Crack Tip Energy Release Rate During Fast Fracture

Talk given at ICM-6, Kyoto, Japan

Ståhle, P.

CRACK TIP ENERGY RELEASE RATE DURING FAST FRACTURE

P. STÅHLE

Department of Technology,Uppsala University,
Box 534, 751 21 Uppsala, Sweden.

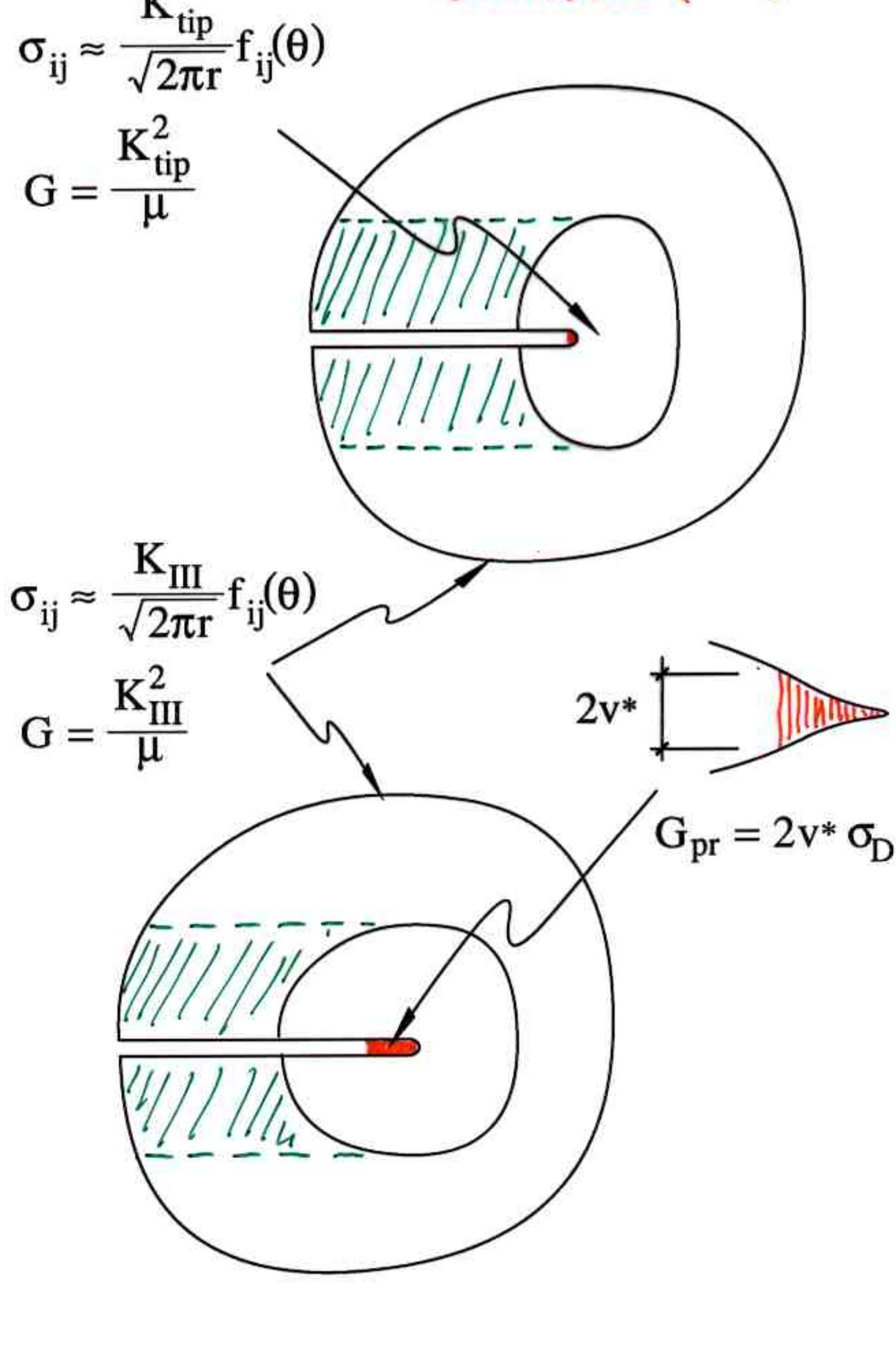
- Visco-plastic Model ; $n<3$

- Dynamic Steady-State

- Mode III

- Asymptotic Field
vs.
Line Model for Process Region

Brickstad (1983)



$$\mu\frac{\partial\dot{w}}{\partial x} = \dot{\tau}_{xz} + \dot{\gamma}_o\tau_Y\left(\frac{\tau}{\tau_Y} - 1\right)^n \frac{\tau_{xz}}{\tau}$$

$$\mu\frac{\partial\dot{w}}{\partial y} = \dot{\tau}_{yz} + \dot{\gamma}_o\tau_Y\left(\frac{\tau}{\tau_Y} - 1\right)^n \frac{\tau_{yz}}{\tau}$$

$$\frac{\partial\dot{\tau}_{xz}}{\partial x} + \frac{\partial\dot{\tau}_{yz}}{\partial y} = \mu m^2\frac{\partial^2\dot{w}}{\partial x^2}$$

$$m = \frac{\dot{a}}{c_s} \qquad \text{and} \qquad \alpha = (1 - m^2)^{\frac{1}{2}}$$

$$\frac{\partial^2\dot{w}}{\partial x^2} + \frac{\partial^2\dot{w}}{\alpha^2\partial y^2} =$$

$$\frac{\lambda\dot{a}\tau_t^2}{\alpha^2K_{III}^2}\left\{\frac{\partial}{\partial x}\left[\left(\frac{\tau}{\tau_Y} - 1\right)^n \frac{\tau_{xz}}{\tau}\right] + \frac{\partial}{\partial y}\left[\left(\frac{\tau}{\tau_Y} - 1\right)^n \frac{\tau_{yz}}{\tau}\right]\right\}$$

