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Crack Tip Energy Release Rate During Fast Fracture

Talk given at ICM-6, Kyoto, Japan

Ståhle, P.

1991

Document Version:

Publisher's PDF, also known as Version of record

[Link to publication](#)

Citation for published version (APA):

Ståhle, P. (1991). Crack Tip Energy Release Rate During Fast Fracture: Talk given at ICM-6, Kyoto, Japan. Eget.

Total number of authors:

1

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

CRACK TIP ENERGY RELEASE RATE DURING FAST FRACTURE

P. STÅHLE

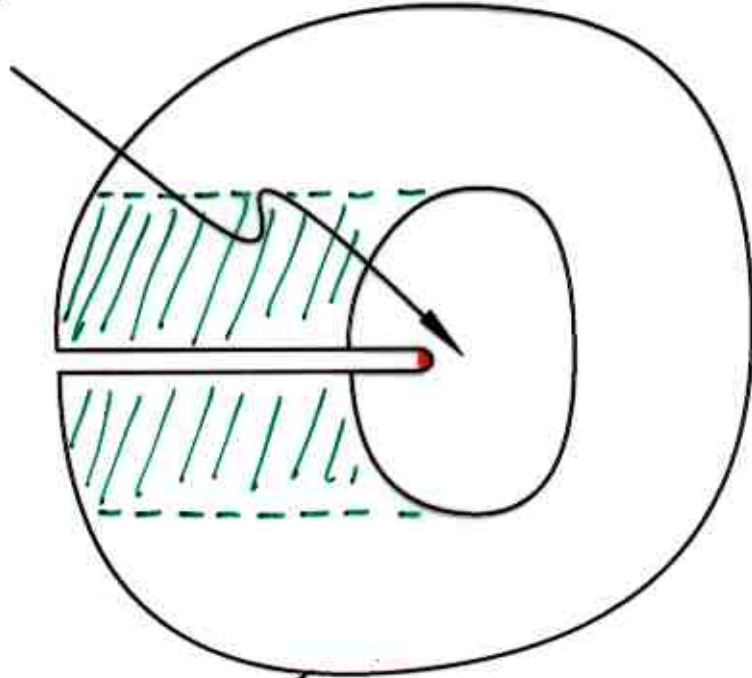
Department of Technology, Uppsala University,
Box 534, 751 21 Uppsala, Sweden.

- Visco-plastic Model ; $n < 3$
- Dynamic Steady-State
- Mode III
- Asymptotic Field
vs.
Line Model for Process Region

Brickestad (1983)

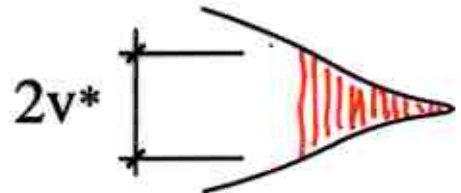
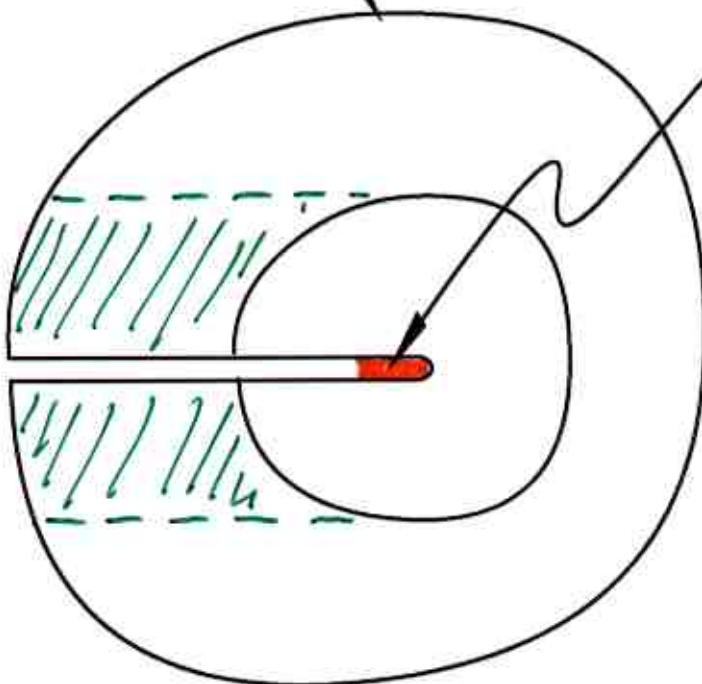
$$\sigma_{ij} \approx \frac{K_{\text{tip}}}{\sqrt{2\pi r}} f_{ij}(\theta)$$

