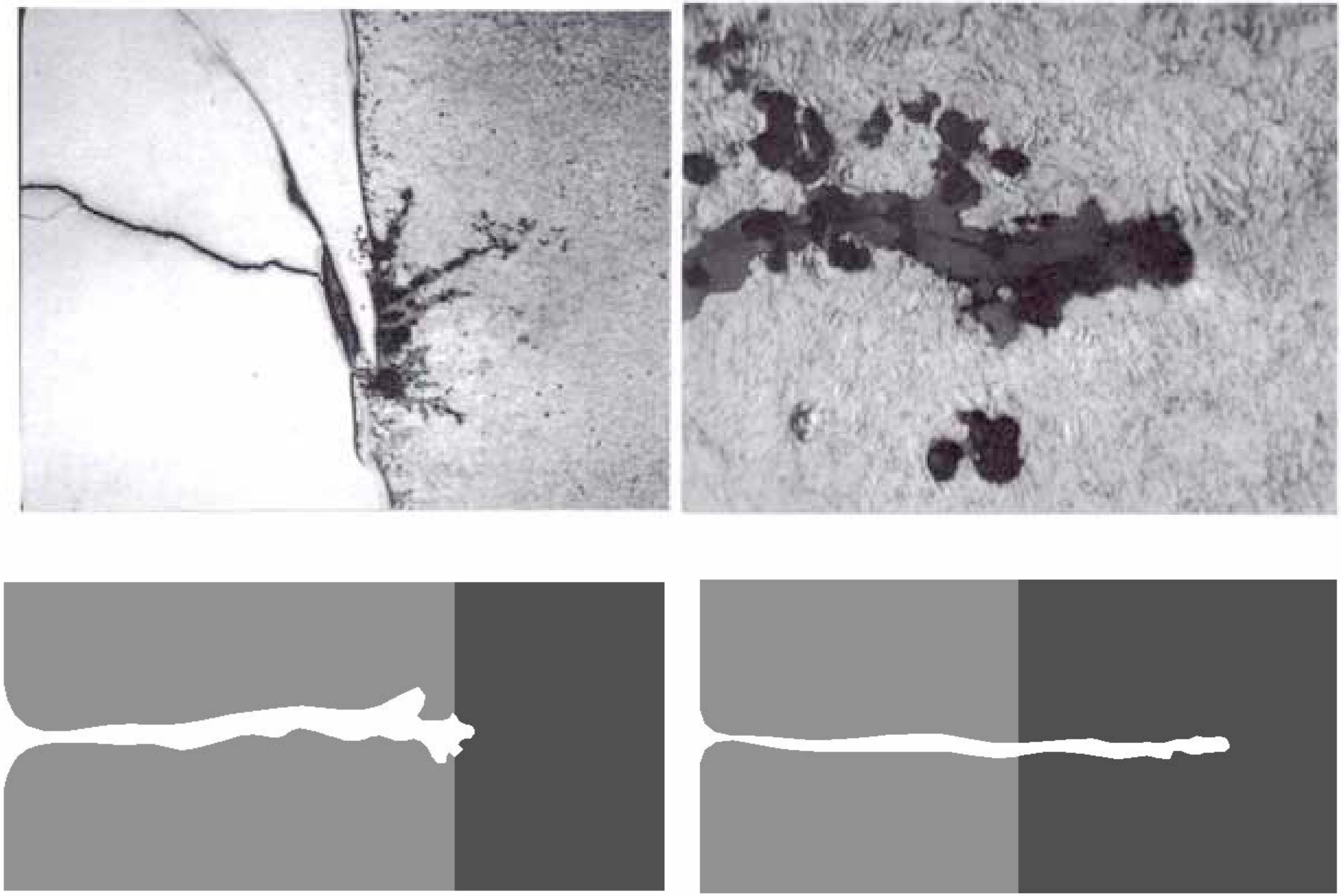


Crack seem to follow a mode I path

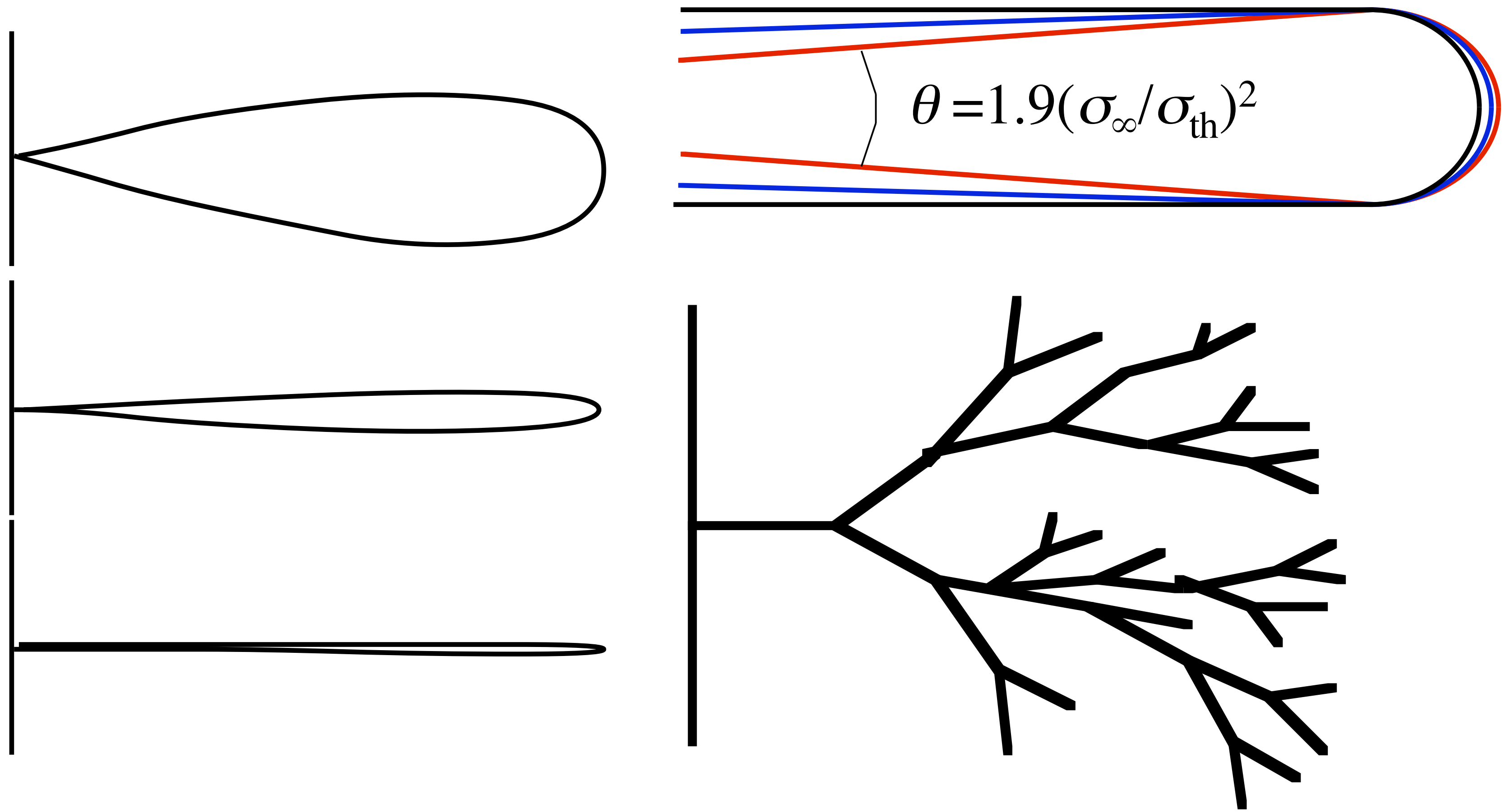




