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## On the spectral properties of the exterior Calderón operator

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In this paper we characterize the spectrum of the exterior Calderón operator which maps the tangential scattered electric field to the tangential scattered magnetic field on the boundary of a scattering obstacle,  $\Omega$ , which is assumed to be an open and bounded domain in  $\mathbb{R}^3$  with a simply connected Lipschitz boundary  $\Gamma$ . We denote the simply connected exterior of the domain  $\Omega$  by  $\Omega_e = \mathbb{R}^3 \setminus \Omega$ . Consider the following exterior problem where the trace of the scattered electric field on the boundary is given by a fixed vector  $\mathbf{m} \in H^{-1/2}(\text{div}, \Gamma)$ ,

$$\begin{aligned}
 & 1) (\mathbf{E}_{\text{sc}}, \mathbf{H}_{\text{sc}}) \in H_{\text{loc}}(\text{curl}, \overline{\Omega}_e) \times H_{\text{loc}}(\text{curl}, \overline{\Omega}_e) \\
 & 2) \begin{cases} \nabla \times \mathbf{E}_{\text{sc}}(\mathbf{x}) = ik\mathbf{H}_{\text{sc}}(\mathbf{x}) \\ \nabla \times \mathbf{H}_{\text{sc}}(\mathbf{x}) = -ik\mathbf{E}_{\text{sc}}(\mathbf{x}) \end{cases} \quad \mathbf{x} \in \Omega_e \\
 & 3) \begin{cases} \hat{\mathbf{x}} \times \mathbf{E}_{\text{sc}}(\mathbf{x}) - \mathbf{H}_{\text{sc}}(\mathbf{x}) = o(1/x) \\ \text{or} \\ \hat{\mathbf{x}} \times \mathbf{H}_{\text{sc}}(\mathbf{x}) + \mathbf{E}_{\text{sc}}(\mathbf{x}) = o(1/x) \end{cases} \quad \text{as } x \rightarrow \infty, \quad \text{uniformly w.r.t. } \hat{\mathbf{x}} \\
 & 4) \boldsymbol{\gamma}(\mathbf{E}_{\text{sc}}) = \mathbf{m} \in H^{-1/2}(\text{div}, \Gamma)
 \end{aligned} \tag{1}$$

where  $x = |\mathbf{x}|$ . The wave number  $k = \omega/c$  is assumed to be a positive constant, where  $\omega$  is the angular frequency of the fields, and  $c$  is the speed of light in the exterior medium. The trace operator  $\boldsymbol{\gamma}$  on  $C(\overline{\Omega}_e; \mathbb{C}^3)$  is given by  $\boldsymbol{\gamma}(\mathbf{u}) = \hat{\mathbf{v}} \times \mathbf{u}|_{\partial\Omega}$ . In the case that  $\mathbf{u}$  belongs to  $H_{\text{loc}}(\text{curl}, \overline{\Omega}_e)$ , the fields have traces on  $\Gamma$  belonging to  $H^{-1/2}(\text{div}, \Gamma)$ , more precisely we have  $(\boldsymbol{\gamma}(\mathbf{E}_{\text{sc}}), \boldsymbol{\gamma}(\mathbf{H}_{\text{sc}})) \in H^{-1/2}(\text{div}, \Gamma) \times H^{-1/2}(\text{div}, \Gamma)$ , see [1] for the definition and the properties of the trace operators in  $H_{\text{loc}}(\text{curl}, \overline{\Omega}_e)$ . Problem (1) has a unique solution [2, 3, 4] and the exterior Calderón operator  $\mathbf{C}^e$  is defined as

$$\mathbf{C}^e : \mathbf{m} \mapsto \boldsymbol{\gamma}(\mathbf{H}_{\text{sc}}), \quad H^{-1/2}(\text{div}, \Gamma) \rightarrow H^{-1/2}(\text{div}, \Gamma),$$

where  $\mathbf{m} = \boldsymbol{\gamma}(\mathbf{E}_{\text{sc}})$  and the fields  $\mathbf{E}_{\text{sc}}$  and  $\mathbf{H}_{\text{sc}}$  satisfy (1). The point spectrum of the exterior Calderón operator is  $P\sigma(\mathbf{C}^e) = \{-i, i\}$ , i.e., eigenvalues  $\lambda = \pm i$  and eigenvectors  $\mathbf{m} \in H^{-1/2}(\text{div}, \Gamma)$  satisfy

$$\mathbf{C}^e \mathbf{m} = \lambda \mathbf{m}$$

where  $\mathbf{m}_{\tau n}^{\pm} = \mathbf{Y}_{\tau n} \mp i\mathbf{C}^e \mathbf{Y}_{\tau n}$  and  $\mathbf{Y}_{\tau n}$  are the generalized spherical harmonics defined in [5].

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