$$G = \frac{K_{\text{tip}}^2}{\mu}$$



$$\sigma_{ij} \approx \frac{K_{\text{III}}}{\sqrt{2\pi r}} f_{ij}(\theta)$$

$$G = \frac{K_{\text{III}}^2}{\mu}$$



$$G_{\text{pr}} = 2v^* \sigma_D$$

$$\mu \frac{\partial \dot{w}}{\partial x} = \dot{\tau}_{xz} + \dot{\gamma}_0 \tau_Y \left(\frac{\tau}{\tau_Y} - 1 \right)^n \frac{\tau_{xz}}{\tau}$$

$$\mu \frac{\partial \dot{w}}{\partial y} = \dot{\tau}_{yz} + \dot{\gamma}_0 \tau_Y \left(\frac{\tau}{\tau_Y} - 1 \right)^n \frac{\tau_{yz}}{\tau}$$

$$\frac{\partial \dot{\tau}_{xz}}{\partial x} + \frac{\partial \dot{\tau}_{yz}}{\partial y} = \mu m^2 \frac{\partial^2 \dot{w}}{\partial x^2}$$

$$m = \frac{\dot{a}}{c_s} \quad \text{and} \quad \alpha = (1 - m^2)^{\frac{1}{2}}$$

$$\frac{\partial^2 \dot{w}}{\partial x^2} + \frac{\partial^2 \dot{w}}{\alpha^2 \partial y^2} = \frac{\lambda \dot{a} \tau_t^2}{\alpha^2 K_{III}^2} \left\{ \frac{\partial}{\partial x} \left[\left(\frac{\tau}{\tau_Y} - 1 \right)^n \frac{\tau_{xz}}{\tau} \right] + \frac{\partial}{\partial y} \left[\left(\frac{\tau}{\tau_Y} - 1 \right)^n \frac{\tau_{yz}}{\tau} \right] \right\}$$

$$\lambda = \frac{\dot{\gamma}_0 K_{III}^2}{\dot{a} \tau_Y \mu}$$

$$\dot{w} = \dot{w}_0 + \lambda \dot{w}_1 + O(\lambda^2) \dot{w}_2$$

$$\tau_{xz} = \tau_{xz0} + \lambda \tau_{xz1} + O(\lambda^2) \tau_{xz2}$$

$$\tau_{yz} = \tau_{yz0} + \lambda \tau_{yz1} + O(\lambda^2) \tau_{yz2}$$



Consumed energy alt.
analytically determined

Sharp Crack Tip

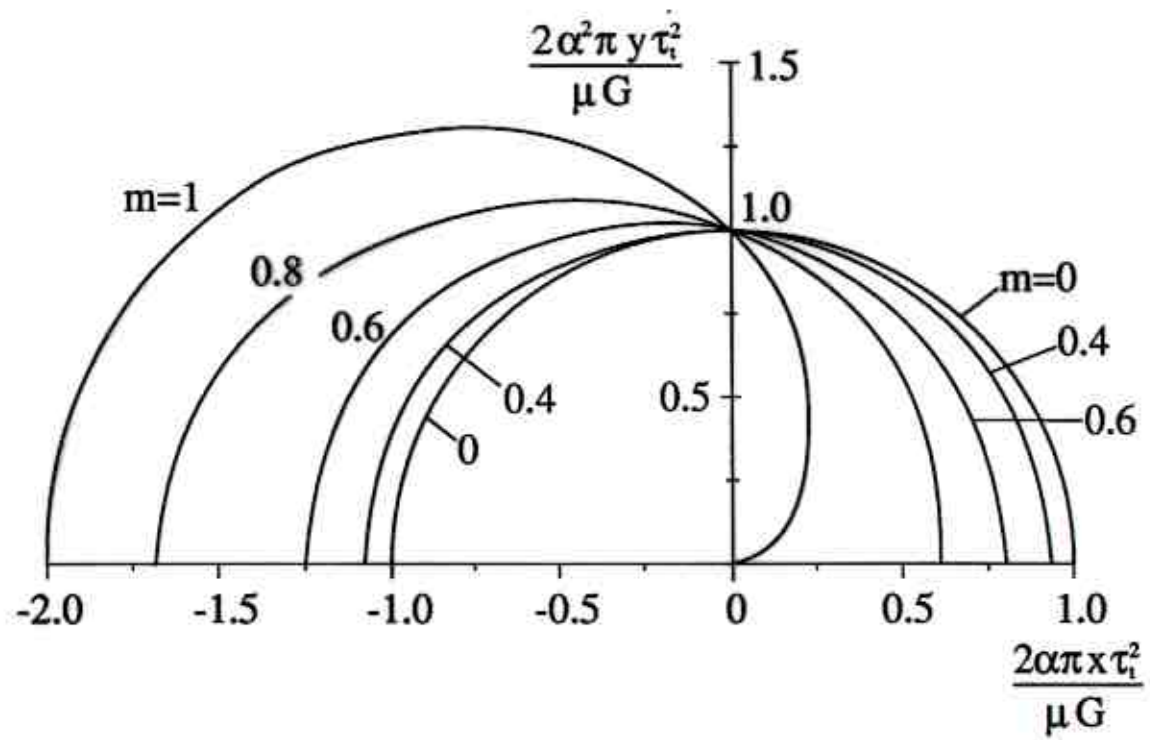
$$\tau_{xz0} = \frac{K_{III}}{\alpha\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right) + O(r)^{-\frac{3}{2}}$$