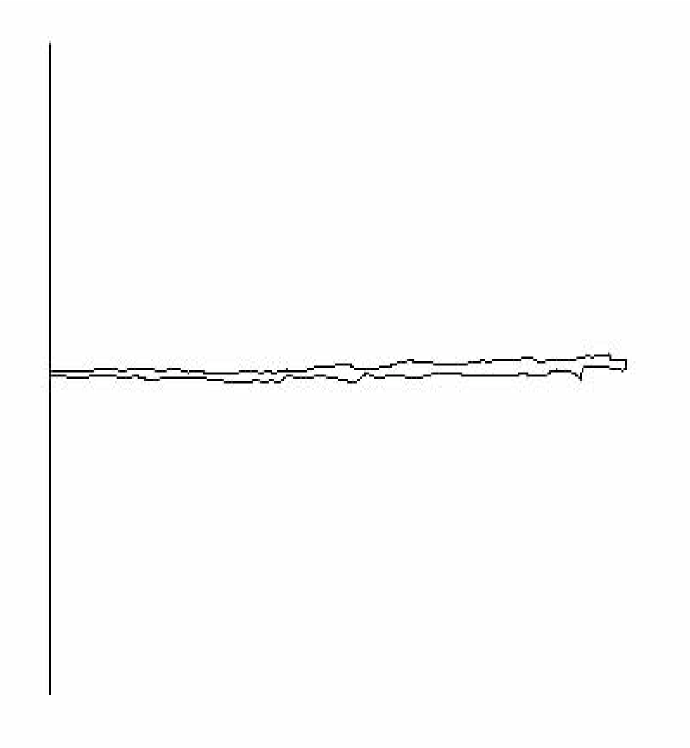
From Cladding to Grey Material



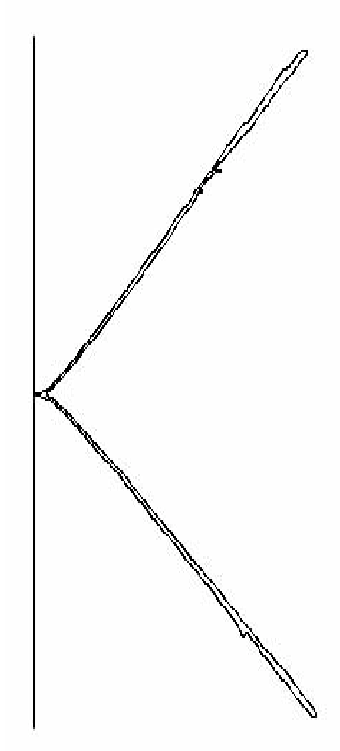
Missing lengthparameter => Selfsimilar growth