$$\lambda = \frac{\dot{\gamma}_o K_{III}^2}{\dot{a}\tau_Y\mu}$$

$$\dot{w} = \dot{w}_0 + \lambda\dot{w}_1 + O(\lambda^2)\dot{w}_2$$

$$\tau_{xz} = \tau_{xz0} + \lambda\tau_{xz1} + O(\lambda^2)\tau_{xz2}$$

$$\tau_{yz} = \tau_{yz0} + \lambda\tau_{yz1} + O(\lambda^2)\tau_{yz2}$$

Consumed energy alt.
analytically determined

Sharp Crack Tip

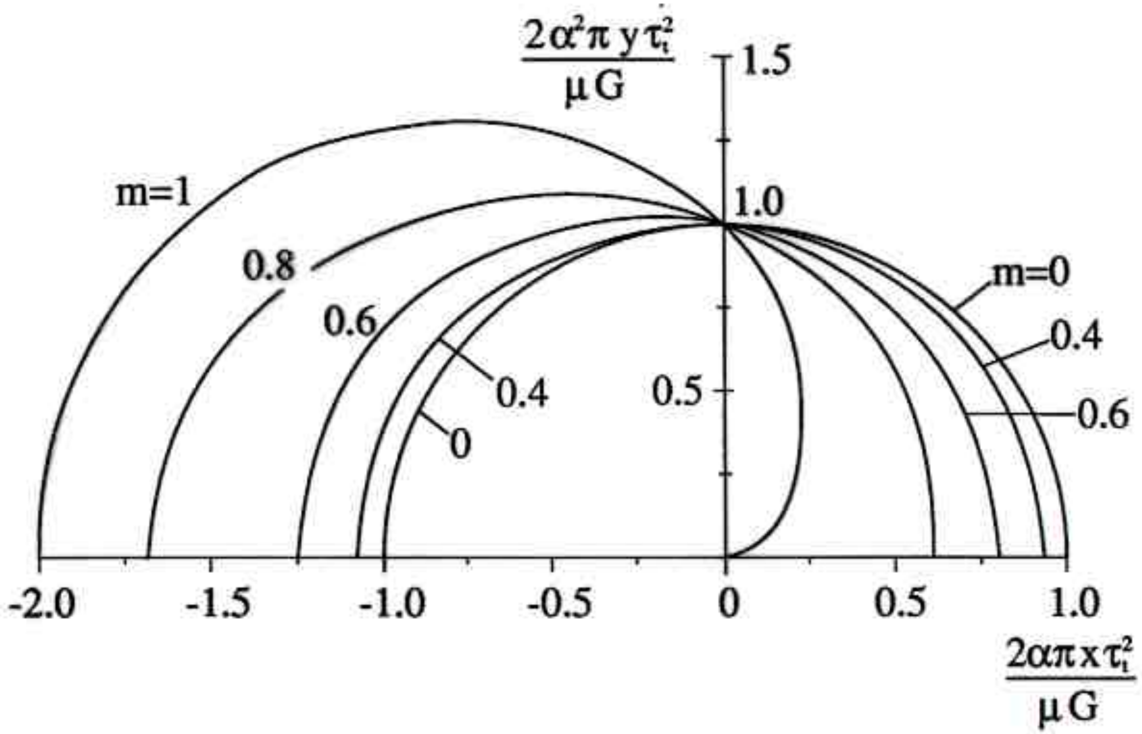
$$\tau_{xz0} = \frac{K_{III}}{\alpha\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right) + \mathcal{O}(r)^{-\frac{3}{2}}$$

$$\tau_{yz0} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) + \mathcal{O}(r)^{-\frac{3}{2}}$$

$$\frac{\partial^2 \dot{w}_1}{\partial x^2} + \frac{\partial^2 \dot{w}_1}{\alpha^2 \partial y^2} =$$
$$\frac{\dot{a}\tau_t^2}{\alpha^2 K_{III}^2} \frac{\partial}{\partial x}\left[\left(\frac{\tau_0}{\tau_Y} - 1\right)^n \frac{\tau_{xz0}}{\tau_0}\right] + \frac{\partial}{\partial y}\left[\left(\frac{\tau_0}{\tau_Y} - 1\right)^n \frac{\tau_{yz0}}{\tau_0}\right]$$

Hodograph Transform ->

Series Expansion in τ_{xz}-τ_{yz} plane



$\frac{2\alpha^2\pi y\tau_i^2}{\mu G}$
1.5
1.0
0.5
m=1
0.8
0.6
0.4
0
m=0
0.4
0.6
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0
$\frac{2\alpha\pi x\tau_i^2}{\mu G}$

n=1

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_o R} - \frac{1}{3} \right) \hat{r}^{-\frac{1}{2}} + \hat{r}^{\frac{1}{2}} - \frac{2}{3}\hat{r} \right\} \Gamma$$

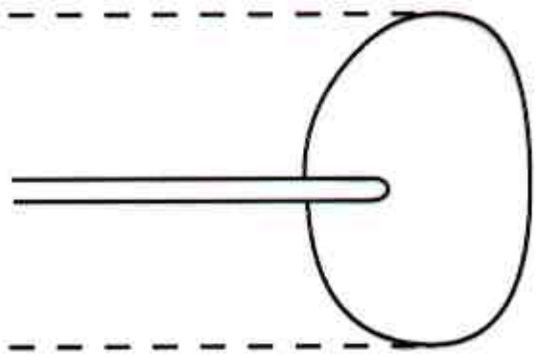
n=2

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_o R} - \frac{2}{3} \right) \hat{r}^{-\frac{1}{2}} + 2 - 2\hat{r}^{\frac{1}{2}} + \frac{2}{3}\hat{r} \right\} \Gamma$$