$$\tau_{yz0} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) + O(r)^{-\frac{3}{2}}$$

$$\frac{\partial^2 \dot{w}_1}{\partial x^2} + \frac{\partial^2 \dot{w}_1}{\alpha^2 \partial y^2} =$$
$$\frac{\dot{\alpha} \tau_I^2}{\alpha^2 K_{III}^2} \frac{\partial}{\partial x} \left[\left(\frac{\tau_0}{\tau_Y} - 1 \right)^n \frac{\tau_{xz0}}{\tau_0} \right] + \frac{\partial}{\partial y} \left[\left(\frac{\tau_0}{\tau_Y} - 1 \right)^n \frac{\tau_{yz0}}{\tau_0} \right]$$

Hodograph Transform ->

Series Expansion in τ_{xz} - τ_{yz} plane



n=1

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_0 R} - \frac{1}{3} \right) \hat{r}^{-\frac{1}{2}} + \hat{r}^{\frac{1}{2}} - \frac{2}{3} \hat{r} \right\} \Gamma$$

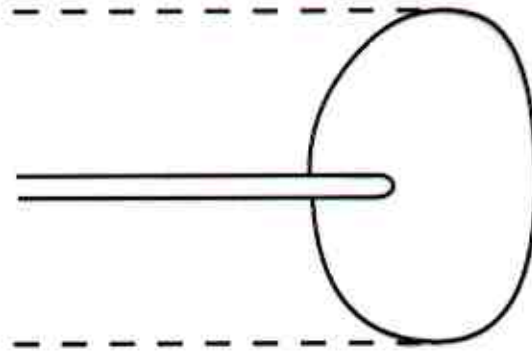
n=2

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_0 R} - \frac{2}{3} \right) \hat{r}^{-\frac{1}{2}} + 2 - 2\hat{r}^{\frac{1}{2}} + \frac{2}{3} \hat{r} \right\} \Gamma$$

n=5/2

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_0 R} - \frac{5 \tan^{-1}(z)}{4} \right) \hat{r}^{-\frac{1}{2}} + \left(\frac{11}{4} - \frac{13}{6} \hat{r}^{\frac{1}{2}} + \frac{2}{3} \hat{r} \right) z \right\} \Gamma$$

$$z = \left[\hat{r}^{-\frac{1}{2}} - 1 \right]^{\frac{1}{2}}, \quad R = \frac{K_{III}^2}{2\pi\tau_Y^2} \quad \text{and} \quad \Gamma = \frac{R\dot{\gamma}_0\tau_Y}{\mu} \sin\left(\frac{\theta}{2}\right)$$