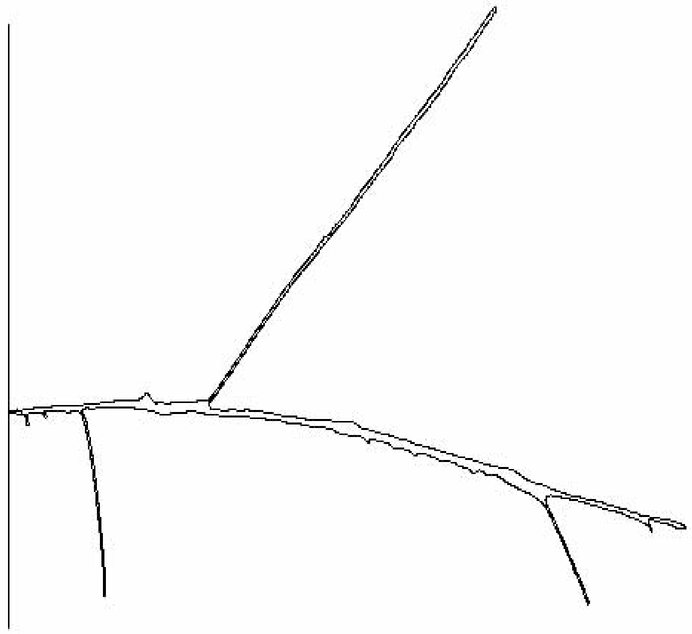
$\varepsilon_f/\varepsilon_\infty = 0.3,\ \sigma_x/\sigma_y = 0$



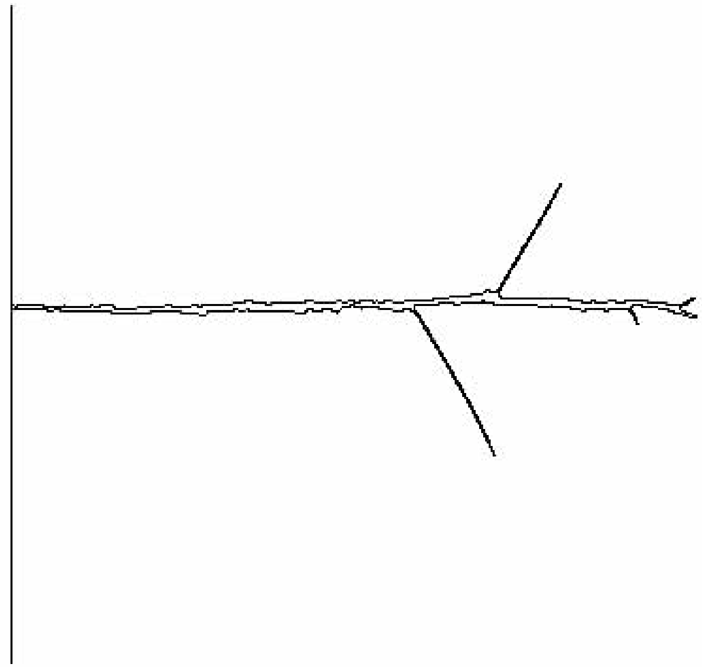
$\varepsilon_f/\varepsilon_\infty = 0.3,\ \sigma_x/\sigma_y = 1.1$

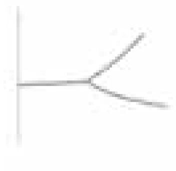
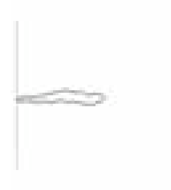
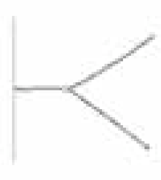


$\varepsilon_f/\varepsilon_\infty = 0.6,\ \sigma_x/\sigma_y = 0.9$



$\varepsilon_f/\varepsilon_\infty = 1.5,\ \sigma_x/\sigma_y = 0.7$



	$\varepsilon_f/\varepsilon_\infty = 0$	0.3	0.5	0.75	1	1.2
$\sigma_x/\sigma_y = 0$						
0.5						
0.7						
0.9						

Contributions to the free energy (Ginzburg & Landau, 1950)

$$\mathcal{F} = \mathcal{F}_{el} + \mathcal{F}_{ch} + \mathcal{F}_{gr}$$

Volume totals:

Elastic energy $$\mathcal{F}_{el} = \int \frac{G(\psi)}{2} (\nabla w)^2 dV$$

Chemical energy $$\mathcal{F}_{ch} = \int U(\psi)\, dV$$

Gradient energy $$\mathcal{F}_{gr} = \int \frac{g_b}{2} (\nabla \psi)^2 dV$$

Antiplane deformation => Two free variables

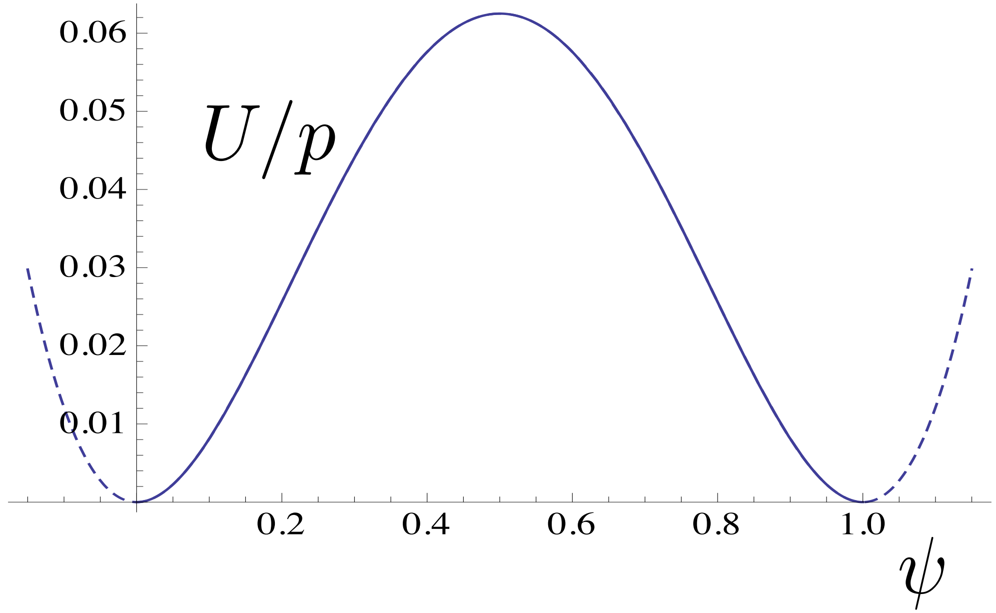
Displacements w and phase (density) ψ

$$\frac{\partial \psi}{\partial t} = -L_\psi \frac{\delta \mathcal{F}}{\delta \psi} \quad , \qquad \frac{\partial w}{\partial t} = -L_w \frac{\delta \mathcal{F}}{\delta w}$$

Ginzburg, Landau (50)

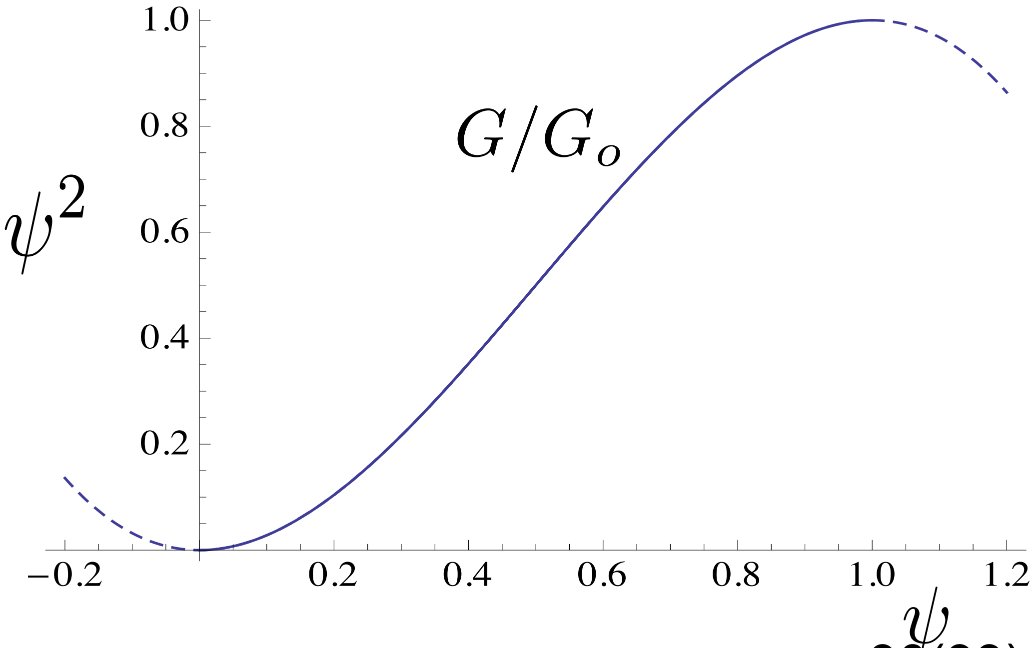
Double-well chemical potential

$$U(\psi) = p\,\psi^2(1-\psi)^2$$



Shear modulus

$$G(\psi) = G_o(\psi)(-2\psi + 3)\psi^2$$



Evolution of the phase

$$\frac{\partial \psi}{\partial t} = -L_\psi[\frac{1}{2}G'(\psi)(\nabla w)^2 + p\psi(\psi^2 - 1) - g_b \triangle \psi]$$

Evolution of the displacements

$$\frac{\partial w}{\partial t} = L_w \nabla \cdot [G(\psi)\nabla w]$$

At equilibrium: $\nabla \cdot [G(\psi)\nabla w] = 0$

One dimension (Ginzburg, Landau)

$$g_b\psi_o'' - p\psi_o(\psi_o^2 - 1) = 0$$

solution

$$\psi_o = \tanh(x/\sqrt{2g_b})$$

With mechanical loading

$$g_b\psi'' - p\psi(\psi^2 - 1) + \kappa p(\psi^2 - 1) - \frac{c}{L_\psi}\psi' = 0$$

Seek perturbation solution:

$$\psi(x) = \psi_o(x) + \omega f(x) \quad \text{as} \quad \omega = c/L_\psi G_o(\nabla w)^2\sqrt{pg_b} \to 0$$