n=5/2

$$\dot{w} = \left\{ \left(\frac{\dot{a}}{\dot{\gamma}_o R} - \frac{5\tan^{-1}(z)}{4} \right) \hat{r}^{-\frac{1}{2}} + \left(\frac{11}{4} - \frac{13}{6}\hat{r}^{\frac{1}{2}} + \frac{2}{3}\hat{r} \right) z \right\} \Gamma$$

$$z = \left[\hat{r}^{-\frac{1}{2}} - 1 \right]^{\frac{1}{2}}, \; R = \frac{K_{III}^2}{2\pi\tau_Y^2} \text{ and } \Gamma = \frac{R\dot{\gamma}_o\tau_Y}{\mu}\sin(\frac{\theta}{2})$$



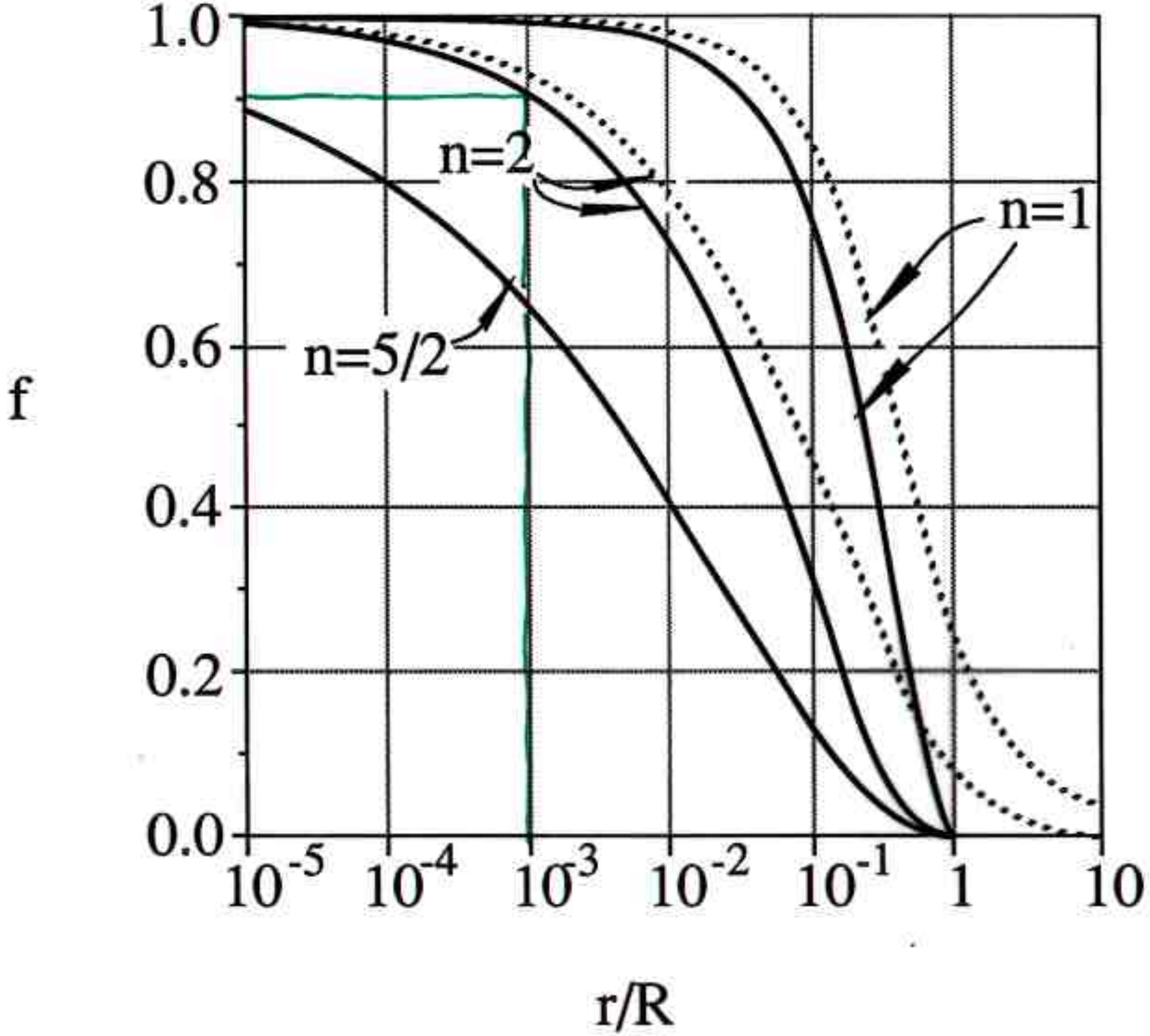
$$\sigma_{ij} \approx \frac{K_{tip}}{\sqrt{2\pi r}} f_{ij}(\theta) + \bar{\sigma}_{ij}$$