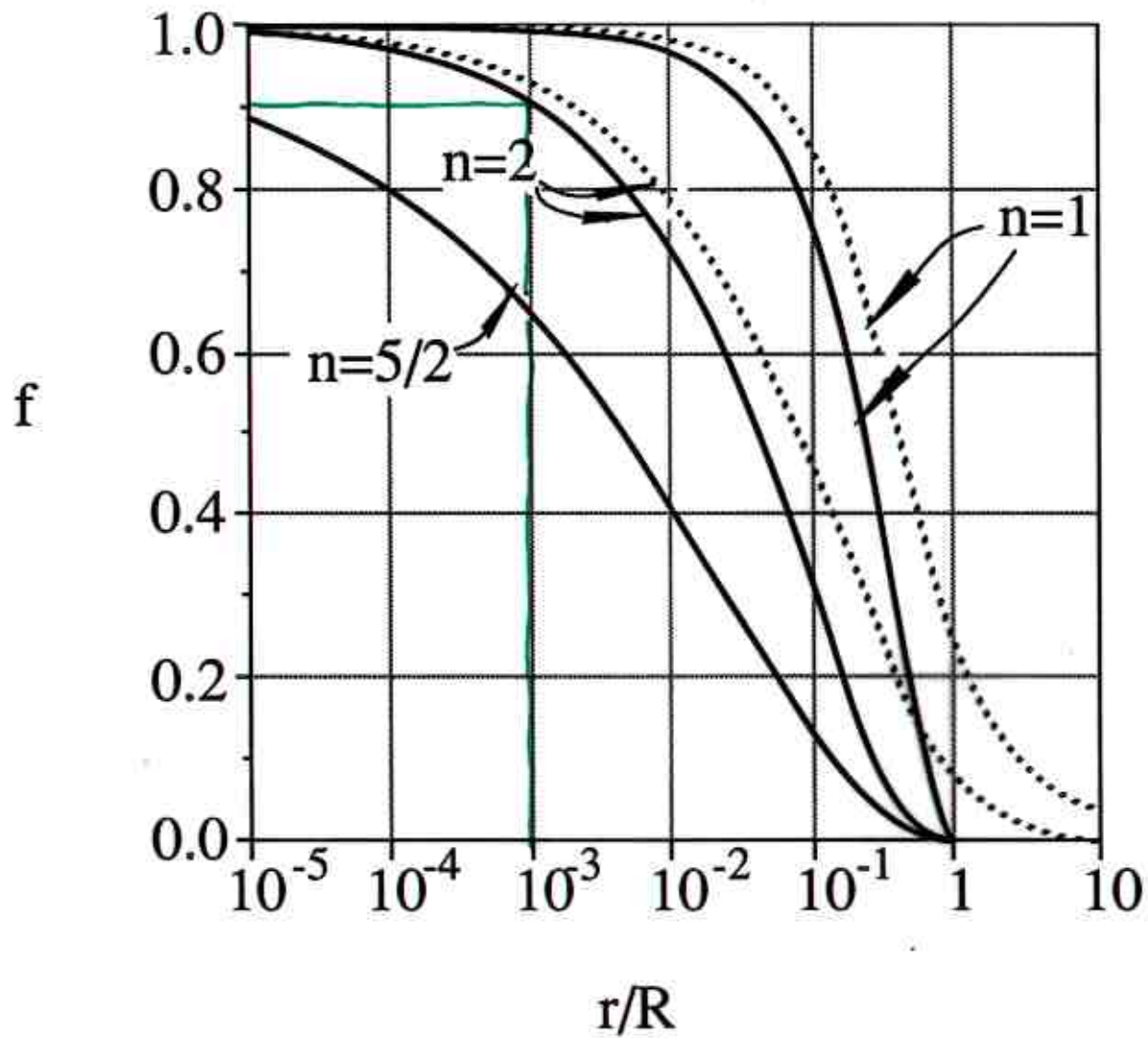
$$\sigma_{ij} \approx \frac{K_{tip}}{\sqrt{2\pi r}} f_{ij}(\theta) + \bar{\sigma}_{ij}$$

$$\bar{\sigma}_{ij} \rightarrow r^{\frac{1}{2}} \quad \text{for} \quad n = 1$$

$$\bar{\sigma}_{ij} \rightarrow r^0 \quad n = 2$$

$$\bar{\sigma}_{ij} \rightarrow r^{-\frac{1}{4}} \quad n = \frac{5}{2}$$

$$\left(\bar{\sigma}_{ij} \rightarrow r^{-\frac{1}{2}} \log(r) \quad n = 3 \right)$$



sharp crack tip

— $m=0$

⋯ $m=0.5$

$$f(r) = \frac{\tilde{K}_{III}(r) - K_{III}}{K_{tip} - K_{III}}$$

$$\tilde{K}_{III}(r) = \frac{\mu(2\pi r)^{\frac{1}{2}} \dot{w}}{a \sin(\frac{\theta}{2})}$$

Line Model for Process Region

$$\tau_{xz0} = \frac{\tau_D}{\alpha\pi} \cosh^{-1} \left(\frac{d + [d^2 + r^2 - 2rd \cos(\theta)]^{1/2}}{r} \right)$$

$$\tau_{yz0} = \frac{\tau_D}{\pi} \cos^{-1} \left(\frac{d - [d^2 + r^2 - 2rd \cos(\theta)]^{1/2}}{r} \right)$$