The result

$$\psi_o' = \psi_o^2 - 1 = \frac{1}{\sqrt{2g_b}} \mathrm{sech}^2(x/\sqrt{2g_b}) \quad ,$$

gives

$$(\kappa - \frac{c}{L_\psi}) \frac{p}{\sqrt{2g_b}} \mathrm{sech}^2(x/\sqrt{2g_b}) = 0 \quad ,$$

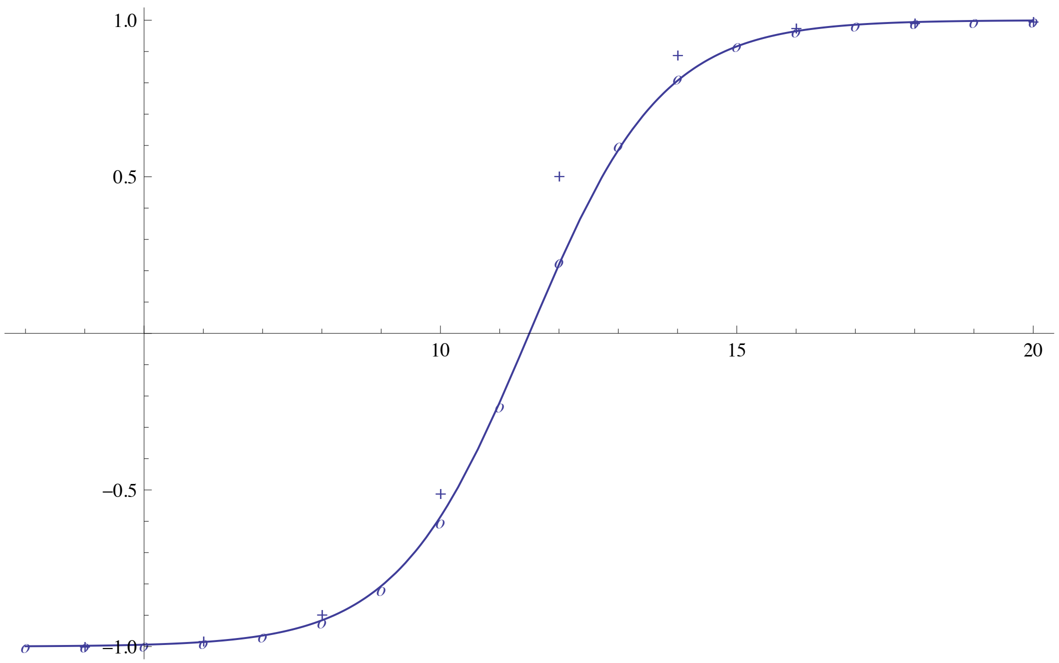
i.e. the speed of the corroding edge

$$c = L_\psi \kappa = \frac{3}{4} p L_\psi G_o (\nabla w)^2 \sqrt{\frac{2g_b}{p}} \quad .$$