$$\bar{\sigma}_{ij} \rightarrow r^{\frac{1}{2}} \qquad \text{for} \qquad n = 1$$

$$\bar{\sigma}_{ij} \rightarrow r^{0} \qquad n = 2$$

$$\bar{\sigma}_{ij} \rightarrow r^{-\frac{1}{4}} \qquad n = \frac{5}{2}$$

$$\left(\bar{\sigma}_{ij} \rightarrow r^{-\frac{1}{2}} \log(r) \qquad n = 3 \right)$$



$$f(r) = \frac{\tilde{K}_{III}(r) - K_{III}}{K_{tip} - K_{III}}$$

$$\tilde{K}_{III}(r) = \frac{\mu(2\pi r)^{\frac{1}{2}}\,\dot{w}}{\dot{a}\sin(\frac{\theta}{2})}$$

Line Model for Process Region

$$\tau_{xz0} = \frac{\tau_D}{\alpha\pi}\cosh^{-1}\left(\frac{d + \left[d^2 + r^2 - 2rd\cos(\theta)\right]^{1/2}}{r}\right)$$

$$\tau_{yz0} = \frac{\tau_D}{\pi}\cos^{-1}\left(\frac{d - \left[d^2 + r^2 - 2rd\cos(\theta)\right]^{1/2}}{r}\right)$$