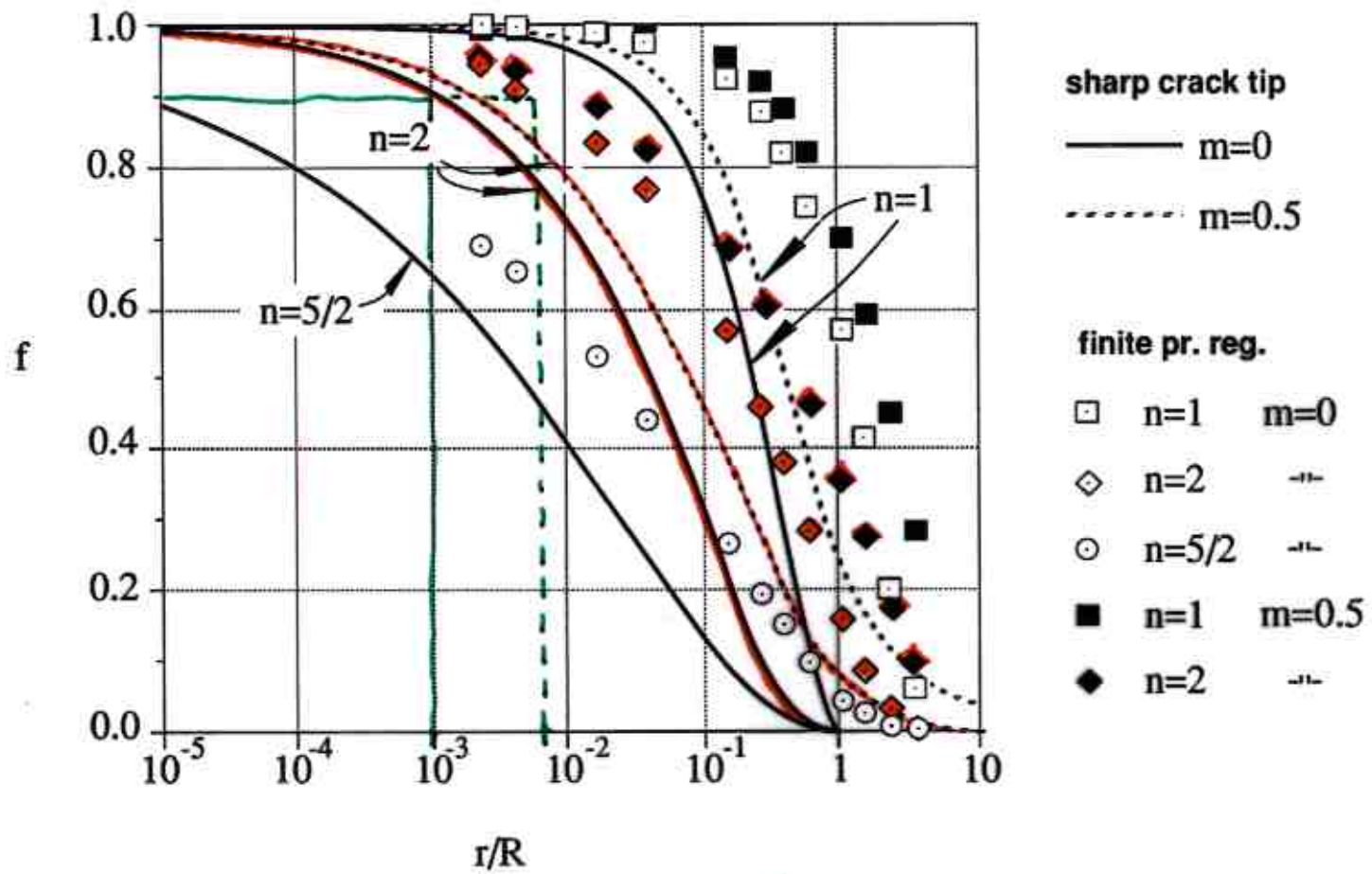
$$d = \frac{\pi}{8} \left(\frac{K_{III}}{\tau_D} \right)^2$$

(FREMUND & HUTCHINSON 86)

$$G_{tip} = G - \frac{1}{\dot{a}} \int_A (\tau_{xz} \dot{\gamma}_{xz}^p + \tau_{yz} \dot{\gamma}_{yz}^p) dA - \int_{-R_{max}}^{R_{max}} U_e^* dy$$

$$G/G_{pr} = 1 + \delta g(m, n) \frac{\dot{\gamma}_0 (\mu \rho)^{1/2} G_{pr}}{3\tau_t^2}$$

$$f(d) = \frac{G_{pr}^{1/2}(d) - G^{1/2}}{G_{pr}^{1/2}(0) - G^{1/2}}$$



One condition for using an asymptotic field instead of a detailed model of the process is that the process region is sufficiently embedded in the region where the asymptotic field dominates.

What does sufficiently mean?

It could mean a process region
even larger than the region
of dominance.