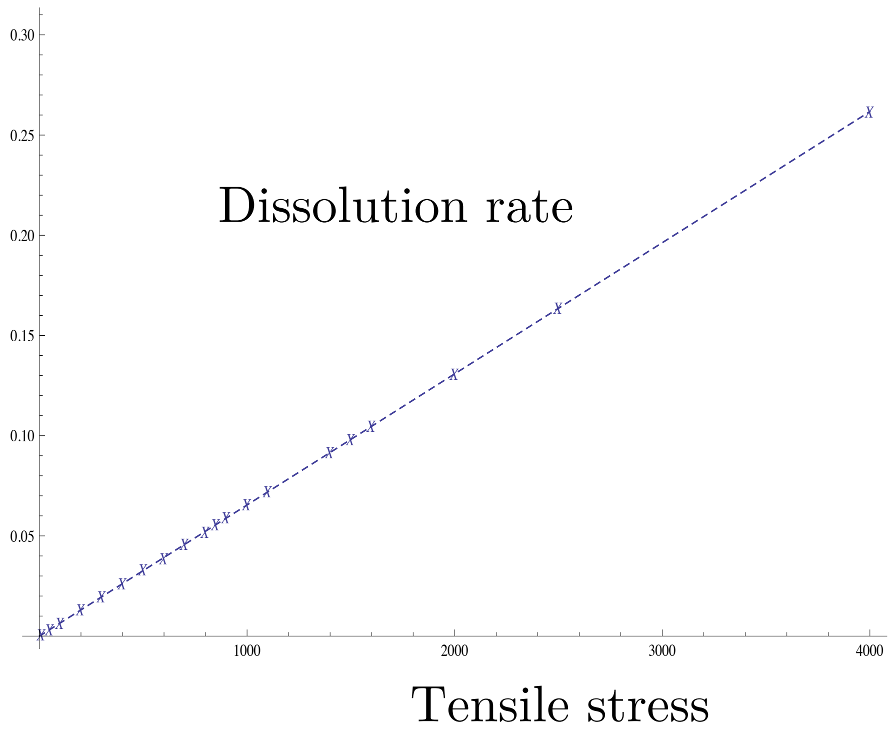
Steady state solution

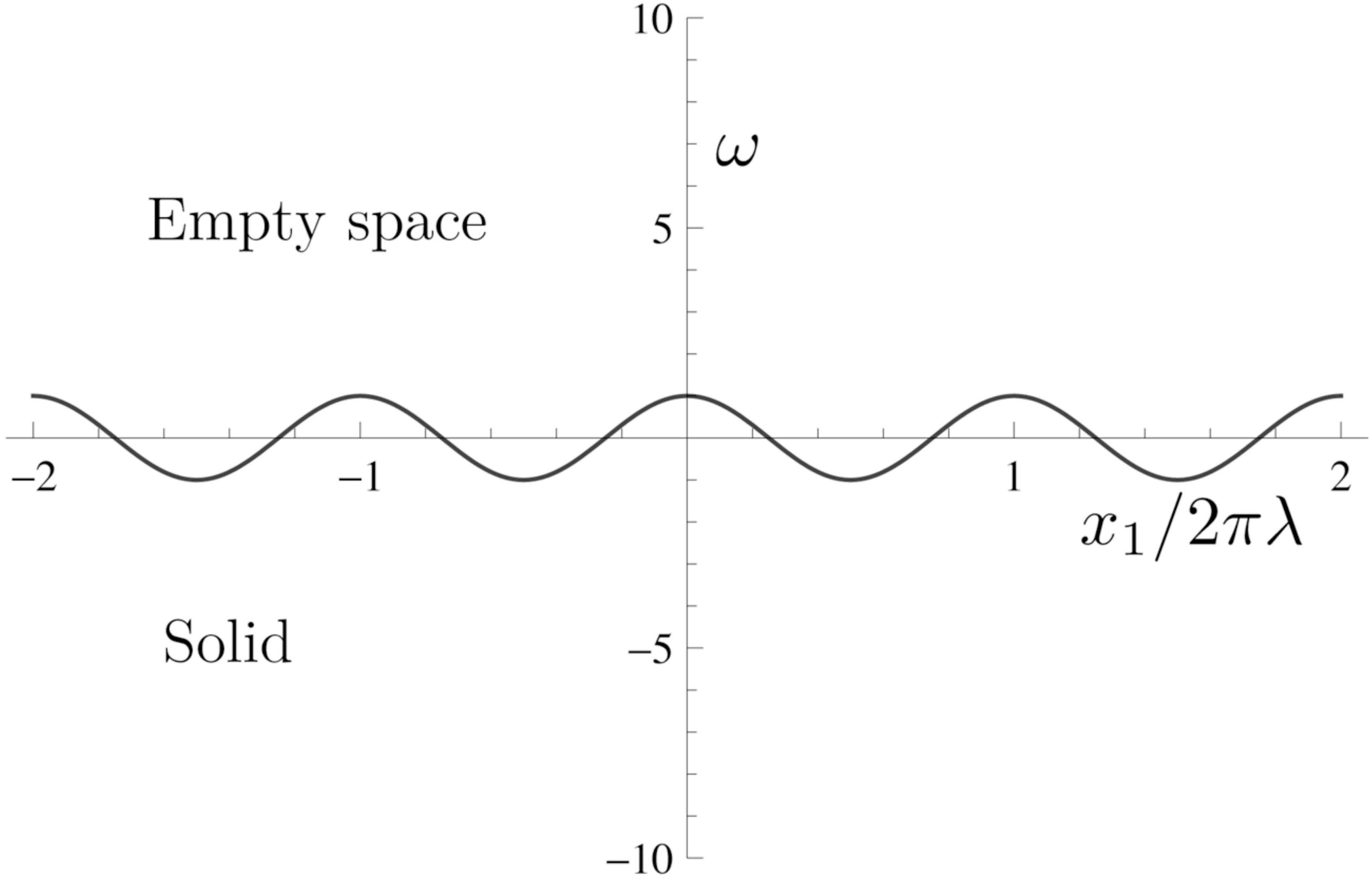
$$\psi = -\tanh\left(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t\right)$$