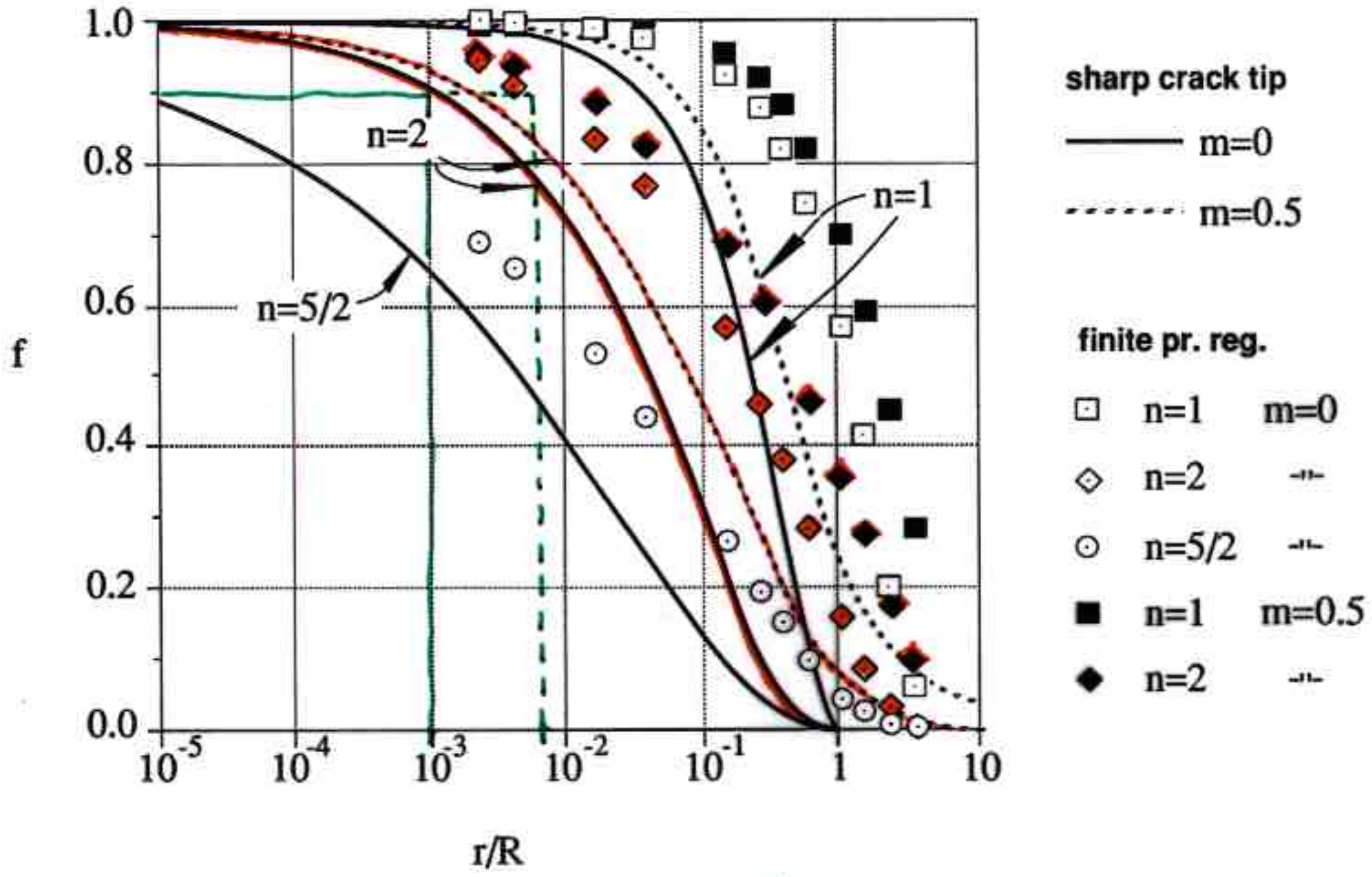
$$d = \frac{\pi}{8}\left(\frac{K_{III}}{\tau_D}\right)^2$$

(FREUND & HUTCHINSON 86)

$$G_{tip} = G - \frac{1}{\dot{a}}\int_A \left(\tau_{xz}\dot{\gamma}^p_{xz} + \tau_{yz}\dot{\gamma}^p_{yz}\right)dA - \int_{-R_{max}}^{R_{max}} U^*_e dy$$

$$G/G_{pr} = 1 + \delta g(m,n)\frac{\dot{\gamma}_o(\mu\rho)^{\frac{1}{2}}G_{pr}}{3\tau_t^2}$$

$$f(d) = \frac{G_{pr}^{\frac{1}{2}}(d) - G^{\frac{1}{2}}}{G_{pr}^{\frac{1}{2}}(0) - G^{\frac{1}{2}}}$$



1.0
0.8
0.6
0.4
0.2
0.0
f
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
r/R
n=2
n=1
n=5/2
sharp crack tip
m=0
m=0.5
finite pr. reg.
n=1 m=0
n=2 -"-
n=5/2 -"-
n=1 m=0.5
n=2 -"-

One condition for using an asymptotic field instead of a detailed model of the process is that the process region is <u>sufficiently embedded</u> in the region where the asymptotic field dominates.

What does sufficiently mean?

It could mean a process region
even larger than the region
of dominance.