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On the spectral properties of the exterior Calderón operator

Gerhard Kristensson(1), Ioannis G. Stratis(2), Niklas Wellander*(3), and Athanasios N. Yannacopoulos(4)
(1) Lund University, P.O. Box 118, SE-221 00 Lund, Sweden
(2) National and Kapodistrian University of Athens, Panepistimioupolis, GR-157 84 Zographou, Athens, Greece
(3) Swedish Defense Research Agency, FOI, P.O. Box 1165, SE-581 11 Linköping, Sweden
(4) Athens University of Economics and Business, Patision 76, GR-104 34 Athens, Greece

In this paper we characterize the spectrum of the exterior Calderón operator which maps the tangential scattered electric field to the tangential scattered magnetic field on the boundary of a scattering obstacle, Ω, which is assumed to be an open and bounded domain in $\mathbb{R}^3$ with a simply connected Lipschitz boundary Γ. We denote the simply connected exterior of the domain Ω by $\Omega_\mathrm{e} = \mathbb{R}^3 \setminus \overline{\Omega}$. Consider the following exterior problem where the trace of the scattered electric field on the boundary is given by a fixed vector $\boldsymbol{m} \in H^{-1/2}(\mathrm{div},\Gamma)$,

$$
\begin{aligned}
&1)\ (\boldsymbol{E}_\mathrm{sc},\boldsymbol{H}_\mathrm{sc}) \in H_\mathrm{loc}(\mathrm{curl},\overline{\Omega}_\mathrm{e}) \times H_\mathrm{loc}(\mathrm{curl},\overline{\Omega}_\mathrm{e})\\
&2)\ \begin{cases} \nabla\times\boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) = \mathrm{i}k\boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) \\ \nabla\times\boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) = -\mathrm{i}k\boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) \end{cases} \qquad \boldsymbol{x}\in\Omega_\mathrm{e}\\
&3)\ \begin{cases} \hat{\boldsymbol{x}}\times\boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) - \boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) = o(1/x) \\ \text{or} \\ \hat{\boldsymbol{x}}\times\boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) + \boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) = o(1/x) \end{cases} \qquad \text{as } x\to\infty, \quad \text{uniformly w.r.t. } \hat{\boldsymbol{x}}\\
&4)\ \boldsymbol{\gamma}(\boldsymbol{E}_\mathrm{sc}) = \boldsymbol{m} \in H^{-1/2}(\mathrm{div},\Gamma)
\end{aligned}
\tag{1}
$$

where $x = |\boldsymbol{x}|$. The wave number $k = \omega/c$ is assumed to be a positive constant, where ω is the angular frequency of the fields, and c is the speed of light in the exterior medium.The trace operator $\boldsymbol{\gamma}$ on $C(\overline{\Omega_\mathrm{e}};\mathbb{C}^3)$ is given by $\boldsymbol{\gamma}(\boldsymbol{u}) = \hat{\boldsymbol{\nu}}\times\boldsymbol{u}|_{\partial\Omega}$. In the case that $\boldsymbol{u}$ belongs to $H_\mathrm{loc}(\mathrm{curl},\overline{\Omega}_\mathrm{e})$, the fields have traces on Γ belonging to $H^{-1/2}(\mathrm{div},\Gamma)$, more precisely we have $(\boldsymbol{\gamma}(\boldsymbol{E}_\mathrm{sc}),\boldsymbol{\gamma}(\boldsymbol{H}_\mathrm{sc})) \in H^{-1/2}(\mathrm{div},\Gamma)\times H^{-1/2}(\mathrm{div},\Gamma)$, see [1] for the definition and the properties of the trace operators in $H_\mathrm{loc}(\mathrm{curl},\overline{\Omega}_\mathrm{e})$. Problem (1) has a unique solution [2, 3, 4] and the exterior Calderón operator $\mathbf{C}^\mathrm{e}$ is defined as

$$
\mathbf{C}^\mathrm{e} : \boldsymbol{m} \mapsto \boldsymbol{\gamma}(\boldsymbol{H}_\mathrm{sc}), \qquad H^{-1/2}(\mathrm{div},\Gamma) \to H^{-1/2}(\mathrm{div},\Gamma),
$$

where $\boldsymbol{m} = \boldsymbol{\gamma}(\boldsymbol{E}_\mathrm{sc})$ and the fields $\boldsymbol{E}_\mathrm{sc}$ and $\boldsymbol{H}_\mathrm{sc}$ satisfy (1). The point spectrum of the exterior Calderón operator is $P\sigma(\mathbf{C}^\mathrm{e}) = \{-\mathrm{i},\mathrm{i}\}$, *i.e.*, eigenvalues $\lambda = \pm\mathrm{i}$ and eigenvectors $\boldsymbol{m} \in H^{-1/2}(\mathrm{div},\Gamma)$ satisfy

$$
\mathbf{C}^\mathrm{e}\boldsymbol{m} = \lambda\boldsymbol{m}
$$

where $\boldsymbol{m}_{\tau n}^{\pm} = \boldsymbol{Y}_{\tau n} \mp \mathrm{i}\mathbf{C}^\mathrm{e}\boldsymbol{Y}_{\tau n}$ and $\boldsymbol{Y}_{\tau n}$ are the generalized spherical harmonics defined in [5].

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