Transition Rate vs. Distance



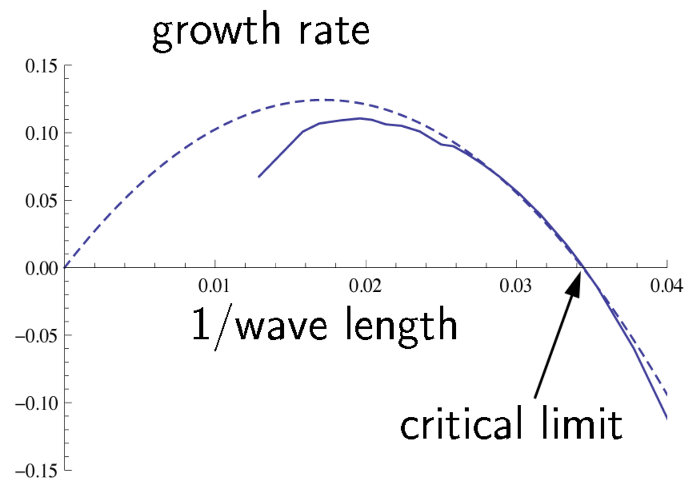
Dissolution Rate vs. Tensile Stress

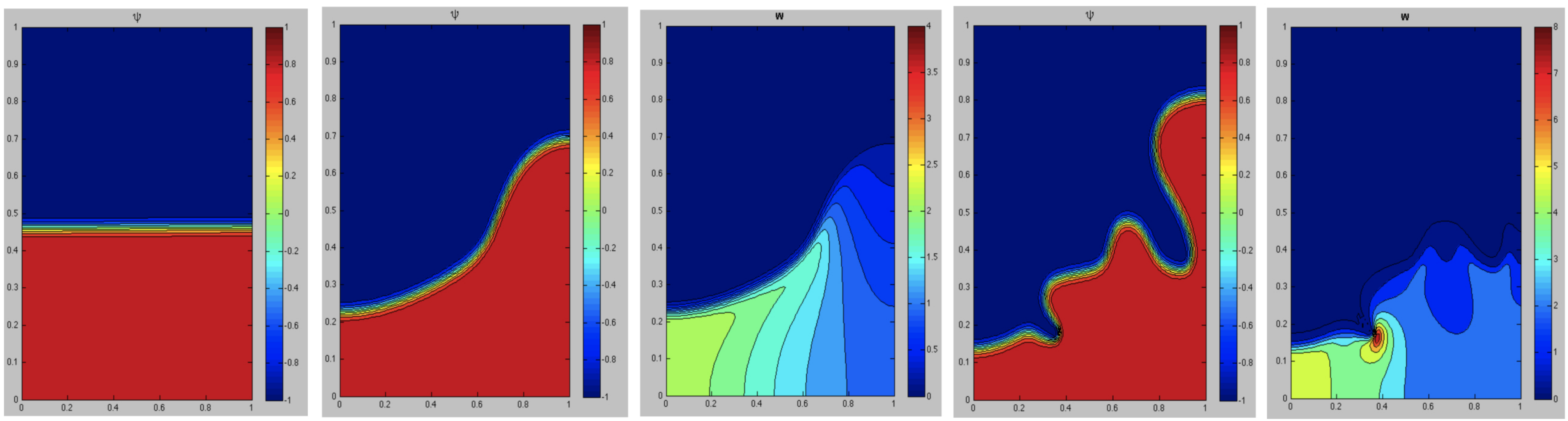



10
ω
Empty space
5
−2
−1
1
2
$x_1/2\pi\lambda$
Solid
−5
−10

$$(\frac{\mathrm{d}}{\mathrm{d}x_2} - 2\beta)(f' + f^2 - 1) = 0$$

$$\psi = -\tanh(\sqrt{\frac{p}{2g_b}}x_2 + \frac{3}{4}L_\psi G_o(\nabla w)^2\sqrt{\frac{2g_b}{p}}t)$$

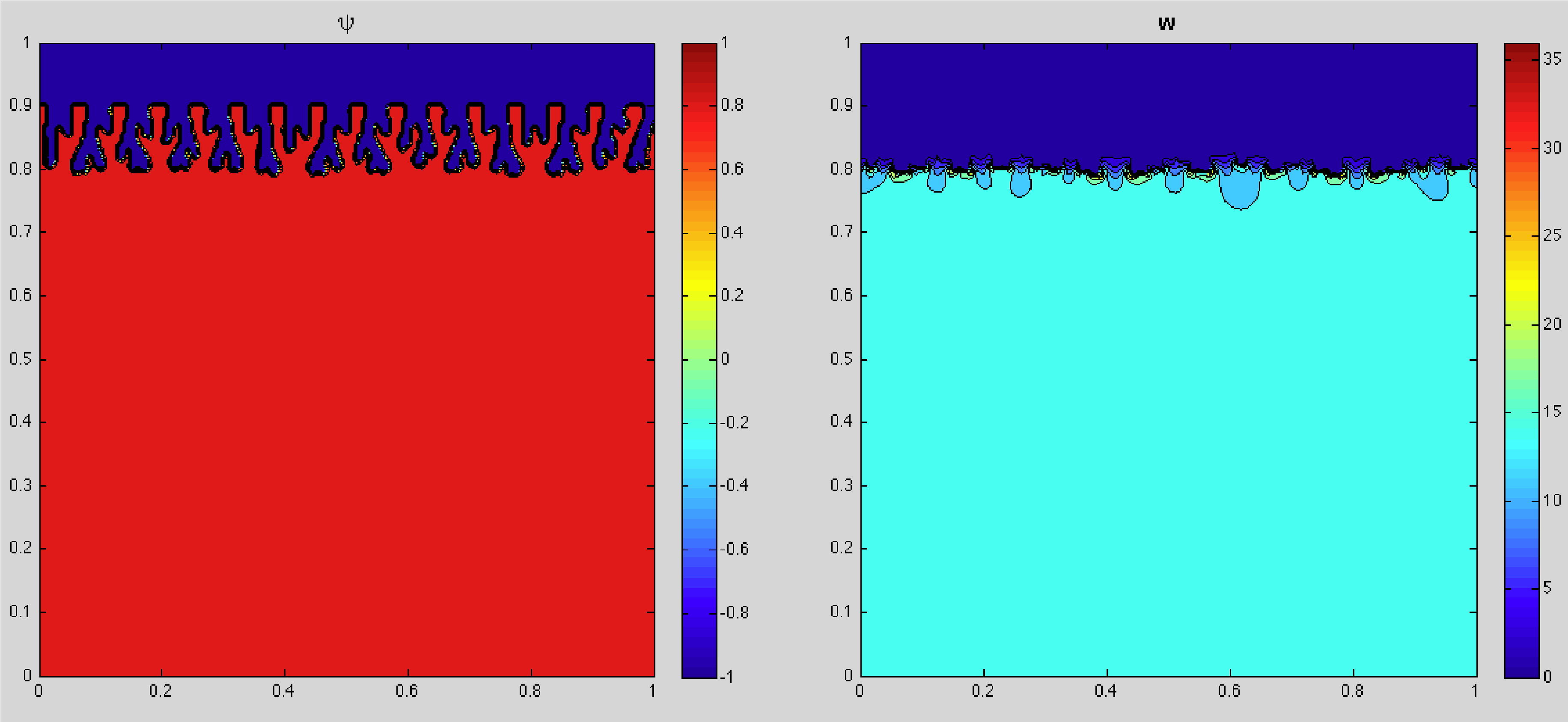




Red is remaining material

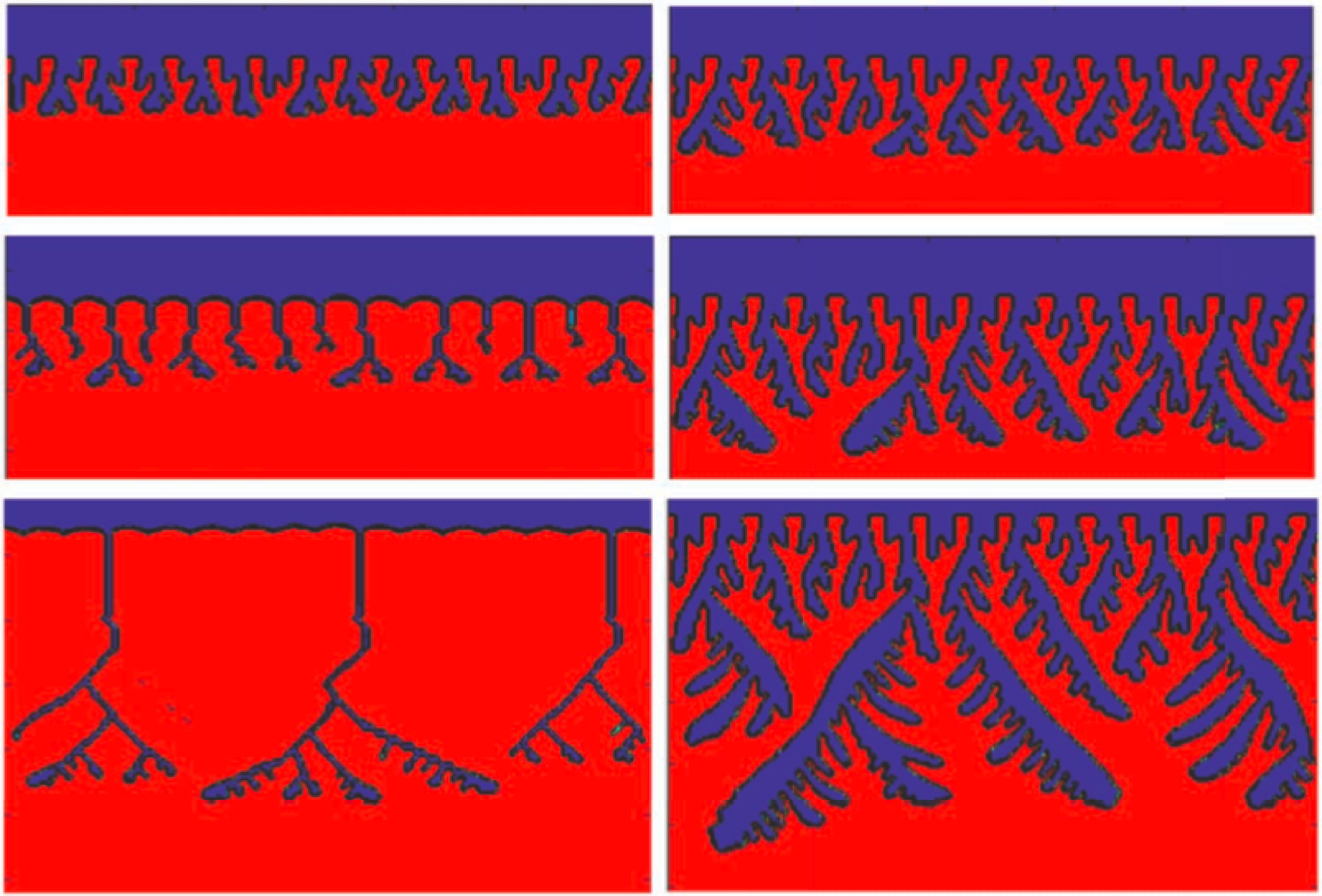
Effective Stress

Effective Stress



Red is remaining material

Effective stress



Without general corrosion | with general corrosion

Summary

Stress corrosion can be modelled as a moving bondary problem

Surface instability, formation cracks and crack growth are captured

Branching occurs as the blunted crack front become unstable

Phase field modelling simplifies the analysis

Time dependent solution to the Ginzburg-Landau equation found